An Experiment in Probability Estimation

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➤ While the activity of marketing research can be fruitfully viewed within a statistical decision theoretic model, relatively little is known concerning the descriptive aspects of how people—managers or consumers—revise probabilities in the light of new information. This paper reports the results of a behavioral study in probability revision, and the implications of these findings for the operational use of decision theoretic concepts in prescriptive and descriptive choice-making models.

The study of personalistic probability and statistical decision theory has been given new impetus by the emergence of three significant trends, all of which appear relevant to the design and administration of marketing research. The first of these trends, exemplified in recent papers by Alexander [1], Boyd and Britt [2], Green [4] and Roberts [8], concerns the viewpoint that the design and conduct of marketing research be considered in terms of the cost versus potential informational value of this activity. As such, the decision of whether or not to do marketing research (and, if so, how much to do) is considered within a broader framework involving (a) the anticipated costs of wrong decisions, with and without the research; (b) the prior information of the user of the research; (c) the anticipated relevance and reliability of the research results; and (d) the cost of alternative research investigations. These concepts, however, are all components of the statistical decision theory model.

A second trend related to the personalistic probability process is the growing use of this framework in the study of consumer decision behavior. Peters [6] has summarized much of the appropriate literature dealing with the subjective evaluation of odds and descriptive choice behavior under differential risks.

Finally, there appears to be mounting interest in the design of overall corporate and military intelligence systems which utilize the framework of statistical decision theory. Edwards and Phillips [3] have played an important role in the conceptualization of such systems and the design of prototype models, in which the subjective estimates of human operators are combined with the computational capacity and speed of the computer in the processing of probabilistic information for strategic-level decision making. In part, Bayesian statistics constitute a normative theory of how one should revise probability judgments in the light of new information. In this article, however, concern is with how subjects actually revise initial (or prior) probabilities on the basis of sample data and the *descriptive* adequacy of the Bayesian model, as a first approximation, in explaining this behavior.

Some attempts to see how well (or poorly) subjects act like Bayesian statisticians have been reported recently. Edwards and Phillips [3] have described a rather complex experiment (simulating operations at a radar console) in which subjects were required to estimate posterior probabilities for four hypotheses based on a series of stimulus dots which were shown on a visual display board. Shuford [10] has suggested the Bayesian model as a conceptual framework for studying certain learning tasks. Green, Halbert and Minas [5] have described a rather elaborate experiment involving a betting situation where subjects could reduce the cost of uncertainty by purchasing additional information before placing their bets.

The experiment reported in this article represents an attempt to construct a simple exercise which still retains the essentials of Bayesian posterior probability estimation. (In part, it represents a simple analogue of the more complicated experiment of Edwards and Phillips.) The objective was to see how well naive subjects could estimate posterior probabilities as a function of the following experimental variables:

1. Prior probabilities: 2 levels—75-25; 50-50[1]

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[1] For convenience, from this point on split probabilities will be designated with a hyphen between them rather than with decimal points, for example, 75–25, rather than 0.75:0.25.

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2. Conditional probabilities: 2 levels—60-40; 80-20
3. Sample size: 4 levels—1, 3, 6, and 9 items

In addition it is wished to determine qualitatively the manner in which subjects weighed prior and sample evidence when conflicts arose between these two sources of information.

RESEARCH DESIGN

Subjects for this experiment consisted of 32 graduate student volunteers, none of whom were versed in decision theory. The experimental game was first designed for one set of prior probabilities (75-25) and played by 24 subjects. An additional game was then designed (using prior probabilities of 50-50) and played by eight subjects; the results of both exercises are summarized here. Each subject was given an initial stake of $10.00 and told that his actual payoff at the end of the game would be proportional to his terminal asset position. Subjects were then given verbally the following instructions (illustrated for the 75-25 prior probability version):

