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Short Note

An example of seismic time picking by third-order bicoherence

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INTRODUCTION

Conventional seismic time-delay estimation relies on the crosscorrelation that quantifies the similarities between two measurements in the second-order time domain. When the noise correlation in the measurements is considerable, the correlation peak can be substantially distorted, resulting in imprecise and even biased estimation of the time delay. The synthetic data computed by Ikelle et al. (1993) and Ikelle and Yung (1994) in their studies of wave propagation through random media provide a good example of data with considerable noise correlation. In picking the arrival times in this data set, we found that the crosscorrelation technique suffers both from the severely restricted signal bandwidth and from the presence of coda. Here we present an alternative approach involving high-resolution nonparametric time-delay estimation in the third-order domain.

THIRD-ORDER BICOHERENCE-CORRELATION

Given three zero-mean, real random signals $x(t)$, $y(t)$, and $z(t)$ and their respective Fourier transforms $X(\omega)$, $Y(\omega)$, and $Z(\omega)$, the cross-bispectrum in the third-order domain is defined as (Nikias and Raghuveer, 1987; Nikias and Mendel, 1990)

$$B_{xyz}(\omega_1, \omega_2) = E[Y(\omega_1)Z(\omega_2)X^*(\omega_1 + \omega_2)], \quad (1)$$

where $E[\cdot]$ is the expectation operator and X^* is the complex conjugate of X. In addition, the bicoherence, which is a normalized cross-bispectrum, is given by

$$b_{xyz}(\omega_1, \omega_2) = \frac{B_{xyz}(\omega_1, \omega_2)}{\sqrt{P_{yy}(\omega_1)P_{zz}(\omega_2)P_{xx}(\omega_1 + \omega_2)}}, \quad (2)$$

where

$$\begin{aligned} P_{xx}(\omega) &= E[X(\omega)X^*(\omega)], \\ P_{yy}(\omega) &= E[Y(\omega)Y^*(\omega)], \\ P_{zz}(\omega) &= E[Z(\omega)Z^*(\omega)], \end{aligned} \quad (3)$$

are the power spectra of the signals. Practical procedures for computing cross-bispectrum and bicoherence can be found in Nikias and Raghuveer (1987) and Nikias and Pan (1988).

One approach to estimate the delay between two signals, $x(t)$ and $y(t)$, in the third-order domain is to evaluate the bicoherence ratio $\Gamma_{xyx}(\omega_1, \omega_2)$ and the bicoherence-correlation $\lambda_{xy}(\tau_1)$. The bicoherence ratio is defined as

$$\Gamma_{xyx}(\omega_1, \omega_2) = \frac{b_{xyx}(\omega_1, \omega_2)}{b_{xxx}(\omega_1, \omega_2)}. \quad (4)$$

The bicoherence-correlation is obtained by summing up along the frequencies ω_2 and taking the 1-D inverse Fourier transform

$$\lambda_{xy}(\tau_1) = \mathcal{F}^{-1}[\Lambda_{xy}(\omega_1)], \quad (5)$$

where

$$\Lambda_{xy}(\omega_1) = \sum_{\omega_2} \Gamma_{xyx}(\omega_1, \omega_2) \quad (6)$$

and $\mathcal{F}^{-1}$ is the inverse Fourier transform operator. The delay is then estimated by locating the lag at the peak of the bicoherence-correlation. A similar idea employing a ratio of bispectra, instead of bicoherences, has been proposed by Sato and Sasaki (1977) for reconstructing holography images.

Frequency-domain interpretations and simulated examples of time-delay estimation based on crosscorrelation, coherence-correlation, bispectral-correlation, and bicoherence-correlation are provided in the Appendix.

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ESTIMATION OF ARRIVAL TIMES

Data set

Figure 1 shows one of the synthetic data sets computed by Ikelle et al. (1993) in their study of wave propagation through random media. A 2-D model of the earth is used. It consists of large- and small-scale inhomogeneities. The large-scale inhomogeneities represent the mean properties of the earth that are overlaid by random variations characterized by an ellipsoidal autocorrelation function. Finite-difference modeling techniques with an explosive source were employed to simulate seismic wave propagation through this model. The wavefield was recorded with 91 receivers uniformly distributed along a quarter of a circle centered at the explosive source. An important and interesting issue for study in this simulation is the anisotropic behavior resulting from random variations, in other words, the dependence of relative arrival times as a function of source-receiver directions. Accurate time picking from such data sets is an essential prerequisite of this study.

