



(Time–)Frequency Analysis of EEG Waveforms

Niko Busch

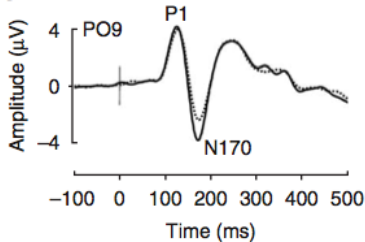
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From ERP waveforms to waves

- ERP analysis:

- time domain analysis: **when** do things (amplitudes) happen?
- treats peaks and troughs as single events.



- Frequency domain (spectral) analysis (Fourier analysis):

- magnitudes and frequencies of waves — no time information.
- peaks and troughs are not treated as separate entities.

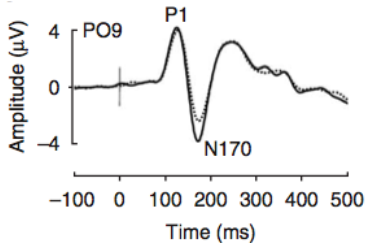
- Time–frequency analysis (wavelet analysis):

- when do which frequencies occur?

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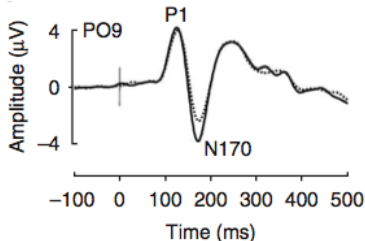
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Why bother?

(Time–)Frequency analysis complements signal analysis:

- neurons are oscillating.
- analysis of signals with trial-to-trial jitter.
- analysis of longer time periods.
- analysis of pre-stimulus and spontaneous signals.
- necessary for sophisticated methods (coherence, coupling, causality, etc.).

Parameters of waves

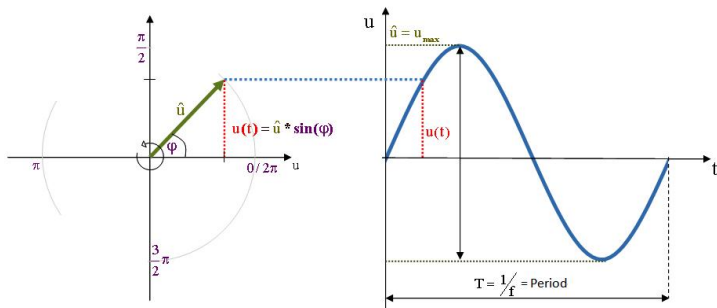
Oscillations regular repetition of some measure over several cycles.

Wavelength length of a single cycle (a.k.a. period).

Frequency $\frac{1}{\text{wavelength}}$ — the speed of change.

Phase current state of the oscillation — angle on the unit circle. Runs from 0° ($-\pi$) — 360° (π)

Magnitude (permanent) strength of the oscillation.



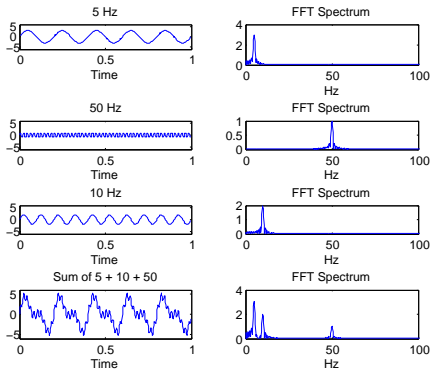
How to disentangle oscillations

Jean Joseph Fourier (1768—1830): *„An arbitrary function, continuous or with discontinuities, defined in a finite interval by an arbitrarily capricious graph can always be expressed as a sum of sinusoids“.*



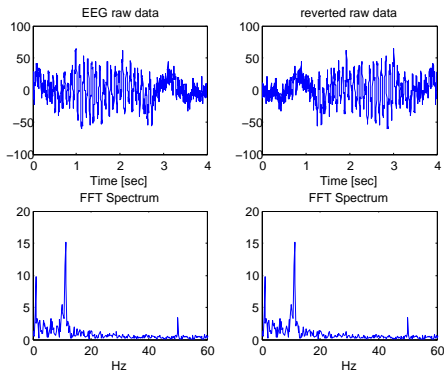
The discrete Fourier transform

- The DFT transforms the signal from the time domain into the frequency domain.
- Requires that the signal be **stationary**.