> The object of this experiment is to see how well people can intuitively revise initial probabilities about the type of card deck from which they are sampling. We will conduct 24 repetitions of this game, using decks of fifty 3" x 5" cards. Each card will be marked either with an X or just left blank.
>
> In each trial of the game you will be presented with four decks of these cards. (Subject was then shown a sample deck of cards.) Three of the four decks will always be blank type decks, and the other deck will always be an X type deck. Each deck is identified by a cover card which I will remove after you have selected one of the decks. You will then be given the opportunity to choose a certain number of cards from the deck which you have chosen. A blank type deck is one in which most of the cards in the deck are blank cards. Similarly, an X type deck is one in which most of the cards are X cards. I'll explain in a moment what I mean by most.
>
> The three blank type and one X type decks constitute your initial probabilities of 3 out of 4, or .75 of choosing a blank type deck on any one trial and .25 of choosing an X type deck. That is, if you were presented with these four decks of cards a large number of times, and if each time you randomly selected a deck as a blank type deck, on the average, 3 out of 4 times your choice would be correct.
>
> You will notice to your right a scale with a movable indicator. (Subject was shown a 10-inch scale and movable indicator.) This device is for your use in revising probabilities. The scale ranges from zero to one in intervals of .05. Since your prior probability of choosing a blank type deck on any one trial is .75, the indicator will be set on this reading before each trial. When revising probabilities based on the sample of cards which you have chosen from the deck of your choice, simply move the indicator to the zero or 5 interval nearest to what you think the revised probability should be.
>
> During the first twelve trials, we will be using what we call 60-40 decks. That is, if the deck is a blank type deck, 60 percent (or exactly 30 out of the 50 cards) will be blank cards, while the remaining 40 percent (or exactly 20 out of the 50 cards) will be X cards. Similarly, if the deck is an X type deck, 60 percent (or 30 out of the 50 cards) will be X cards, while the remaining 40 percent (or 20 out of the 50 cards) will be blank cards. The cards within each deck will be thoroughly shuffled and decks will be rearranged on the carrying board after each trial.
>
> When you are presented with a set of four decks of cards, the experimenter will ask you to select a sample of cards from any single one of the four decks. The sample size will be one of the following: 1, 3, 6, or 9 cards. You may sample within that one deck in any way you please in obtaining your sample cards. On the basis of the number of blank cards (if any) and X cards (if any) drawn, we want you to revise your prior probability of .75 *to the probability that you now feel that the deck you are sampling from is a blank type deck.* You are *not* to express your revised probability that you are sampling from an X type deck. You are invited to use the scale and indicator for this purpose. To see if you understand this procedure, let us suppose that on the basis of a particular sample outcome, you feel that there is a strong possibility that the deck from which you sampled is an X type deck. In which direction would you move the indicator? (If the subject revised his probability down from .75 and acknowledged an understanding of this phase of the game, further explanation at this stage was not given.)
>
> When you have revised your probability of a blank type deck, the scorer will record your selection and compare it with the true revised probability. If you are correct to the nearest .05 probability mark, you will have $1.00 added to your assets. If you are .05 points off, in either direction, you will receive $.75; .10 points off, $.50; .15 points off, $.25; and if .20 points off, there will be no payoff. Beyond .20 points off, you will have to pay the scorer $.25 for each .05 points beyond the .20 level.
>
> Before starting the actual game we will run through three trials just for practice and we will tell you what the true revised probability is, based on the results of your particular sample. In the real game, however, you will not be told what the true probabilities are until the entire game is over. Incidentally, these funds are not our personal money, so you should not feel reluctant about trying to win as much money as you can. Are there any questions before we start?

Following completion of the first 12 trials (excluding the practice set), the subject was given these instructions for the second twelve:

> The second, and final, 12 trials of this game will be played exactly as the first set with only one exception. This time we will be using 80-20 decks instead of 60-40 decks. That is, if the deck is a blank type deck, 80 percent (or exactly 40 out of the 50 cards) will be blank cards, while the remaining 20 percent (or exactly 10 out of the 50 cards) will be X cards. Similarly, if the deck is an X type deck, 80 percent (or 40 out of the 50 cards) will be X cards, while the remaining 20 percent (or 10 out of the 50 cards) will be blank cards. Again, this 80-20 percent breakdown constitutes the reliability of the sample information. Payoffs will be the same as before, and we will go through three practice trials before starting the real game. Again you are asked to express your revised prob-

ability that the deck you are sampling from is a blank type deck.

