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BUSM3021
*Business Analytics*
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# Homework 3

*Due 11/4/2021*

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1. (50pts)

Sphero produces toy robots for various companies. Due to a large new order, Sphero predicts that demand for its R2-D2 robot will be 12,500 units in the next quarter. This is a substantial increase relative to previous quarters and will challenge Sphero's manufacturing capabilities. The R2-D2 robot consist of three key components: a housing unit, a motor unit, and a circuitry unit. Sphero is convinced that it will have to subcontract some of these units and purchase a number of these components from outside vendors because its internal capacity is limited.

The table below shows both the production cost per unit for each component when the robot is produced internally and the cost per unit for each component when purchased from an outside vendor:

| ***Component*** | ***Vendor Cost*** | **Internal Cost** |
|---|---|---|
| *Housing* | $12.10 | $8.10 |
| *Motor* | $25 | $23 |
| *Circuitry* | $19.50 | $14.50 |

Sphero's manufacturing facility has three departments: Assembly, Quality Control (QC), and Distribution. If produced internally, each component requires a certain amount of time (in hours/unit) in each of these three departments. The following table shows these time requirements; the final row of the table shows the total number of hours available for R2-D2 production in the upcoming quarter.

| ***Component*** | ***Assembly*** | ***QC*** | **Distribution** |
|---|---|---|---|
| *Housing* | 0.07 | 0.09 | 0.06 |
| *Motor* | 0.08 | 0.02 | 0.05 |
| *Circuitry* | 0.04 | 0.06 | 0.04 |
| **Capacity** | **1650** | **1650** | **1650** |

a. Formulate a Linear Program that can help Sphero meet its predicted demand most efficiently. Clearly define the decision variables, the objective function, and the constraints.

b. To avoid overreliance on outside vendors for an individual component, Sphero also wants to ensure that the fractions of each component purchased from outside vendors are within 15% of each other. Adjust your formulation to incorporate this.

2. (50pts)

A startup company must pay off accounts payable that it has in each of the next five months:

| *Month* | *1* | *2* | *3* | *4* | *5* |
|---|---|---|---|---|---|
| *Accounts Payable* | $17,000 | $7,000 | $4,000 | $6,500 | $12,000 |

At the start of the first month, the company has a $13,500 cash reserve, as well as a total of $48,000 in various certificates of deposit (CDs). Specifically, the startup has $15,000 in CD A, $8,000 in CD B, and $25,000 in CD C.

To pay off its accounts payable, the startup will have to withdraw some of its CDs. However, it will have to pay a penalty for any CD sold prior to the end of month 5. The penalty for selling $1 worth of each of CD is shown in the following table:

| | *Month of Sale* | | | | |
|---|---|---|---|---|---|
| | *1* | *2* | *3* | *4* | *5* |
| *CD A* | $0.07 | $0.06 | $0.04 | $0.03 | $0.02 |
| *CD B* | $0.17 | $0.15 | $0.12 | $0 | $0 |
| *CD C* | $0.34 | $0.34 | $0.32 | $0.30 | $0 |

The startup wants to minimize the cost of meeting its accounts payable (that is, the total amount of penalties paid), assuming that all accounts must be paid on time. Construct a Linear Programming formulation to help the company meet its objective.

Formulate a Linear Program that can the startup plan its payments. Clearly define the decision variables, the objective function, and the constraints.

problem 1

(A) LP setup

decision variables:

$X_{VH}$: number of units a vendor supplies for the housing component

$X_{VM}$: number of units a vendor supplies for the motor component

$X_{VC}$: number of units a vendor supplies for the circuit component

$X_{IH}$: number of units internal warehouse supplies for the housing component

$X_{IM}$: number of units internal warehouse supplies for the motor component

$X_{IC}$: number of units internal warehouse supplies for the circuit component

objective function:

minimize cost to make the R2D2 robot. represented by the function:

$12.10X_{VH} + 25X_{VM} + 19.50X_{VC} + 8.10X_{IH} + 23X_{IM} + 14.50X_{IC}$

## constraints:

(1) set of constraints based on the time requirements in hr/units to internally producing R2D2

assembly capacity

$$0.07X_{IH} + 0.08X_{IM} + 0.04X_{IC} \leq 1650$$

quality control capacity

$$0.09X_{IH} + 0.02X_{IM} + 0.06X_{IC} \leq 1650$$

distribution capacity

$$0.06X_{IH} + 0.05X_{IM} + 0.04X_{IC} \leq 1650$$

(2) set on constraints to ensure demand of product is met regardless of which warehouse (vendor or internal) produces each part of R2D2

atleast 12500 housing components are made

$$X_{VH} + X_{IH} \geq 12500$$

atleast 12500 motor components are made

$$X_{VM} + X_{IM} \geq 12500$$

atleast 12500 circuit components are made

$$X_{VC} + X_{IC} \geq 12500$$

problem 1

(B) sphero wants to ensure the fractions of each component purchased from outside vendors are within 15% of each other

