Tutorial - 4

Ans1 $T(n) = 3T(n/2) + n^2$

$a = 3,\ b = 2$ $\qquad f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3}$

comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (case 3)

$\therefore$ according to master Theorem

$T(n) = \Theta(n^2)$.

Ans2 $T(n) = 4T(n/2) + n^2$

$a = 4,\ b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (case 2)

$\therefore$ according to master Theorem

$T(n) = \Theta(n^2 \log n)$

Ans3 $T(n) = T(n/2) + 2^n$

$a = 1,\ b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (case 3)

$\therefore$ According to master Theorem

$T(n) = \Theta(2^n)$

Ans4) $T(n) = 2^n T(n/2) + n^3$

$\therefore$ Master's Theorem is not applicable as a is function of n.

Ans 5 $T(n) = 16T(n/4) + n$

$a = 16,\ b = 4$, $f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (case 1)

$T(n) = \Theta(n^2)$

Ans 6 $T(n) = 2T(n/2) + n\log n$

$a = 2,\ b = 2$, $f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$

$\therefore$ According to masters $T(n) = \Theta(n\log n)$

Ans7 $T(n) = 2T(n/2) + n/\log n$

$a = 2,\ b = 2,\ f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

$\therefore$ According to master theorem

$T(n) = \Theta(n)$

Ans8

$$T(n) = 2T(n/4) + n^{0.51}$$

$a = 2,\ b = 4,\ f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

$\therefore$ According to master's Theorem $T(n) = \Theta(n^{0.5})$

Ans9

$$T(n) = 0.5T(n/2) + 1/n$$

$\therefore$ Master's not applicable as $a < 1$

Ans10

$$T(n) = 16T(n/4) + n!$$

$a = 16,\ b = 4,\ f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$ $\quad n^2 < n!$

According to masters,

$T(n) = \Theta(n!)$

Ans11

$$T(n) = 4T(n/2) + \log n$$

$a = 4,\ b = 2$ $\quad f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

According to masters, $T(n) = \Theta(n^2)$

Ans12 $T(n) = sqrt(n) + n/2 + \log n$

Master's not applicable as a is not constant.

Ans13 $T(n) = 3T(n/2) + n$

$a = 3, b = 2$ $f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ According to master's Theorem,

$T(n) = O(n^{\log_2 3})$

Ans14 $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, b = 3$ $f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

According to master Theorem, $T(n) = \theta(n)$

Ans15 $T(n) = 4T(n/2) + cn$

$a = 4, b = 2$ $f(n) = c * n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c * n$

$\therefore$ According to masters Theorem,

$T(n) = \theta(n^2)$

Ans16 $T(n) = 3T(n/4) + n\log n$

$a = 3 \quad b = 4$, $f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

$\therefore$ According to masters theorem,
$T(n) = \Theta(n\log n)$

Ans17 $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3$ $f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\Theta(n) = \Theta(n/2)$

$\therefore$ According to masters Theorem,
$T(n) = \Theta(n\log n)$

Ans18 $T(n) = 6T(n/3) + n^2\log n$

$a = 6$
$b = 3$
$f(n) = n^2\log n$ $n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2\log n$

$\therefore$ According to master's Theorem $T(n) = \Theta(n^2\log n)$

Ans 19

$$T(n) = 4T(n/2) + n \log n$$

$a = 4,\ b = 2$ $\qquad f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

$\therefore$ According to masters Theorem

$T(n) = \Theta(n^2)$

Ans 20 $\quad T(n) = 64T(n/8) - n^2 \log n$

Masters Theorem is not applicable as $f(n)$ is not increasing function.

Ans 21 $\quad T(n) = 7T(n/3) + n^2$

$a = 7$ $\qquad f(n) = n^2$

$b = 3$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

According to masters, $T(n) = \Theta(n^2)$

Ans 22 $\quad T(n) = T(n/2) + n(2 - \cos n)$

Master's theorem isn't applicable since regularity condition is isolated in case 3.