Prior to the start of the first phase of the game, the subject was given three practice trials. He was shown a rectangular tray with compartments in which were placed four decks of cards. The subject then selected one of the decks. The experimenter stated the number of cards to sample (1, 3, 6, or 9, the number determined randomly), after which the subject selected the appropriate number of cards from the deck of his choice. He observed these cards and then revised the prior probability of .75 with respect to a blank deck, by suitable manipulation of the scale indicator. After each trial in the practice set, the subject was told the correct probability, as based on his particular sample results; the actual game was then begun.

After each trial the cards were returned to the deck and a new tray with a new set of decks was placed before the subject. The deck used earlier was shuffled and returned to the old tray. The decks were then rearranged by another experimenter (concealed from the subject) while the next trial was being conducted; in this way one tray was being used for play while another tray was being set up for the next trial. The experiment was conducted with a minimum of conversation. At the conclusion of the entire game, subjects were paid as promised and requested not to discuss any aspects of the game with others.

A total of 24 games (each consisting of 24 trials) was played in the above manner. In half of the games the 60-40 decks were presented first and in half of the games the 80-20 decks were presented first. A second group of eight games was then conducted, the only change being that two blank and two X type decks were presented to the subject and, hence, the appropriate prior probabilities were 50-50. Instructions were suitably modified to reflect this change. In all games, each sample size of 1, 3, 6, and 9 cards was replicated three times, following the same preselected randomized sequence, resulting in 12 trials each, for the 60-40 and 80-20 deck proportions, or a total of 24 trials per game over 32 subjects.

EXPERIMENTAL RESULTS

The experimental game format described in the previous section is summarized as follows:

Group	*Prior probabilities*	*Deck proportions*	*Sample sizes*	*Replications*
1 (24 subjects)	75-25	60-40	1, 3, 6, 9	3
	75-25	80-20	1, 3, 6, 9	3
2 (8 subjects)	50-50	60-40	1, 3, 6, 9	3
	50-50	80-20	1, 3, 6, 9	3

The calculation of Bayesian posterior probabilities for each possible sample result is illustrated in the appendix. (In these games the hypergeometric distribution is used to find the appropriate conditional probabilities since samples are being drawn without replacement from a finite distribution.)

Three response measures were computed from the data:

(a) Actual probabilities (as estimated by subjects) averaged over all subject-replications, designated as P_a for each set of conditions.

(b) The averaged signed deviation, (actual probability minus computed probability) designated as

$$\beta = \sum_{i=1}^{n} \frac{(P_{ai} - P_c)}{n}$$

for each set of conditions. This measure provides an estimate of bias for the n subject-trials ($i = 1, 2, \ldots, n$).

(c) A measure of dispersion over all subject-trials, designated

$$\sigma = \left[\sum_{i=1}^{n} \frac{(P_{ai} - P_c)^2}{n} \right]^{\frac{1}{2}}$$

for each set of conditions. This measure provides an estimate of the variability of subjects' estimates, on the average.