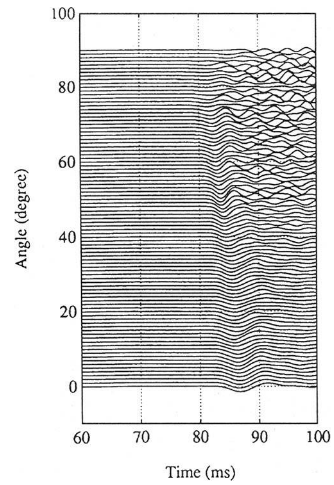

FIG. 1. Seismograms corresponding to wave propagation through a 2-D random medium.

Three signal characteristics that introduce complications in picking arrival times in the data set in Figure 1 are as follows:

1) Signal bandwidth—the explosive source signature is band limited to the first 600 Hz, which covers only 2% of the sampling frequency at 30 kHz.
2) Pulse-broadening effect—the seismic pulse is broadened when propagating through the dispersive random medium.
3) Seismic coda—multiple scattering in the random medium during wave propagation generates coda that act as a correlated noise source in the first-arrival-time estimation.

Results of time picking[1]

Figure 2 shows the crosscorrelation and the bicoherence-correlation between adjacent traces of the data in Figure 1. The increase in the resolution of the delay peaks in the

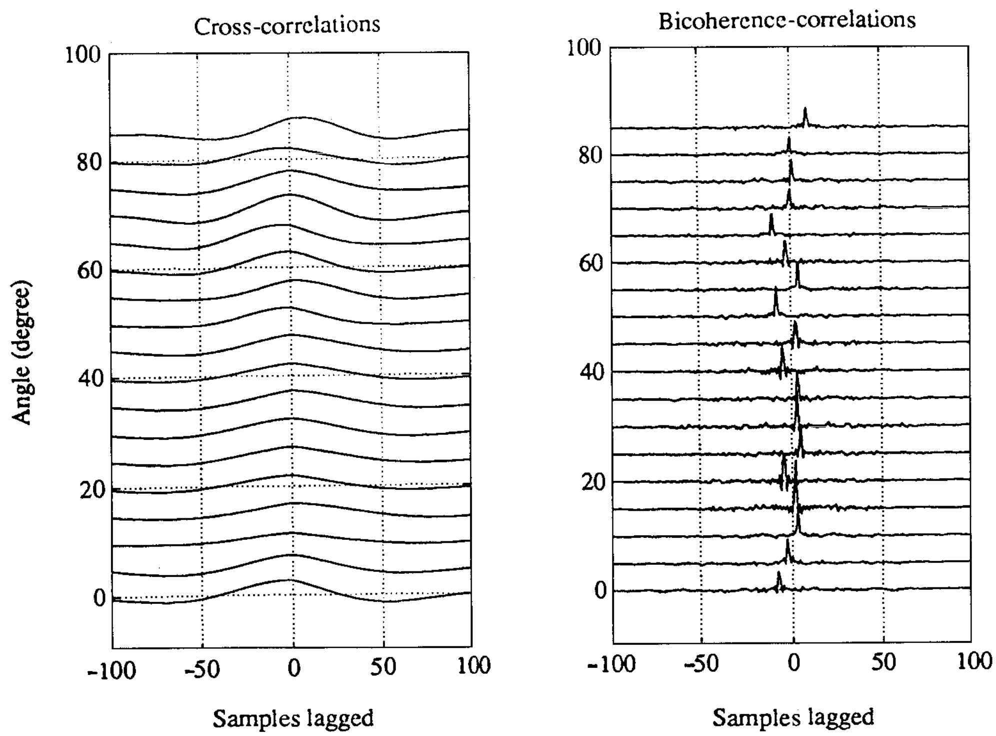

FIG. 2. Crosscorrelation and bicoherence-correlation between adjacent receivers of the data in Figure 1.

