

1. (a) Let the number of full-time consultants working in the morning, afternoon, and evening shifts be x_1 , x_2 , x_3 respectively.

Let the number of part time consultants working from 8am to noon= y_1

Let the number of part time consultants working from noon to 4pm= y_2

Let the number of part time consultants working from 4pm to 8pm= y_3

Let the number of part time consultants working from 8pm to midnight= y_4

According to the given conditions:

Minimum number of employees working from 8am to noon=4

i.e., $x_1 + y_1 \geq 4$

Similarly considering other conditions

$x_1 + x_2 + y_2 \geq 8$

$x_2 + x_3 + y_3 \geq 10$

$x_3 + y_4 \geq 6$

Considering the requirement that at least one full-time consultant must be on duty for every part time consultant on duty, this gives us the following constraints:

$x_1 \geq y_1$ for 8am to noon shift

$x_1 + x_2 \geq y_2$ for noon to 4pm shift

$x_2 + x_3 \geq y_3$ for 4pm to 8pm shift

$x_3 \geq y_4$ for 8pm to midnight shift

Amount earned by a full-time consultant in a day= $14 \times 8 = \$112$

Amount earned by a part time consultant in a day= $12 \times 4 = \$48$

So total cost to be minimized= $112(x_1 + x_2 + x_3) + 48(y_1 + y_2 + y_3 + y_4)$

As the pay rate for part time consultants is lesser so it would be ideal to maximize the number of part time consultants

According to the given conditions the minimum number of consultants required to work in a day= 28

Considering at least one full-time consultant is required for every part time consultant in every of the 4 shifts so the minimum number of full-time employees required is $28/4 = 7$

Now adding all the above conditions:

$x_1 + y_1 + x_1 + x_2 + y_2 + x_2 + x_3 + y_3 + x_3 + y_4 \geq 28$

$2(x_1 + x_2 + x_3) + (y_1 + y_2 + y_3 + y_4) \geq 28$

$2 \times 7 + (y_1 + y_2 + y_3 + y_4) \geq 28$

That implies the number of part time employees required should be ≥ 14

$$\text{Minimum cost} = 112 \times 7 + 48 \times 14 = 784 + 672 = \text{\$1456}$$

Number of consultants paid to work full-time = 7

Number of consultants paid to work part time = 14

(b) From the first part, total number of consultants paid to work full-time = 7

Amount paid to one full-time consultant working for 8 hours = $14 \times 8 = \$112$

If a lunch break is given to all full-time consultant for 1 hour, the amount per consultant becomes = $112 - 14 = \$98$

So, the new amount paid to the 7 consultants is = $98 \times 7 = \$686$

Thus, the new minimum cost for both full-time and part time consultants = $\$686 + 672 = \text{\$1358}$

