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Paper and More

January 10, 1994

Kevin Brown grudgingly lowered the volume on Miles Davis' "Round About Midnight"[1] and resumed his work – reviewing his expansion plans for augmenting Paper and More's sole store in Jamaica, Vermont with at least eight more stores over the next two and a half years. In one month Brown would be meeting with venture capitalist funding broker Mathew Abraham, who in preliminary talks had emphasized the importance of establishing how store operations would scale with the expansion. The rough plans under Brown's pencil included ideas for siting stores, marketing, working with suppliers, product assortment, and store inventory management.

Fresh out of business school and recently married, Brown had opened the first P&M store in 1991 using his personal finances. Operated as an office supply store that catered to small businesses and home office users, P&M supplemented with printer supplies the many different paper products it sold. These included paper for home and office printing and writing, text and cover stocks (for external correspondence), and such other products as forms, envelopes, file folders, index cards, tags, tickets, and pressure-sensitive and other special papers. Brands included International Paper, Crane, Xerox, Georgia Pacific, Weyerhaeuser, and Fox River. Brown's expansion plans included introducing P&M's own store brand of paper.

Brown and Abraham, who, united by a love for Miles Davis, had been friends in business school, had met at the end of October 1993 to discuss the expansion plans for P&M. Abraham advised Brown that his proposals needed to address (i) up-front store costs, that is, the costs of opening each new store, including siting, storefront development, and inventory; (ii) economies of scale associated with opening each store; and (iii) the potential to successfully operate in different markets. Explaining that the way in-store inventory was managed would have implications for up-front costs and operating potential and the way the supply chain was managed for economies of scale, Abraham suggested that were Brown successful in getting funding, a management team with a track record in inventory management should be brought in. Brown insisted, though, that because inventory management was core to his business model, he needed to better understand the dynamics of matching inventory to demand.

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[1] Legacy Records, 1956.

Professor Noel Watson prepared this case but would like to acknowledge that Professor Ananth Raman and Professor Marshall Fisher (Wharton School) provided instrumental guidance. HBS cases are developed solely as the basis for class discussion. Certain details are fictional. Cases are not intended to serve as endorsements, sources of primary data, or illustrations of effective or ineffective management. In order to utilize this case, please acquire a copy of the related spreadsheet, Courseware 604-703.

Store Operations

For the past two years Brown had sought to create and preserve goodwill by maintaining high inventory levels for printing supplies and paper. P&M assessed inventory levels and placed orders for paper products every other Saturday. The P&M store was conveniently located within three hours of its product distributors in Vermont and New Hampshire, enabling orders dispatched by the end of the business day (3 p.m. on Saturday) to be received by the 10 a.m. store opening on Monday (the store was closed on Sundays).

Considering how inventory might be managed more systematically in anticipation of expansion, Brown realized that there were advantages to the current store location that might not carry over to other locations. He nevertheless wondered whether his experience to date might help him gain a better appreciation for inventory-demand dynamics. For example, two years' worth of historical sales information for paper products might be used to simulate store performance under different assumptions about inventory policies. Brown subsequently undertook a series of analyses.

Note: For the following analyses, please use the Excel models Paper and More distributed with the case. The various worksheets are identified as to which analysis they correspond. The data tables and Solver routines have already been preset in the worksheets so as to minimize the effort required to run the Excel analysis tools. Saving often is highly advised when performing these analyses.

Analysis #1

Brown decided to analyze sales for a staple brand for office and home printing: Great White Multi-Use 20-lb. 8½" x 11" 90 Bright from International Paper. Brown did not make much money on this stock-keeping unit (SKU), but he thought the product was important because it helped drive overall store traffic. The product's wholesale price was $20.00 per ream (500 sheets), which P&M retailed for $21. Brown estimated his annual inventory-carrying costs for paper products at 25% of the product's wholesale price.