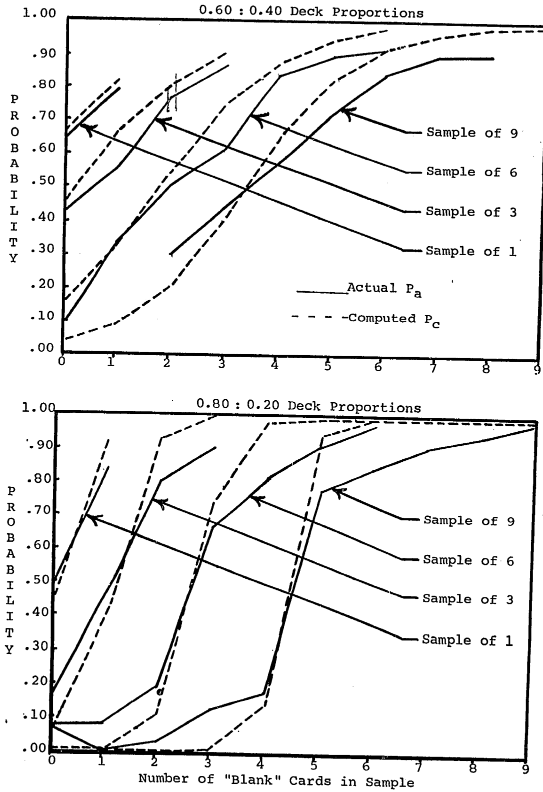
Quantitative Aspects of Probability Revision

Figure 1 shows, for each sample size, the mean actual vs. computed probabilities, as a function of the number of blank cards in the sample, for the 75-25 prior probability case. Similarly, Figure 2 shows the appropriate data for the 50-50 prior probability case. The most striking phenomenon indicated by these graphs is the tendency for subjects to underestimate extremely high posterior probabilities (as sample size increased) and to overestimate extremely low posterior probabilities. This tendency is most pronounced in the cases dealing with 80-20 deck proportions in which the discriminatory content (ability of a sample observation to distinguish between a blank and X deck) is high.

Overall, subjects appear to be slightly underestimating Bayesian posterior probabilities as indicated by the bias measure β. However, there does not appear to be a reduction in estimating bias as sample size increases. Generally, subjects appear to exhibit less information bias in the case of 50-50 prior probabilities and 60-40 deck proportions. In this set of conditions the computed posterior probabilities are, of all conditions, closest to probabilities as estimated from the proportion of blank cards in the sample. These results are only suggestive, however, inasmuch as aggregated bias figures reflect sampling error in the occurrence of the various possible outcomes for each sample size.

Table 1 summarizes, by type of condition and sample size, the computed variability measure σ, for the experiment. Looking first at variability by condition, one can note from the table that both prior probabilities and deck proportions influence dispersion. Over all sample

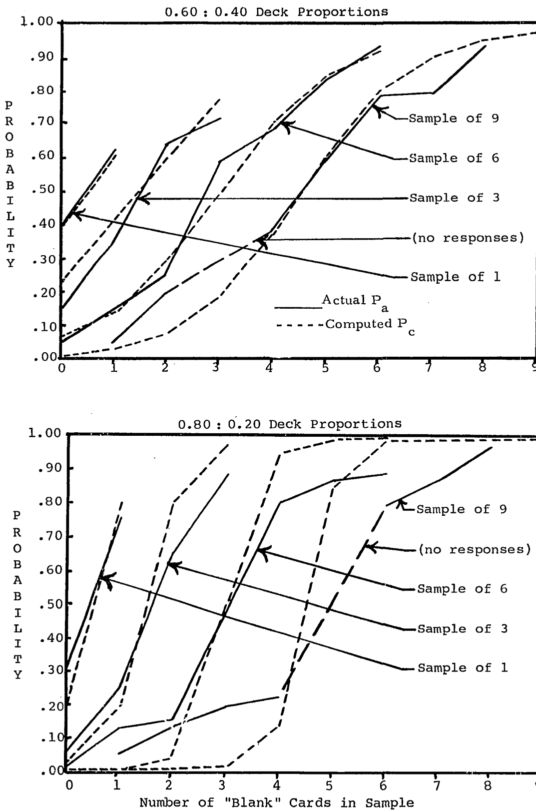
Figure 1
MEAN POSTERIOR PROBABILITY RESPONSES FOR 75-25 PRIOR PROBABILITIES



sizes, the lowest subject-trial variability was associated with the 50-50 prior distribution, 60-40 deck proportions.

