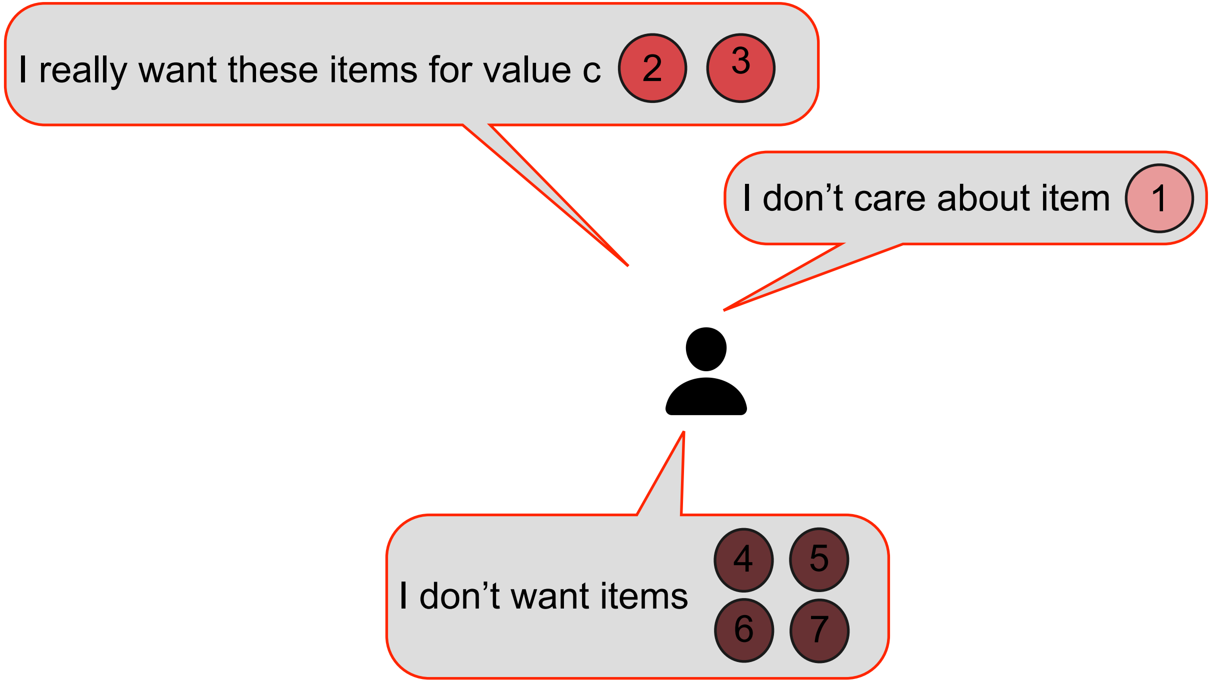
Mixed Manna Allocation Framework

{-1,0,c} ONSUB Valuations

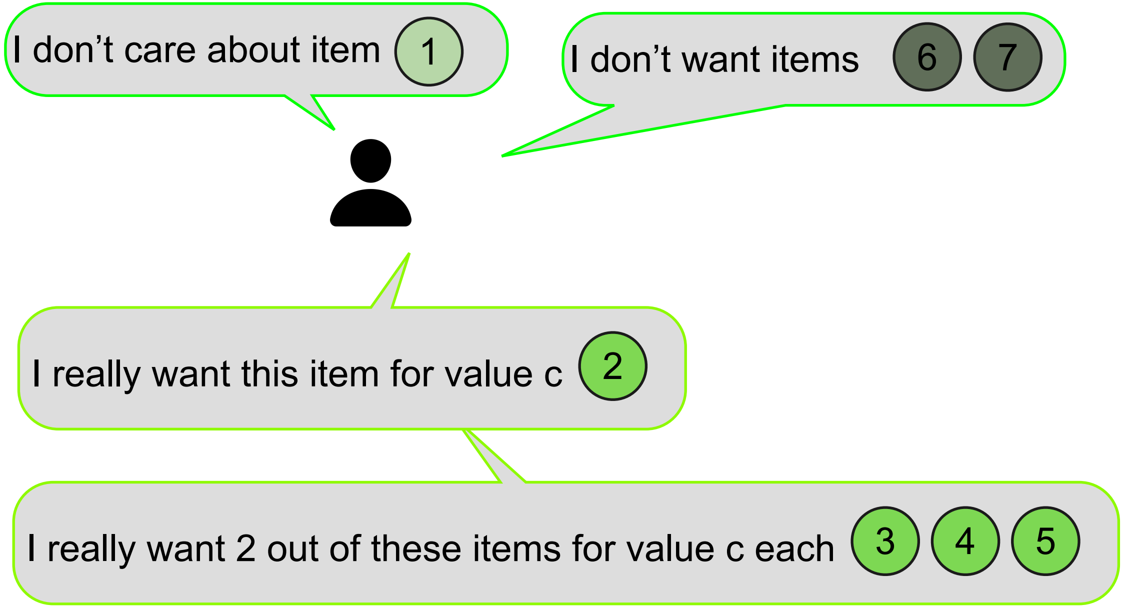


Agents have preferences over items-

- **Goods** you want (value c)
- Items you’re **indifferent** to (value 0)
- **Chores** you don’t want (value -1)

Agent : $v(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}|$

{-1,0,c} ONSUB Valuations



Items can be valued c individually, but lose their value together

Similarly, items can be valued at 0, and can become chores

With all 3 items, I still get a value of 2c

Agent : $v(S) = c|S \cap o_2| + c\,min\{|S \cap \{o_3, o_4, o_5\}|, 2\} - |S \cap \{o_6, o_7\}|$

Setup with notation:

2 agents, 7 objects

Valuation function $v : 2^o \to \mathbb{R}$

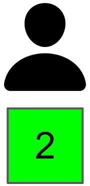
Agent 1 : $v_1(S) = c\,|\,S \cap o_2\,| + cmin\{\,|\,S \cap \{o_3, o_4, o_5\}\,|\,,2\} - |\,S \cap \{o_6, o_7\}\,|$

Agent 2 : $v_2(S) = c\,|\,S \cap o_2, o_3\,| - |\,S \cap \{o_4, o_5, o_6, o_7\}\,|$