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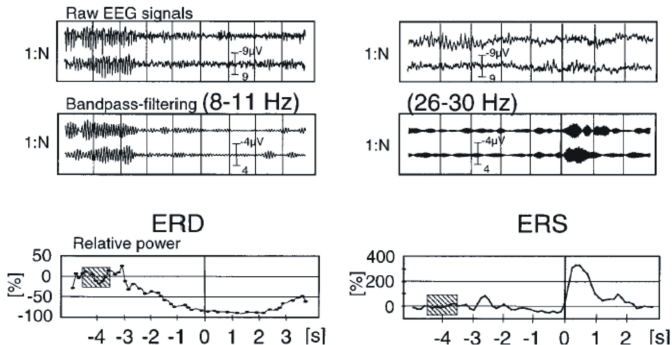


The discrete Fourier transform

- The DFT transforms the signal from the time domain into the frequency domain.
- Requires that the signal be **stationary**.
- Non-stationary signals:
 - When does the 10 Hz oscillation occur?
 - DFT does not give time information.
 - Time information is not necessary for stationary signals
 - Frequency contents do not change — all frequency components exist all the time.
 - How to investigate event-related spectral **changes** in brain signals?

Event-related synchronisation / desynchronisation

- Cut the signal in two time windows and assume stationarity in each half.
- $ERD/ERS^1 = \frac{\text{poststimulus power} - \text{baseline power}}{\text{baseline power}} * 100$

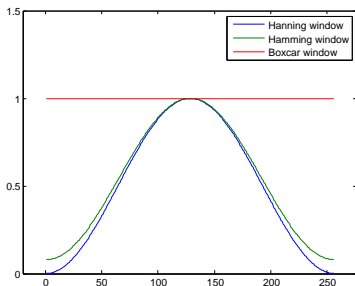


- But why not use even smaller windows?
- ⇒ Windowed FFT / Short term Fourier transform.

¹Pfurtscheller & Lopes da Silva (1999). Clin Neurophysiol

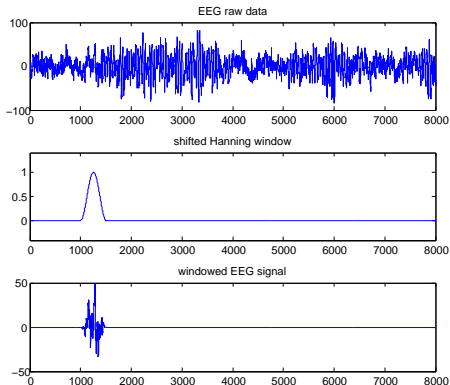
The short term Fourier transform (STFT) I

- Assume that some portion of a non-stationary signal is stationary.
- Important parameters:
 - window function (Hamming, Hanning, Rectangular, etc.)
 - window overlap
 - window length: width should correspond to the segment of the signal where its stationarity is valid.



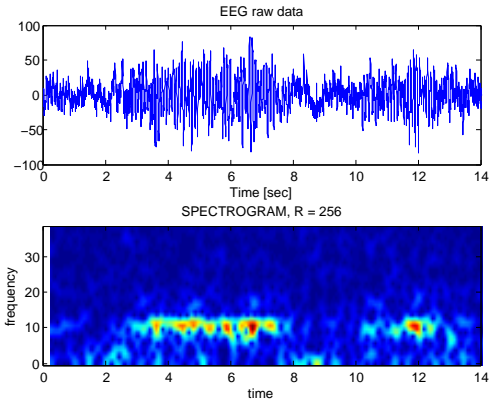
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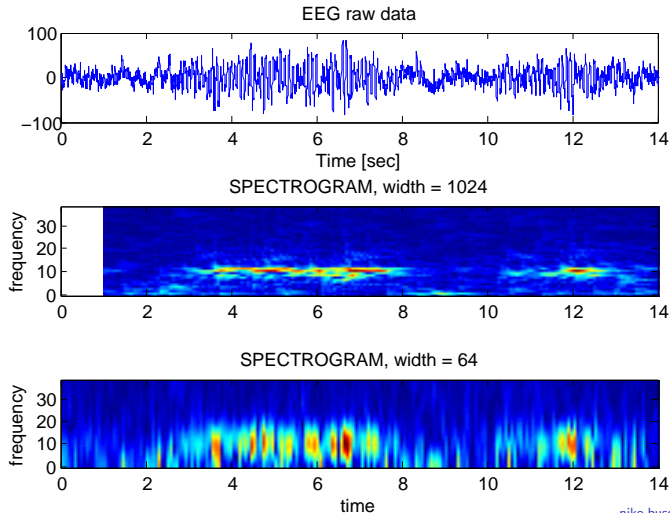
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The short term Fourier transform (STFT) II

- Window length affects resolution in time and frequency
 - short window: good time resolution, poor frequency resolution.
 - long window: good frequency resolution, poor time resolution.