Variability increased slightly as the prior probabilities changed from 50-50 to 75-25, and reflected the fact that some subjects relied too heavily on the sample information. As deck proportions changed from 60-40 to 80-20, however, increased variability over subject-trials indicated that some subjects did not appreciate the impact of information of relatively high discriminatory content on the computation of posterior probabilities. This tendency, particularly pronounced in the case of 80-20 deck proportions, accounts in large measure for the high variability shown in Table 1.

Figure 2
MEAN POSTERIOR PROBABILITY RESPONSES FOR 50-50 PRIOR PROBABILITIES



Finally, contrary to prior expectations, increasing sample size does not appear to reduce the variability measure over all subject-trials. It is again clear that subjects are not able (or willing) to extract all of the information contained in the data. At least this appears to be the case for all of the sample sizes used in the experiment.

Qualitative Aspects of Probability Revision

Of particular importance to Bayesian posterior probability computation is the case where prior and sample information are in conflict, this conflict being resolved by application of Bayes' Theorem. In the case of the conditions involving 75-25 prior probabilities, a sample may be drawn in which the number of X cards exceeds the number of blank cards. For example, given 75-25 priors, deck proportions of 60-40, and a sample size of three, one might observe the outcomes: (a) three X cards; (b) two X cards. In each of these specific outcomes, *if* the prior information regarding the more frequent occurrence of blank decks is disregarded one would assign, based on sample outcomes only, a probability of less than .50 to the occurrence of a blank deck. In the case of three X cards out of a total of three, the computed posterior probability with respect to blank would be less than .5 (as a matter of fact, .46), since the sample findings tend to swamp the priors. In the second case, however, the computed posterior probability with respect to blank is .67, indicating that a sample outcome of two X cards out of three is insufficient to reduce the computed posterior probability with respect to blank below .5.

Out of a total of 576 trials where prior probabilities of 75-25 were given, 178 subject-trials resulted in sample outcomes where the number of X cards exceeded the number of blank cards drawn. That is, the sample findings went against the prior probabilities. Table 2 summarizes how subjects behaved under these circumstances.

Of the 178 subject-trials resulting in prior-sample information conflict, subjects estimated a posterior probability with respect to blank below .5 (that is, went against the priors) in 74 instances; in 58 of these 74 instances application of the Bayesian model also would have resulted in computed posterior probabilities of less than .5. In 104 of the 178 trials subjects went against the sample findings and estimated a posterior probability with respect to blank which was above .5; in 66 of these 104 instances application of the Bayesian model also would have resulted in computed posterior probabilities exceeding 0.5.

Thus, overall, subjects made approximately 70 percent qualitatively correct decisions when prior and sample information were in conflict. As might be expected, however, marked differences in performance occurred between deck proportions 60-40 and 80-20. In the 60-40 case, subjects came closer, qualitatively, to computed outcomes when they elected to go against the sample findings. In the 80-20 case, in which the discriminatory content of the information was high, subjects could make errors only when they went against the sample findings. In general, as would be expected, and can be noted from Table 2, subjects made fewer errors in direction as the difference between the computed posterior probability and .5 increased.

Table 1

VARIABILITY MEASURE BY TYPE OF CONDITION AND SAMPLE SIZE

Sample size	75–25 priors: 60-40 deck proportions	75–25 priors: 80-20 deck proportions	50-50 priors: 60-40 deck proportions	50-50 priors: 80-20 deck proportions
1	0.243	0.295	0.100	0.273
3	0.177	0.368	0.068	0.151
6	0.186	0.226	0.264	0.258
9	0.175	0.175	0.209	0.178
All samples	0.195	0.266	0.160	0.215

Table 2

EFFECT OF PRIOR AND CONDITIONAL PROBABILITIES ON POSTERIOR PROBABILITY RESPONSES

(for 75-25 prior probabilities and sample breakdowns with a greater number of X cards than blank cards)