Current Valuation implementation

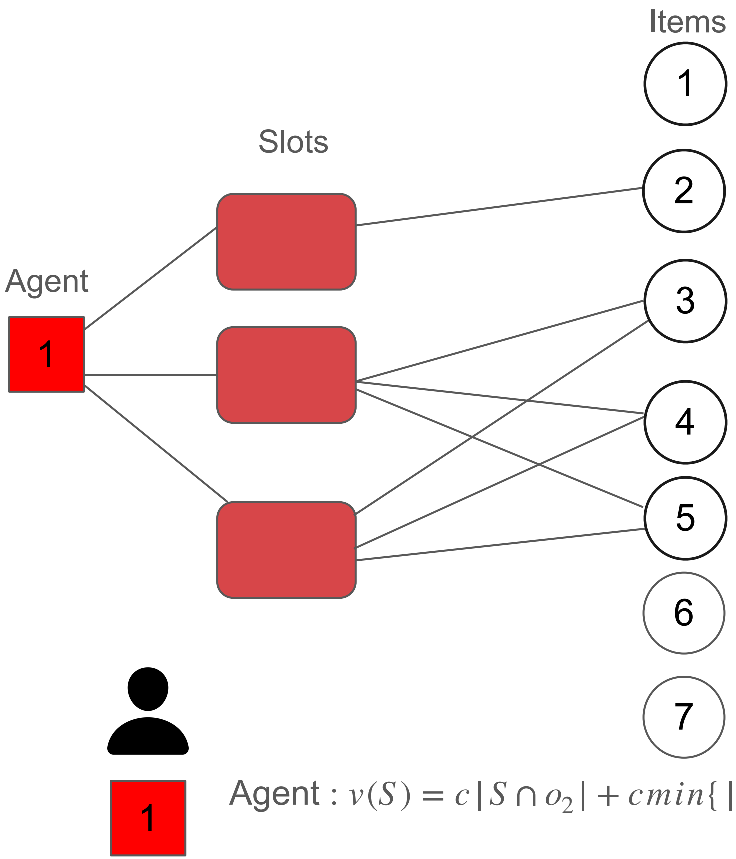
Agents have desired items list:

- desired_items_0=$[o_1, o_2, o_3]$
- desired_items_c=$[o_2, o_3]$
- Rest valued at -1



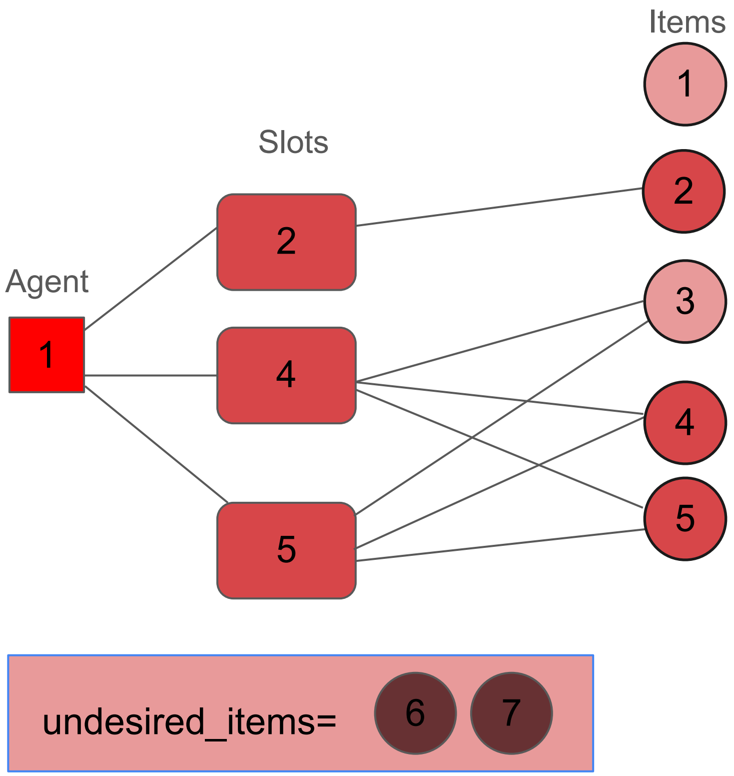
Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}|$

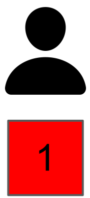
Value function implementation



Agent : $v(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}|, 2\} - |S \cap \{o_6, o_7\}|$

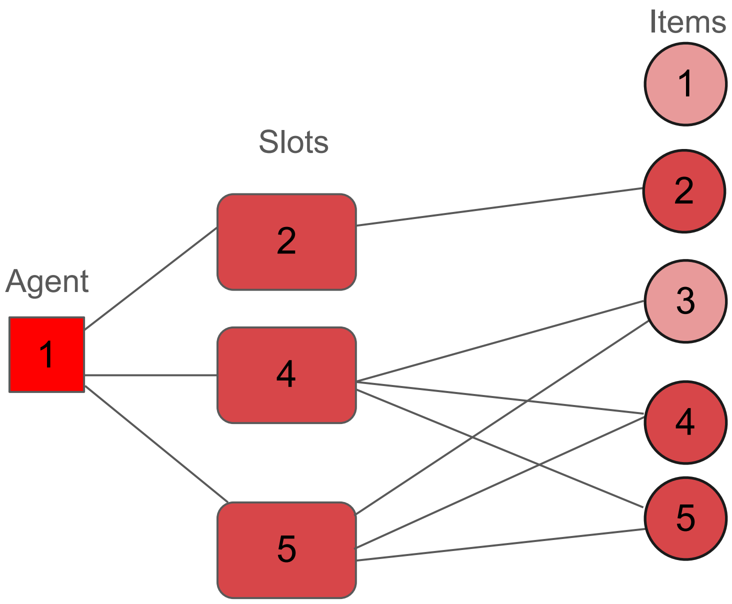
Value function implementation



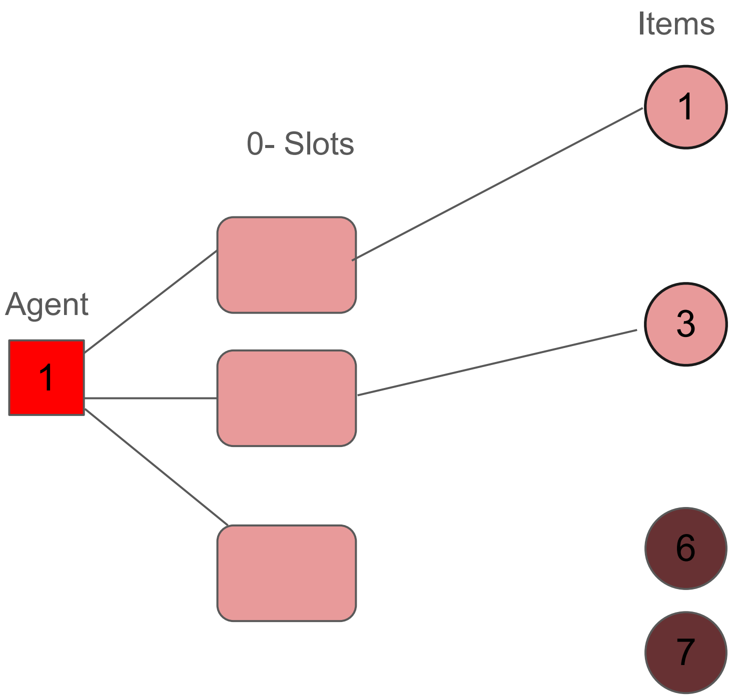


Agent : $v(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}\,|$

Value function implementation



Compute bipartite matching for c and 0 valued items



With the new “list” of 0 valued items, compute a secondary bipartite matching for 0 and -1 valued items

Given preferences over items by agents, run mixed manna allocation algorithm to compute allocations

Phase 1: Run yankee swap with all items valued at 0 and c as “goods”

Phase 2: Redistribute items between agents considering differences in 0 and c valued items