[1]The "normalized" second-order approach based on coherence-correlation also has been applied. Like the crosscorrelation, it fails to suppress the correlated coda effects and results in erroneous time-delay estimates.

bicoherence-correlation is impressive; the conventional crosscorrelation has suffered from the narrow-band source signature. The incremental delays between adjacent traces are then combined and accumulated. The resulting relative delays are shown in Figure 3. These moveouts predicted from the crosscorrelation and the bicoherence-correlation are overlaid individually on the original data in Figure 4. It is clear that, unlike the bicoherence ratio approach, the predicted moveout computed via the crosscorrelation fails to follow the trajectory of the first arrivals.

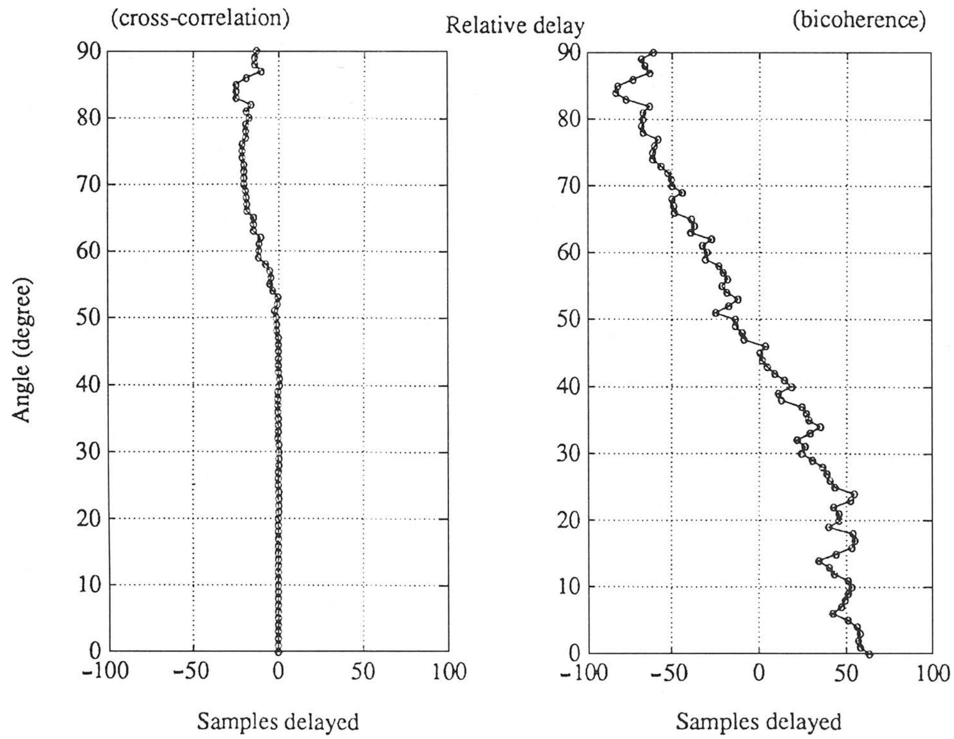

FIG. 3. Relative delays obtained from the crosscorrelation and bicoherence-correlation displayed in Figure 2.

