

Homework 6 answers

Due 12/02/2021

1. (50pts)

A small firm is evaluating three possible investments. Profits from each investment will be affected by the market conditions. The investor has developed a payoff table for three possible market conditions during the next five years.

	Market Conditions		
	<i>Poor</i>	<i>Average</i>	<i>Good</i>
Probability	40%	40%	20%
Investment A	-8	15	20
Investment B	2	8	6
Investment C	6	6	5

- a) Determine the optimal investment using an Expected Value criterion.

	<i>Poor</i>	<i>Average</i>	<i>Good</i>	Expected Value
Probability	40%	40%	20%	
Investment A	-8	15	20	6.8
Investment B	2	8	6	5.2
Investment C	6	6	5	5.8

Hence, the optimal choice is Investment A.

- b) A consultant approaches the investor and claims to be able to predict the economic future with perfect certainty. What is the maximum that the investor would be willing to pay for this information?

$$EV_{wPI} = 0.4 \cdot 6 + 0.4 \cdot 15 + 0.2 \cdot 20 = 12.4$$

$$EVPI = 12.4 - 6.8 = 5.6$$

Hence, the maximum the investor would be willing to pay is 5.6.

Another private consulting company periodically issues a 5-year forecast of market conditions. Experience has shown that this forecast is reasonably good, but not perfect, as shown by the following probabilities:

- $P(\text{positive forecast} \mid \text{good market conditions}) = 0.8$
- $P(\text{negative forecast} \mid \text{good market conditions}) = 0.2$
- $P(\text{positive forecast} \mid \text{average market conditions}) = 0.5$
- $P(\text{negative forecast} \mid \text{average market conditions}) = 0.5$
- $P(\text{positive forecast} \mid \text{poor market conditions}) = 0.1$
- $P(\text{negative forecast} \mid \text{poor market conditions}) = 0.9$

- c) The firm purchases the 5-year forecast but is disappointed to learn that the forecast is negative concerning market conditions. How should the investor revise its probability estimates for the market conditions given the negative forecast? How will this change the firm's decision?

We first calculate the joint probabilities:

	Poor	Average	Good	Total
Positive	0.04	0.2	0.16	0.4
Negative	0.36	0.2	0.04	0.6

Next, we calculate the posterior probabilities:

$$P(\text{poor market conditions} \mid \text{negative forecast}) = 0.36/0.6 = 3/5$$

$$P(\text{average market conditions} \mid \text{negative forecast}) = 0.2/0.6 = 1/3$$

$$P(\text{good market conditions} \mid \text{negative forecast}) = 0.04/0.6 = 1/15$$

Given the negative forecast, the expected values will therefore be:

	<i>Poor</i>	<i>Average</i>	<i>Good</i>	
Probability	0.6	0.333	0.067	Expected Value
Investment A	-8	15	20	1.53
Investment B	2	8	6	4.27
Investment C	6	6	5	5.93

Hence, the firm's decision will change to Investment C.

2. (50pts)

Amazon is trying to determine whether to build a distribution center near Fresno or near Henderson. The cost of building a distribution center is \$20 million near Fresno and \$40 million near Henderson.

However, if Amazon builds near Fresno and an earthquake occurs there during the next 3 years, construction will be terminated and Amazon will lose \$20 million (and will still have to build a distribution center near Henderson). Amazon believes there is a 20% chance that an earthquake will occur near Fresno during the next 5 years. For \$900,000, a geologist can be hired to analyze the fault shifts near Fresno. The geologist will either predict that an earthquake will occur or that an earthquake will not occur. The geologist's past record indicates that she will predict an earthquake on 90% of the occasions for which an earthquake will occur and no earthquake on 85% of the occasions for which an earthquake will not occur.

- a) Identify the alternatives, states of nature, and payoff table if the geologist is not hired.

	Earthquake	No Earthquake
Build Fresno	-60	-20
Build Henderson	-40	-40

- b) Determine the optimal alternative using an expected value criterion.

$$EMV(\text{Build Fresno}) = 0.2(-60) + 0.8(-20) = -28$$

$$EMV(\text{Build Henderson}) = -40$$

Hence, the optimal alternative is to build in Fresno.

- c) Find the expected value of perfect information.

$$EVwPI = 0.2(-40) + 0.8(-20) = -24$$

$$EVPI = 4$$

- d) Find the posterior probabilities of the respective states of nature for each of the geologist's predictions.