For the past two years the P&M store held an average inventory level of about 80 reams of Great White, although the actual inventory level varied considerably due to product sales, demand uncertainty, and inconsistent ordering policies. Saturday's biweekly stocktaking confirmed that the product had always been in stock and thus had a 100% in-stock rate (i.e., the ratio of weeks during which the item did not stock out to the total number of weeks was 100%). An example of one type of service level measure retailers used, in-stock rate was often referred to as a "Type 1" service level. Fill rate, the percentage of demand satisfied from in-stock inventory, sometimes referred to as a "Type 2" service level, was also 100% for Great White over the two-year period. Although a 100% fill rate was ideal from a customer service perspective, Brown knew that he was paying a premium to hold inventory to meet that service level. Brown believed that a 98% fill rate (Type 2) was acceptable customer service, and sought to devise a simple, but consistent inventory policy to meet that goal.

Brown considered establishing a consistent order-up-to point policy, so that each time he checked his inventory levels, he would order enough to bring his inventory up to a target order-up-to level. Since customer loyalty for Great White was high, he further assumed that there was no substitution to another product in the event of a stock-out, so if customers found this product out of stock, they failed to purchase and their sales were lost to P&M.

To estimate inventory-carrying costs he assumed a uniform daily demand rate over a two-week period and used that assumption as follows. If the product did not stock out, the average inventory for the two-week period was half the sum of the starting and ending inventories. Otherwise, the average inventory over the entire two-week period was the product of the fraction of the two-week period that the product was not stocked out and the average inventory over this period. The assumption of a uniform daily demand rate implied that the fraction of time that the product was not stocked out was equal to the ratio of actual sales (starting inventory) to demand and the average inventory over this period was simply half of the starting inventory (since the ending inventory was zero). For example, if starting inventory was 20 and the product stocked out at the end of the first week, then average inventory over a two-week period was five (since the product was not stocked out for half the period and had an average of 20/2 = 10 units of inventory in the first week). Having calculated these two-week average inventory totals for each two-week period in the two year history, the inventory holding cost over the entire two year period was equal to the sum of all the two-week average inventories * 12 days/300 days * 25% * cost of the product. Here 12 days was the number of business days in a two-week period and 300 days the number of business days in the year (the store was closed on Sundays and closed for two weeks in January.)

Brown wondered how much he could reduce inventory if his goal was to achieve a 98% fill rate. The inventory carrying cost data would give him a sense of the savings he might realize from lower inventory levels. Finally, Brown wondered how he might explicitly optimize the product's net profit.

Performing Analysis #1

The worksheet "Analysis #1" shows the historical demand data for the Great White and simulates performance metrics (including costs, profits, and service levels) for the order-up-to point entered by the user.

In the worksheet, a table with column heading has already been started for you in columns N through R. Also, formula references to the in-stock rate, fill rate, total inventory cost, and net profits have been inserted in row 2 of columns O through R so that Excel's Data Table command can be used.

a) Construct in the worksheet "Analysis #1" a table giving in-stock rate, fill rate, total inventory costs, and net profit for various order-up-to points using the following instructions.

<u>Instructions for creating Data Table</u>:

- Enter the numbers 0 through 85 in cells N3–N88. This is the range of values for the order-up-to points.
- Highlight cells N2–R88.
- From the drop-down menu, select "Data," then "Table."
- Enter K3 in "Column input cell," leave "Row input cell" blank, and hit OK.
- Hit the F9 key to calculate the different metrics for each order-up-to level[2].

Questions:

i) What order-up-to point gives a 98% in-stock rate (Type 1 service)?

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[2] This step may be unnecessary depending on your computer's configuration. If the F9 key does not recalculate the measures, try locking the F-Lock button on your keyboard and hit F9 again.

ii) A 98% fill rate (Type 2 service)?

iii) Why are they not the same?

iv) Why is there a tendency for one to be higher than the other?

v) What order-up-to point maximizes the net profit on Great White?

b) The order-up-to point that maximizes the profit for Great White can also be determined by considering the logic of the newsvendor problem, in which a newsvendor faces uncertain daily demand and must decide upon an order quantity before the beginning of the day and is unable to replenish supplies during the day. The cost of overage (underage) is the cost to the newsvendor of one extra unit (one lost sale) at the end of the day. (See Appendix A.) The optimal order-up-to point is based on the ratio of the cost of underage to the sum of the costs of overage and underage. This ratio is referred to as the critical fractile and is the optimal in-stock rate that maximizes profits. The smallest inventory order-up-to point that gives this in-stock rate (or higher) is the optimal inventory order-up-to point.

