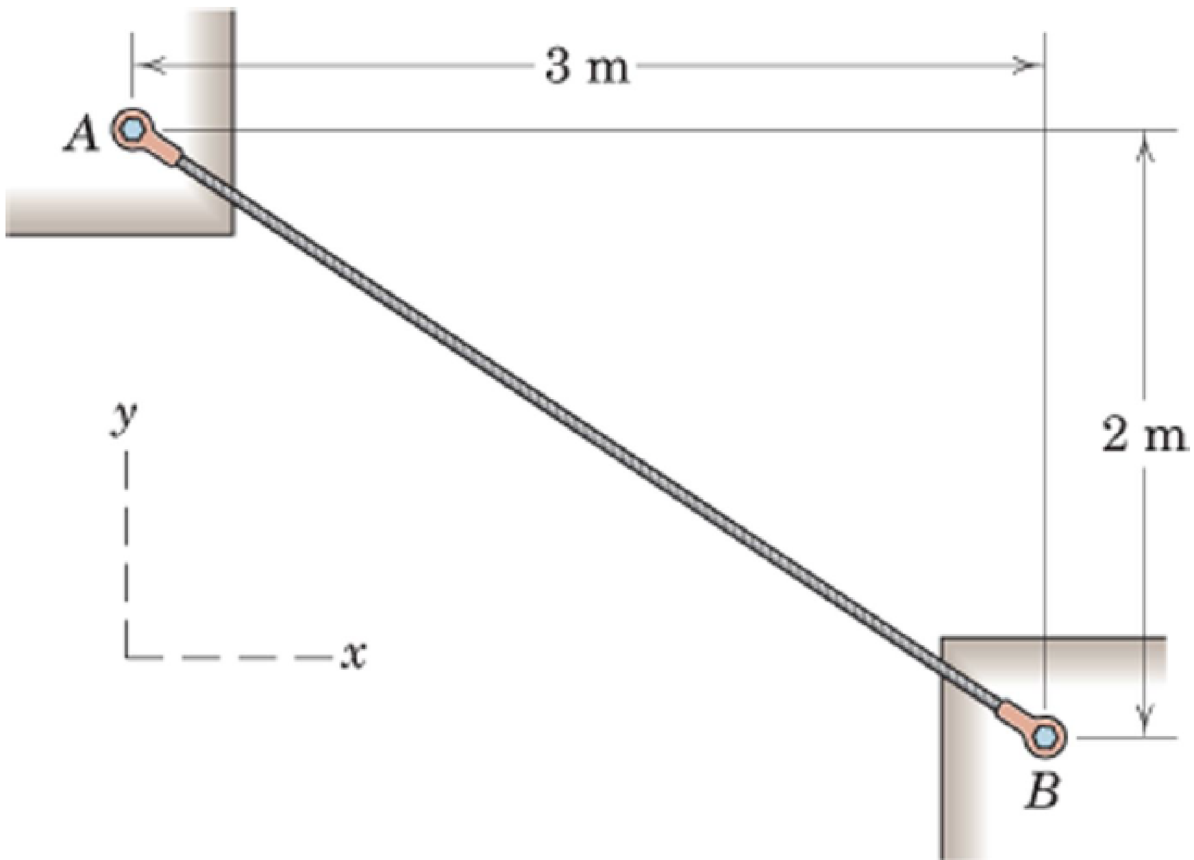
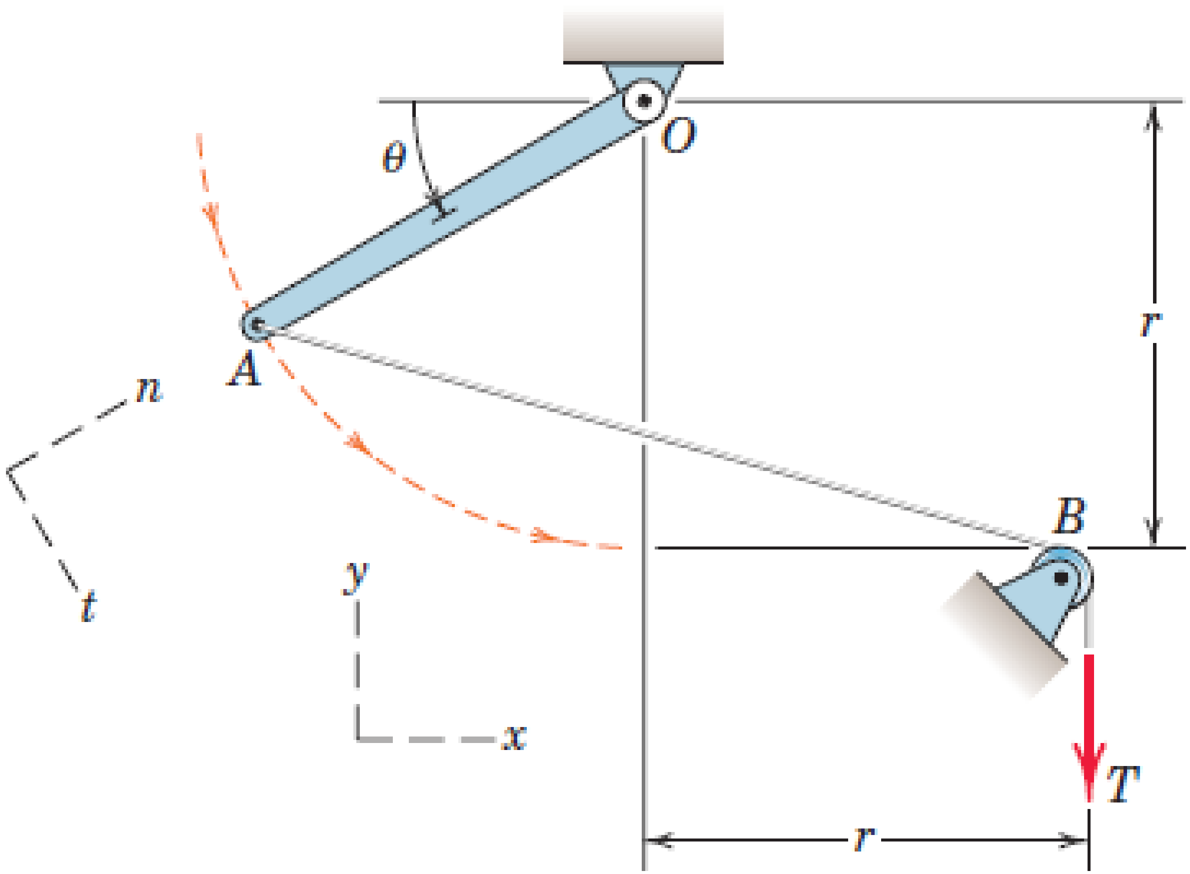
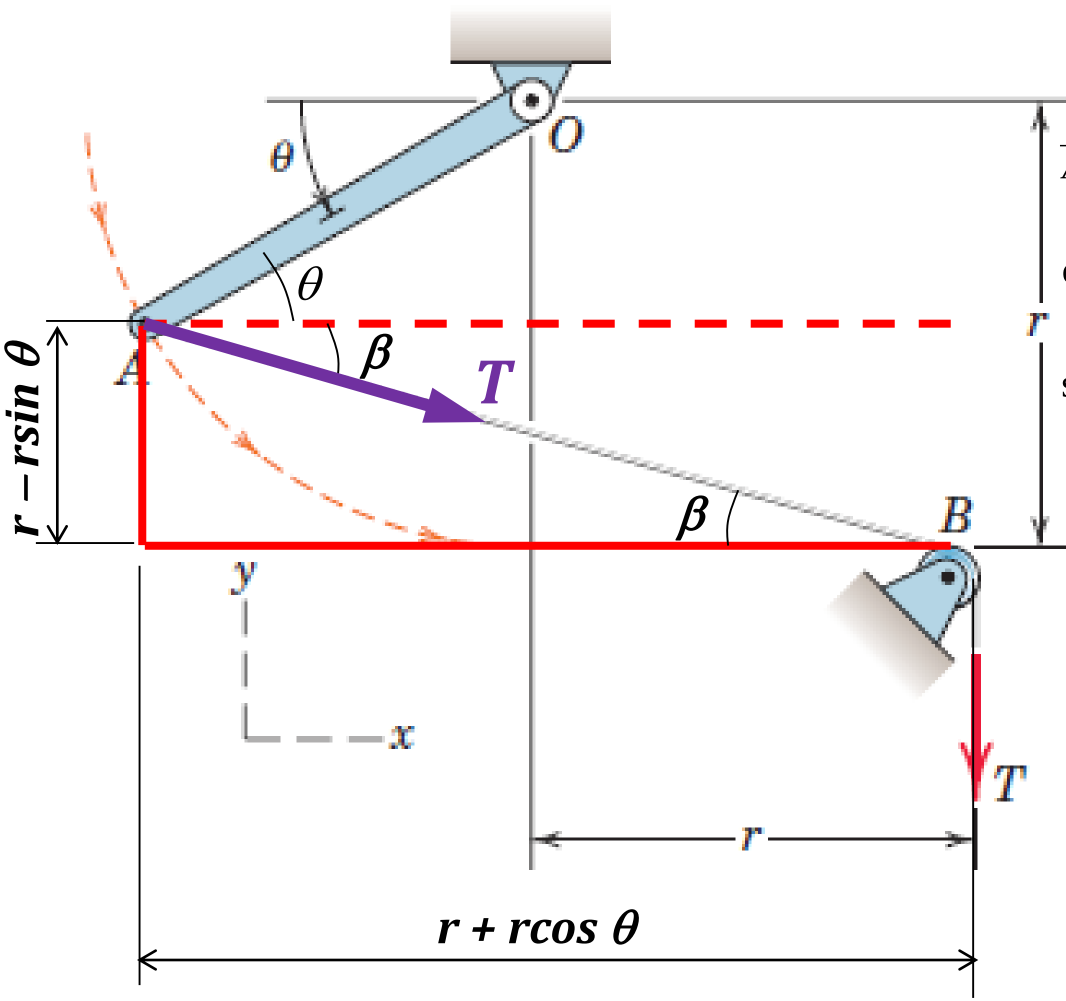
1. A cable stretched between the fixed supports A and B is under a tension T of 900 N. Express the tension as a vector using the unit vectors $\vec{i}$ and $\vec{j}$, first, as a force $\vec{T}_A$ acting on A and second, as a force $\vec{T}_B$ acting on B.