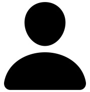
2. (a) Pareto improving paths to X_0^C

2. (b)(c) Exchange paths

Phase 3: Greedily allocate “chores” based on current allocation

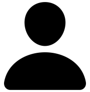
Yankee Swap over items valued at 0 and above

Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}\,|, 2\} - |S \cap \{o_6, o_7\}\,|$



1 $\beta_1^0 =$ 1 2 3 4 5 $\cancel{6}$ $\cancel{7}$

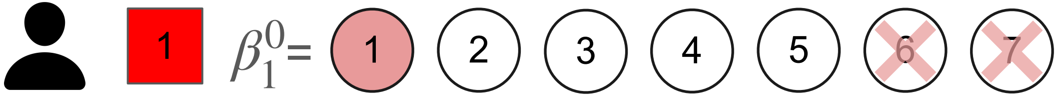
Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}\,|$



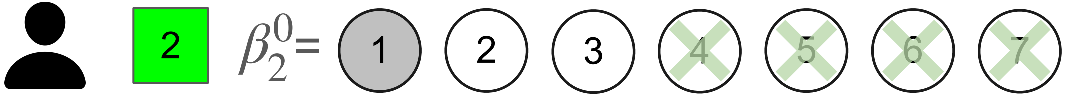
2 $\beta_2^0 =$ 1 2 3 $\cancel{4}$ $\cancel{5}$ $\cancel{6}$ $\cancel{7}$

Yankee Swap over items valued at 0 and above

Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}\,|, 2\} - |S \cap \{o_6, o_7\}\,|$

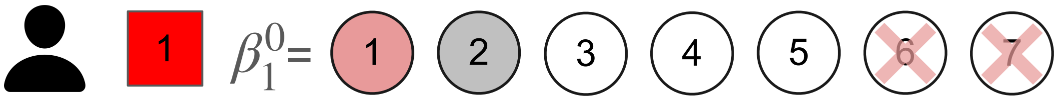


Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}\,|$

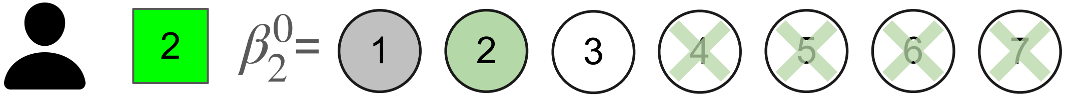


Yankee Swap over items valued at 0 and above

Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}\,|, 2\} - |S \cap \{o_6, o_7\}\,|$

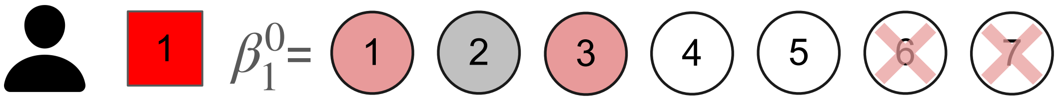


Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}\,|$

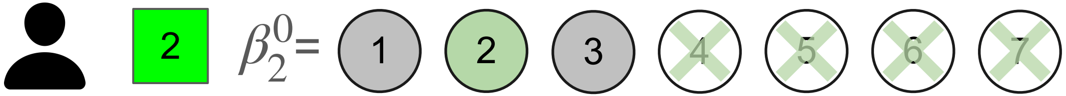


Yankee Swap over items valued at 0 and above

Agent 1 : $v_1(S) = c|S \cap o_2| + c\,min\{|S \cap \{o_3, o_4, o_5\}|, 2\} - |S \cap \{o_6, o_7\}|$

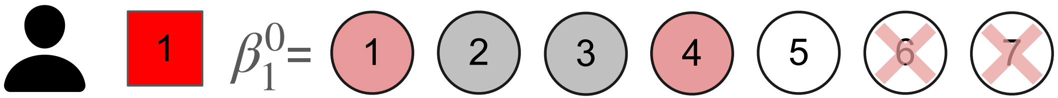


Agent 2 : $v_2(S) = c|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}|$

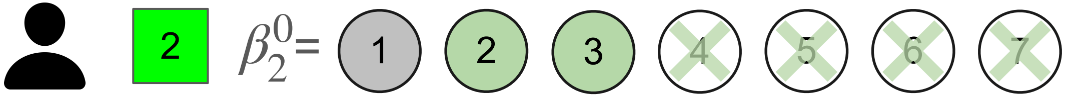


Yankee Swap over items valued at 0 and above

Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}|\,, 2\} - |S \cap \{o_6, o_7\}|$

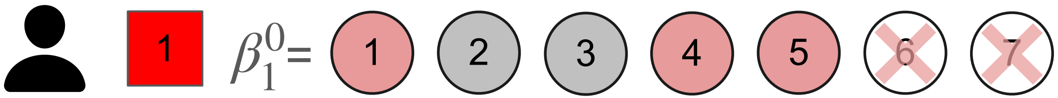


Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}|$

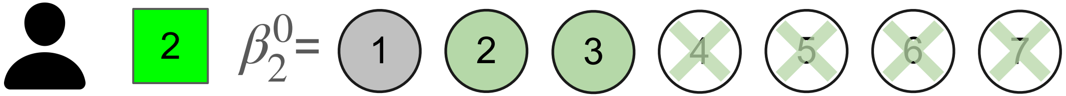


Yankee Swap over items valued at 0 and above

Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}|\,, 2\} - |S \cap \{o_6, o_7\}|$



Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}|$



$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Given preferences over items by agents, run mixed manna allocation algorithm to compute allocations

Phase 1: Run yankee swap with all items valued at 0 and c as “goods”

Phase 2: Redistribute items between agents considering differences in 0 and c valued items