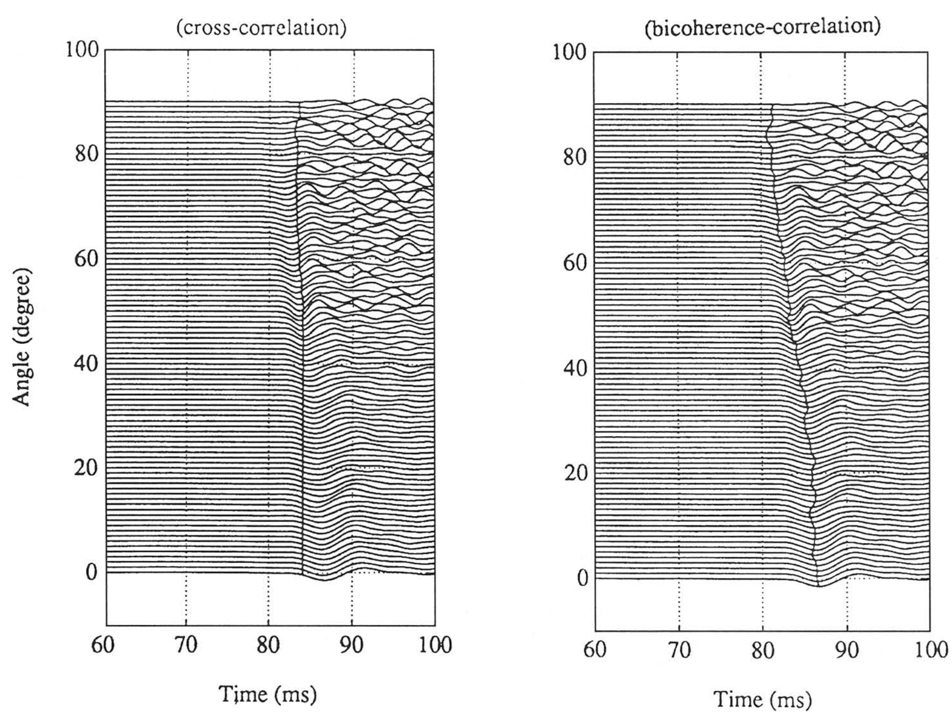

FIG. 4. Moveouts predicted by the crosscorrelation and bicoherence-correlation being overlaid on the original data.

Conventional crosscorrelation has suffered badly from the narrow-band source signature as well as from the corruption of the correlated coda. On the other hand, the combination of inherent signal prewhitening and Gaussian noise canceling capability in the bicoherence-correlation (see the Appendix for a discussion of these properties) has produced high-resolution correlation peaks that track the first-arrival times.

CONCLUSION

In conclusion, we have used the bicoherence-correlation in the third-order domain to pick arrival times in a seismic data set affected by pulse-broadening effects and a considerable correlated coda. Unlike the conventional crosscorrelation method, this method has succeeded in tracking the moveout of the first arrivals.

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APPENDIX

INTERPRETATIONS OF TIME-DELAY ESTIMATION BASED ON SECOND- AND THIRD-ORDER CUMULANTS

Problem formulation

Estimation of the time delay between measurements $x(t)$ and $y(t)$ from two spatially distributed receivers can be mathematically formulated as

$$\begin{aligned} x(t) &= f(t) \otimes s(t) + m(t), \\ y(t) &= h(t) \otimes s(t-\tau_0) + n(t), \end{aligned} \tag{A-1}$$

where $\otimes$ denotes time convolution. In the arrival-time-picking application, $x(t)$ and $y(t)$ are the receiver signals, $s(t)$ is the source wavelet, and $f(t)$ and $h(t)$ represent the impulse response functions of the random medium. $m(t)$ and $n(t)$ are corrupting noises assumed to be uncorrelated with the signal $s(t)$ even though the two noise sources may be mutually correlated.

The objective is to estimate the delay τ_0 with limited knowledge of both the signal $s(t)$ and the propagation media $f(t)$ and $h(t)$ and to do this in the presence of corrupting, possibly correlated, noises.

Second-order techniques

Cross-correlation.—The conventional procedure for determining the time delay τ_0 relies on the crosscorrelation function

$$r_{xy}(\tau) = E[x(t)y(t+\tau)] = \mathcal{F}^{-1}[P_{xy}(\omega)], \tag{A-2}$$

where $\mathcal{F}^{-1}$ is the inverse Fourier transform operator and the cross-spectrum $P_{xy}(\omega)$ is given by

$$P_{xy}(\omega) = E[X^*(\omega)Y(\omega)]. \tag{A-3}$$

Here, $r_{xy}(\tau)$ quantifies the similarities between $x(t)$ and $y(t)$ at various time lags and is expected to peak at the proper time delay. The argument τ that maximizes $r_{xy}(\tau)$ is often taken to be an estimate of τ_0.

On the basis of the model described in equation (A-1), the cross-spectrum can be partitioned into the components

$$P_{xy}(\omega) = \underbrace{F^*(\omega)H(\omega)}_{\text{media}}\ \underbrace{P_{ss}(\omega)}_{\text{source}}\ \underbrace{e^{-j\omega\tau_0}}_{\text{delay}} + \underbrace{P_{mn}(\omega)}_{\text{noise}}. \tag{A-4}$$

$e^{-j\omega\tau_0}$, equal to with a unit amplitude and a linear phase relationship (slope $-\tau_0$); this corresponds to a delta function at τ_0 [i.e., $\delta(t-\tau_0)$] in the time domain. This ideal delay contribution is nevertheless "colored" by the propagation media [i.e., $F^*(\omega)H(\omega)$], the source signature [i.e., $P_{ss}(\omega)$], and the correlated noises [i.e., $P_{mn}(\omega)$].

