Type II Error and Power Calculations

Recall that in hypothesis testing you can make two types of errors

- **Type I Error** – rejecting the null when it is true.
- **Type II Error** – failing to reject the null when it is false.

The probability of a Type I Error in hypothesis testing is predetermined by the significance level.

The probability of a Type II Error cannot generally be computed because it depends on the population mean which is unknown. It can be computed at, however, for given values of μ, σ^2, and n.

The **power** of a hypothesis test is nothing more than 1 minus the probability of a Type II error. Basically the power of a test is the probability that we make the right decision when the null is not correct (i.e. we correctly reject it).

Example: Consider the following hypothesis test

$$H_0 : \mu \geq 30$$
$$H_a : \mu < 30$$

Assume you have prior information $\sigma^2 = 10,0000$ so that in a sample of 100

$$\sigma^2_{\bar{X}} = \frac{\sigma^2}{n} = \frac{10,000}{100} = 100 \Rightarrow \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = 10$$

What we would like to now is calculate the probability of a Type II error conditional on a particular value of μ. Lets assume that $\mu = 26$, but we could choose any value such that the null is not correct. Lets also assume that the significance level for the test is 0.05.

We know

1. This is a left tailed test
2. We will fail to reject the null (commit a Type II error) if we get a Z statistic greater than -1.64.
3. This -1.64 Z-critical value corresponds to some $\bar{X}$ critical value $\left(\bar{X}_{critical}\right)$, such that

$$P(z - stat \geq -1.64) = P\left(\bar{X} \geq \bar{X}_{critical} \mid \begin{matrix} \mu = 30 \\ \sigma_{\bar{X}} = 10 \end{matrix}\right) = 0.95$$

We can find the value of $\bar{X}_{critical}$ by solving the following equation

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$$-1.64 = Z_{critical} = \frac{\overline{X}_{critical} - \mu_0}{\sigma_{\overline{X}}} \Rightarrow$$

$$-1.64 = \frac{\overline{X}_{critical} - 30}{10} \Rightarrow \overline{X}_{critical} = 13.6$$

So I will incorrectly fail to reject the null as long as a draw a sample mean that greater than 13.6. To complete the problem what I now need to do is compute the probability of drawing a sample mean greater than 13.6 given $\mu = 26$ and $\sigma_{\overline{X}} = 10$. Thus, the probability of a Type II error is given by

$$P\left(\overline{X} > 13.6 \middle| \begin{matrix} \mu = 26 \\ \sigma_{\overline{X}} = 10 \end{matrix}\right) = P\left(Z > \frac{13.6 - 26}{10}\right) = P\left(Z > -1.24\right) = 0.8925$$

and the power of the test is 0.1075.

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