2. (a) Pareto improving paths to X_0^C

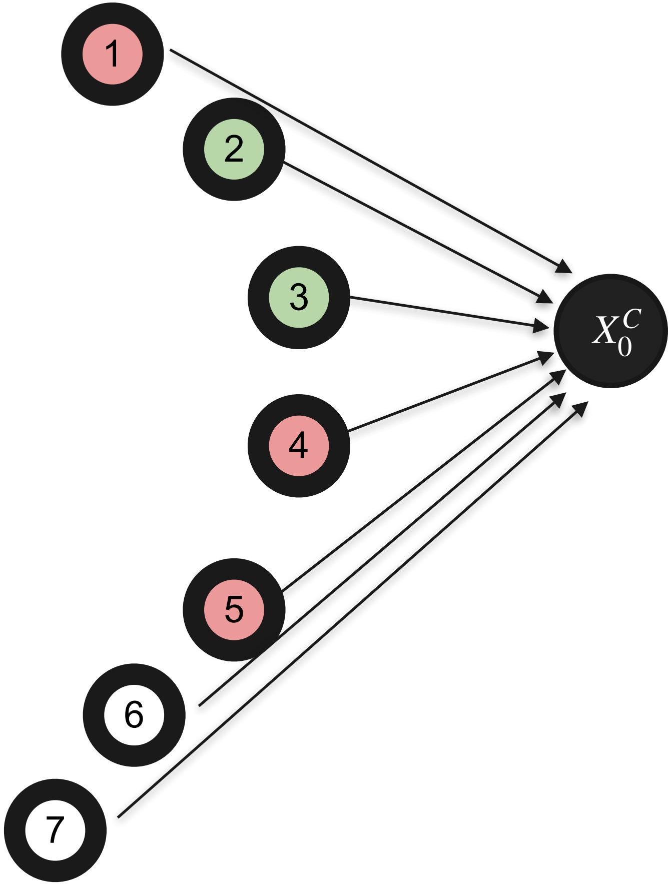
2. (b)(c) Exchange paths

Phase 3: Greedily allocate “chores” based on current allocation

Allocation matrices

$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

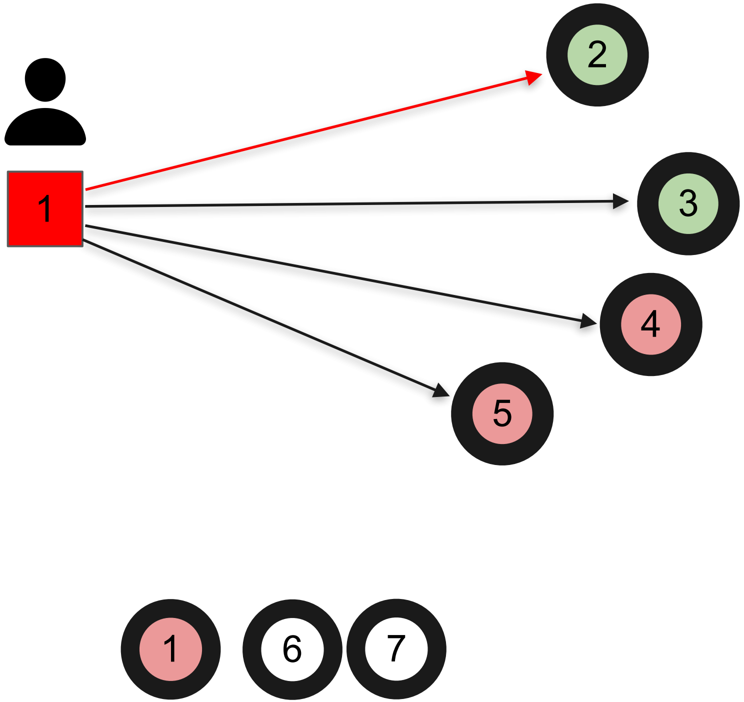
Initialization



$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

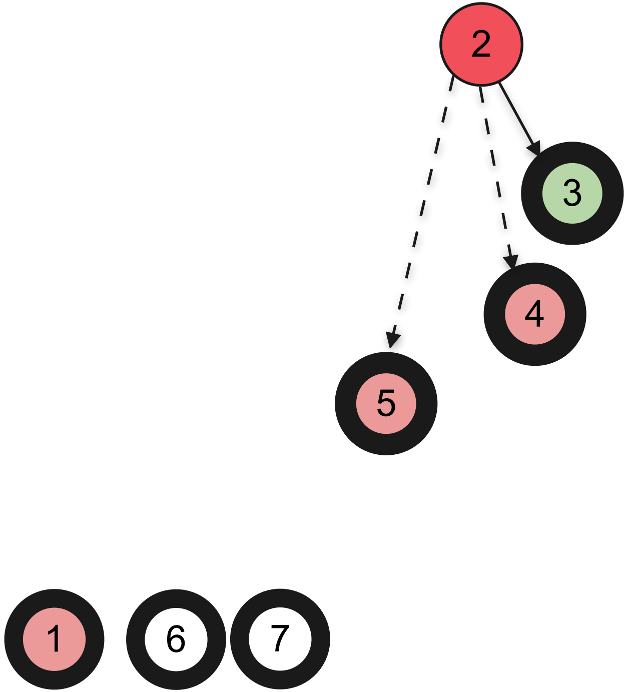
$$X^c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Agent 1



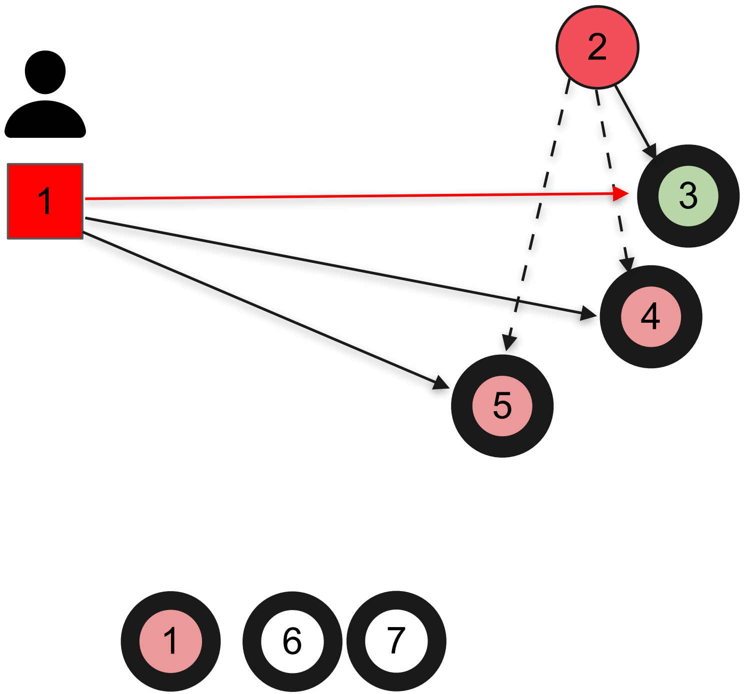
$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Agent 1