Coherence-correlation.—To suppress the effects caused by the transmission media and the source signal, the cross-spectrum is normalized by the individual autospectra, resulting in the complex coherence function

$$\gamma_{xy}(\omega) = \frac{P_{xy}(\omega)}{\sqrt{P_{xx}(\omega)P_{yy}(\omega)}}. \tag{A-5}$$

Combining equations (A-1), (A-3), (A-4), and (A-5), $\gamma_{xy}(\omega)$ can be expressed as

$$\gamma_{xy}(\omega) = \underbrace{\frac{G(\omega)}{|G(\omega)|}}_{\text{media}}\ \underbrace{\mathcal{R}(\omega)}_{\text{S/N ratio}}\ \underbrace{e^{-j\omega\tau_0}}_{\text{delay}} + \underbrace{\frac{P_{mn}(\omega)}{\sqrt{P_{xx}(\omega)P_{yy}(\omega)}}}_{\text{noise}}, \tag{A-6}$$

where

$$G(\omega) = \frac{H(\omega)}{F(\omega)} \tag{A-7}$$

and $\mathcal{R}(\omega)$ is a combined measure of the signal-to-noise ratios $\mathcal{R}_x(\omega)$ [in $x(t)$] and $\mathcal{R}_y(\omega)$ [in $y(t)$], written as

$$\mathcal{R}(\omega) = \frac{1}{\sqrt{\left(1+\frac{1}{\mathcal{R}_x(\omega)}\right)\left(1+\frac{1}{\mathcal{R}_y(\omega)}\right)}}. \tag{A-8}$$

As in the cross-spectrum, the complex coherence function embodies the delay information $e^{-j\omega\tau_0}$. A coherence-correlation can be obtained by taking the inverse Fourier transform of $\gamma_{xy}(\omega)$.

The coherence approach provides two attractive properties over the conventional crosscorrelation:

1) The amplitude effects of the propagation media are normalized.
2) $\gamma_{xy}(\omega)$ depends on the signal-to-noise ratio $\mathcal{R}(\omega)$ rather than directly on the signal spectrum $P_{ss}(\omega)$. In the limit, when the corrupting noises are negligible, $\mathcal{R}(\omega)$ will become unity irrespective of the spectral contents of the source signature, and the delay impulse $\delta(t-\tau_0)$ can be reconstructed.

Nevertheless, the coherence-correlation is still incapable of eliminating the noise correlation term.

The coherence-correlation technique is also known as the smoothed coherence transform, Knapp and Carter (1976).

Third-order techniques

One major advantage of third-order over conventional second-order approaches is that if the signals $x(t)$ and $y(t)$ are Gaussian or, in general, symmetrically distributed then

$$B_{xyx}(\omega_1,\omega_2) = b_{xyx}(\omega_1,\omega_2) = 0, \quad \text{for all } \omega_1 \text{ and } \omega_2, \tag{A-9}$$

where $B_{xyx}(\omega_1,\omega_2)$ is the cross-bispectrum [equation (1)] and $b_{xyx}(\omega_1,\omega_2)$ is the bicoherence [equation (2)].

Corrupting measurement noise is often caused by multiple independent sources and is generally considered to be Gaussian. Therefore, in contrast to the second-order domain, Gaussian noise does not "color" the correlation peak in the third-order domain. However, the signals contained in the measurement must be non-Gaussian with an asymmetrical probability distribution. Most seismic source wavelets are non-Gaussian. Nonetheless, if there is good reason to believe that the source wavelet is indeed symmetrically distributed, the following proposed algorithms can be extended to higher orders (e.g., fourth-order cumulants and trispectra; Mendel, 1991).

Bispectral-correlation.—One procedure proposed by Sato and Sasaki (1977) for reconstructing holography images is to form a ratio between the cross-bispectrum $B_{xyx}(\omega_1,\omega_2)$ and the auto-bispectrum $B_{xxx}(\omega_1,\omega_2)$, written as

$$\beta_{xyx}(\omega_1,\omega_2) = \frac{B_{xyx}(\omega_1,\omega_2)}{B_{xxx}(\omega_1,\omega_2)}. \tag{A-10}$$

Combining equations (1), (A-1), and (A-10), the bispectral ratio can be rewritten as

$$\beta_{xyx}(\omega_1,\omega_2) = \underbrace{G(\omega_1)}_{\text{media}}\underbrace{e^{-j\omega_1\tau_0}}_{\text{delay}}, \tag{A-11}$$

which eliminates completely the correlation of the Gaussian noise. However, the ratio is still colored by the medium transfer function.