2. Determine the *x*-*y* components of the tension *T* which is applied to point *A* of the bar *OA*. Neglect the effects of the small pulley at *B*. Assume that *r* and θ are known. Also determine the *n*-*t* components of the tension *T* for *T*=100 N and θ=35º.





x-y coordinates

$$\overline{AB} = \sqrt{(r - r\sin\theta)^2 + (r + r\cos\theta)^2} = r\sqrt{3 + 2\cos\theta - 2\sin\theta}$$

$$\cos\beta = \frac{r + r\cos\theta}{r\sqrt{3 + 2\cos\theta - 2\sin\theta}} = \frac{1 + \cos\theta}{\sqrt{3 + 2\cos\theta - 2\sin\theta}}$$

$$\sin\beta = \frac{r - r\sin\theta}{r\sqrt{3 + 2\cos\theta - 2\sin\theta}} = \frac{1 - \sin\theta}{\sqrt{3 + 2\cos\theta - 2\sin\theta}}$$

$$T_x = T\cos\beta = T\frac{1 + \cos\theta}{\sqrt{3 + 2\cos\theta - 2\sin\theta}}$$

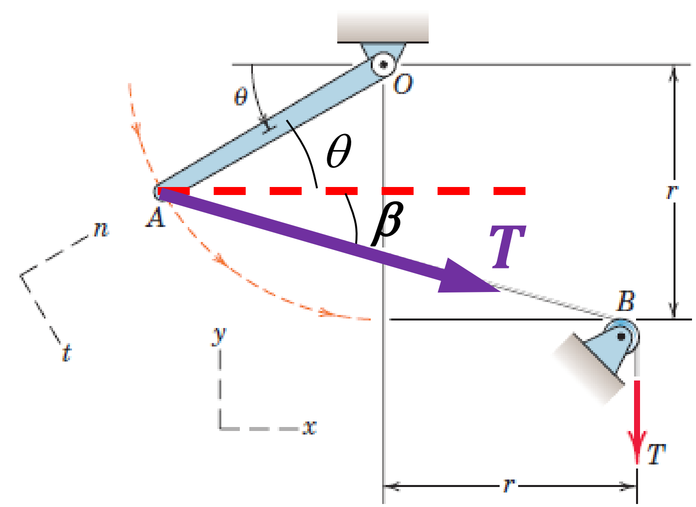
$$T_y = -T\sin\beta = T\frac{\sin\theta - 1}{\sqrt{3 + 2\cos\theta - 2\sin\theta}}$$

n-t coordinates (for θ=35° and T=100 N)

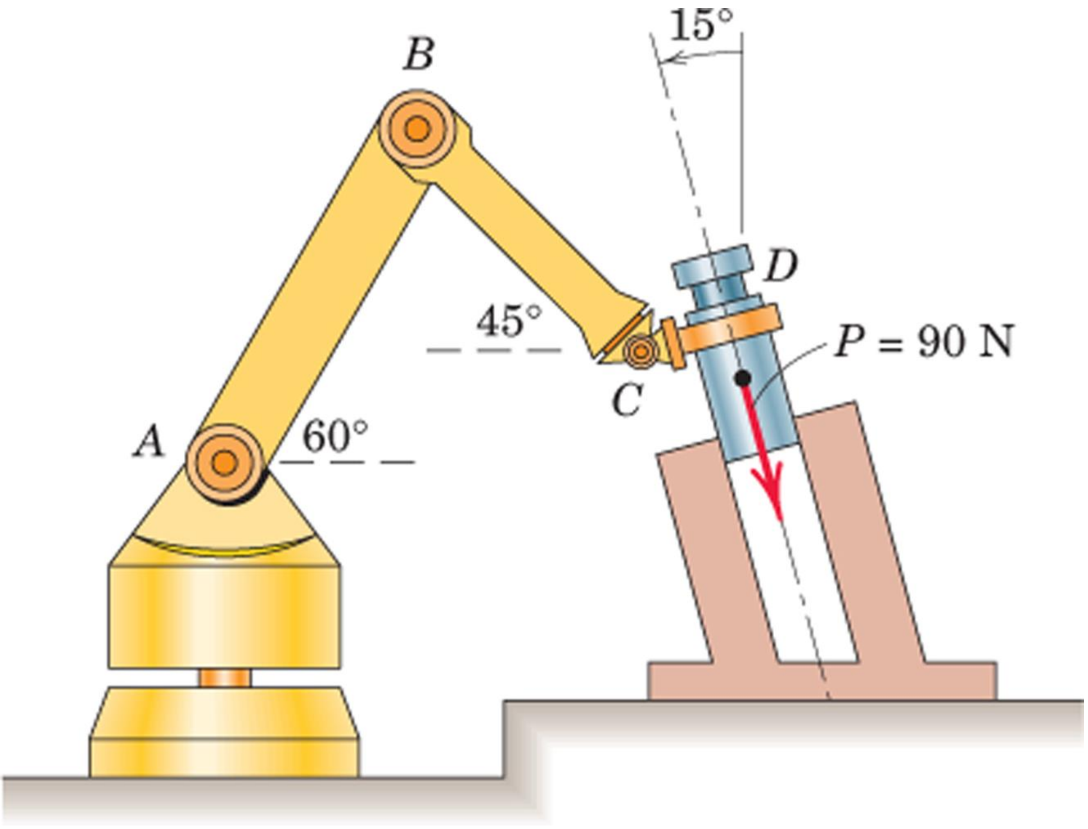
$$\beta = \arctan\left(\frac{1 - \sin 35}{1 + \cos 35}\right) = 13.19^o$$

$$T_n = T\cos(\theta + \beta) = 100\cos(35 + 13.19) = 66.67\ N$$

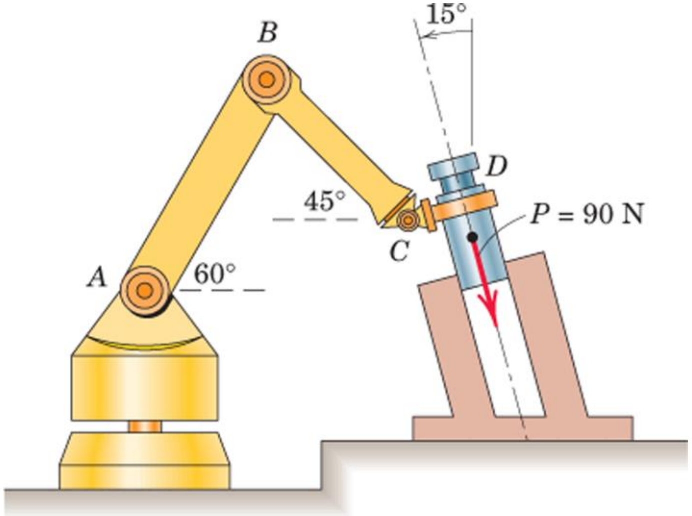
$$T_t = T\sin(\theta + \beta) = 100\sin(35 + 13.19) = 74.54\ N$$



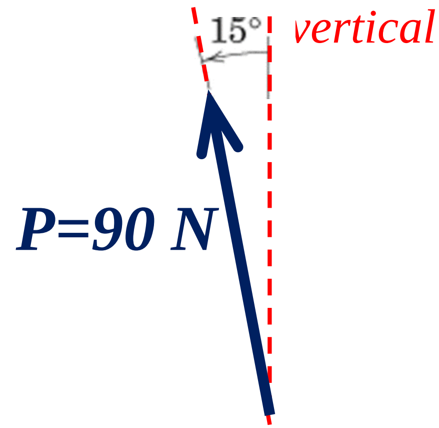
3. In the design of the robot to insert the small cylindrical part into a close-fitting circular hole, the robot arm must exert a 90 N force ***P*** on the part parallel to the axis of the hole as shown. Determine the components of the force which the part exerts on the robot along axes **(a)** parallel and perpendicular to the arm *AB*, and **(b)** parallel and perpendicular to the arm *BC*.



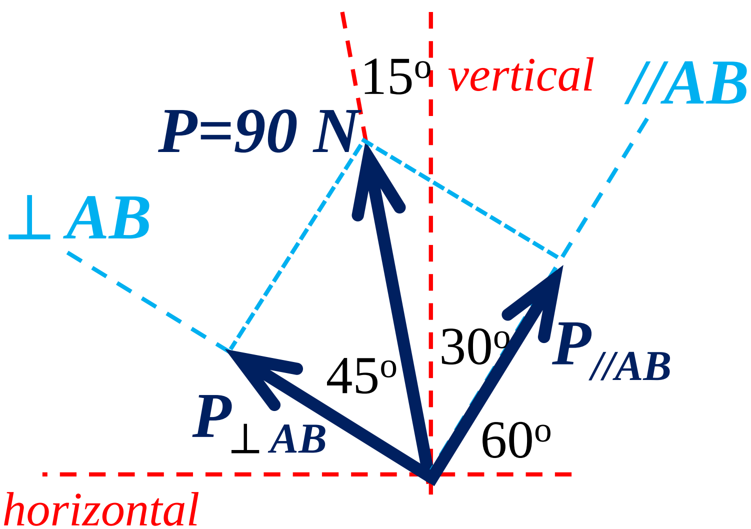
the robot arm must exert a 90 N force ***P*** on the part