$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

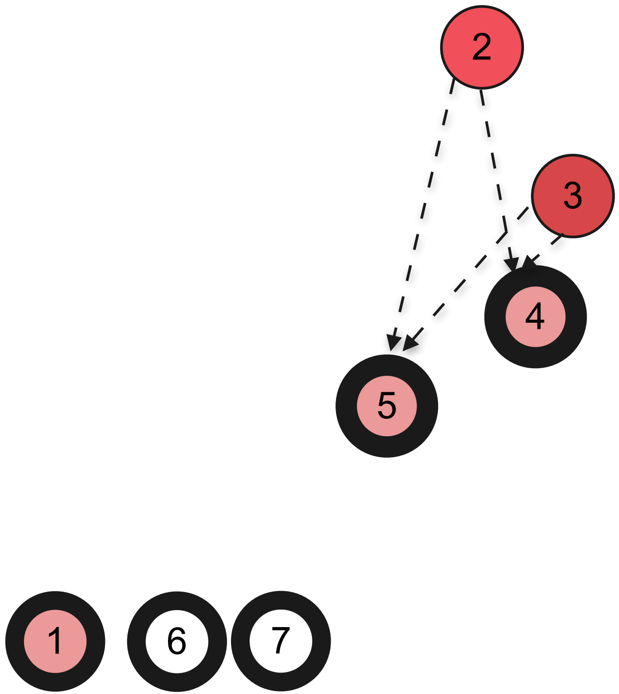
Agent 1



$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

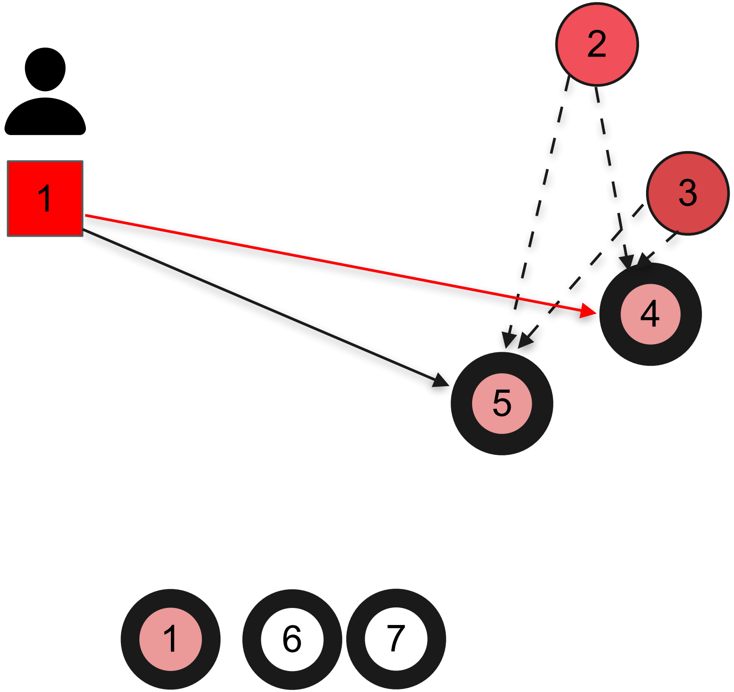
$$X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Agent 1



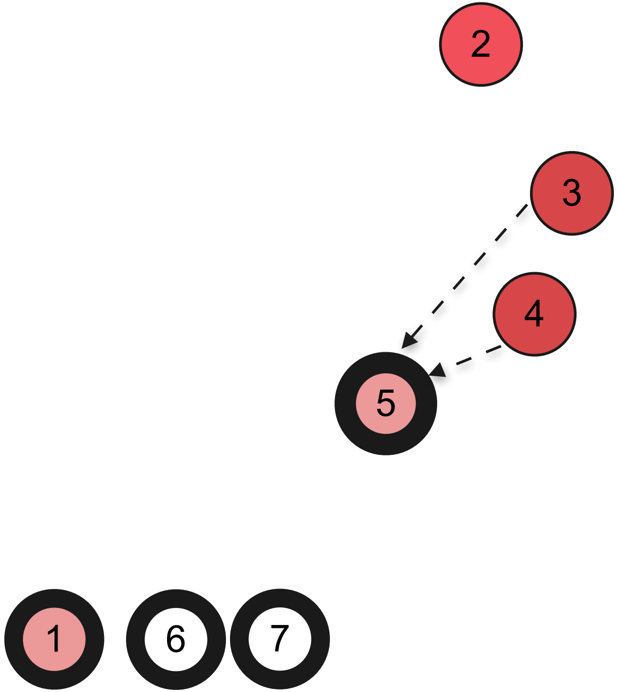
$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Agent 1



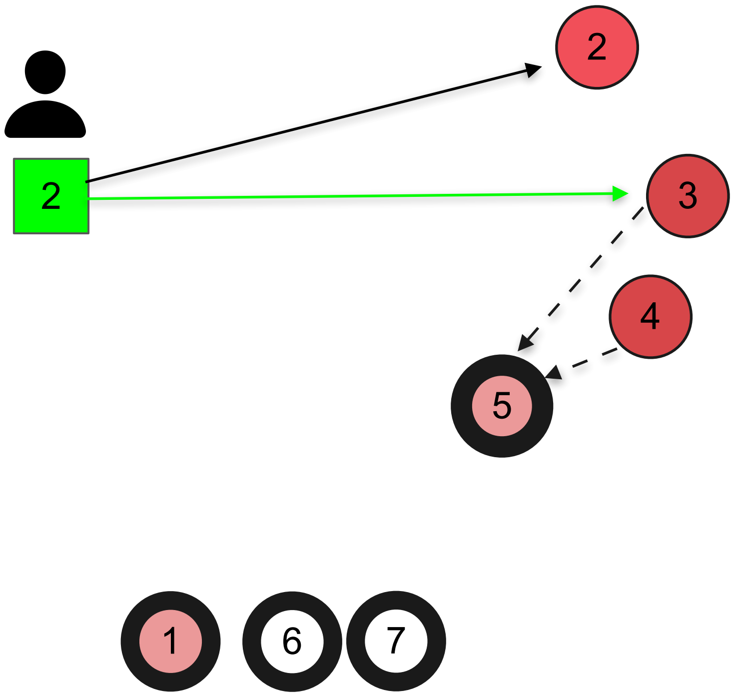
$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Agent 1



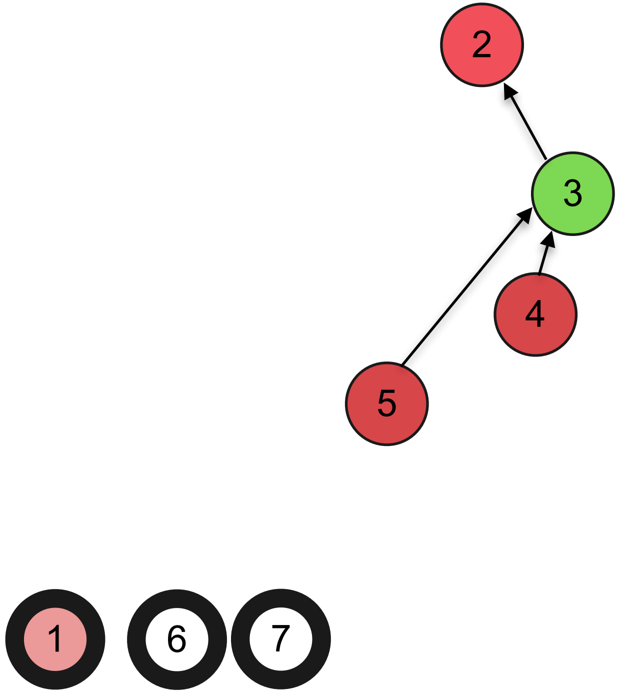
$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Agent 2



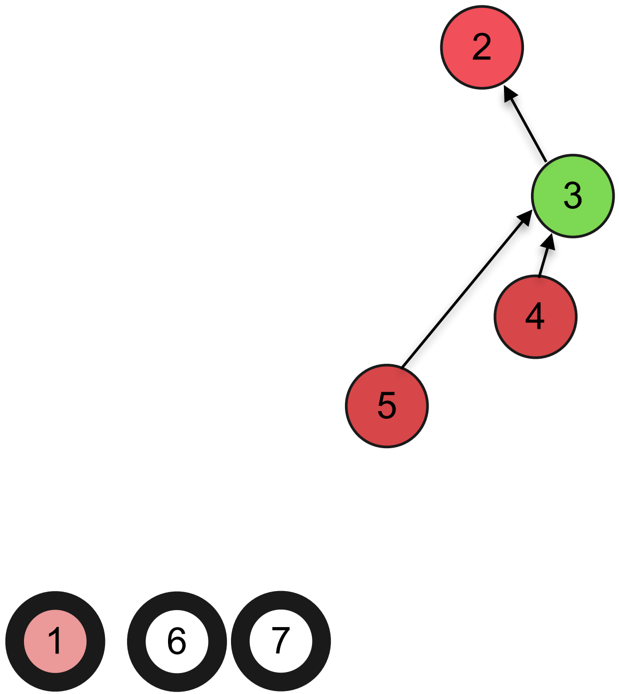
$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Agent 2



$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Agent 2

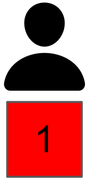


No more paths to sync node X_0^C

Onto phase 2b!