Questions:

i) Evaluate Brown's proposal that he should use the holding cost for one unit for the two-week period as the cost of overage in the newsvendor model. Calculate this overage cost for the Great White SKU.

ii) Evaluate Brown's proposal that he should use the margin on one unit (= retail price – wholesale price) as the cost of underage.

iii) Calculate the critical fractile based on the underage and overage costs given (i) and (ii) above. This gives the optimal in-stock rate for those costs. The corresponding optimal order-up-to point is the smallest order-up-to point that has an in-stock rate equal to or higher than the critical fractile. Use your table of order-up-to points and in-stock rates to find the optimal order-up-to point based on your calculated critical fractile.

iv) The number calculated from (iii) does not match the order-up-to point determined in #1a v). Brown wonders if his cost of underage depends on his order-up-to point. Evaluate Brown's thought that if his order-up-to point is high enough the cost of underage is close to his margin minus the holding cost, but as his order-up-to point gets lower and lower the cost of underage should get closer to just the margin.

v) Based on (iv), the margin calculated in (ii) is the maximum value of the cost of underage. Calculate the minimum value of the cost of underage and the corresponding order-up-to point. What is the maximum difference in net profits over the range of order-up-to points from the order point calculated in (iii) to the order-up-to point calculated here?

c) What type of inventory policy, fill-rate service level, in-stock rate service level, or profit maximization criteria should Brown choose? What strategic product characteristics or overall retailing strategy could influence his decision?

d) The foregoing analysis suggests a process for managing individual product inventory: *Periodically determine the inventory order-up-to point that meets the required inventory policy for that product on historical data and use this order-up-to point for the future.* What concerns about the efficacy of the process might Brown still have, and how might he try to address them using his data?

Analysis #2

a) The worksheet "Analysis #2" is the same as "Analysis #1," showing the historical demand data for the Great White SKU.

i) What are the mean and standard deviation of the demand for the SKU? (The normal distribution is completely specified once the mean and standard deviation are known.)

ii) Generate a histogram of the observed demand values for the Great White using the following instructions.

<u>Instructions for creating Histogram:</u>

- Enter the numbers 10, 20, 30, . . . , 80 in cells K26–K33.
- From the drop-down menu select "Tools" then "Data Analysis." (If "Data Analysis" is not available from the Tools menu, first select "Tools" then "Add-ins" from the drop-down menu and add the "Analysis Toolpak.")
- From the menu in the "Data Analysis" dialog box that appears choose "Histogram" and hit OK.
- Put cells B2–B51 in the "Input Range" and cells K26–K33 in the "Bin Range."
- Check the "Cumulative Percentage" and the "Chart Output" checkboxes. Then hit OK.
- The histogram, both a table and a chart, will be generated in a new worksheet. (A general note on creating histograms in Excel is located in Appendix B.)

iii) Compare the cumulative frequencies of your histogram with those of the histogram of a normal distribution with mean 40 and standard deviation 20 shown in **Exhibit** 1. (The histogram is also copied into columns Q – S in the Analysis #2 worksheet.) Comment on the fit of the normal distribution to the demand for Great White.

b) The inverse cumulative function of a demand distribution takes an in-stock rate and returns the order-up-to point that gives this in-stock rate.[3] Create a table of order-up-to points for given in-stock rates using the NORMINV() function and the mean and standard deviation calculated in a) (above) along with the following instructions.