force which the part exerts on the robot

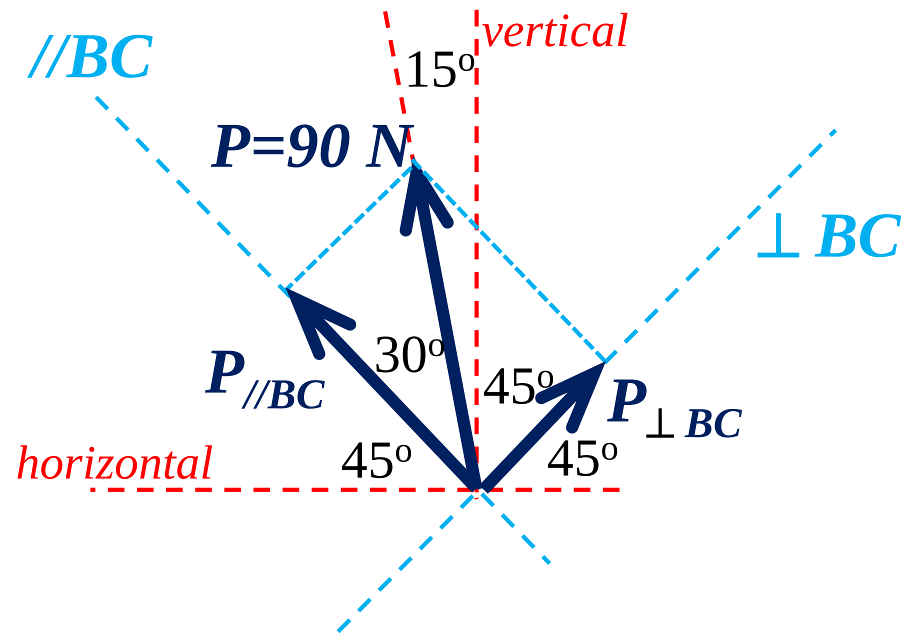


(a) parallel and perpendicular to the arm *AB*



$P_{//AB} = P_{\perp AB} = 90\cos 45 = 63.64\ N$

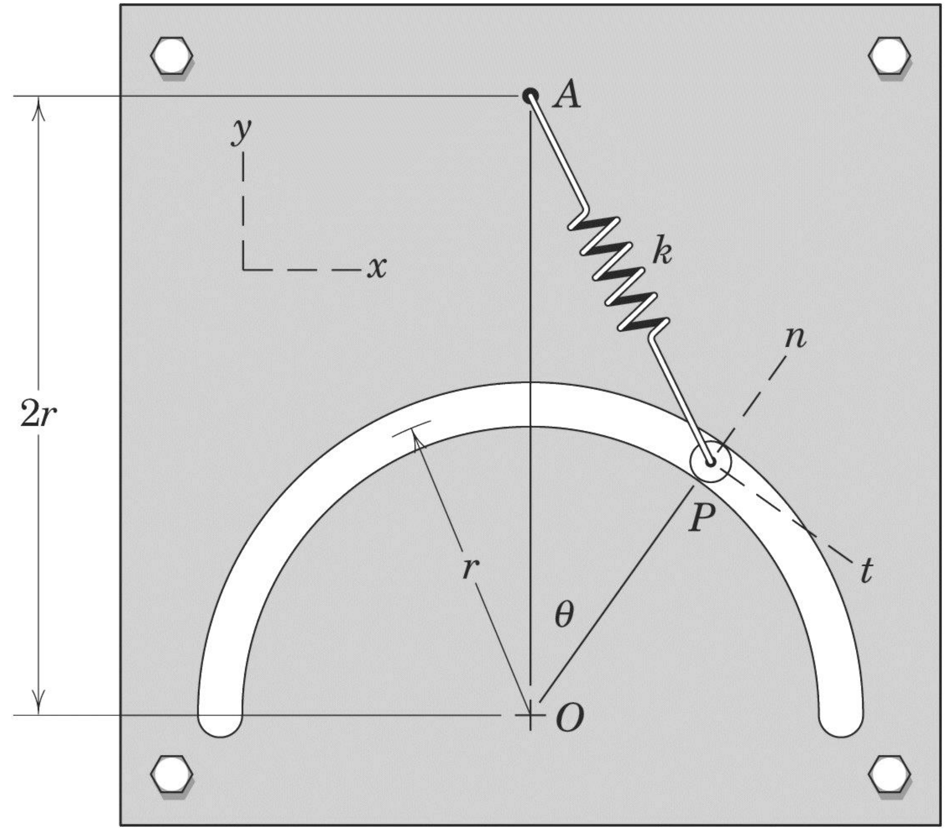
(b) parallel and perpendicular to the arm *BC*.



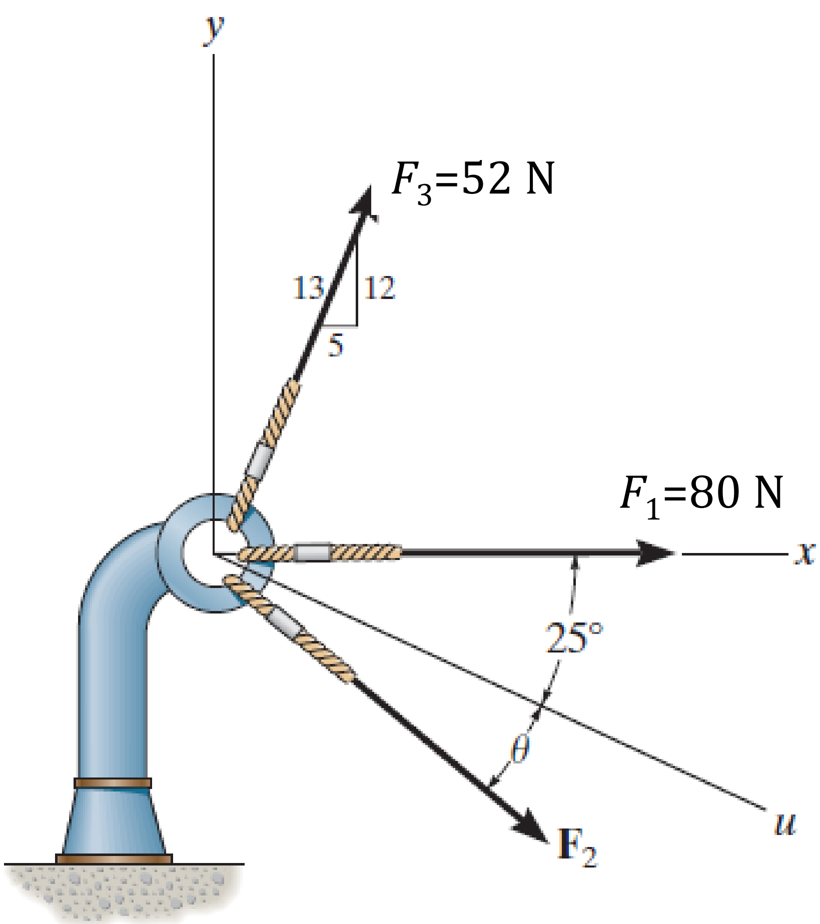
$P_{//BC} = 90\cos 30 = 77.94\ N$

$P_{\perp BC} = 90\sin 30 = 45\ N$

4. The unstretched length of the spring is r. When pin P is in an arbitrary position θ, determine the x- and y-components of the force which the spring exerts on the pin. Evaluate your answer for r=400 mm, k=1.4 kN/m and θ=40°.



5. Three forces act on the bracket. Determine the magnitude and direction θ of F_2 so that the resultant force is directed along the positive u axis and has a magnitude of 50 N.

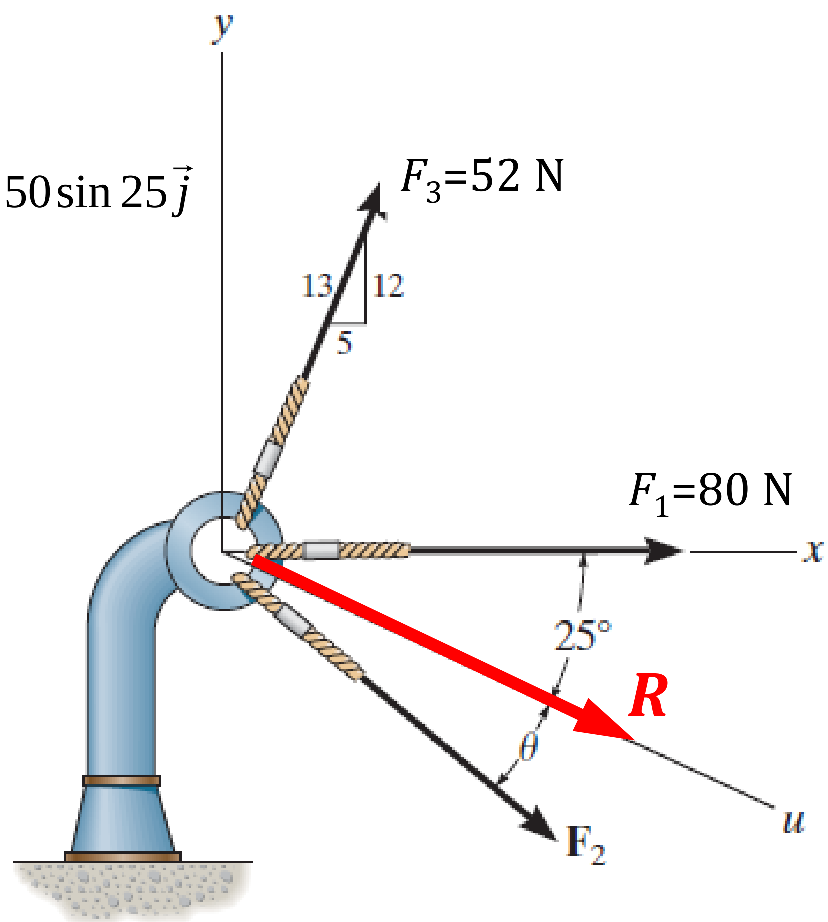


if R=50 N θ =? F₂ =?

Resultant

$$\vec{R} = R\cos 25\vec{i} - R\sin 25\vec{j} \quad \Rightarrow \quad \vec{R} = 50\cos 25\vec{i} - 50\sin 25\vec{j}$$

$$\underline{\vec{R} = 45.315\vec{i} - 21.13\vec{j}}$$



$$\vec{F}_1 = 80\vec{i}$$

$$\vec{F}_2 = F_2\cos(\theta+25)\vec{i} - F_2\sin(\theta+25)\vec{j}$$

$$\vec{F}_3 = 52\frac{5}{13}\vec{i} + 52\frac{12}{13}\vec{j} = 20\vec{i} + 48\vec{j}$$

$$\sum\vec{F} = \vec{R} = \sum F_x\vec{i} + \sum F_y\vec{j}$$

$$\sum F_x = 80 + F_2\cos(\theta+25) + 20 = 45.315$$

$$F_2\cos(\theta+25) = -54.685 \quad 1$$

$$\sum F_y = -F_2\sin(\theta+25) + 48 = -21.13$$

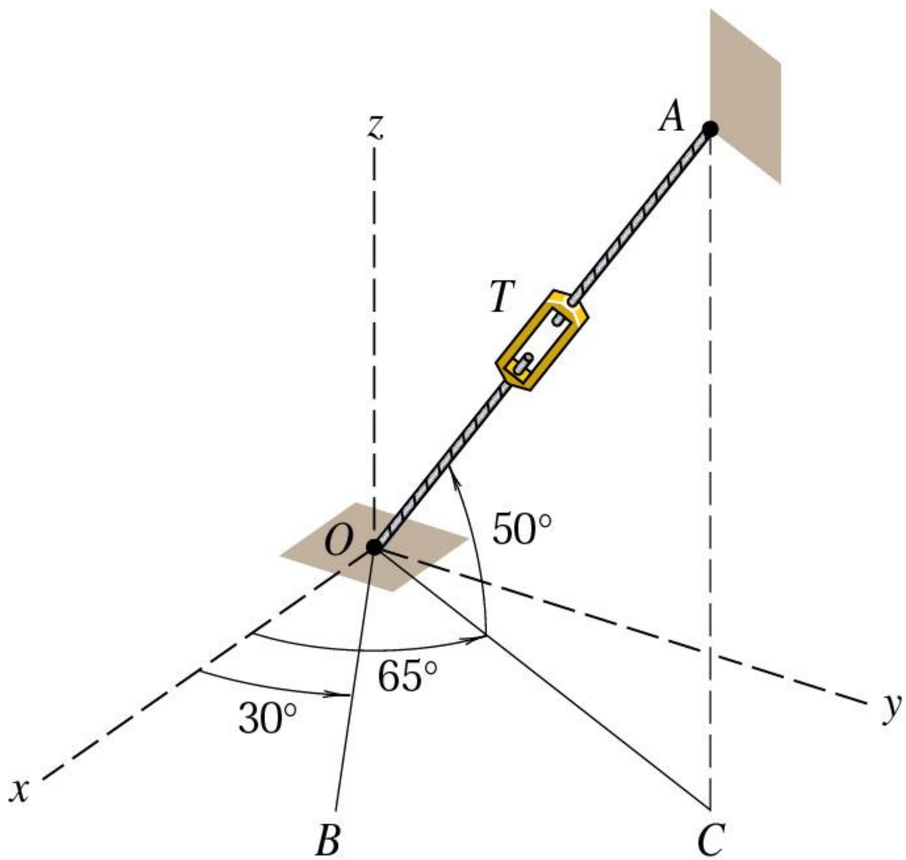
$$F_2\sin(\theta+25) = 69.13 \quad 2$$

$$\frac{2}{1} \quad \frac{\cancel{F_2}\sin(\theta+25)}{\cancel{F_2}\cos(\theta+25)} = \frac{69.13}{-54.685}$$

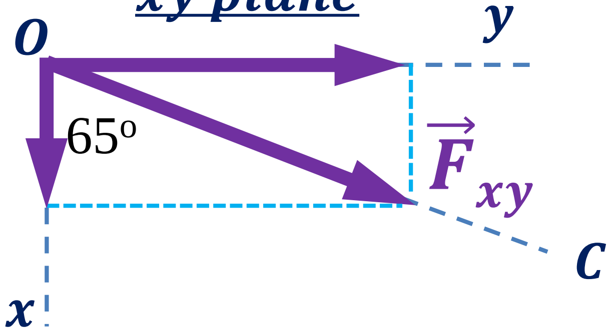
$$\tan(\theta+25) = -1.264$$

$$\boxed{\theta = 103.35^\circ \qquad F_2 = 88.14\ N}$$

6. The turnbuckle *T* is tightened until the tension in cable *OA* is 5 kN. Express the force $\vec{F}$ acting on point *O* as a vector. Determine the projection of $\vec{F}$ onto the *y*-axis and onto line *OB*. Note that *OB* and *OC* lies in the *x*-*y* plane.

