1. (a) Let the number of full-time consultants working in the morning, afternoon, and evening shifts be x1, x2, x3 respectively.

   Let the number of part time consultants working from 8am to noon= y1
   Let the number of part time consultants working from noon to 4pm= y2
   Let the number of part time consultants working from 4pm to 8pm= y3
   Let the number of part time consultants working from 8pm to midnight= y4

   According to the given conditions:
   Minimum number of employees working from 8am to noon=4
   i.e., x1+y1>=4

   Similarly considering other conditions
   X1+x2+y2>=8
   X2+x3+y3>=10
   X3+y4>=6

   Considering the requirement that at least one full-time consultant must be on duty for every part time consultant on duty, this gives us the following constraints:
   X1>=y1 for 8am to noon shift
   X1+x2>=y2 for noon to 4pm shift
   X2+x3>=y3 for 4pm to 8pm shift
   X3>=y4 for 8pm to midnight shift

   Amount earned by a full-time consultant in a day= 14*8= $112
   Amount earned by a part time consultant in a day= 12*4= $48

   So total cost to be minimized= 112*(x1+x2+x3) + 48*(y1+y2+y3+y4)

   As the pay rate for part time consultants is lesser so it would be ideal to maximize the number of part time consultants

   According to the given conditions the minimum number of consultants required to work in a day= 28

   Considering at least one full-time consultant is required for every part time consultant in every of the 4 shifts so the minimum number of full-time employees required is 28/4= 7

   Now adding all the above conditions:
   X1+y1+x1+x2+y2+x2+x3+y3+x3+y4>=28
   2(x1+x2+x3) + (y1+y2+y3+y4)>=28
   2*7 + (y1+y2+y3+y4)>=28
   That implies the number of part time employees required should be >=14

Minimum cost= 112*7 + 48*14= 784 + 672= **$1456**

Number of consultants paid to work full-time= 7
Number of consultants paid to work part time= 14

(b) From the first part, total number of consultants paid to work full-time= 7
Amount paid to one full-time consultant working for 8 hours= 14*8= $112

If a lunch break is given to all full-time consultant for 1 hour, the amount per consultant becomes= 112-14= $98

So, the new amount paid to the 7 consultants is= 98*7= $686

Thus, the new minimum cost for both full-time and part time consultants= $686 + 672= **$1358**

2. Let the number of collegiate backpacks produced= x
   Let the number of mini backpacks produced= y

   Total profit z= 32x + 24y

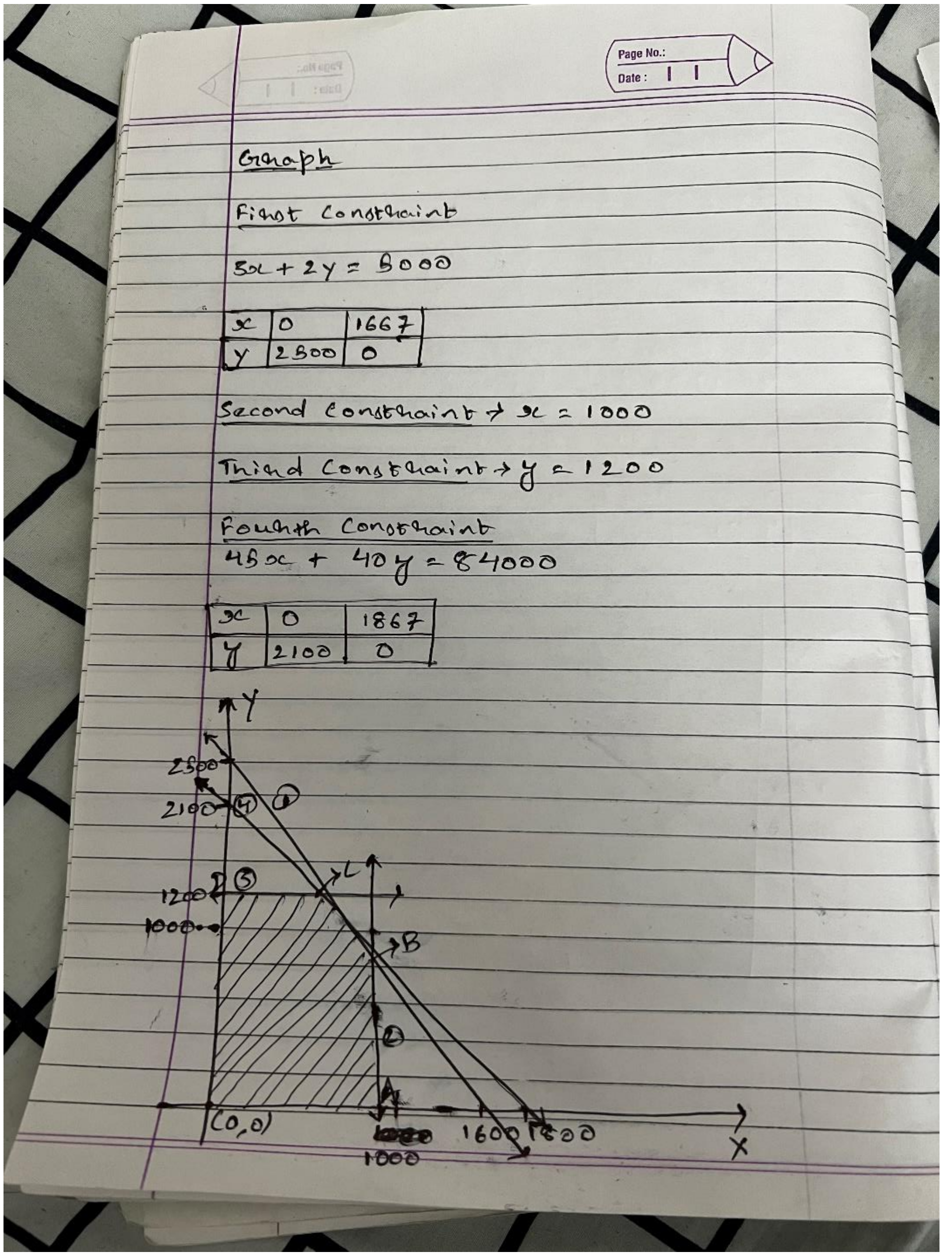
   Total nylon fabric required= 3x + 2y
   Therefore, according to the given condition, 3x + 2y<= 5000