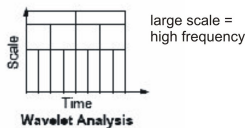
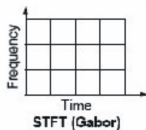
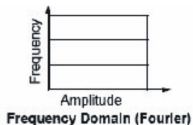


Uncertainty principle

- Werner Heisenberg (1901—1976):
 - Energy and location of a particle cannot be both known with infinite precision.
 - a result of the wave properties of particles (not the measurement).
- Applies also to time–frequency analysis:
 - We cannot know what spectral component exists at any given time **instant**.
 - What spectral components exist at any given **interval** of time?
- Spectral/temporal resolution trade off cannot be avoided — but it can be optimised.
 - Good frequency resolution at low frequencies.
 - Good time resolution at high frequencies.



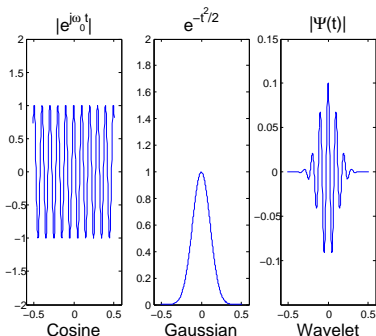
From STFT to wavelets



- STFT: fixed temporal & spectral resolution
 - Analysis of high frequencies \Rightarrow insufficient temporal resolution.
 - Analysis of low frequencies \Rightarrow insufficient spectral resolution.
- Wavelet analysis:
 - Analysis of high frequencies \Rightarrow narrow time window for better time resolution.
 - Analysis of low frequencies \Rightarrow wide time window for better spectral resolution.

What is a wavelet?

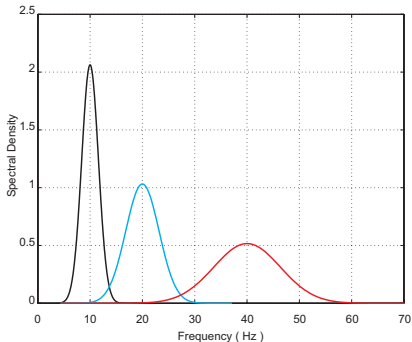
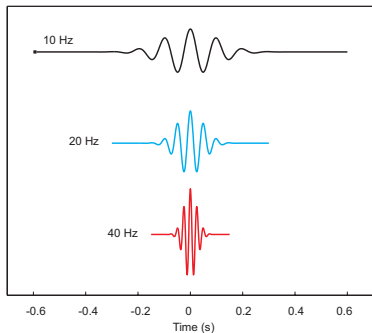
Motherwavelet



- Zero mean amplitude.
- Finite duration.
- Mother wavelet: prototype function ($f =$ sampling frequency).
- Wavelets can be scaled (compressed) and translated.

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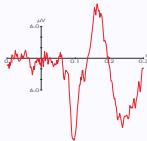
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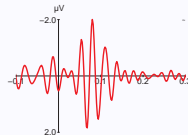
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Wavelet transform of ERPs

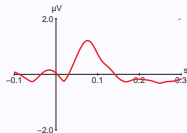
ERP



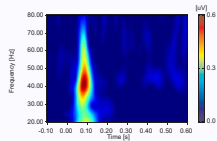
Bandpass-filtered ERP



Wavelet transformed ERP



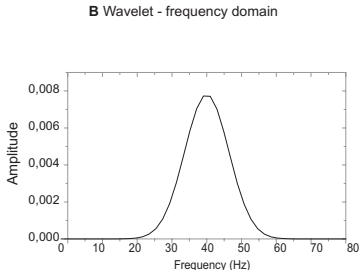
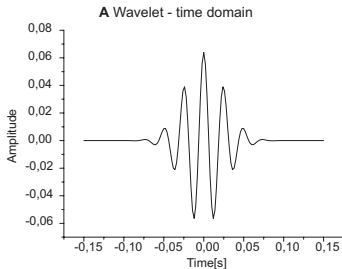
Gamma-Band: ca. 30 – 80 Hz



... but be careful:

- any signal can be **represented** as oscillations w. time-frequency analysis
- but it does not imply that the signal **is** oscillatory!