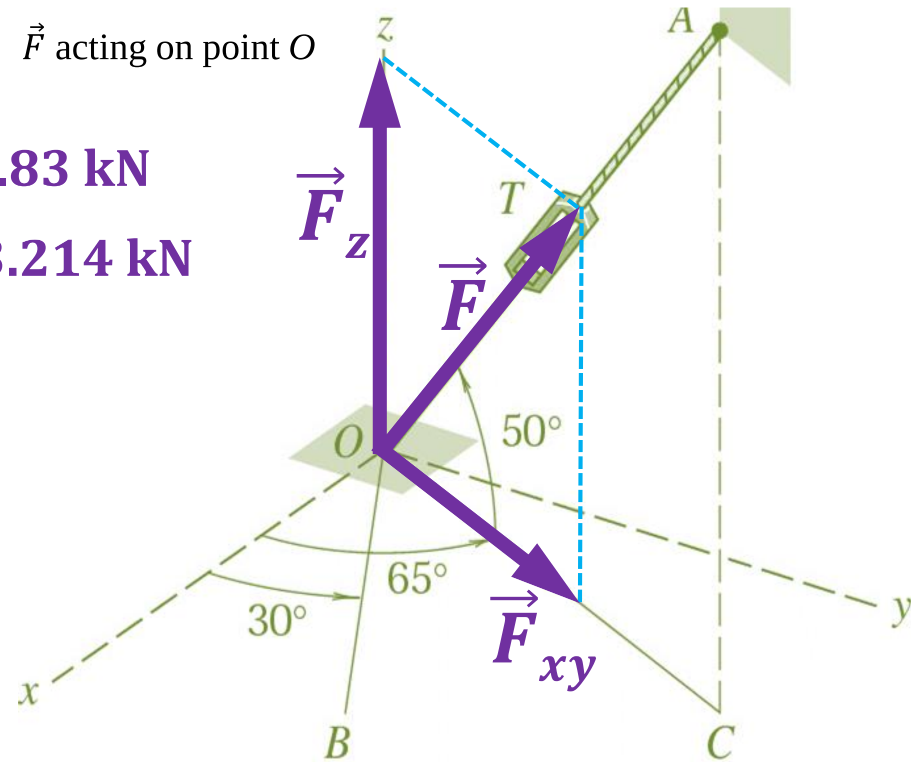


xy plane



$F_z = 5\sin50 = 3.83$ kN

$F_{xy} = 5\cos50 = 3.214$ kN



$F_y = F_{xy}\sin65 = 3.214\sin65 = 2.91$ kN

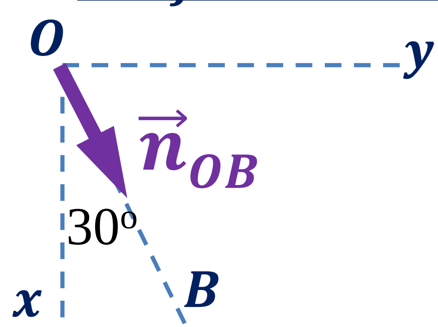
$F_x = F_{xy}\cos65 = 3.214\cos65 = 1.358$ kN

$$\boxed{\vec{F} = 1.358\vec{i} + 2.91\vec{j} + 3.83\vec{k}}$$

Projection onto y − axis

$$F_y = \vec{F}\cdot\vec{j} = 2.91\ kN$$

Projection onto line OB



Unit vector of line OB

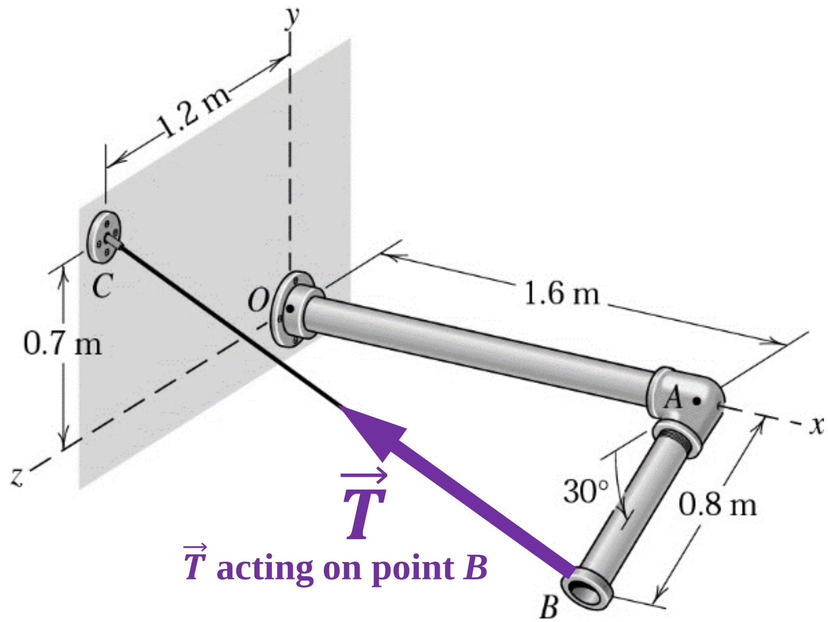
$$\vec{n}_{OB} = \cos30\vec{i} + \cos60\vec{j}$$

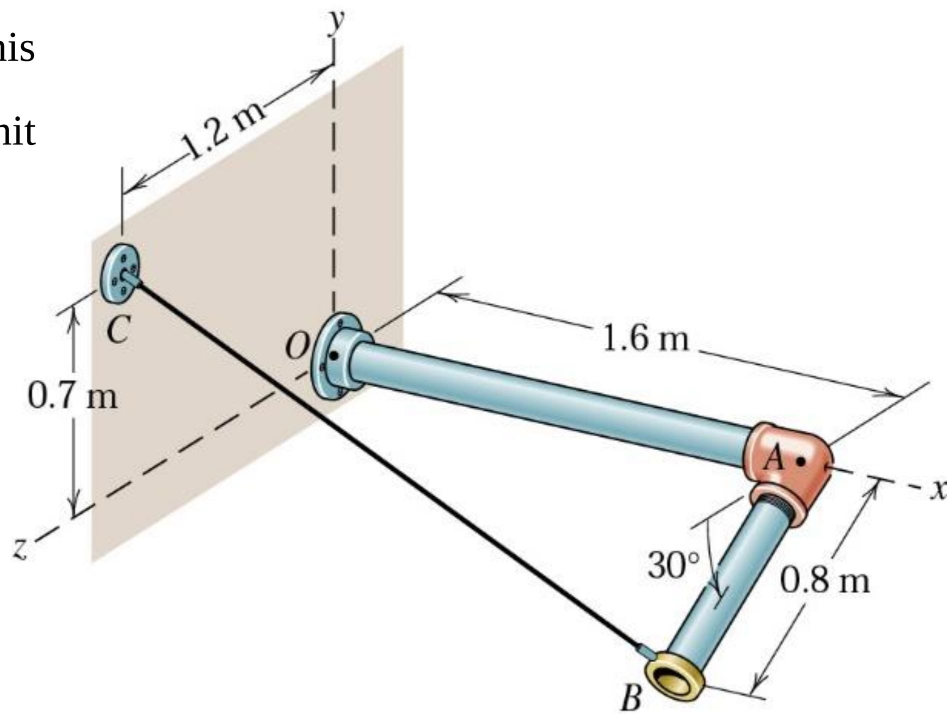
$$F_{OB} = \vec{F}\cdot\vec{n}_{OB} = \left(1.358\vec{i} + 2.91\vec{j} + 3.83\vec{k}\right)\cdot\left(0.866\vec{i} + 0.5\vec{j}\right)$$

$$F_{OB} = (1.358)(0.866) + (2.91)(0.5)$$

$$F_{OB} = 2.63\ kN$$

7. The cable BC carries a tension of 750 N. Write this tension as a force $\vec{T}$ acting on point B in terms of the unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$. The elbow at A forms a right angle.





$$\vec{T} = T\vec{n}_{BC} = T\frac{\vec{r}_{C/B}}{\left|\vec{r}_{C/B}\right|}$$

The coordinates of points B and C are $B\ (1.6; -0.8sin30; 0.8cos30)\ B\ (1.6; -0.4; 0.693)\ ,\ C\ (0;\ 0.7; 1.2)$

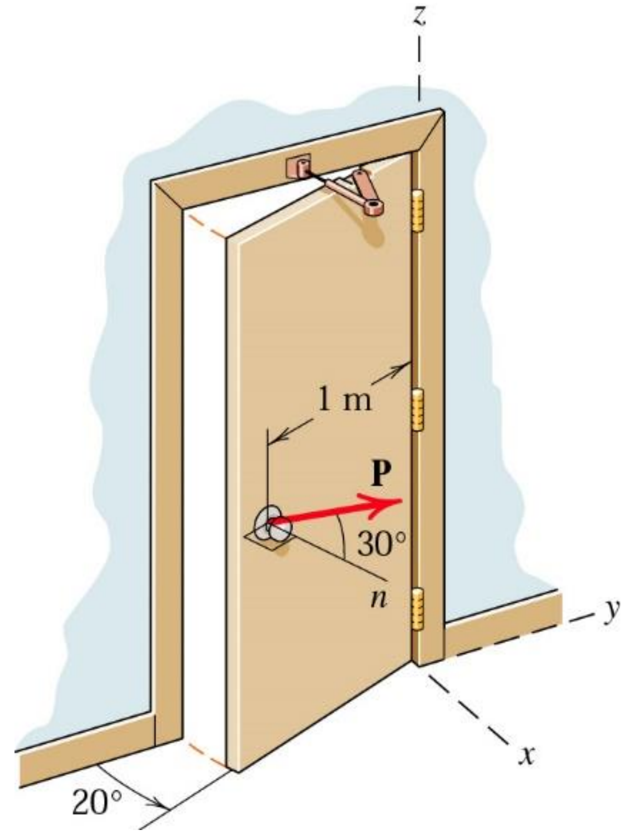
The position vector BC is $\vec{r}_{C/B} = -1.6\vec{i} + 1.1\vec{j} + 0.507\vec{k} \quad \Rightarrow \quad \left|\vec{r}_{C/B}\right| = \overline{BC} = \sqrt{1.6^2 + 1.1^2 + 0.507^2} = 2\ m$