Sample	Blank cards drawn	X cards drawn	60-40 Conditional probabilities: Responses against priors	60-40 Conditional probabilities: Responses with priors	80-20 Conditional probabilities: Responses against priors	80-20 Conditional probabilities: Responses with priors
1	0	1	4	26[a]	8[a]	16
3	0	3	5[a]	3	7[a]	1
	1	2	8	17[a]	5[a]	9
6	0	6	1[a]	0	5[a]	0
	1	5	2[a]	1	4[a]	1
	2	4	4	9[a]	3[a]	1
9	0	9	0[a]	0	3[a]	1
	1	8	0[a]	0	3[a]	0
	2	7	1[a]	0	4[a]	0
	3	6	1[a]	3	5[a]	1
	4	5	0	14[a]	1[a]	1
Total			26	73	48	31

[a] Subject-trials in which option chosen was in accordance with Bayesian model.

In the 398 out of 576 trials where the sample contained at least as many blank cards as X (that is, where the prior probability regarding a blank deck was reinforced), subjects almost invariably estimated a posterior probability equal to or exceeding .75, as would be expected if Bayesian procedures were employed. This suggests that subjects behaved qualitatively in accordance with the model when prior and sample information were not in conflict.

In the foregoing presentation of results, the reader will note the absence of the usual tests of significance, analysis of variance computations and so forth. The presentation of results has been restricted to descriptive statistics for two major reasons. First, due to the self-selected nature of the subjects (graduate student volunteers) it is not apparent what the appropriate statistical universe is; any computation of *F*-ratios, *etc.*, would be strictly an empirical procedure. Second, this small-scale experiment was conducted as an exploratory investigation where the objective was as much to develop additional hypotheses about human inference as it was to test hypotheses asserted prior to the study.

DISCUSSION

Probably the most interesting finding of this study, which concurs with the results of the Edwards and Phillips experiment, is that subjects are reluctant, or unable, to use all of the certainty contained in the sample in revising prior probabilities; thus this leads to over or underestimation of extremely low or extremely high posterior probabilities [7]. This suggests that application of Bayes' Theorem could lead to rather marked differences in the results of combining prior and sample information. Throughout the experiment subjects appear to regress toward the .50 posterior probability level. Interestingly enough, however, subjects' estimates (as a group) are not highly biased over all sample outcomes. That is, they appear to overestimate low probabilities to about the same extent as they underestimate high posterior probabilities.

If one is interested in the qualitative aspects of probability revision, it does appear that subjects tend to revise probabilities in the direction that would be indicated by the use of Bayes' Theorem, but here again, this tendency is not without exceptions. In the case of 80-20 deck proportions, for example, two-thirds of the subjects did not perceive that the presence of a single X card in a sample of one was sufficient evidence to reduce the probability of a blank deck below .5.

Experiments by Shuford [11] and others suggest that subjects can estimate displayed probabilities (relative frequencies or numerosity) quite well. If this experiment is any indication, however, it appears as though subjects experience extreme difficulty in *manipulating* probabilities. As Edwards and Phillips suggest, this inability may be due as much to cognitive limitations as to motivational reasons.

In future experiments of this general class, it would be fruitful to consider additional levels of prior and conditional probabilities. Subjects may exhibit regression toward extremes, and it would be interesting to see at what point subjects treat information as almost certain versus almost valueless in revising prior probabilities. Also, little research has been done on the extent to which subjects exhibit a kind of price-quality imputation. Subjects may act as though information which costs more is more reliable.

It would appear that effective utilization of the statistical decision model (as either a first approximation to a description of choice behavior, or in eliciting prior probabilities for normative application of the model) will require more detailed study of the regression effect, as described above. Although no overall bias was found in this experiment—subjects tended to overestimate to about the same extent, on the average, as they underestimated—it would be of interest to determine under what circumstances this symmetry no longer holds. That is, the assumption of independence of tastes and beliefs, a cornerstone of expected utility theory, may, in certain instances, be a poor description of behavior. All of which suggests that a prescriptive model for probability revision may lead to less distortion in the combination of prior and sample evidence. As Edwards and Phillips [3] suggest, man's role as estimator of conditional probabilities may be coupled with the computer's role in probability revision for maximum effectiveness in the processing of probabilistic information.

