The Hilbert transform H of a signal u is defined as

$$H(u)(t) := (u(s) \star \frac{1}{\pi s})(t)$$

$$= p.v. \int_{-\infty}^{\infty} u(s)h(t-s)ds$$

$$= \frac{1}{\pi} p.v. \int_{-\infty}^{\infty} \frac{u(s)}{t-s} ds$$

Property:

$$H(H(u))(t) = -u(t)$$

Relationship to the Fourier transform:

$$F(H(u))(w) = (-\jmath \cdot sgn(w))F(u)(w)$$

where

$$F(u)(w) := \frac{1}{2\pi} \int_{-\infty}^{+\infty} u(t)e^{-\jmath w t} dt$$

is the Fourier transform of the signal u and

$$sgn(w) := \frac{w}{\|w\|}$$

is the signal of $w \in \Im$. Therefore, my conclusion is

$$H(u)(t) = F^{-1}(w \mapsto -\jmath \cdot sgn(w) \cdot F(u)(w))(t)$$