<u>Instructions for creating table of fill rates:</u>

- The table should be placed in columns N and O. In column N, list the in-stock rates being considered, which range from 1% to 99%, in cells N2 to N100.
- In column O, calculate the smallest order-up-to point that has the considered in-stock rate or greater. For example, =NORMINV(0.2,10,5) returns the order-up-to point that has in-stock rate 0.2 (20%) for a normal demand distribution with mean 10 and standard deviation 5. If the order-up-to point returned is a fraction, then round up to the nearest integer to get the

[3] More formally, a cumulative distribution function takes a value as input and returns the probability that a draw from the distribution will be smaller than or equal to this value. The NORMDIST command in Excel provides the cumulative distribution for the normal distribution. The inverse of a cumulative distribution function takes a probability as input and returns the value with which a draw from the distribution will be smaller than or equal to this value, with the input probability.

smallest order-up-to point. The Excel command =ROUNDUP(x,0) rounds up x to the nearest integer. (It is not necessary to use the Data Table command here.)

c) The order-up-to point that generates a particular fill rate can be calculated using a simulation, although some look-up tables exist for normal and other common distributions. The worksheet "Analysis #2(c)" generates random numbers drawn from a normal distribution based on the mean and standard deviation the user inputs in cells H6 and H7.[4] Given an order-up-to point entered by the user in cell H2, the worksheet calculates and reports in cell H3 the corresponding fill rate, that is, the percentage of total demand that is met from in-stock inventory.

Construct a table of fill rates given inventory order points using the following instructions.

<u>Instructions for creating table of order-up-to points:</u>

- In worksheet "Analysis #2(c)," enter in cells H6 and H7 the mean and standard deviation calculated in Analysis #2(a).
- In the worksheet, a table with column heading has already been started for you in columns J and K. Also, a formula reference to the fill rate has been inserted in row 2 of column K so that Excel's Data Table command can be used.
- Enter numbers 0 thru 85 in cells J3–J88. These are the range of values for the order-up-to points.
- Highlight cells J2–K88.
- From the drop-down menu, select "Data," then "Table."
- Enter H2 in "Column input cell," leave "Row input cell" blank, and hit OK.
- Hit the F9 key to calculate the fill rate for each order-up-to level.

d) Calculate the order-up-to point that has in-stock rate just greater than or equal to the critical fractile described in #1(b-iii) using the method described in Analysis #2(b).

Analysis #3

The worksheet "Analysis #3(a)" enumerates the historical data for Fox River Writing Paper.

a) Construct a line plot of sales in all 49 periods using the following instructions.

<u>Instructions for constructing line plot:</u>

- Highlight cells B4–B53.
- From the drop-down menu select "Insert," then "Chart."
- Highlight "Line" under the "Chart Type" options, leave the "Chart Subtype" as is, hit "Next," and then hit "Next" again.

[4] The worksheet comes with 5000 random numbers between 0 and 1 already generated. These numbers are then converted to draws from a normal distribution with the mean and standard deviation that is input by the user.

- Fill in the appropriate labels for the axes and then hit "Finish."

i) Have Excel add a polynomial trendline to the plot using the following instructions.

Instructions for adding a polynomial trendline:

- Highlight the chart and then choose "Chart" and "Add Trendline" from the main drop-down menu.
- In the "Add Trendline" dialog box, highlight the polynomial option, leaving the order at 2.
- Click on the option tab in the dialog box and click on both "Display equation on chart" and "Display R-squared value on chart." (These are displayed on the chart in a text box that can be moved around so that it can be clearly seen.)
- Click OK to add trendline.

ii) Using similar instructions to (i) above, have Excel add a logarithmic trendline to the plot. Compare the R-square of this trendline with that of (i). Which trendline explains more of the variance seen in the data?

b) Brown tries to use the polynomial trendline of question (a) to decide his order-up-to point for period 50. He would like to use the approach in Analysis #2. The trendline allows Brown to estimate the center (mean) of period 50's demand; however, he needs to estimate a standard deviation about this center and verify that the distribution about the center is approximated by a normal distribution.

i) What is the forecasted mean of period 50's demand based on the polynomial trendline?

ii) Calculate the standard deviation of sales over the period as in Analysis #2(a). If the polynomial fit is to be used to forecast period 50's demand, why is the calculated standard deviation an overestimate of the variation about the forecast?

iii) Brown decides to use the actual deviation around the polynomial forecasts for the periods 1–49 to generate his standard deviation for period 50's forecast.