The unit vector of $\vec{T}$ $(\vec{r}_{BC})$ ***is*** $\vec{n}_{C/B} = \vec{n}_{BC} = \vec{n}_T = \dfrac{-1.6\vec{i} + 1.1\vec{j} + 0.507\vec{k}}{2} = -0.797\vec{i} + 0.548\vec{j} + 0.253\vec{k}$

Tension $\vec{T}$ acting on point B in vector form

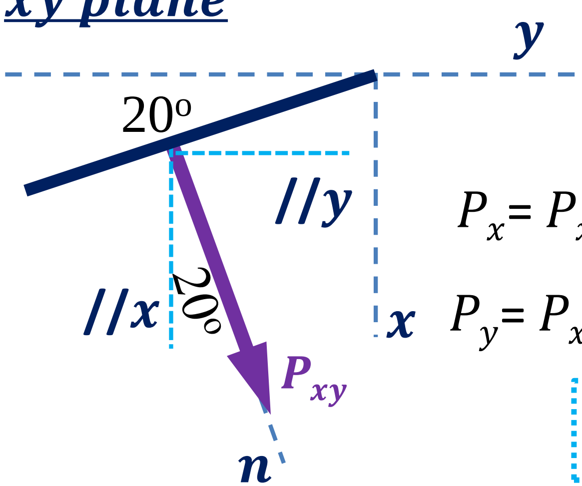
$$\vec{T} = T\vec{n}_{BC} = 750\left(-0.797\vec{i} + 0.548\vec{j} + 0.253\vec{k}\right) = \underline{-597.75\vec{i} + 411\vec{j} + 189.75\vec{k}}$$

8. In opening a door which is equipped with a heavy-duty return mechanism, a person exerts a force P of magnitude 40 N as shown. Force P and the normal n to the face of the door lie in a vertical plane. Express P as a vector and determine the angles θ_x, θ_y and θ_z which the line of action of P makes with the positive x-, y- and z-axes.



$P_z = 40\sin 30 = 20$ N $P_{xy} = 40\cos 30 = 34.64$ N

xy plane



$P_x = P_{xy}\cos 20 = 34.64\cos 20 = 32.55$ N

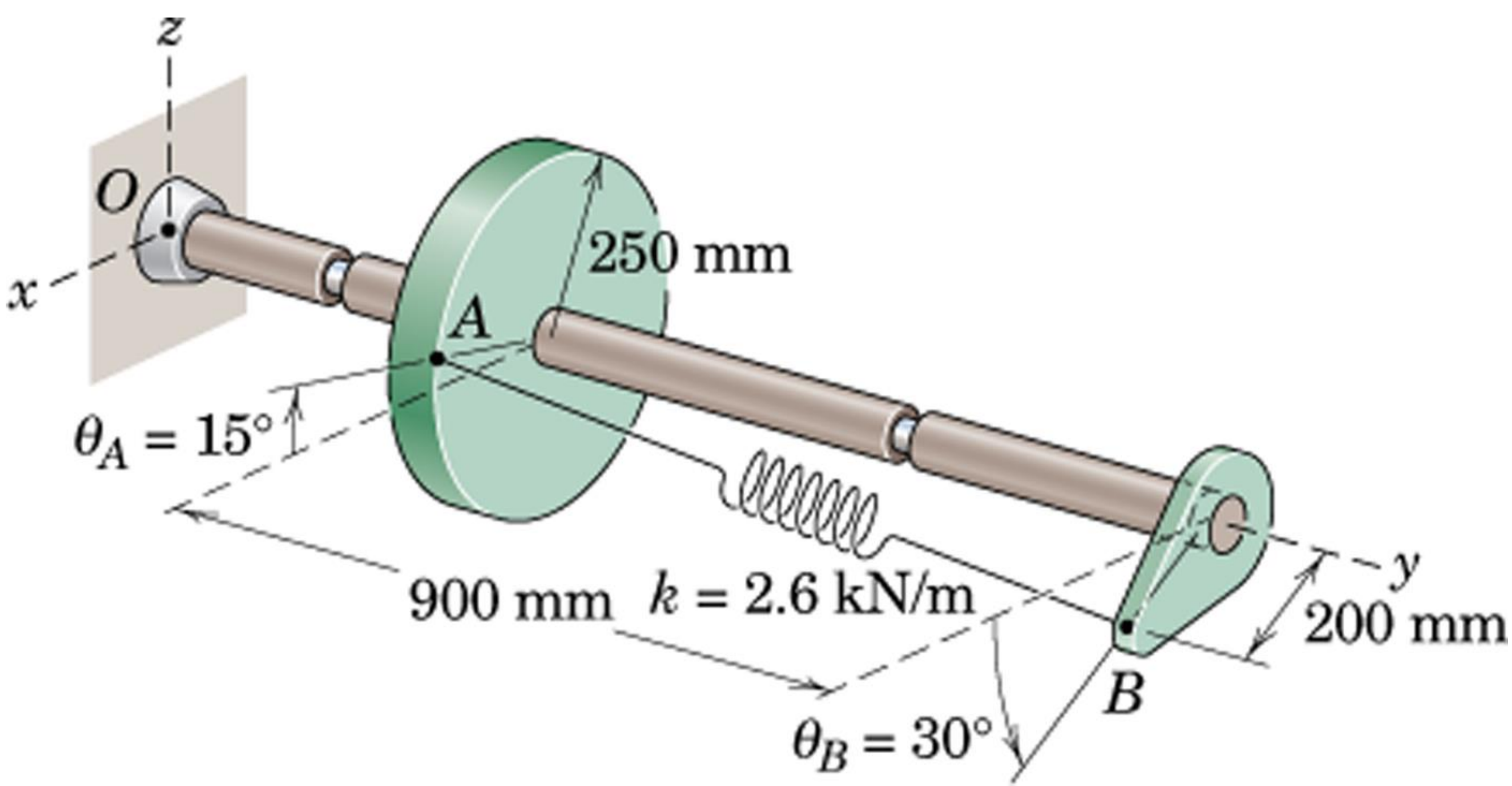
$P_y = P_{xy}\sin 20 = 34.64\cos 20 = 11.848$ N

$$\boxed{\vec{P} = 32.55\vec{i} + 11.848\vec{j} + 20\vec{k}}$$

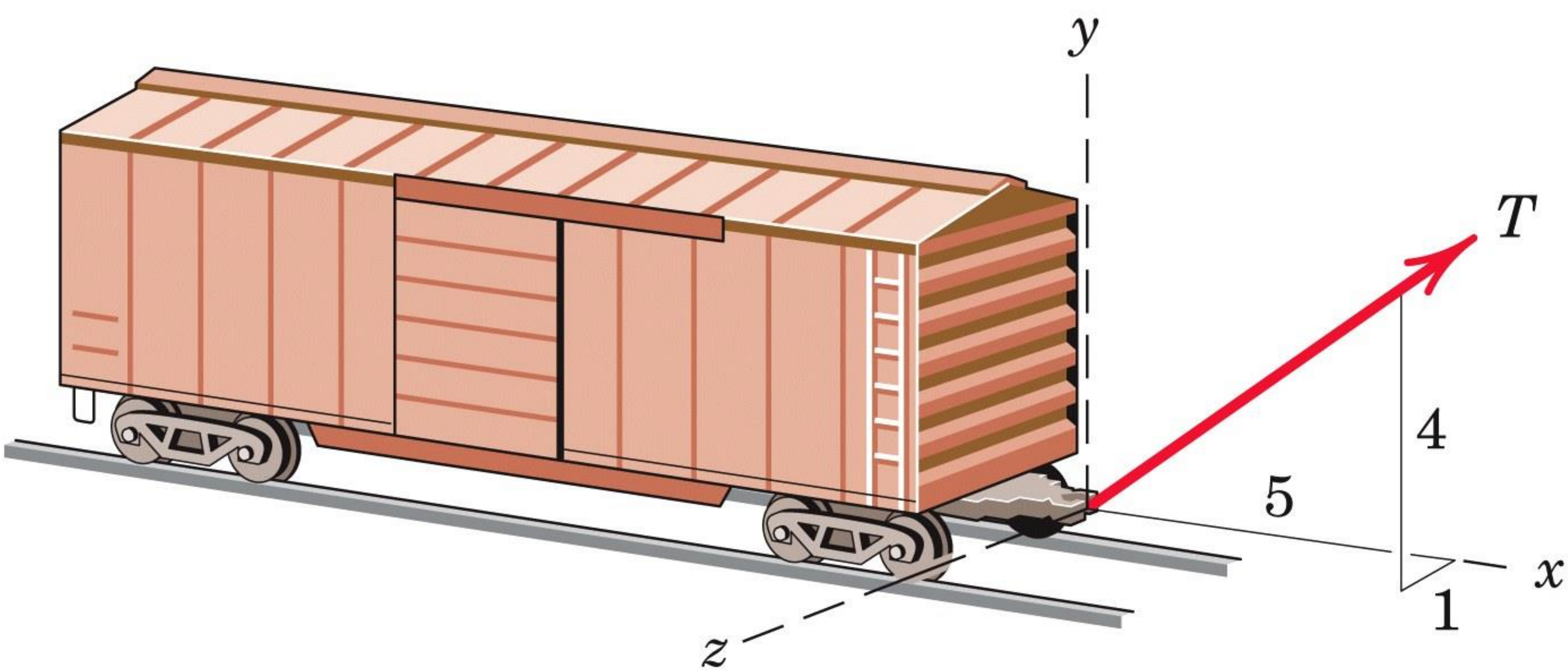
angles θ_x, θ_y and θ_z which the line of action of P makes with the positive x-, y- and z-axes

$$\theta_x = a\cos\left(\frac{32.55}{40}\right) = 35.536^\circ \qquad \theta_y = a\cos\left(\frac{11.848}{40}\right) = 72.77^\circ \qquad \theta_z = a\cos\left(\frac{20}{40}\right) = 60^\circ$$

9. The spring of constant k = 2.6 kN/m is attached to the disk at point A and to the end fitting at point B as shown. The spring is unstretched when θ_A and θ_B are both zero. If the disk is rotated 15° clockwise and the end fitting is rotated 30° counterclockwise, determine a vector expression for the force which the spring exerts at point A.