(1) $$\frac{\chi_{VH} - \chi_{VM}}{1250} \leq 0.15$$

(2) $$\frac{\chi_{VM} - \chi_{VH}}{1250} \leq 0.15$$

(3) $$\frac{\chi_{VH} - \chi_{VC}}{1250} \leq 0.15$$

(4) $$\frac{\chi_{VC} - \chi_{VH}}{1250} \leq 0.15$$

(5) $$\frac{\chi_{VM} - \chi_{VC}}{1250} \leq 0.15$$

(6) $$\frac{\chi_{VC} - \chi_{VM}}{1250} \leq 0.15$$

problem 2 LP setup

decision variables:

$X_{A1}$ : amount (in dollars) withdrawn from CD A to pay off account payable principle for month 1

$X_{A2}$ : amount (in dollars) withdrawn from CD A to pay off account payable principle for month 2

$X_{A3}$ : amount (in dollars) withdrawn from CD A to pay off account payable principle for month 3

$X_{A4}$ : amount (in dollars) withdrawn from CD A to pay off account payable principle for month 4

$X_{A5}$ : amount (in dollars) withdrawn from CD A to pay off account payable principle for month 5

$X_{B1}$ : amount (in dollars) withdrawn from CD B to pay off account payable principle for month 1

$X_{B2}$ : amount (in dollars) withdrawn from CD B to pay off account payable principle for month 2

$X_{B3}$ : amount (in dollars) withdrawn from CD B to pay off account payable principle for month 3

$X_{B4}$ : amount (in dollars) withdrawn from CD B to pay off account payable principle for month 4

$X_{B5}$ : amount (in dollars) withdrawn from CD B to pay off account payable principle for month 5

$X_{C1}$ : amount (in dollars) withdrawn from CDC to pay off accounts payable principle for month 1

$X_{C2}$ : amount (in dollars) withdrawn from CDC to pay off accounts payable principle for month 2

$X_{C3}$ : amount (in dollars) withdrawn from CDC to pay off accounts payable principle for month 3

$X_{C4}$ : amount (in dollars) withdrawn from CDC to pay off accounts payable principle for month 4

$X_{C5}$ : amount (in dollars) withdrawn from CDC to pay off accounts payable principle for months

$X_{R1}$ : amount (in dollars) of cash reserve used to pay off accounts payable principle for month 1

$X_{R2}$ : amount (in dollars) of cash reserve used to pay off accounts payable principle for month 2

$X_{R3}$ : amount (in dollars) of cash reserve used to pay off accounts payable principle for month 3

$X_{R4}$ : amount (in dollars) of cash reserve used to pay off accounts payable principle for month 4

$X_{R5}$ : amount (in dollars) of cash reserve used to pay off accounts payable principle for months

## objective function

to minimize the cost of meeting accounts payable, note there is no cost to use cash reserve thus the decision variables with regards to cash are not included

$$0.07X_{A1} + 0.06X_{A2} + 0.04X_{A3} + 0.03X_{A4} + 0.02X_{A5} +$$
$$0.11X_{B1} + 0.15X_{B2} + 0.12X_{B3} + 0.00X_{B4} + 0.00X_{B5} +$$
$$0.34X_{C1} + 0.34X_{C2} + 0.32X_{C3} + 0.30X_{C4} + 0.00X_{C5}$$

## constraints:

(1) 17000 dollars must be paid off in month 1
↳ in principle

$$X_{A1} + X_{B1} + X_{C1} + X_{R1} = 17000$$

(2) 7000 dollars must be paid off in month 2
↳ in principle

$$X_{A2} + X_{B2} + X_{C2} + X_{R2} = 7000$$

(3) 4000 dollars must be paid off in month 3
↳ in principle

$$X_{A3} + X_{B3} + X_{C3} + X_{R3} = 4000$$

(4) 6500 dollars must be paid off in month 4

$$X_{A4} + X_{B4} + X_{C4} + X_{R4} = 6500$$

(5) 12000 dollars must be paid off in months

$$X_{A5} + X_{B5} + X_{C5} + X_{R5} = 12000$$

Note: I have assumed the cost of pulling from a CD is absorbed by that CD
ie if I draw 10 dollars from CD1 for month 1, the 0.10 cent fee is also deducted from CD1

(6) only up to 13500 dollars available to draw from cash reserve

$$X_{R1} + X_{R2} + X_{R3} + X_{R4} + X_{R5} \leq 13500$$

(7) only up to 15000 dollars available to draw from CD A

$$1.07X_{A1} + 1.06X_{A2} + 1.04X_{A3} + 1.04X_{A4} + 1.02X_{A5} \leq 15000$$

(8) only up to 8000 dollars available to draw from CDB

$$1.17X_{B1} + 1.15X_{B2} + 1.12X_{B3} + X_{B4} + X_{B5} \leq 8000$$

(9) only up to 25000 dollars available to draw from CDC

$$1.34X_{C1} + 1.34X_{C2} + 1.32X_{C3} + 1.30X_{C4} + X_{C5} \leq 25000$$