Instructions for generating polynomial forecasts and their errors:

- Use the formula for the polynomial trendline to calculate the forecasts for periods 1–49 in the column next to sales.
- Create a column of the errors (sales-forecast) for each of the periods 1–49.
- What is the mean and standard deviation of these errors?
- Using the following bins –15,-10,-5,0,5,10,15 generate a histogram of the errors as in #2(c) and compare its cumulative frequencies with those of **Exhibit 2**, which shows a histogram for the normal distribution with mean 0 and standard deviation 8.
- Comment on the fit.

iv) If an approximate critical fractile for the Fox River Writing Paper is 0.8, calculate the order-up-to point that maximizes profit.

c) Brown considers that the sales of Fox River Writing Paper may be explained by the fact that the product was a new product brought on the market in period 1. Therefore, the period during which the product seems to be trending upwards is a "ramp-up" period, after which the sales of the product become more stable with no trend. If Brown thinks that this "ramp-up" was from periods 1 to 25, describe the approach you would take to setting an order-up-to point in period 50.

Analysis #4

Brown realized that with any order point that resulted in a fill rate less than 100% there would be periods when supply was less than demand and sales would be lost. He wondered how to estimate the level of lost sales and what changes he might need to make to inventory management to adjust for lost sales.

Performing Analysis #4

a) Consider the data in the worksheet "Analysis #4(a)" which reports observed sales for a specialty envelope together with beginning and ending inventory

i) Which of the service levels, in-stock rate or fill rate, can Brown determine from the product's historical data?

ii) Are optimal profits being realized from the product?

b) One approach to dealing with historical demand in which lost sales is suspected, is to directly estimate the lost sales based on when the product stocked out. With point-of-sale (POS) records, which show the timing of the last sale, Brown realized that he could estimate at what point in the week the product had stocked out, assuming no shrinkage[5]. How could this information be used to estimate lost sales in a period?

c) For the specialty envelope, Brown does not have complete POS records. He wonders if he can use the service level data that he does have to still estimate lost sales. For this approach he will use the Great White data in which there were no lost sales to see if it is possible to estimate lost sales using the service level data he has on the specialty envelope. The following instructions allow a table to be constructed of in-stock rate (Type 1 service level), fill rate (Type 2 service level), % error in mean sales, and % error in standard deviation of sales given inventory order points. Here:

$$\%\text{ error in mean of sales} = \frac{\text{Average of true demand} - \text{Average of observed sales}}{\text{Average of observed sales}}$$

$$\%\text{ error in std. dev. of sales} = \frac{\text{Std. dev. of true demand} - \text{Std. dev. of observed sales}}{\text{Std. dev. of observed sales}}$$

<u>Instructions for constructing the table:</u>

- In the worksheet "Analysis #4(c)," a table with column heading has already been started for you in columns N through R. Also, formula references have also been inserted in row 2 of columns O through R.

[5] Shrinkage is the loss of inventory over time due to damage, misplacement, or theft.

- Enter numbers 0 through 85 in cells N3–N88. These are the range of values for the order-up-to points.
- Highlight cells N2–R88.
- From the drop-down menu, select "Data," then "Table."
- Enter H2 in "Column input cell," leave "Row input cell" blank, and hit OK.
- Hit the F9 key.

Assuming that the relationship shown in the table between in-stock rate and fill rate holds in Analysis #4(a), determine the fill rate and total lost sales for the specialty envelope product after determining its Type 1 service level. If the table you have generated has more than one entry for an in-stock rate, use the average of the results over this in-stock rate.

i) What is its mean of actual demand?

ii) The standard deviation of actual demand?

iii) If Brown assumed demand for the specialty envelope to be normally distributed, what order-up-to point that gives a 98% fill rate would have maximized profit for the special envelope?