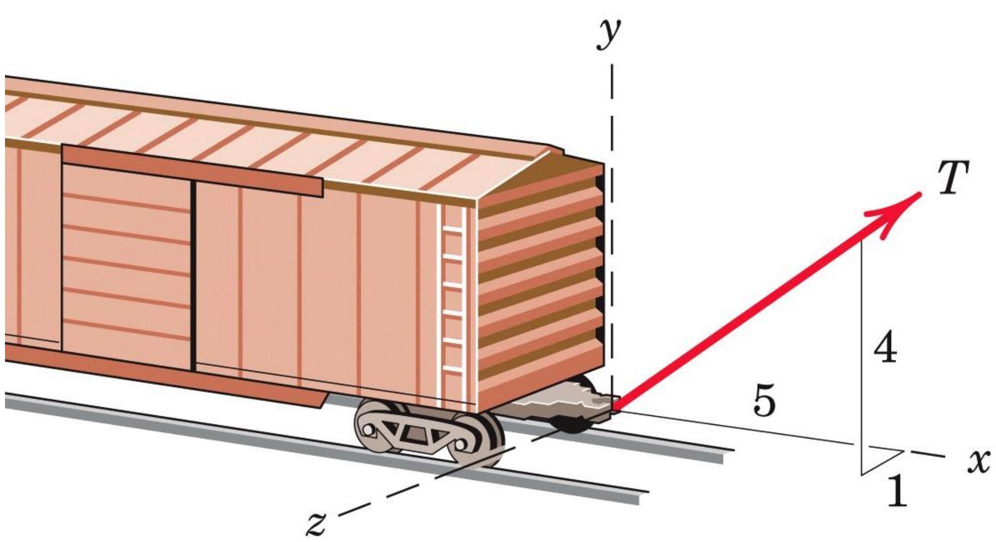


10. An overhead crane is used to reposition the boxcar within a railroad car-repair shop. If the boxcar begins to move along the rails when the x-component of the cable tension reaches 3 kN, calculate the necessary tension T in the cable. Determine the angle θ_{xy} between the cable and the vertical x-y plane.



T_x=3 kN, calculate tension T, the angle θ_{xy} between the cable and the vertical x-y plane.

Unit vector of $\vec{T}$



$$\vec{n}_T = \frac{5\vec{i} + 4\vec{j} + \vec{k}}{\sqrt{5^2 + 4^2 + 1^2}}$$

$$\vec{n}_T = 0.77\vec{i} + 0.617\vec{j} + 0.154\vec{k}$$

$\vec{T}$ in vector form

$$\vec{T} = T\left(0.77\vec{i} + 0.617\vec{j} + 0.154\vec{k}\right)$$

x-component of $\vec{T}$

$$0.77T = 3 \qquad \Rightarrow \qquad T = 3.896\ kN$$ ***(magnitude of $\vec{T}$)***

y and z-components of $\vec{T}$

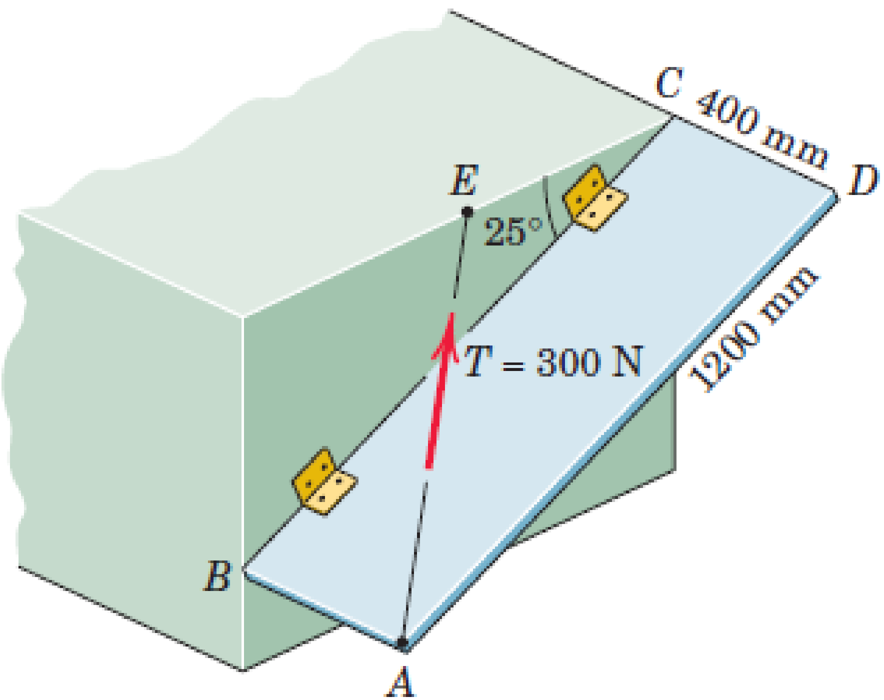
$$T_y = 0.617(3.896) = 2.4\ kN \qquad\qquad T_z = 0.154(3.896) = 0.6\ kN$$

$$\boxed{\vec{T} = 3\vec{i} + 2.4\vec{j} + 0.6\vec{k}}$$

$$T_{xy} = \sqrt{T_x^2 + T_y^2} = \sqrt{3^2 + 2.4^2} = \underline{3.84\ kN}$$

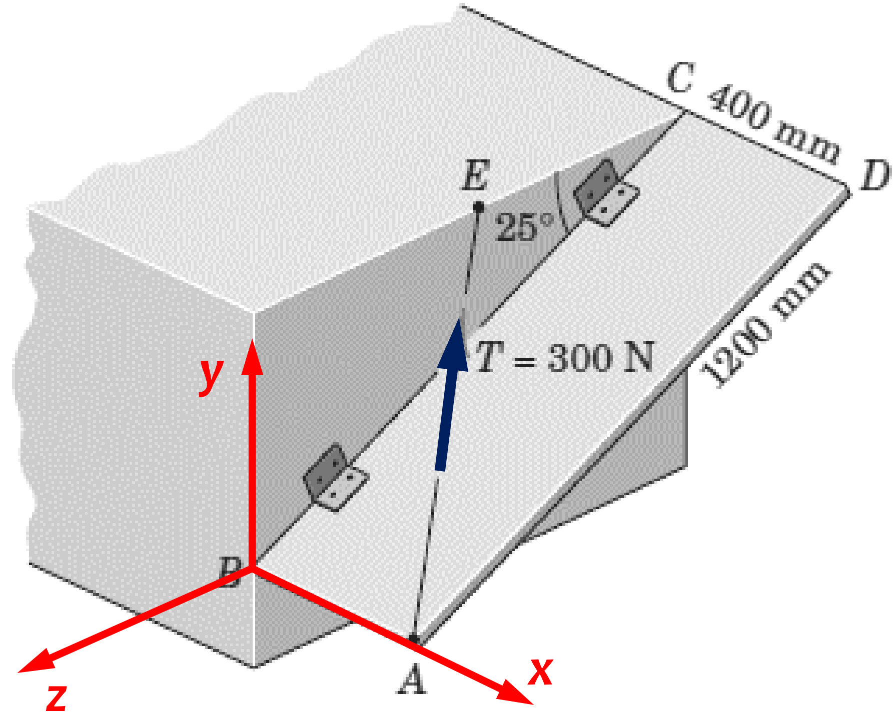
$$\cos\theta_{xy} = \frac{T_{xy}}{T} = \frac{3.84}{3.89} = 0.896 \qquad \Rightarrow \qquad \underline{\theta_{xy} = 9.598^\circ}$$

11. The rectangular plate is supported by hinges along its side *BC* and by the cable *AE*. If the cable tension is 300 N, determine the projection onto line *BC* of the force exerted on the plate by the cable. Note that *E* is the midpoint of the horizontal upper edge of the structural support.



If T=300 N, determine the projection onto line BC of the force exerted on the plate by the cable.

Coordinates of points A, B, C and E with respect to coordinate system



$\boldsymbol{A}\ (400, 0, 0) \quad \boldsymbol{B}\ (0, 0, 0)$

$\boldsymbol{C}\ (0, 1200sin25, -1200cos25)$

$\boldsymbol{C}\ (0, 507.14, -1087.57)$

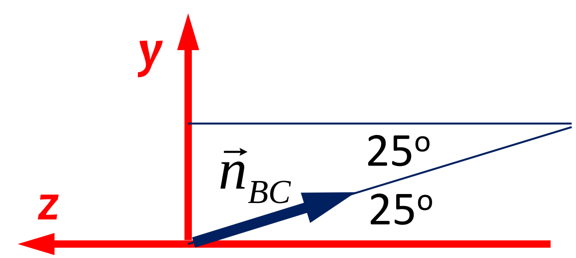
$\boldsymbol{E}\ (0, 1200sin25, -600cos25)$

$\boldsymbol{E}\ (0, 507.14, -543.78)$

$$\vec{T} = T\vec{n}_T = 300\left(\frac{-400\vec{i} + 507.14\vec{j} - 543.78\vec{k}}{844.33}\right)$$

$$\vec{T} = -142.12\vec{i} + 180.19\vec{j} - 193.21\vec{k}$$

Unit vector of line BC



$$\vec{n}_{BC} = -\cos 25\vec{k} + \sin 25\vec{j} = 0.423\vec{j} - 0.906\vec{k}$$

Projection of T onto line BC

$$T_{BC} = \vec{T}\cdot\vec{n}_{BC} = \left(-142.12\vec{i} + 180.19\vec{j} - 193.21\vec{k}\right)\cdot\left(0.423\vec{j} - 0.906\vec{k}\right)$$

$$\boxed{T_{BC} = 251.26\ N}$$

12. The y and z scalar components of a force are 100 N and 200 N, respectively. If the direction cosine $l=cos\theta_x$ of the line of action of the force is –0.5, write $\vec{F}$ as a vector.

$$F_y = 100\ N \qquad F_z = 200\ N \qquad l = -0.5$$

$$l^2 + m^2 + n^2 = 1 \quad \Rightarrow \quad m^2 + n^2 = 1 - 0.5^2 \quad \Rightarrow \quad m^2 + n^2 = 0.75$$

$$\sqrt{F_y^2 + F_z^2} = 223.61\ N \ \left(= F_{yz}\right)$$

$$F_y = F\cos\theta_y = Fm \qquad F_z = F\cos\theta_z = Fn$$

$$F\sqrt{0.75} = 223.61 \quad \Rightarrow \quad F = 258.2\ N$$

$$F_x = F\cos\theta_x = -129.1\ N$$

$$\boxed{\vec{F} = -129.1\vec{i} + 100\vec{j} + 200\vec{k}}$$

13. Determine the parallel and normal components of force $\vec{F}$ in vector form with respect to a line passing through points A and B.

