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EXPERIMENT NO.4

Aim: Write a program to demonstrate Euler's Totient Function

Theory:

Euler's totient function, denoted by $\varphi(n)$, is a mathematical function that counts the number of positive integers less than or equal to n that are relatively prime to n. In other words, it gives the count of numbers between 1 and n that share no common factors with n except for 1. Two numbers are considered relatively prime if their greatest common divisor (GCD) is 1.

Properties of phi:

<u>Prime Numbers</u>: For a prime number p, $\varphi(p)$ = p- 1. This is because all positive integers less than a prime number are coprime to it. Example: Let's take the prime number p = 7. $\varphi(7)$ = 7-1 = 6. There are 6 positive integers less than 7 that are coprime to 7: {1,2, 3, 4, 5, 6}.

<u>Euler's Totient Function</u>: Euler's Totient Function $\varphi(n)$ calculates the count of positive integers less than or equal to n that are coprime to n. When n is a product of two distinct primes p and q, the formula for $\varphi(n)$ simplifies to $\varphi(n)$ = (p- 1) * (q- 1). Example: Let's take two distinct prime numbers, p = 5 and q = 7. n =p*q=5*7=35. $\varphi(35)$ = (5-1) *(7-1) = 4 *6 = 24. There are 24 positive integers less than or equal to 35 that are coprime to 35.

<u>Powers of Primes</u>: For a prime power p^k, where k is a positive integer, φ(p^k) = p^k- p^(k-1). This is because every multiple of p up to p^k is not coprime to p^k except p itself and its powers up to p^(k-1). Example: Let's take the prime number p = 2 and k = 3. p^k = 2^3 = 8. $\varphi(8)$ = 2^3-2^(3-1) = 8-4 = 4. There are 4 positive integers less than or equal to 8 that are coprime to 8: {1,3, 5, 7}

Code:

```
import java.util.Scanner;

public class TotientCalculator {
    static boolean isPrime(int n) {
        if (n <= 1) {
            return false;
        }
        for (int i = 2; i < n; i++) {
            if (n % i == 0) {
                return false;
            }
        }
        return true;
    }

    static int gcd(int a, int b) {
        if (b == 0) {
            return a;
        }
        return gcd(b, a % b);
    }

    public static void main(String[] args) {
```

```java
        Scanner scanner = new Scanner(System.in);
        int n;
        System.out.println("Enter the number n:");
        n = scanner.nextInt();
        int phi;
        int answer = n;
        int originalN = n;
        if (isPrime(n)) {
            phi = n - 1;
            System.out.println("phi(" + originalN + ") = " + phi);
        } else {
            for (int i = 2; i * i <= n; i++) {
                if (n % i == 0) {
                    while (n % i == 0) {
                        n /= i;
                    }
                    answer -= answer / i;
                }
            }
            if (n > 1) {
                answer -= answer / n;
            }
            System.out.println("phi(" + originalN + ") = " + (answer));
        }
        System.out.println("Relatively prime numbers of " + originalN + ":
");
```

```
        for (int i = 1; i < originalN; i++) {
            int gcdNo = gcd(originalN, i);
            if (gcdNo == 1) {
                System.out.print(i + " ");
            }
        }
    }
}
```

Output:

Output

```
java -cp /tmp/6iHa3fys4c TotientCalculator
Enter the number n:
17
phi(17) = 16
Relatively prime numbers of 17:
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
```

Output

```
java -cp /tmp/y646vsrhzC TotientCalculator
Enter the number n:
39
phi(39) = 24
Relatively prime numbers of 39:
1 2 4 5 7 8 10 11 14 16 17 19 20 22 23 25 28 29 31 32 34 35 37 38
```

Output Clear

```
java -cp /tmp/gFkMnVvUJq TotientCalculator
Enter the number n:
169
phi(169) = 156
Relatively prime numbers of 169:
1 2 3 4 5 6 7 8 9 10 11 12 14 15 16 17 18 19 20 21 22 23 24 25 27 28 29 30 31 32 33 34
    35 36 37 38 40 41 42 43 44 45 46 47 48 49 50 51 53 54 55 56 57 58 59 60 61 62 63
    64 66 67 68 69 70 71 72 73 74 75 76
```

Example:

Example:

1. $n = 17$.
case 1: 17 is a prime number. For prime number p, $\phi(p) = p-1$.

$\therefore \phi(17) = 16$

$\therefore$ There are 16 positive integers less than 17 that are co-prime to 17.
$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$

2. Case 2: $n = 39$.
39, is a product of two distinct primes p & q, where $p = 13$ & $q = 3$.

$\therefore \phi(n) = (p-1) * (q-1)$

$\phi(39) = (13-1) * (3-1) = 12 * 2 = 24$

$\therefore$ There are 24 positive integers less than 39 that are co-prime to 39.

3. Case 3: $n = 169$

$\phi(169) = (13)^2 - (13)^1$

$= 156.$

$\therefore$ There are 156 positive integers less than 169 that are coprime to 169.