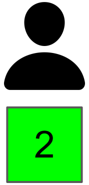
$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Allocation after Phase 2a



Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}\,|, 2\} - |S \cap \{o_6, o_7\}\,|$

$F_{\beta_1^c}(X^c, 1) = \phi$

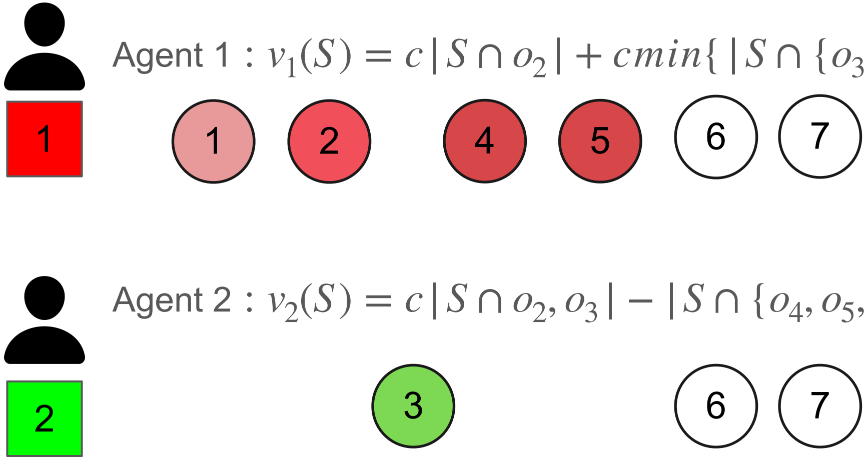


Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}\,|$

$F_{\beta_2^c}(X^c, 2) = o_2$

Phase 2b:

Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}\,|, 2\} - |S \cap \{o_6, o_7\}\,|$



$F_{\beta_1^c}(X^c, 1) = \phi$

Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}\,|$

$F_{\beta_2^c}(X^c, 2) = o_2$

Phase 2b:

If for some agent i, path from $F_{\beta_i^c}(X^c, i)$ to X_j^c such that

- $|X_i^c| < |X_j^c| + 1$(one agent has significantly more items valued at c)
- $|X_i^c| = |X_j^c| + 1$and i < j (agent with lower index get priority)

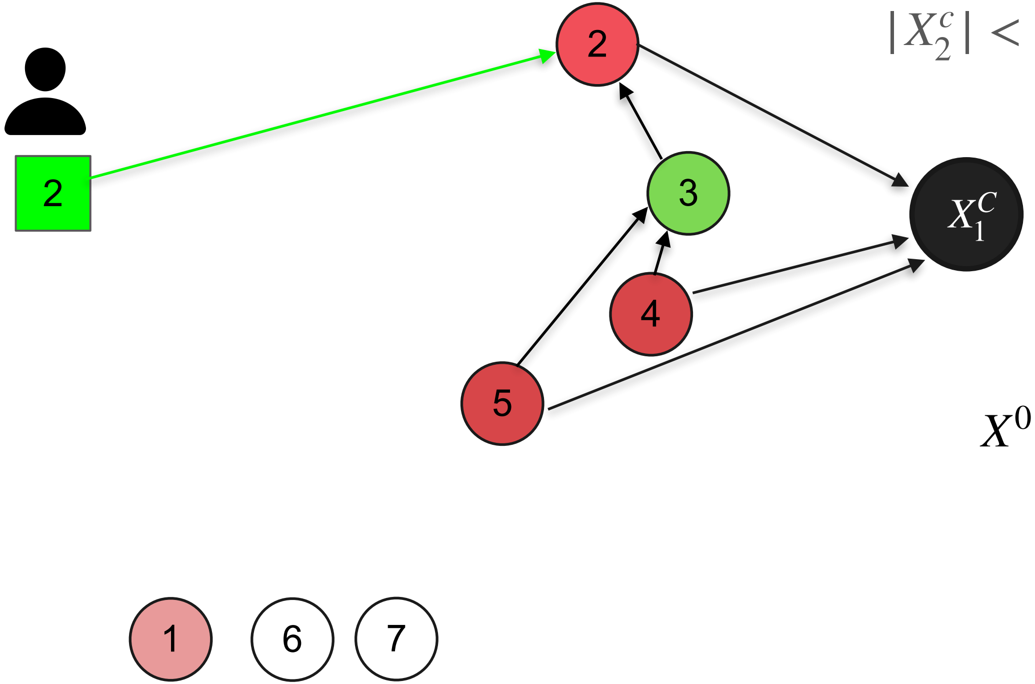
- n agents
- Iterate and find values of each agent- list=[3c,c…] (todo)

- f()- same as 2A
- add start node and edge to f() (same add agent in 2A)
- Add new sync node
- Check list and for every higher valued agent add edge from items in their bundle to sync node
- Find shortest path (same as prev)
- Update allocation matrix (needs a change for last item maybe?)
- Update exchange graph (should be the same?)

Redistributing items between agents

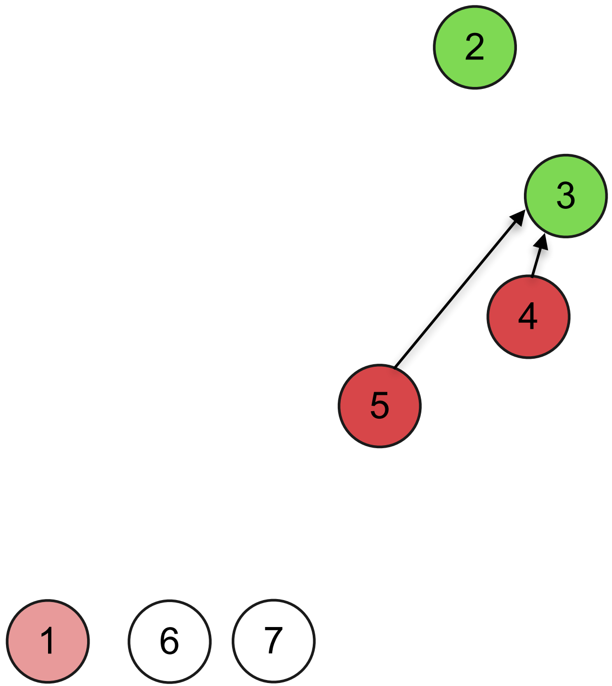
Path from $F_{\beta_2^c}(X^c, 2) = \{o_2\}$ to $X_1^c = \{o_2, o_4, o_5\}$