In summary it appears that subjects' revision of prior probabilities, in the light of sample data, fails to utilize all of the information in the sample. In contrast to subjects' ability to estimate displayed probabilities rather well, their ability to *process* probabilities intuitively yields results which differ quantitatively rather markedly from the revisions implied by application of the Bayesian model. This would suggest that those marketing research studies which are relevant to the use of Bayesian statistics could benefit from utilizing the formal procedures which underlie this model of information processing.

APPENDIX

The Bayesian model provides essentially a framework for choosing among alternative courses of action under conditions of partial ignorance, where the opportunity exists to gather additional information (via experimentation) about the likelihood of alternative descriptions of the world, or states of nature, being the true state.

In the two-alternative experiment summarized in this paper, we have two possible states of nature:

H_1: Deck is of type blank

H_2: Deck is of type X

Let:

z = prior probability that H_1 is true,

$1 - z$ = prior probability that H_2 is true,

y = a random variable whose probability distributions, if H_i is true, are $p_i(y)$ with $i = 1, 2$.

Given a value of the random variable y, we can determine the probability of H_1 (or H_2), conditional on the sample result, by application of Bayes' Theorem:

$$P_y(H_1) = \frac{zp_1(y)}{zp_1(y) + (1 - z)p_2(y)}, \tag{1}$$

where:

P_y (H_1) = the probability (posterior) that H_1 is true, given that a particular value of y was observed,

p_1 (y) = the probability of observing a specific value of y given that H_1 is true,

p_2 (y) = the probability of observing a specific value of y given that H_2 is true.

The Hypergeometric Distribution

In the experimental game considered in this article a Bayesian player would need to compute the conditional probability (generalized as $p_i(y)$ above) of r successes in a sample of n drawn without replacement from a finite population of size N [9]:

$$(2) \quad P_h(r \mid n, R, N) = \binom{R}{r}\binom{N-R}{n-r} \Big/ \binom{N}{n},$$

where:

N = size of population,
R = number of successes in population.

For computational purposes, the above formula can be expressed as:

$$(3) \quad P_h(r \mid n, R, N) = \frac{n!}{r!\,(n-r)!} \cdot \frac{R!\,(N-R)!\,(N-n)!}{(R-r)!\,[(N-R)-(n-r)]!\,N!}.$$

Computation of Posterior Probabilities

As an illustration, suppose as Bayesian players of the experimental game described earlier, two blank cards were drawn out of a sample of six for the case of 75-25 priors and 60-40 deck proportions with respect to a blank type deck. What is the posterior probability $P_p(H_1)$ that the deck is a blank type, given $r = 2$; $n = 6$; $N = 50$; $R(H_1) = 30$; and $R(H_2) = 20$? First, using the formula for the hypergeometric probability distribution, compute the conditional probability of obtaining two blank cards out of a sample of six cards:

$$(4) \quad P_h[r \mid n, R(H_1), N] = \frac{6!}{(2!)(6-2)!} \cdot \frac{30!\,(50-30)!\,(50-6)!}{28!\,[(50-30)-(6-2)]!\,50!} = 0.133.$$

If sampling from an X type deck, the appropriate conditional probability of obtaining two blank cards out of a sample of six would be:

$$(5) \quad P_h[r \mid n, R(H_2), N] = \frac{6!}{(2!)(6-2)!} \cdot \frac{20!\,(50-20)!\,(50-6)!}{18!\,[(50-20)-(6-2)]!\,50!} = 0.328.$$

The following tabulation shows how the Bayesian player could compute the relevant posterior probabilities, for $r = 2$, $n = 6$:

State	*Prior probabilities*	*Conditional probabilities*	*Joint probabilities*	*Posterior probabilities*
Blank deck	.75	.133	.099	.55
X deck	.25	.328	.082	.45
			.181	

In a similar manner, the computed probabilities P_c were obtained for all sample outcomes possible under the experiment and are shown (as dotted lines) in Figures 1 and 2.

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