2. Let the number of collegiate backpacks produced = x

Let the number of mini backpacks produced = y

$$\text{Total profit } z = 32x + 24y$$

$$\text{Total nylon fabric required} = 3x + 2y$$

Therefore, according to the given condition, $3x + 2y \leq 5000$

Also, $x \leq 1000$ and $y \leq 1200$

$$\text{Total labor required} = 45x + 40y$$

$$\text{Available labor} = 35 \times 40 \times 60 = 84000 \text{ minutes}$$

$$\text{So } 45x + 40y \leq 84000$$

Formulating the LP

$$\text{Maximize } Z = 32x + 24y$$

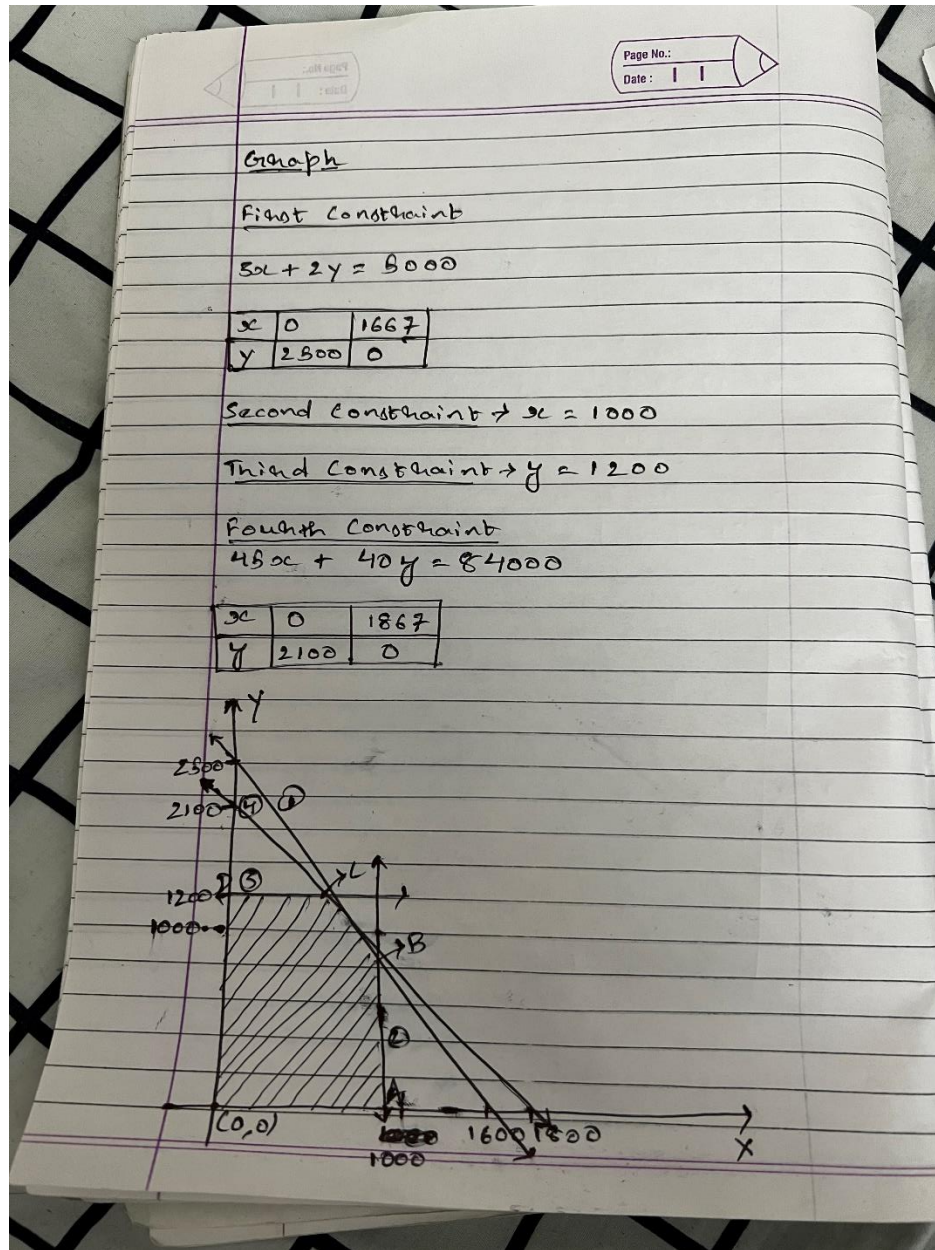
$$\text{Subject to, } 3x + 2y \leq 5000$$

$$x \leq 1000$$

$$y \leq 1200$$

$$45x + 40y \leq 84000$$

$$x, y \geq 0$$



From the graph the coordinates for the shaded area can be found as follows:

$X=0, y=1200$

$X=1000, y=0$

$X=800, y=1200$

$X=1000, y=950$

The maximum profit can be determined when $x=1000$ and $y=950$

So, Maximize $Z = 32x + 24y = 32 \cdot 1000 + 24 \cdot 950 = 32000 + 22800 = \text{\$54800}$

3. (a) Let the number of large products made by plant 1 be x_1

Let the number of large products made by plant 2 be x_2

Let the number of large products made by plant 3 be x_3

Let the number of medium products made by plant 1 be y_1

Let the number of medium products made by plant 2 be y_2

Let the number of medium products made by plant 3 be y_3

Let the number of small products made by plant 1 be z_1

Let the number of small products made by plant 2 be z_2

Let the number of small products made by plant 3 be z_3

The decision variables can be defined as follows:

$$x_1 + y_1 + z_1 \leq 750$$

$$x_2 + y_2 + z_2 \leq 900$$

$$x_3 + y_3 + z_3 \leq 450$$

$$20x_1 + 15y_1 + 12z_1 \leq 13000$$

$$20x_2 + 15y_2 + 12z_2 \leq 12000$$

$$20x_3 + 15y_3 + 12z_3 \leq 5000$$

$$x_1 + x_2 + x_3 \leq 900$$

$$y_1 + y_2 + y_3 \leq 1200$$

$$z_1 + z_2 + z_3 \leq 750$$

Where $x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3 \geq 0$

(b) The linear programming model can be formulated as follows:

Maximize A, where $A = 420(x_1 + x_2 + x_3) + 360(y_1 + y_2 + y_3) + 300(z_1 + z_2 + z_3)$

(c) `library(lpSolveAPI)`

```
b <- make.lp(0,3,verbose = "neutral")
```

```
add.constraint(b, c(1,1,1), "<=", 750 )
```

```
add.constraint(b, c(1,1,1), "<=", 900)
```

```
add.constraint(b, c(1,1,1), "<=", 450)
```

```
add.constraint(b, c(20,15,12), "<=", 13000)
```

```
add.constraint(b, c(20,15,12), "<=", 12000)
```

```
add.constraint(b, c(20,15,12), "<=", 5000)
```

```
add.constraint(b, c(1,1,1), "<=", 900)
```

```
add.constraint(b, c(1,1,1), "<=", 1200)
```

```
add.constraint(b, c(1,1,1), "<=", 750)
```

```
b.col <- c("P 1","P 2","P 3")
```

```
b.row <- c("Y1l","Y1m","Y1s","Y2l", "Y2m","Y2s","Y3l","Y3m","Y3s")
```

```
dimnames(b) <- list(b.row,b.col)
```

```
b
```

```
solve(b)
```

```
get.objective(b)
```

```
get.variables(b)
```