   Also, x<=1000 and y<=1200

   Total labor required= 45x + 40y
   Available labor= 35*40*60= 84000 minutes
   So 45x + 40y <=84000

   Formulating the LP

   Maximize Z= 32x + 24y
   Subject to, 3x + 2y<=5000
   X<=1000
   Y<=1200
   45x + 40y<=84000
   X, y>=0



From the graph the coordinates for the shaded area can be found as follows:

X=0, y=1200

X=1000, y=0

X=800, y=1200

X=1000, y=950

The maximum profit can be determined when x=1000 and y=950

So, Maximize Z= 32x + 24y= 32*1000 + 24*950= 32000 + 22800= **$54800**

3. (a) Let the number of large products made by plant 1 be x1

Let the number of large products made by plant 2 be x2

Let the number of large products made by plant 3 be x3

Let the number of medium products made by plant 1 be y1

Let the number of medium products made by plant 2 be y2

Let the number of medium products made by plant 3 be y3

Let the number of small products made by plant 1 be z1

Let the number of small products made by plant 2 be z2

Let the number of small products made by plant 3 be z3

The decision variables can be defined as follows:

X1+y1+z1<=750

X2+y2+z2<=900

X3+y3+z3<=450

20x1+15y1+12z1<=13000

20x2+15y2+12z2<=12000

20x3+15y3+12z3<=5000

X1+x2+x3<=900

Y1+y2+y3<=1200

Z1+z2+z3<=750

Where x1, x2, x3, y1, y2, y3, z1, z2, z3>=0

(b) The linear programming model can be formulated as follows:

Maximize A, where A= 420(x1+x2+x3) + 360(y1+y2+y3) + 300(z1+z2+z3)

(c)
```
library(lpSolveAPI)
b <- make.lp(0,9,verbose = "neutral")
b

## Model name:
##   a linear program with 9 decision variables and 0 constraints

add.constraint(b, c(1,1,1,0,0,0,0,0,0), "<=", 750 )
add.constraint(b, c(0,0,0,1,1,1,0,0,0), "<=", 900)
add.constraint(b, c(0,0,0,0,0,0,1,1,1), "<=", 450)
add.constraint(b, c(20,15,12,0,0,0,0,0,0), "<=", 13000)
add.constraint(b, c(0,0,0,20,15,12,0,0,0), "<=", 12000)
add.constraint(b, c(0,0,0,0,0,0,20,15,12), "<=", 5000)
add.constraint(b, c(1,1,1,0,0,0,0,0,0), "<=", 900)
add.constraint(b, c(0,0,0,1,1,1,0,0,0), "<=", 1200)
add.constraint(b, c(0,0,0,0,0,0,1,1,1), "<=", 750)
add.constraint(b, c(6, 6, 6, -5, -5, -5, 0, 0, 0), "=", 0)
add.constraint(b, c( 3, 3, 3, 0, 0, 0, -5, -5, -5), "=", 0)

set.objfn(b, c(420,360,300,420,360,300,420,360,300))
lp.control(b, sense='max')

b.col <- c("P 1","P 2","P 3","p 4","p 5","p 6","p 7","p 8","p 9")
b.row <- c("Y1l","Y1m","Y1s","Y2l", "Y2m","Y2s","Y3l","Y3m","Y3s","%C1","%C2"
)
dimnames(b) <- list(b.row,b.col)

b
```