$|X_2^c| < |X_1^c| + 1$



$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

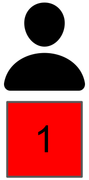
Redistributing items between agents



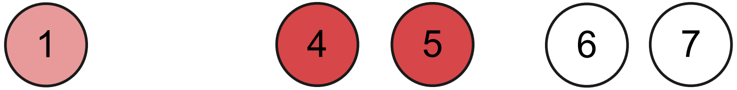
$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

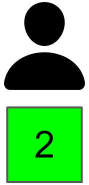
$$X^c = \begin{bmatrix} 0 & 0 & 1 \\ \boxed{0} & \boxed{1} & \boxed{0} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Allocation after Phase 2b



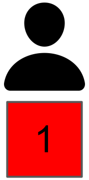
Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}|\,, 2\} - |S \cap \{o_6, o_7\}|$



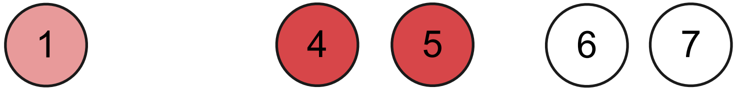


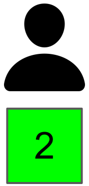
Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}|$

Iteration 2: Back to Phase 2a



Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}\,|, 2\} - |S \cap \{o_6, o_7\}\,|$



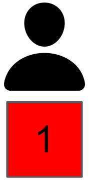


Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}\,|$

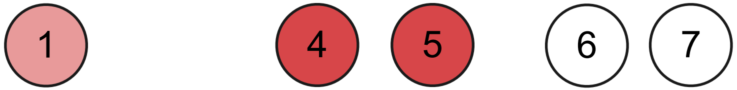
No items which have an edge to X_0^C exist.

Phase 2a done!

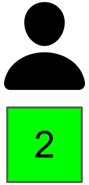
Iteration 2: Phase 2b



Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}\,|, 2\} - |S \cap \{o_6, o_7\}\,|$



$F_{\beta_1^c}(X^c, 1) = o_2$



Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}\,|$

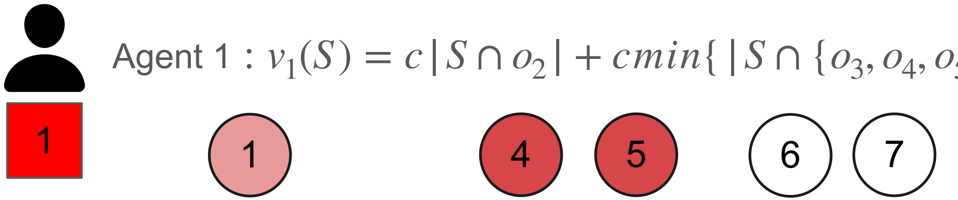
$F_{\beta_2^c}(X^c, 2) = \phi$

No agent has a significantly higher valued bundle

Stage 2b done!

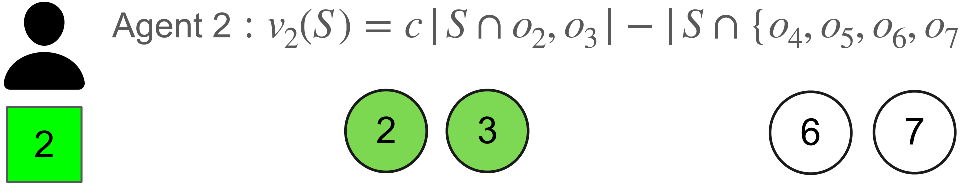
Iteration 2: Phase 2b

Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}|\,, 2\} - |S \cap \{o_6, o_7\}|$



$F_{\beta_1^c}(X^c, 1) = o_2$

Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}|$



$F_{\beta_2^c}(X^c, 2) = \phi$

Phase 2b:

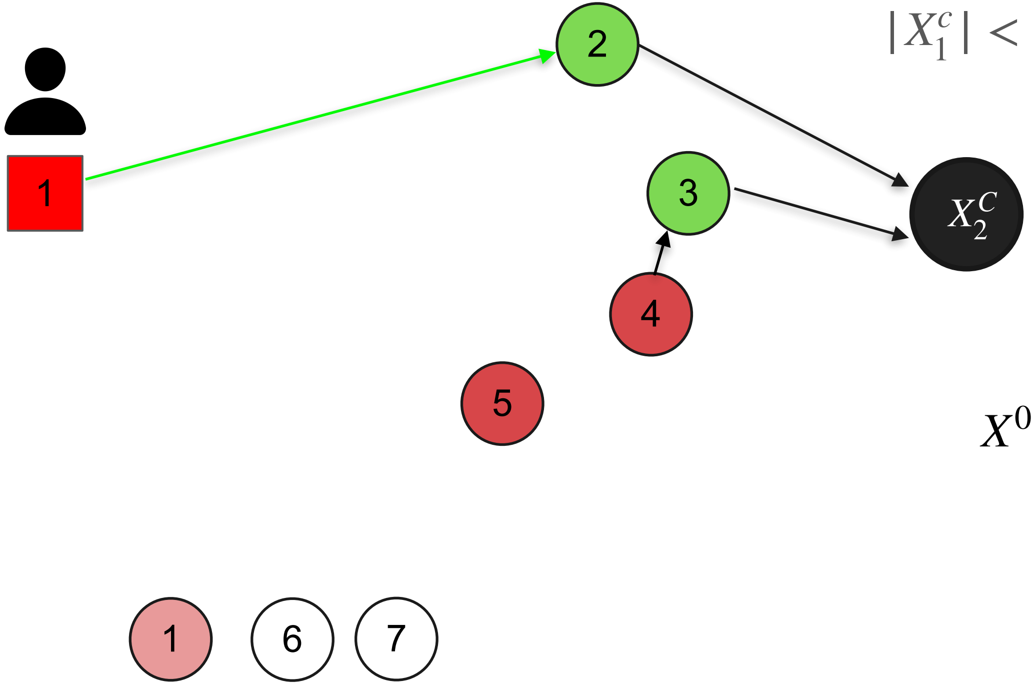
If for some agent i, path from $F_{\beta_i^c}(X^c, i)$ to X_j^c such that

- $|X_i^c| < |X_j^c| + 1$(one agent has significantly more items valued at c)
- 0+1<2
- $|X_i^c| = |X_j^c| + 1$and i < j (agent with lower index get priority)

Redistributing items between agents

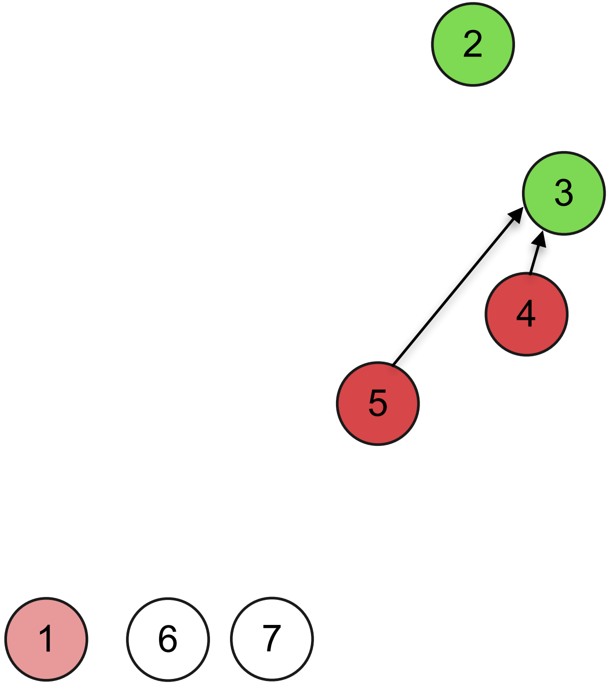
Path from $F_{\beta_1^c}(X^c,1) = o_2$ to $X_2^c = \{o_2, o_3\}$