The bispectral-correlation is obtained by summing up $\beta_{xyx}(\omega_1,\omega_2)$ along ω_2 and then taking the 1-D inverse Fourier transform.

Bicoherence-correlation.—Using the same tactics as for second-order coherence-correlation, the filtering effects of the propagation media for bispectral-correlation can be reduced by using the bicoherence ratio $\Gamma_{xyx}(\omega_1,\omega_2)$ [equation (4)]. The corresponding bicoherence-correlation $\lambda_{xy}(\tau_1)$ in the time domain is given by equation (5).

Combining equations (2), (4), and (A-1), the bicoherence ratio can be represented as

$$\Gamma_{xyx}(\omega_1,\omega_2) = \underbrace{\frac{G(\omega_1)}{|G(\omega_1)|}}_{\text{media}}\underbrace{\bar{\mathcal{R}}(\omega_1)}_{\text{S/N ratio}}\underbrace{e^{-j\omega_1\tau_0}}_{\text{delay}} \tag{A-12}$$

Not only has the correlated noise been attenuated, but also the amplitude effects of the propagation paths have been suppressed.

$\bar{\mathcal{R}}(\omega_1)$ can be expressed in terms of the signal-to-noise ratios $\mathcal{R}_x(\omega_1)$ and $\mathcal{R}_y(\omega_1)$ in the measurements $x(t)$ and $y(t)$, respectively, as

$$\bar{\mathcal{R}}(\omega_1) = \sqrt{\frac{1+\dfrac{1}{\mathcal{R}_x(\omega_1)}}{1+\dfrac{1}{\mathcal{R}_y(\omega_1)}}}. \tag{A-13}$$

If the two measurements are made in similar noise environments [i.e., $\mathcal{R}_x(\omega_1)\approx\mathcal{R}_y(\omega_1)$], as in seismic time picking, then $\bar{\mathcal{R}}(\omega_1)\approx 1$ and the delay impulse will be preserved in the bicoherence-correlation.

An example

To highlight the effects of correlated noise as well of a narrow-band source signature and a propagation medium on second-order time-delay estimation methods, two measurements, $x(t)$ and $y(t)$ (Figures A-1a and A-1b), are simulated under the following assumptions (all frequencies are normalized with respect to the Nyquist frequency ω_N):

$$\begin{aligned} x(t) &= s(t) + m(t), \\ y(t) &= h(t)\otimes s(t-\tau_0) + n(t). \end{aligned} \tag{A-14}$$

The signal $s(t)$ is an exponentially distributed random signal with a bandwidth between $0.1\omega_N$ and $0.2\omega_N$. The function $h(t)$ is a realization of a low-pass finite-impulse response filter with a cutoff frequency at $0.15\omega_N$. The functions $m(t)$ and $n(t)$ are low-pass Gaussian random noise with a bandwidth of $0.35\omega_N$. $n(t)$ is selected to be a delayed version of $m(t)$ [i.e., $n(t) = m(t-\tau_{mn})$], and the signal-to-noise ratio is −3 dB (i.e., the power of the signal is half that of the noise).

A total of 10 000 samples are simulated for each measurement. The "true" signal delay τ_0 is ten samples, and the noise delay is chosen to be five samples.

The crosscorrelation and the coherence-correlation between $x(t)$ and $y(t)$ are displayed in Figure A-1c and A-1d, respectively. As a result of the low-pass nature of the signal, both the signal delay peak (at ten lagged samples) and the noise delay peak (at five lagged samples) in the crosscorrelation are substantially broadened and intermingled. The coherence-correlation manages to isolate and sharpen the two peaks, but the noise delay still predominates because of the poor signal-to-noise ratio. Hence, in the presence of correlated noise, both second-order approaches provide a biased estimate of the signal delay τ_0.