Priors

$$P(\text{earthquake}) = 0.2$$

$$P(\text{no earthquake}) = 0.8$$

Likelihood ratios

$$P(\text{predict earthquake} | \text{earthquake}) = 0.9$$

$$P(\text{predict no earthquake} | \text{earthquake}) = 0.1$$

$$P(\text{predict no earthquake} | \text{no earthquake}) = 0.85$$

$$P(\text{predict earthquake} | \text{no earthquake}) = 0.15$$

Joint Probabilities

$$P(\text{predict earthquake and earthquake}) = 0.9(0.2) = 0.18$$

$$P(\text{predict no earthquake and earthquake}) = 0.1(0.2) = 0.02$$

$$P(\text{predict no earthquake and no earthquake}) = 0.85(0.8) = 0.68$$

$$P(\text{predict earthquake and no earthquake}) = 0.15(0.8) = 0.12$$

$$P(\text{predict earthquake}) = 0.18 + 0.12 = 0.3$$

$$P(\text{predict no earthquake}) = 0.02 + 0.68 = 0.7$$

Posterior Probabilities

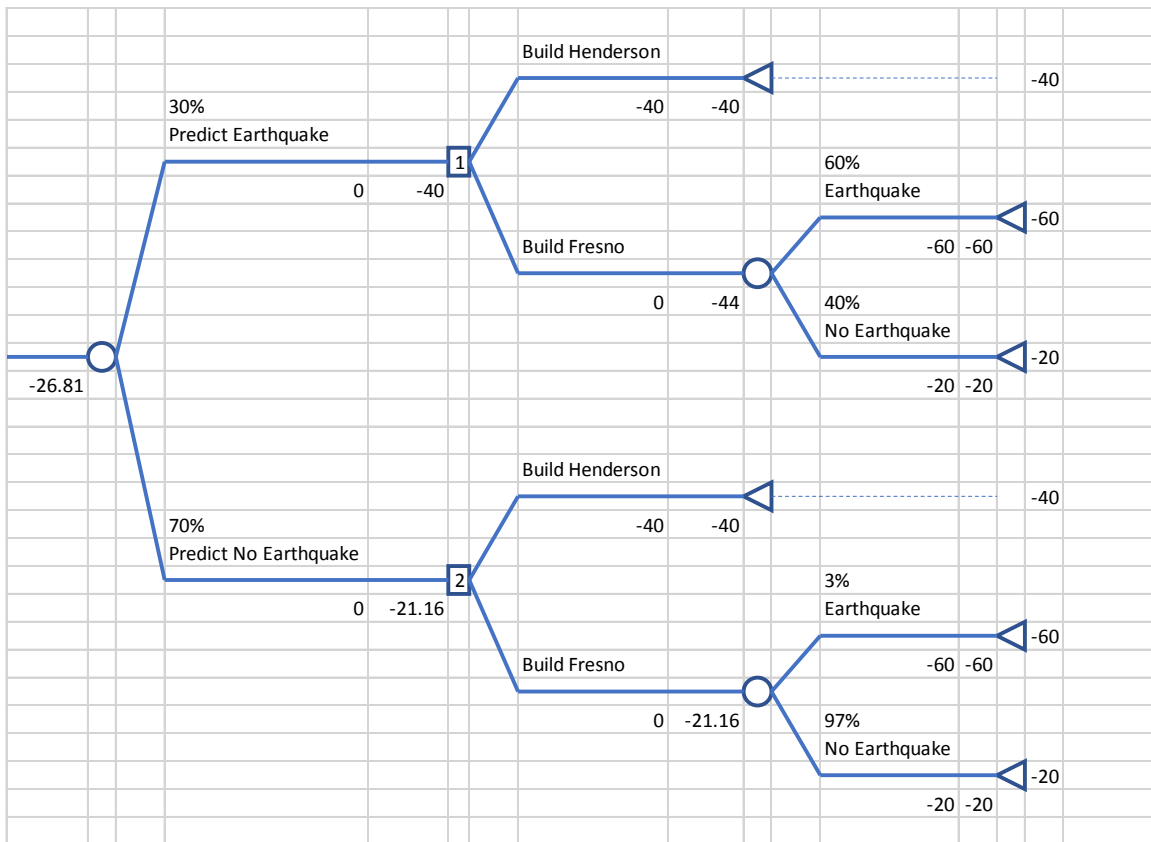
$$P(\text{earthquake} | \text{predict earthquake}) = 0.18/0.3 = 0.6$$

$$P(\text{no earthquake} | \text{predict earthquake}) = 0.12/0.3 = 0.4$$

$$P(\text{earthquake} | \text{predict no earthquake}) = 0.02/0.7 = 0.029$$

$$P(\text{no earthquake} | \text{predict no earthquake}) = 0.68/0.7 = 0.971$$

e) What is the expected value of sample information? Should Amazon hire the geologist?



$$EV_{wSI} = -26.81$$

$$EV_{SI} = 1.19$$

Thus, Amazon should hire the geologist.