Important parameters of a wavelet



Length how many cycles does a wavelet have?

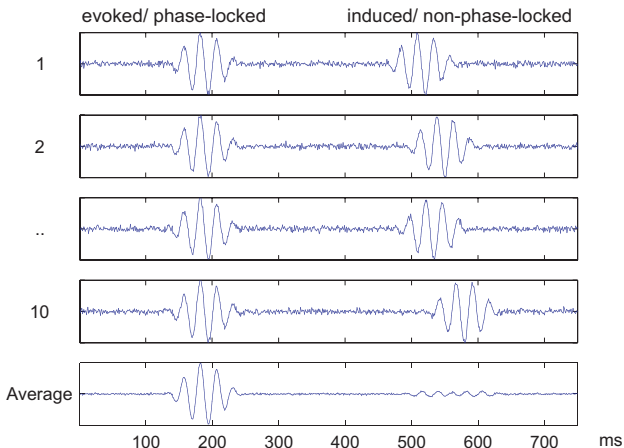
e.g. 40 Hz wavelet (25 ms/cycle), 12 cycles \Rightarrow 250 ms length

σ_t standard deviation in time domain: $\sigma_t = \frac{m}{2\pi * f_0}$

σ_f standard deviation in frequency domain: $\sigma_f = \frac{1}{2\pi * \sigma_t}$

time resolution increases with frequency, whereas frequency resolution decreases with frequency.

Evoked and induced oscillations I

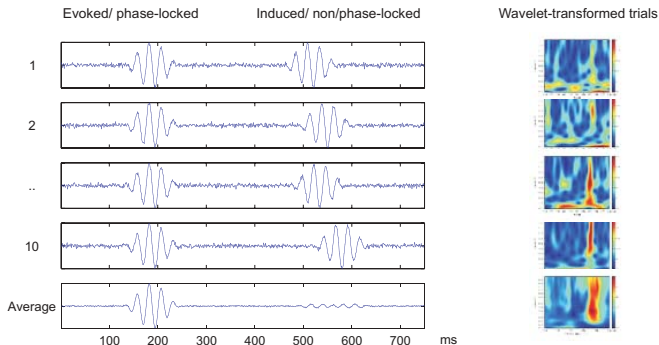


Evoked time-frequency representation of the average of all trials (ERP).

Induced average of time-frequency transforms of single trials.

Evoked and induced oscillations II

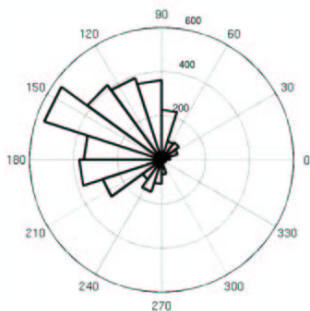
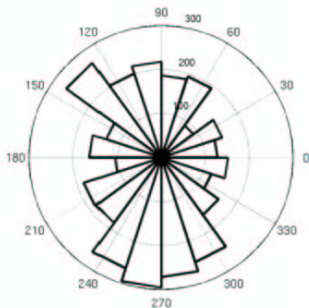
Wavelet analysis of single trials reveals non-phase-locked activity.



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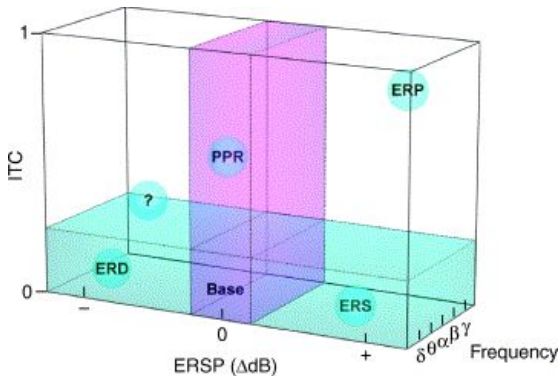
Induced average of time-frequency transforms of single trials.

