

Solution to example given in the document

Given data:

$$n = 10$$

$$\bar{x} = 27$$

$$\text{variance} = 20$$

$$H_0: \mu = 30$$

$$H_a: \mu \neq 30$$

$$\alpha = 0.05$$

Solution:

As standard deviation is given, I'll perform z-test to test the hypothesis

$$SE = \sqrt{\text{variance}}/\sqrt{n} = \sqrt{20}/\sqrt{10} = 1.414$$

$$z_{\text{stat}} = (\bar{x} - \mu_0)/SE = (-3)/1.414 = -2.122$$

$$\text{Therefore, } p\text{-value} = 0.033838$$

As, p-value is between 0.01 and 0.05, we have significant evidence against H_0 .

As, p-value: 0.033838 is less than α : 0.05, we will reject the null hypothesis.

To check whether $\mu < 30$

$$H_0: \mu = 30$$

$$H_a: \mu < 30$$

$$\alpha = 0.05$$

Solution:

$$SE = \sqrt{\text{variance}}/\sqrt{n} = \sqrt{20}/\sqrt{10} = 1.414$$

$$z_{\text{stat}} = (\bar{x} - \mu_0)/SE = (-3)/1.414 = -2.122$$

As this is a left tailed test,

We need to check whether $Z_{\text{stat}} < -Z_{\alpha}$

If true, we reject H_0 else we retain it.

$$\text{Here, } Z_{\alpha} = -1.645$$

$$\text{As, } Z_{\text{stat}}: -2.122 < -Z_{\alpha}: -1.645$$

Therefore, we reject null hypothesis and hold $\mu < 30$ as True.