Analysis #5

Brown considered for multiple products an aggregate service level similar to the Type 1 service level identified in Analysis #2: percentage of SKU-periods in which no stockout occurs. The total number of SKU-periods is the number of SKUs multiplied by the total number of two-week periods under consideration. Each time a SKU stocks out in a period it is considered a SKU-period stockout. (If two SKUs stock out in a period, it is considered as two SKU-period stockouts.)

He considered two products with similar margins but different sales levels: the Great White, and Georgia Pacific Multi-Purpose 20lb 81/2 x 11 90 Bright. The Georgia Pacific paper was also equivalently priced per ream at $20 wholesale and $21 retail. Average demand for the Georgia Pacific product was one quarter that for the Great White.

Performing Analysis #5

a) The worksheet "Analysis #5" shows the historical demand data for both the Great White and Georgia Pacific and simulates their cost, profit, and service level performance given order-up-to points entered by the user. Given the following two order-up-to points per product: Great White (45, 65); Georgia Pacific (10, 18), generate two 2x2 matrices, one for overall net profit (sum of net profits for both products) and the other for the aggregate Type 1 service level. Each row represents different order-up-to points for Georgia Pacific and each column for Great White. Each cell represents, therefore, a unique combination of order-up-to points for both products. In each cell, place the corresponding overall net profit or aggregate Type 1 service level.

b) Consider the following two settings: Setting A—Great White with order-up-to point 45 and Georgia Pacific with order-up-to point 18; Setting B—Great White with order-up-to point 65 and Georgia Pacific with order-up-to point 10. Why is the aggregate Type 1 service level for Setting A higher than the aggregate Type 1 service level for Setting B when the profit for Setting A is lower with lower inventory costs?

Analysis #6

Brown also considered two products with different margins but the same sales levels: the Great White and Hammermill Bond 20lb, 8 1/2 x 11, 92 Bright premium and watermarked. The Hammermill, which wholesaled for $22.00 per ream and retailed for $25.00, was equally as popular as the Great White.

Performing Analysis #6

a) The worksheet "Analysis #6" is similar to worksheet "Analysis #5" but now compares the Great White to the Hammermill. Just as in Analysis #5(a), generate two 2x2 matrices, one for overall net profit (sum of net profits for both products) and another for the aggregate Type 1 service level, for all combinations of the following order-up-to points for Great White and Hammermill: Great White (45, 65); Hammermill (55, 75).

b) Consider the following two settings: Setting A—Great White with order-up-to point 65 and Hammermill with order-up-to point 55; Setting B—Great White with order-up-to point 45 and Hammermill with order-up-to point 75. Why is the aggregate Type 1 service level for Setting A higher than the aggregate Type 1 service level for Setting B when the profit for Setting A is lower with lower inventory costs?

c) Generalize the observations from both Analyses #5b) and #6b). What are the dangers of using the aggregate Type 1 service level to evaluate allocation of inventory to multiple products? Among multiple products that neither substitute for nor complement one another, how should the allocation of inventory be prioritized?

Analysis #7

Brown also considered how to manage products in the face of storage constraints. Inasmuch as it represented Brown's actual inventory at the beginning of a cycle, the inventory order-up-to point also determined his storage needs. Again taking Great White and Hammermill, both of which have the same volume requirement per ream of 190 cubic inches, Brown wondered how the inventory order-up-to point and required volume for each product related to the shelf space provided to each.

Performing Analysis #7

a) The worksheet "Analysis #7" is similar to worksheet "Analysis #6" except that the worksheet now tracks in cubic inches the shelf space allocation for each SKU (given in cells B16 and C16 and equal to order-up-to point * 190) and overall allocation (given in cell D16 and equal to the sum of the individual allocations). The following instructions allow a table to be constructed of the inventory order-up-to points for each SKU that maximizes overall net profit and the overall net profit and the total shelf space allocation given total available shelf space quantities of 17,000, 21,000, and 25,000 cubic inches.