Phase-locking factor (PLF)



- a.k.a. intertrial coherence (ITC) or phase-locking-value (PLV).
- measures phase consistency of a frequency at a particular time across trials.
- $PLF = 1$: perfect phase alignment.
- $PLF = 0$: random phase distribution.

The EEG state space



TRENDS in Cognitive Sciences

- Frequency x phase locking x amplitude changes²
- Evoked and induced activity are extremes on a continuum.
- ERPs cover only small part of the EEG space.

²Makeig, Debener, Onton, Delorme (2004). TICS

Examples 1: spontaneous EEG

- Stimulus pairs are presented at different phases of the alpha rhythm — sequential or simultaneous?³
- If the stimulus pair falls within the same alpha cycle \Rightarrow perceived simultaneity.
- Does the visual system take snapshots at a rate of 10 Hz?

Simultaneity and the alpha rhythm

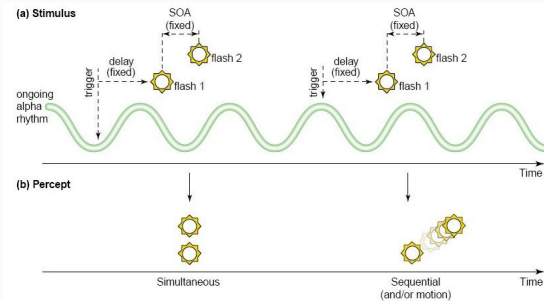


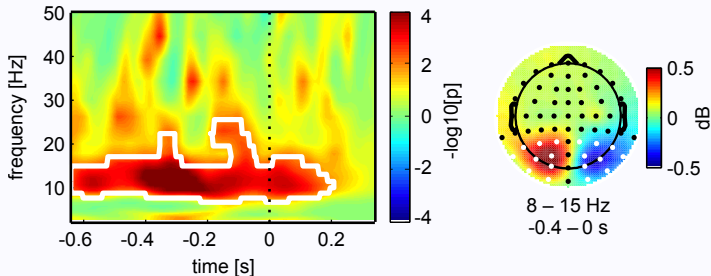
Figure from VanRullen & Koch (2003): Is perception discrete or continuous? TICS

³Varela et al. (1981): Perceptual framing and cortical alpha rhythm.
Neuropsychologia

Examples 2: pre-stimulus EEG power

- Spatial attention to left or right.
- Stronger alpha power over ipsilateral hemisphere.⁴

Attention: ipsi- vs. contra-lateral

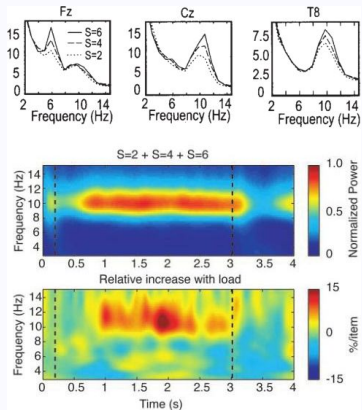


⁴Busch & VanRullen (2010): Spontaneous EEG oscillations reveal periodic sampling of visual attention. PNAS.

Examples 3: analysis of long time intervals

- Sternberg memory task with different set sizes.
- Alpha power increases linearly with set size.⁵

Effect of set size



⁵Jensen et al. (2002): Oscillations in the alpha band (9–12 Hz) increase with memory load during retention in a short-term memory task. Cereb Cortex.

Recommended reading

WWW:

- EEGLAB's time-frequency functions explained: <http://bishoptechbits.blogspot.com/>
- FFT explained: <http://blinkdagger.com/matlab/matlab-introductory-fft-tutorial/>
- Wavelet tutorial <http://users.rowan.edu/~polikar/WAVELETS/WTtutorial.html>

Books:

- Barbara Burke Hubbard: The World According to Wavelets.
- Steven Smith: The Scientist & Engineer's Guide to Digital Signal Processing (<http://www.dspguide.com/>).
- Herrmann, Grigutsch & Busch: EEG oscillations and wavelet analysis. In: Event-related Potentials: A Methods Handbook.

Papers:

- Tallon-Baudry & Bertrand (1999) Oscillatory gamma activity in humans and its role in object representation. TICS.
- Samar, Bopardikar, Rao & Swartz (1999) Wavelet analysis of neuroelectric waveforms: a conceptual tutorial. Brain Lang.

Thank you...

... for your interest !

Please ask questions!!!