Cartesian components of $\vec{F}$

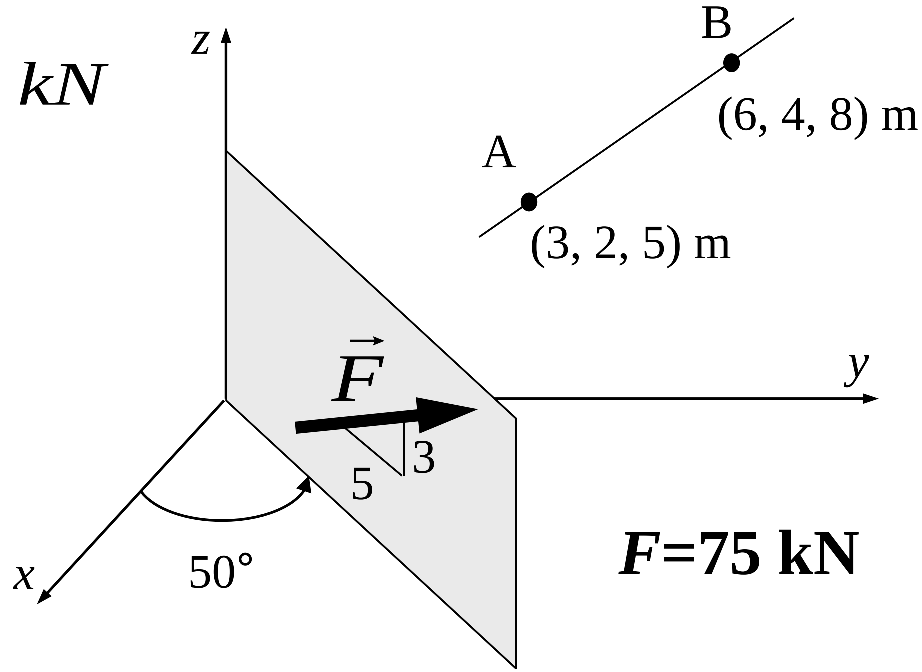
$$F_{xy} = 75\frac{5}{\sqrt{5^2+3^2}} = 64.311\ kN$$

$$F_x = F_{xy}\cos 50 = 41.34\ kN$$

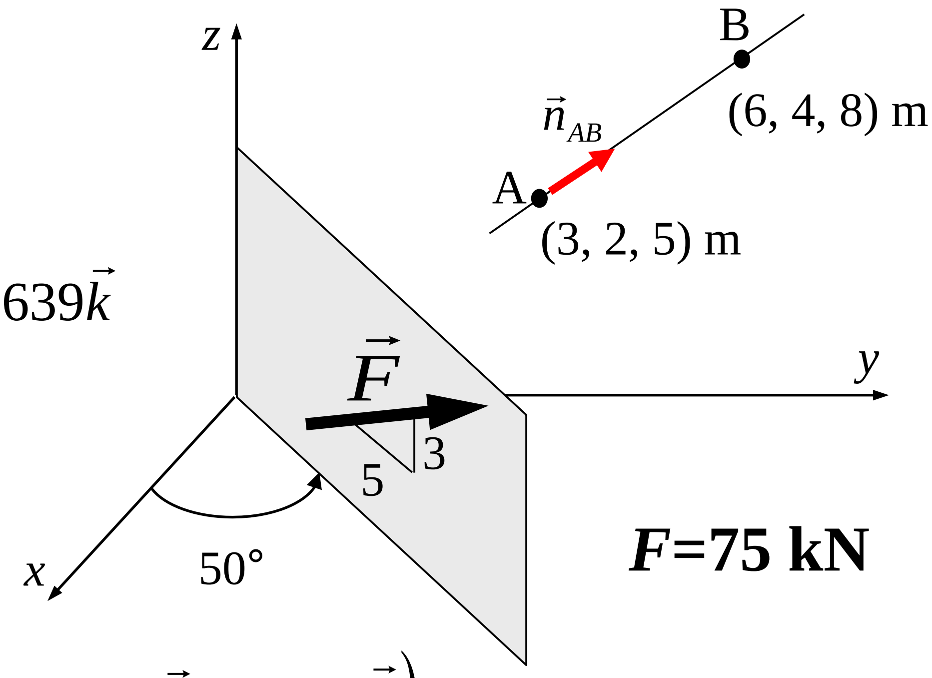
$$F_y = F_{xy}\sin 50 = 49.265\ kN$$

$$F_z = 75\frac{3}{\sqrt{5^2+3^2}} = 38.59\ kN$$

$$\boxed{\vec{F} = 41.34\vec{i} + 49.265\vec{j} + 38.59\vec{k}}$$



$$\vec{F} = 41.34\vec{i} + 49.265\vec{j} + 38.59\vec{k}$$



***F*=75 kN**

Unit vector of line AB

$$\vec{n}_{AB} = \frac{3\vec{i} + 2\vec{j} + 3\vec{k}}{\sqrt{3^2 + 2^2 + 3^2}} = 0.639\vec{i} + 0.426\vec{j} + 0.639\vec{k}$$

Parallel component of $\vec{F}$ to line AB (its scalar value) :

$$F_{//} = \vec{F} \cdot \vec{n}_{AB}$$

$$F_{//} = \left(41.34\vec{i} + 49.265\vec{j} + 38.59\vec{k}\right) \cdot \left(0.639\vec{i} + 0.426\vec{j} + 0.639\vec{k}\right)$$

$$F_{//} = (41.34)(0.639) + (49.265)(0.426) + (38.59)(0.639) = \underline{72.14\ kN}$$

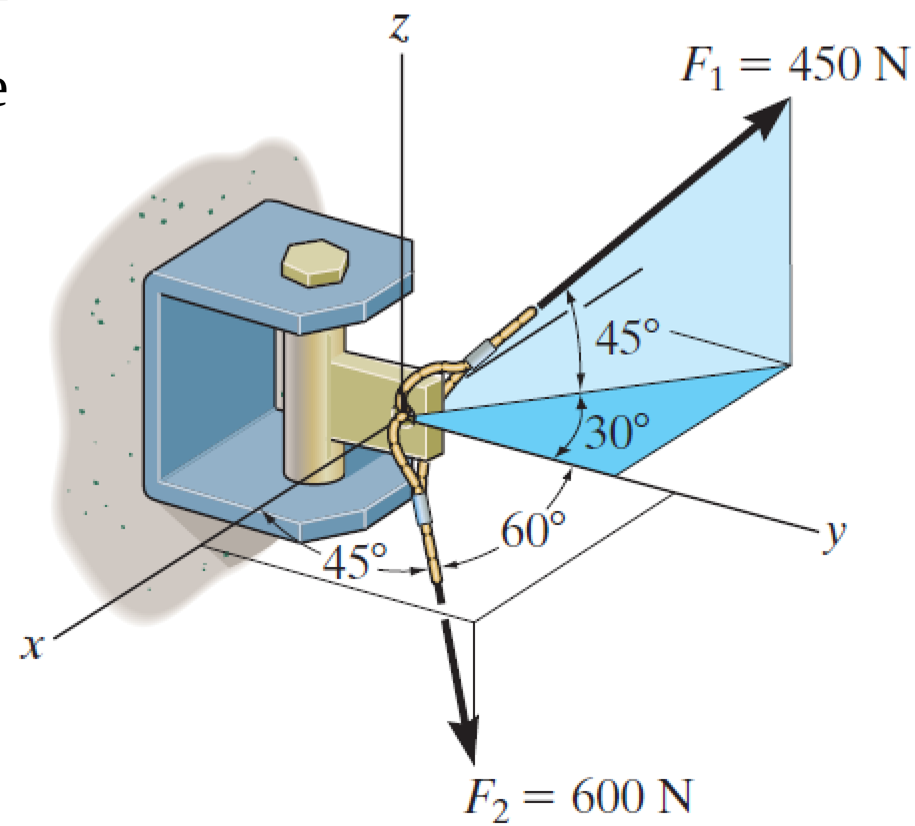
Parallel component of $\vec{F}$ to line AB (in vector form):

$$\vec{F}_{//} = F_{//}\vec{n}_{AB} = 72.14\left(0.639\vec{i} + 0.426\vec{j} + 0.639\vec{k}\right) = \underline{46.14\vec{i} + 30.76\vec{j} + 46.14\vec{k}}$$

Normal component of $\vec{F}$ to line AB (in vector form):

$$\vec{F}_{\perp} = \vec{F} - \vec{F}_{//} = \underline{-4.8\vec{i} + 18.505\vec{j} - 7.55\vec{k}}$$

14. Determine the magnitude and direction angles of the resultant force acting on the bracket.



Resultant

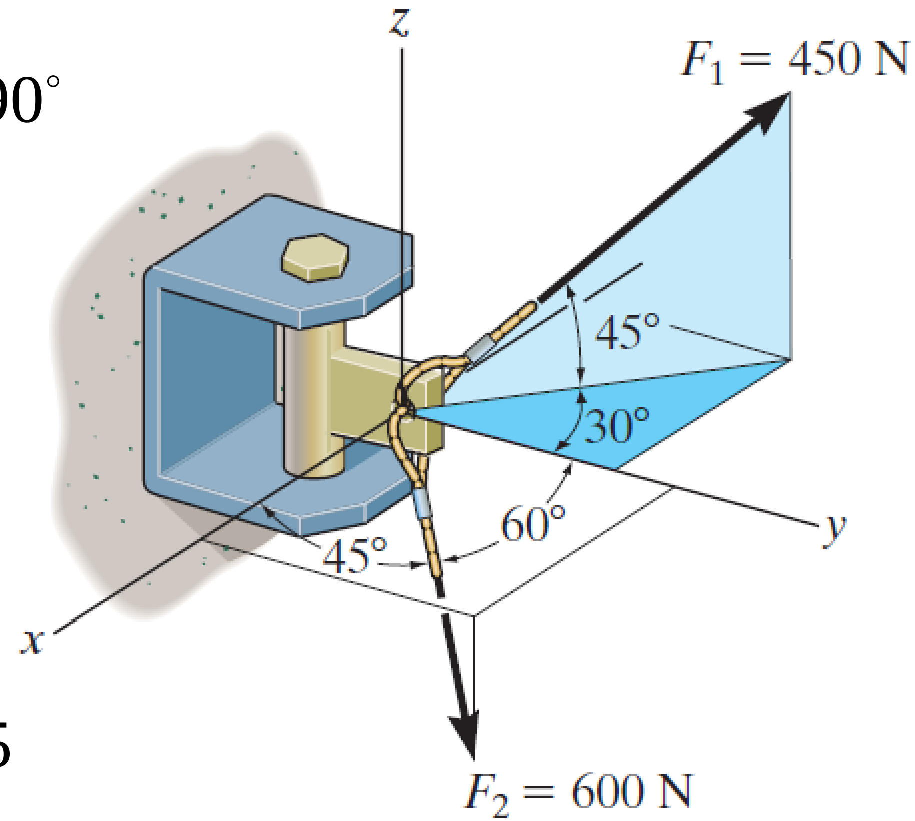
$$\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_1 = -\underbrace{450\cos 45}_{(F_1)_{xy}}\sin 30\vec{i} + \underbrace{450\cos 45}_{(F_1)_{xy}}\cos 30\vec{j} + 450\sin 45\vec{k}$$