<u>Instructions for creating table of inventory order-up-to points:</u>

- In the worksheet "Analysis #7" a table with column headings has already been started for you in columns F through J.
- Enter the values for the total shelf space available, that is, 17,000, 21,000, and 25,000, separately in cell D17 and run Excel's Solver for each value to calculate the required order-up-to points and other information. The Data Table command should not be used here.
- To run Solver choose "Tools" then "Solver" from the drop-down menu. Then hit "Solve" and "OK."[6]
- The optimal order-up-to points are automatically inserted in B3 and C3 by Solver when it is finished. In columns G through J, record the optimal order-up-to points for the Great White and Hammermill SKUs, the overall net profit, and the total shelf space allocation.

b) From your table it is observed that the optimal overall shelf space allocation is much less than 25,000, given 25,000 cubic inches of available space. This implies that the optimal order-up-to points found here are the same as if we had maximized profits for each SKU separately without a shelf space constraint; that is, found their "stand-alone" optimal order-up-to points. Why is this true?

c) Given generalizations from (c) of Analysis #6, why doesn't the optimal allocation for a shelf space constraint of 17,000 cubic inches have Hammermill at its "stand-alone" optimal inventory order-up-to point and allocate the remaining inventory to Great White?

[6] If the pre-programmed Solver routine for this analysis is not available for whatever reason, the user needs to reprogram it using the note in Appendix C. The user should then enter values for the total shelf space available in cell D17 and run Solver each time to calculate the required order-up-to points.

Appendix A: Note on Newsvendor Problem

Consider a newsvendor who faces uncertain daily demand and must decide upon an order quantity before the beginning of the day. The newsvendor maximizes her expected profit from the sale of newspapers. The newsvendor buys her newspapers at a wholesale unit cost of *c* and sells them at price *p*. If there are any newspapers left over at the end of the day, the newsvendor can salvage these newspapers at unit salvage value *s*. If demand is greater than the number of newspapers ordered, then this excess demand is lost.

In order to determine the optimal inventory order-up-to point we consider the costs of an excess newspaper and of a lost sale. The cost of an excess newspaper ordered is $c - s$, the cost of the newspaper offset by the salvage value. We refer to this cost as the overage cost and assign it symbol *Co*. The opportunity cost of lost sale is $p - c$, the lost margin. We refer to this cost as the underage cost and assign it symbol *Cu*. The optimal order-up-to point for the newsvendor can be shown to be the smallest order-up-to point that has an in-stock rate greater than or equal to the ratio $\frac{Cu}{Co + Cu}$. This ratio is referred to as the critical fractile. Note for this newsvendor problem $\frac{Cu}{Co + Cu} = \frac{p - c}{p - s}$.

When demand is normal with mean μ and standard deviation σ, the optimal inventory order-up-to point $= \mu + \Phi^{-1}\left(\frac{Cu}{Co + Cu}\right)\sigma$. Here Φ^{-1} is the inverse cumulative distribution for the standard normal distribution, which has mean 0 and standard deviation 1. This inventory order-up-to point can be calculated in Excel using the formula $NORMINV\left(\frac{Cu}{Co + Cu}, \mu, \sigma\right)$.

Appendix B: Note on Creating Histograms in Excel

A histogram shows the frequencies (either as the number of occurrences or percentage of total number of occurrences) of categories of events based on actual data of occurrences. See **Exhibit 1** for example.

The Excel feature to create histograms is included in the Analysis ToolPak add-in. This add-in must be installed in order to create the histogram. If the Analysis ToolPak has been installed, then the "Data Analysis" option is available from the "Tools" menu. To install the Analysis ToolPak add-in, choose "Add-ins" from the "Tools" drop-down menu. "Analysis ToolPak" should be available from the list of add-ins available. Click on its check-box and then click on "OK."