$|X_1^c| < |X_2^c| + 1$



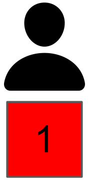
$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Redistributing items between agents

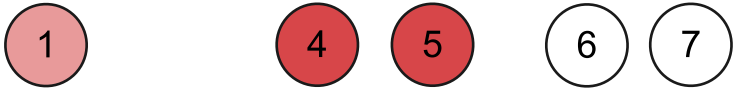


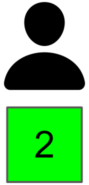
$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Iteration 2:



Agent 1 : $v_1(S) = c\,|\,S \cap o_2\,| + cmin\{\,|\,S \cap \{o_3, o_4, o_5\}\,|\,,2\} - |\,S \cap \{o_6, o_7\}\,|$





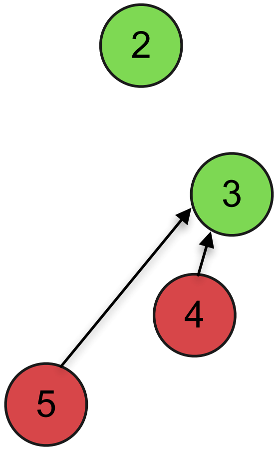
Agent 2 : $v_2(S) = c\,|\,S \cap o_2, o_3\,| - |\,S \cap \{o_4, o_5, o_6, o_7\}\,|$

No paths augmented in current iteration, moving on to stage 3!

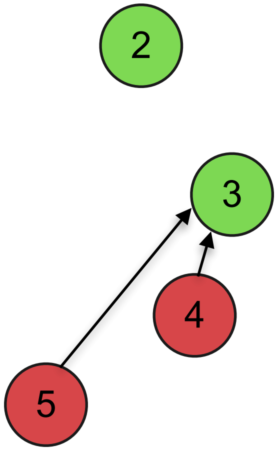
When is phase 2b/2c needed?

- If everyone picks in priority order, there is no agent with **significantly** more items than someone else (since every agent gets 1 turn to pick 1 item)
- Only case one agent gets more is when
  - agent values more items (which is handled by phase 2b)
  - round robin ends on their turn (which could lead to problems with indexing as handled by 2c)

Unclear whether 2b2c
needed if ran in order



Sequential



Random

Given preferences over items by agents, run mixed manna allocation algorithm to compute allocations

Phase 1: Run yankee swap with all items valued at 0 and c as “goods”

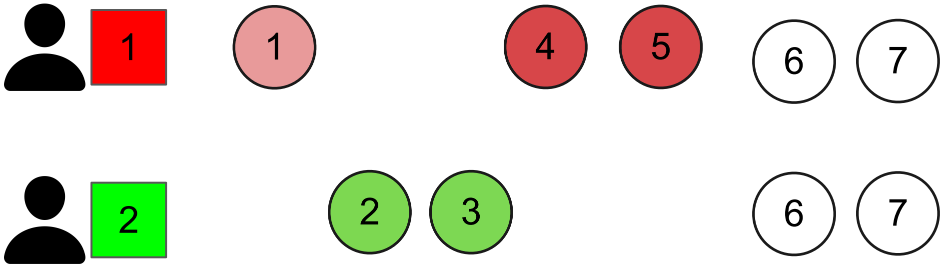
Phase 2: Redistribute items between agents considering differences in 0 and c valued items

2. (a) Pareto improving paths to X_0^C

2. (b)(c) Exchange paths

Phase 3: Greedily allocate “chores” based on current allocation

Greedily allocate items from remaining -1 valued items

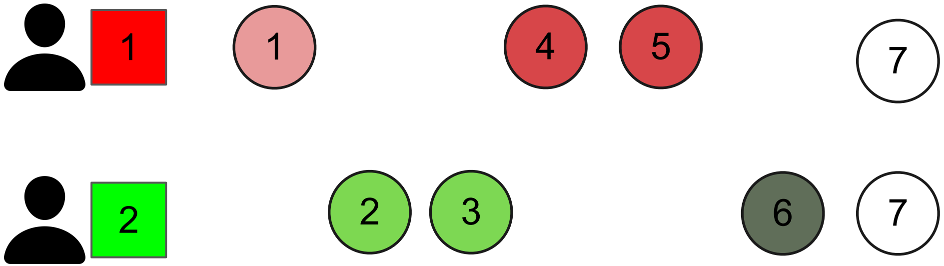


$v_1(S) = 2c$

$v_2(S) = 2c$

$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad X^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Greedily allocate items from remaining -1 valued items

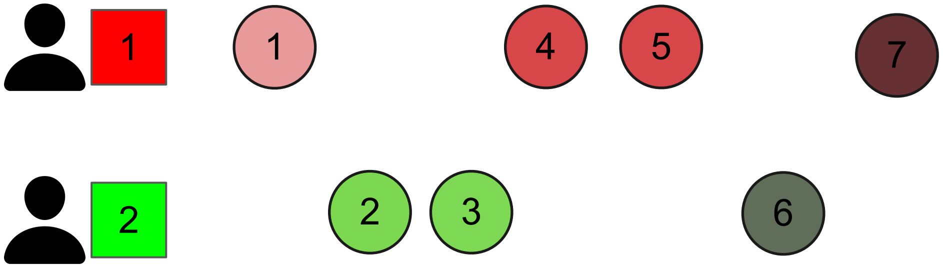


$v_1(S) = 2c$

$v_2(S) = 2c - 1$

$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad X^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Greedily allocate items from remaining -1 valued items



$$v_1(S) = 2c - 1$$

$$v_2(S) = 2c - 1$$

$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad X^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Allocation after Mixed Manna Algorithm

1 Agent 1 : $v_1(S) = c\,|S \cap o_2| + c\,min\{\,|S \cap \{o_3, o_4, o_5\}|, 2\} - |S \cap \{o_6, o_7\}|$

1 4 5 7

2 Agent 2 : $v_2(S) = c\,|S \cap o_2, o_3| - |S \cap \{o_4, o_5, o_6, o_7\}|$

2 3 6

$$X^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad X^c = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad X^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

With additive valuations:

Phase 1: Run yankee swap with all items valued at 0 and c as “goods”

Phase 2: Redistribute items between agents considering differences in 0 and c valued items

2. (a) Pareto improving paths to X_0^C

2. (b)(c) Exchange paths

Phase 3: Greedily allocate “chores” based on current allocation

Future work

- Valuation functions implementation
- Phase 2b/2c
- Implementing agents to scale
- Implementing a larger scale implementation