$$\vec{F}_1 = -159.1\vec{i} + 275.57\vec{j} + 318.2\vec{k}$$

Direction angles for $\vec{F}_2$

$\theta_x = 45^\circ$ $\theta_y = 60^\circ$ $\theta_z > 90^\circ$



Direction cosines

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\left\{ l^2 + m^2 + n^2 = 1 \right\}$$

$$\cos^2 45 + \cos^2 60 + \cos^2\theta_z = 1$$

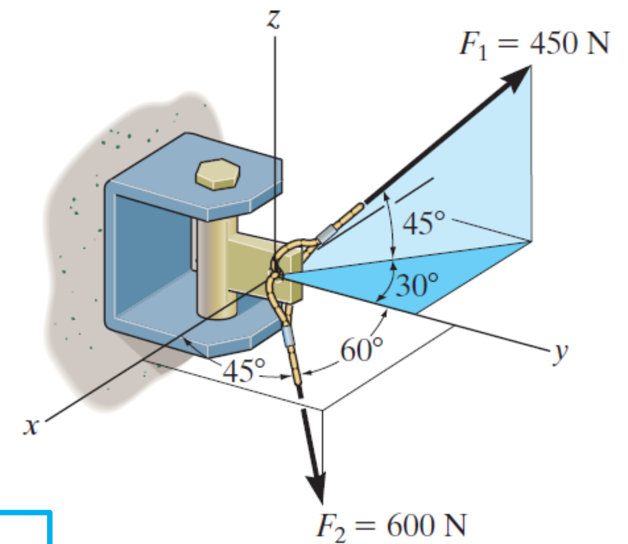
$$\cos^2\theta_z = 0.25 \;\Rightarrow\; \cos\theta_z = \pm 0.5$$

$$\theta_z > 90^\circ \;\Rightarrow\; \cos\theta_z = -0.5 \;\Rightarrow\; \theta_z = 120^\circ$$

$$\vec{F}_2 = 600\cos 45\vec{i} + 600\cos 60\vec{j} + 600\cos 120\vec{k}$$

$$\vec{F}_2 = 424.26\vec{i} + 300\vec{j} - 300\vec{k}$$

$$\vec{F}_1 = -159.1\vec{i} + 275.57\vec{j} + 318.2\vec{k} \qquad \vec{F}_2 = 424.26\vec{i} + 300\vec{j} - 300\vec{k}$$



Resultant $\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2$

$$\vec{R} = (-159.1 + 424.26)\vec{i} + (275.57 + 300)\vec{j} + (318.2 - 300)\vec{k}$$

$$\boxed{\vec{R} = 265.16\vec{i} + 575.57\vec{j} + 18.2\vec{k}}$$

Magnitude of Resultant Force

$$\boxed{|\vec{R}| = R = \sqrt{265.16^2 + 575.57^2 + 18.2^2} = 633.97\ N}$$

Direction Cosines of Resultant Force

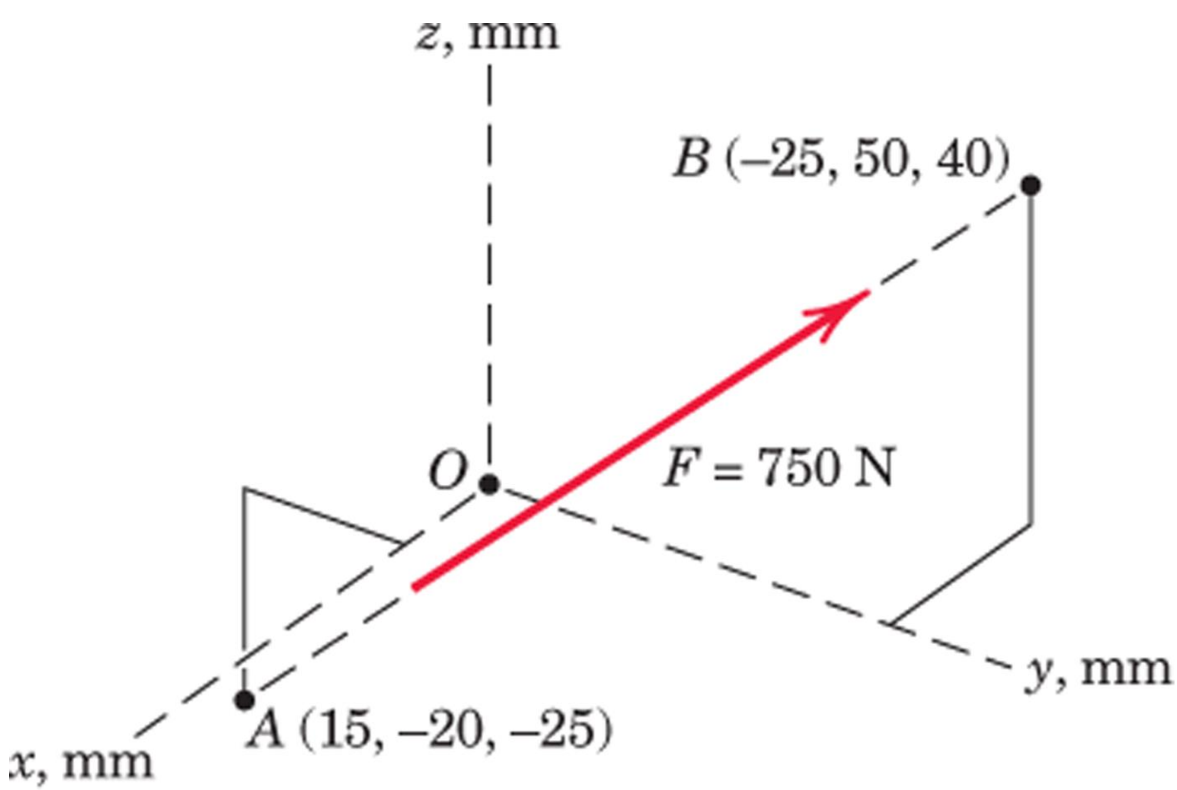
$$\cos\theta_x = \frac{R_x}{R} \qquad \cos\theta_y = \frac{R_y}{R} \qquad \cos\theta_z = \frac{R_z}{R}$$

$$\cos\theta_x = \frac{265.16}{633.97} = \underline{\underline{0.418}} \qquad \cos\theta_y = \frac{575.57}{633.97} = \underline{\underline{0.907}} \qquad \cos\theta_z = \frac{18.2}{633.97} = \underline{\underline{0.029}}$$

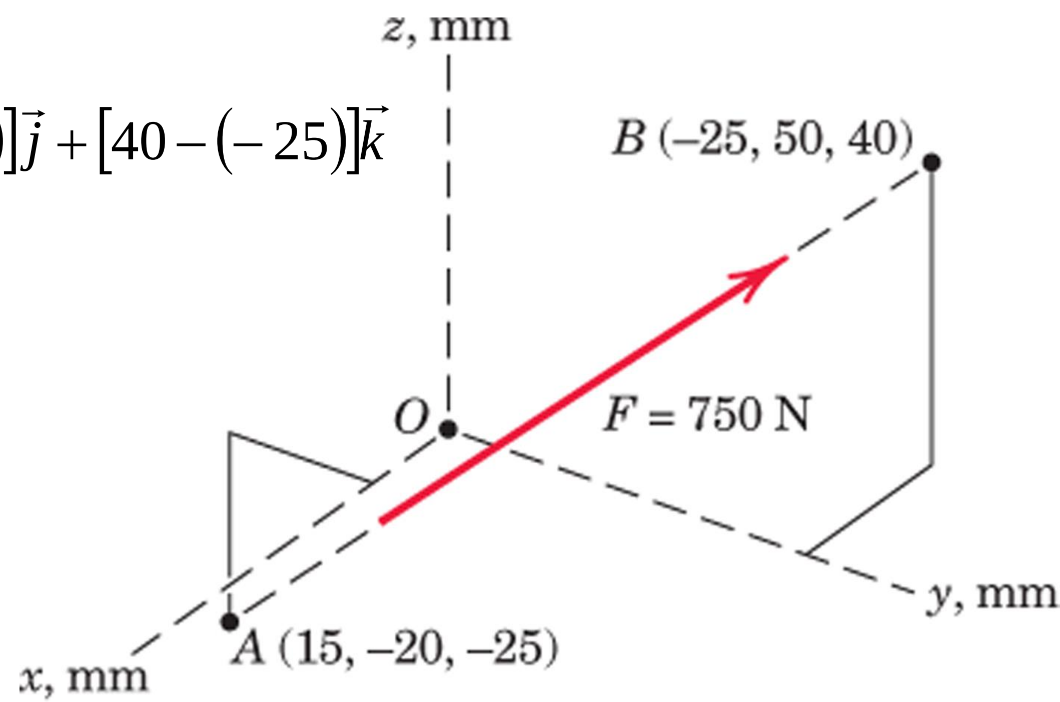
Direction angles for $\overrightarrow{R}$

$$\theta_x = \arccos(0.418) = \underline{\underline{65.3^\circ}} \qquad \theta_y = \arccos(0.907) = \underline{\underline{24.9^\circ}} \qquad \theta_z = \arccos(0.029) = \underline{\underline{88.3^\circ}}$$

15. Express the force $\vec{F}$ as a vector in terms of unit vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$. Determine the direction angles θ_x, θ_y and θ_z which $\vec{F}$ makes with the positive *x*-, *y*-, and *z*-axes.



Position vector



$$\vec{r}_{B/A} = \overrightarrow{AB} = (-25-15)\vec{i} + [50-(-20)]\vec{j} + [40-(-25)]\vec{k}$$

$$\vec{r}_{B/A} = \overrightarrow{AB} = -40\vec{i} + 70\vec{j} + 65\vec{k}$$

Unit vector

$$\vec{n} = \frac{\vec{r}_{B/A}}{|\vec{r}_{B/A}|} = \frac{-40\vec{i} + 70\vec{j} + 65\vec{k}}{\underbrace{\sqrt{40^2 + 70^2 + 65^2}}_{103.56}}$$

$$\vec{n} = -0.386\vec{i} + 0.676\vec{j} + 0.627\vec{k}$$

$$\vec{F} = F\vec{n} = 750(-0.386\vec{i} + 0.676\vec{j} + 0.627\vec{k}) = -289.5\vec{i} + 507\vec{j} + 470.25\vec{k}$$

Direction cosines $l = -0.386$ $\quad m = 0.676$ $\quad n = 0.627$ $\quad \{l^2 + m^2 + n^2 = 1\}$

Direction angles

$$\begin{cases} l = \cos\theta_x = -0.386 & \Rightarrow & \theta_x = 112.7^\circ \\ m = \cos\theta_y = 0.676 & \Rightarrow & \theta_y = 47.47^\circ \\ n = \cos\theta_z = 0.627 & \Rightarrow & \theta_z = 51.17^\circ \end{cases}$$