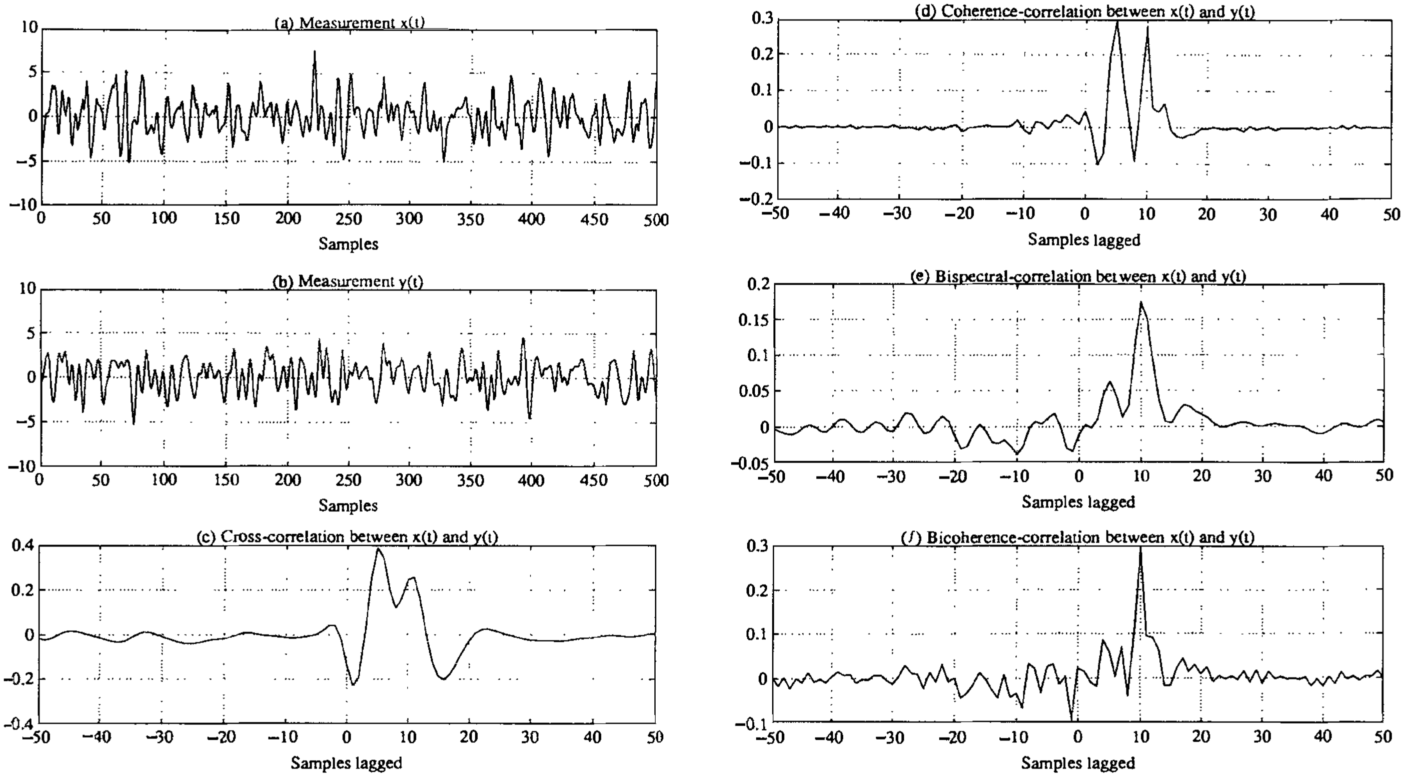

FIG. A-1. Crosscorrelation, coherence-correlation, bispectral-correlation, and bicoherence-correlation between two narrow-band measurements, $x(t)$ and $y(t)$, corrupted by correlated noises: (a) measurement $x(t)$, (b) measurement $y(t)$, (c) crosscorrelation, (d) coherence-correlation, (e) bispectral-correlation, and (f) bicoherence-correlation.

The bispectral-correlation and the bicoherence-correlation between $x(t)$ and $y(t)$ are displayed in Figures A-1e and A-1f, respectively. Compared with the correlations in the second order (Figures A-1c and A-1d), the third-order techniques are shown to be less sensitive to the correlated Gaussian noise (relatively delayed by five samples) and to yield an unbiased representation of the signal delay. In addition, the bicoherence approach is able to counteract the low-pass effects of the transmission medium $h(t)$ and to return a better-defined impulse, which then leads to a more precise estimate of τ_0.