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Assignment 1.

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(1) Mean $E[x+y]$

The expected value of the sum of independent random variables is equal to the sum of their expectations.

$$E[x+y] = E[x] + E[y].$$

x and y being discrete variables.

$$E[x+y] = \sum_x x\,P(x) + \sum_y P(y)$$

x and y being continuous variables.

$$E[x+y] = \int_{-\infty}^{\infty} x\,f(x)\,dx + \int_{-\infty}^{\infty} y\,f(y)\,dy.$$

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Variance.

the variance of an independent variable is the expected value of squared difference b/w the independent variable & expected value.

$var[x+y] = E[((x+y) - E[x+y])^2]$

$= E[(x+y)^2] - (E(x+y))^2$

$= E[x^2] + E[y^2] + 2E[xy] - (E[x])^2 - (E[y])^2 - 2E[x]E[y]$

$Var[x+y] = Var[x] + Var[y] + 2(E[xy] - E[x]E[y])$

(2) a) $H(x) = \sum_{1}^{n} p(x_i) \log_2 \frac{1}{p(x_i)}$

$P(x=a) = 0 + \frac{1}{6} + \frac{1}{6} = 1/3$

$P(x=b) = \frac{1}{6} + 0 + \frac{1}{6} = 1/3$

P [

$H[x] =$

$=$

b) H

P

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...is depend-
...xpected
...ifference
...variable &

$E[(x+y))^2]$

$(x+y))^2$

$E[xy] -$
$)^2 - 2E[x]E[y]$

$(E[xy] -$
$E[y])$

$\log_2 \frac{1}{p(x_i)}$

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$$p(x=c) = \frac{1}{6} + \frac{1}{6} + 0 = 1/3.$$

$$H[x] = \frac{1}{3}\log_2 3 + \frac{1}{3}\log_2 3 + \frac{1}{3}\log_2 3$$

$$= 3 * \frac{1}{3}\log_2 3 = \log_2 3.$$

$$H[x] = 1.58 \text{ bits}$$

b) $$H[y] = \sum_i^4 P(y_i)\log_2 \frac{1}{P(y_i)}$$

$$P(y=a) = 0 + \frac{1}{6} + \frac{1}{6} = 1/3$$

$$P(y=b) = \frac{1}{6} + 0 + \frac{1}{6} = 1/3$$

$$P(y=c) = \frac{1}{6} + \frac{1}{6} + 0 = 1/3$$

$$H[y] = 1.58 \text{ bits.}$$

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c) $x_1 = 1/3$
$x_2 = 1/3$
$x_3 = 1/3$.

$$P\left(\frac{y}{x}\right) = \frac{P(x, y)}{P(x)}$$

Conditional probability matrix.

P(y/x).

		y: a	b	c
x	a	0	1/2	1/2
	b	1/2	0	1/2
	c	1/2	1/2	0

$$H(y/x) = \sum_j \sum_i P(x_j, y_i) \log_2 \frac{1}{P(y_i/x_j)}$$

$$= \frac{1}{6}\log_2 2 + \frac{1}{6}\log_2 2 + 4 \times \frac{1}{6}\log_2 2$$

$$= 6 \times \frac{1}{6} \log_2 2.$$

$$= \log_2 2$$

$H(y/x) = 1$ bits.

d) H

e)

f)

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d) $H[x,y] = \sum_j \sum_i P(x_i, y_i) \log_2 \frac{1}{P(x_j, y_i)}$

$= 6 \times \frac{1}{6} \log_2 6.$

$H[x,y] = 2.58$ bits

e) will ~~be~~ come equal to $H[y/x]$ because of symmetry.

$H[x,y] = 1$ bit.

f) $I(x,y) = H[y) - H(y/x)$

OR.

$= H(x) - H(x/y)$

$= H(x) + H(y) - H(x,y)$

$I[x,y] = 0.58.$

③ mean $E(x)$

$E(x) = \sum x \, P(x)$

$= 0 + 1/5 + 2/5 + 3/5 + 4/5$

$= 10/5$ mean $= 2$

atrix.

$\log_2 \frac{1}{P[y_i/x_j]}$

$\log_2 2$

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Variance.

$var = \Sigma x^2 P(x) - (E(x))^2.$

$= 1/5 + 4/5 + 9/5 + 16/5 - 4.$

$= \frac{30}{5} - 4.$

variance = 2.

Standard deviation.

$\sigma = \sqrt{var(x)} = \sqrt{2}$

$\sigma = 1.41$

4) mean.

$= \Sigma x\, P(x)$

$= 0 + 0.53 + 0.4 + 0.24 + 0.12$

$= 1.29.$

variance.

$var(x) = \Sigma x^2 P(x) - (E(x))^2$

$= 0.53 + 0.8 + 0.72 + 0.45 - 1.6641$

$= 0.8359.$

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(5) $n = 5$.

success $(p) = 1/4 = 0.25$.

failure $(q) = 1 - 0.25 = 0.75$.

using binomial distribution.

$p(x) = {}^nC_r \; p^r q^{n-3}$.

$= {}^5C_3 \; p^3 q^2$.

$= \underline{0.088}$.

mean $\Rightarrow \mu = np = 5 \times 0.25 = 1.25$.

variance $\Rightarrow npq = 5 \times 0.25 \times 0.75$
$= 0.9375$.

(6). A $\rightarrow$ fair coin tossed.
B $\rightarrow$ fake " "
C $\rightarrow$ head comes.

$P(H/A) = 0.5$
$P(H/B) = 1$.

0-12

$)^2$

1.1641

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$P(H) = P(H/A) \times P(A) + P(H/B) \times P(B)$

$= \frac{1}{2} \times \frac{2}{3} + 1 \times \frac{1}{3}$

$= \frac{2}{3}$

$P(H) = 0.67$

(7) average code length.

$L = \sum_{k=1}^{4} P_k l_k$.

$= 0.5 + 0.5 + 0.375 + 0.250 + 0.3744$.

$= 1.999$ bits/symbol.

Entropy.

$H(x) = \sum_{k+1}^{4} P_k \log_2 \frac{1}{P_k}$.

$= 0.5 + 0.5 + 0.375 + 0.25 + 0.3744$.

$H(x) = 1.9994$ bits/symbol.

8) con

Each

$\tilde{x}_n$

x_n

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$\tilde{x}_n$

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8) consider $\{\vec{v_i}\}$ in R^d.

Each datapt. can be written as

$$\vec{x_n} = \Sigma_{i=1}^{d} \alpha_{ni} \vec{v_i}$$

$$x_n = \sum_{i=1}^{d} (x_n^T v_i) v_i$$

our goal is to lower dimensional subspace $m < d$.

$$\tilde{x}_n = \sum_{i=1}^{m} z_{ni} v_i + \sum_{i=m+1}^{d} b_i v_i$$

coefficients z_{ni} depends on the data pt. x_n objective is to minimize

$$J = \frac{1}{N} \sum_{n=1}^{N} \| x_n - \tilde{x}_n \|^2.$$

taking derivative w.r.t z_{n1},

$$z_{nj} = x_n^T v_j \rightarrow \{1 \longrightarrow m.$$

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$B) \times P(B)$

250 +

0.3744.

p_2.

25 + 0.3744.

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To minimize.

$$J = \frac{1}{N} \sum_{n=1}^{N} \sum_{i=M+1}^{d} \left(x_n^T v_i - \bar{x}^T v_i \right)^2$$

$$= \sum_{i=M+1}^{d} v_i^T S v_i$$

$(d-M)$ eigen values.