The Excel histogram function generates either a chart or table of frequencies for categories of numerical occurrences. These categories are referred to as bins and must be input by the user. Consider a column of 100 numbers in cells A1 through A100 for which we would like to create a histogram as in **Exhibit 1**. We would like to know the frequency of the numbers in the following bins:

- less than or equal to 10
- from 11 to 20
- from 21 to 30
- from 31 to 40
- from 41 to 50
- from 51 to 60
- from 61 to 70
- from 71 to 80
- above 80

To create the histogram of the column of 100 values, we perform the following steps:

1. Create the following list in cells C1 through C8: 10, 20, 30, 40, 50, 60, 70, 80. (This list is the list of upper limits for all the bins.)
2. Choose "Data Analysis" from the "Tools" drop-down menu and then "Histogram" from the list of analysis tools.
3. Set Input range to A1–A100 and Bin Range to C1–C8. Click on the button for "New Worksheet Ply" and "Cumulative Percentage." If a chart output is required also click on the box for "Chart Output." Then click "OK."
4. The histogram chart or table shows the frequency as the number of occurrences in the data in cells A1–A100 for each category. The frequency as a percentage can be calculated by dividing the number of occurrences for each category by the total number of occurrences and the cumulative % by totaling the frequency % for all preceding rows.

Appendix C: Instructions for Programming Excel's Solver for Problem 7

Excel's Solver can be used to find the value or set of values that minimize, maximize, or set a defined objective function to a particular value. Solver can also determine this value or set of values so that they meet a number of defined constraints. These constraints are usually of the form: some function of the values being less than, greater than, or equal to another function or number.

Excel's Solver is accessed directly from the "Tools" drop-down menu. However, Solver is packaged with Excel as an add-in and therefore may not be available from the "Tools" menu unless the add-in has been explicitly installed. To install Solver if it is not installed, choose "Add-ins" from the "Tools" drop-down menu. "Solver add-in" should be available from the list of add-ins available. Click on its check-box and then click on "OK."

For Question 7, after putting a shelf space value in cell D17, from the drop-down menu select "Tools" then "Solver" and perform the following:

- Put cell D13 in the "Set Target Cell" box.
- Put cells B3 and C3 in the "By Changing Cells" box.
- Hit "Add."
- Put B3 in the "Cell Reference" box, set the middle box to "int," and leave the "Constraint" box blank. Hit "Add."
- Put C3 in the "Cell Reference" box, set the middle box to "int," and leave the "Constraint" box blank. Hit "Add."
- Put B3 in the "Cell Reference" box, set the middle box to ">=," and put 0 in the "Constraint" box. Hit "Add."
- Put C3 in the "Cell Reference" box, set the middle box to ">=," and put 0 in the "Constraint" box. Hit "Add."
- At the end of the next step it is important to hit "OK" instead of "Add." Put D16 in the "Cell Reference" box, set the middle box to "<=," and put D17 in the "Constraint" box. Hit "OK."
- Hit "Solve."
- When Solver is completed hit "OK." The optimal order-up-to points generated by Solver for the input shelf space are inserted in cells B3 and C3.

At this point Solver has been programmed. To generate optimal order up-to-points for a new shelf space value, enter this value in cell D17 and simply choose "Tools" and "Solver" from the drop-down menus and then click on "Solve" and then "OK."

Exhibit 1 Histogram of Normal Distribution with Mean 40 and Standard Deviation 20

Bin	Frequency	Cumulative
0—10	6.7%	6.7%
11—20	9.2%	15.9%
21—30	15.0%	30.9%
31—40	19.1%	50.0%
41—50	19.1%	69.1%
51—60	15.0%	84.1%
61—70	9.2%	93.3%
71—80	4.4%	97.7%
More	2.3%	100.0%

Exhibit 1 shows the frequency as a percentage. The cumulative percentage shows the cumulative summation of the percentage frequencies as we move down the list of bins.

Source: Casewriter.

Exhibit 2 Histogram of Normal Distribution with Mean 0 and Standard Deviation 8

Bin	Frequency	Cumulative
Less (–15)	3.0%	3.0%
(–15)—(–10)	7.6%	10.6%
(–10)—(–5)	16.0%	26.6%
(–5)—0	23.4%	50.0%
0—5	23.4%	73.4%
5—10	16.0%	89.4%
10—15	7.6%	97.0%
More	3.0%	100.0%

Exhibit 2 shows the frequency as a percentage. The cumulative percentage shows the cumulative summation of the percentage frequencies as we move down the list of bins.

Source: Casewriter.