Some results on the products of connected linearly ordered sets

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Department of Mathematics Miami University

Nov 2014 AMS Southeastern Sectional Meeting

Outline

Some results on the products of connected linearly ordered sets

The Theorem of Eda and Kamijo

- Introduction
 - The Theorem of Eda and Kamijo
 - Aronszajn lines?
- - Do you remember Calc !?
 - Notations
 - = *n*-dim to 1-dim
 - = *n*-dim to *n*-dim
- - Byproduct
 - Proof

Coordinate-wise functions

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Eda and Kamijo

Proof of New Theorem

Do you remember
Calc I?

Notations

n-dim to 1-dir n-dim to n-dir

No analogue of Peano curve Byproduct

Definition

Let $X_0, \ldots, X_{n-1}, Y_0, \ldots, Y_{m-1}$ be sets. We say that a function $f: \prod_{i < n} X_i \to \prod_{j < m} Y_j$ is *coordinate-wise* if and only if there exists $\pi: m \to n$ such that for every j < m, the value of f(x)(j) only depends on $x(\pi(j))$, i.e. there exists $h_j: X_{\pi(j)} \to Y_j$ such that for every $x \in \prod_{i < n} X_i$, $f(x)(j) = h_j(x(\pi(j)))$.

Note that there are so many homeomorphisms $f: \mathbb{R}^2 \to \mathbb{R}^2$ that are not coordinate-wise, for example,

$$f(x,y) = (x+y, x-y)$$

Coordinate-wise functions

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Eda and Kamijo

Proof of New Theorem

Do you remember Calc I?

Notations

a-dim to 1-dim

n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve Byproduct

Definition

Let $X_0, \ldots, X_{n-1}, Y_0, \ldots, Y_{m-1}$ be sets. We say that a function $f: \prod_{i < n} X_i \to \prod_{j < m} Y_j$ is *coordinate-wise* if and only if there exists $\pi: m \to n$ such that for every j < m, the value of f(x)(j) only depends on $x(\pi(j))$, i.e. there exists $h_j: X_{\pi(j)} \to Y_j$ such that for every $x \in \prod_{i < n} X_i$, $f(x)(j) = h_j(x(\pi(j)))$.

Note that there are so many homeomorphisms $f: \mathbb{R}^2 \to \mathbb{R}^2$ that are not coordinate-wise, for example,

$$f(x,y) = (x+y, x-y)$$

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct

Definition

Let $X_0, \ldots, X_{n-1}, Y_0, \ldots, Y_{m-1}$ be sets. We say that a function $f: \prod_{i < n} X_i \to \prod_{j < m} Y_j$ is *coordinate-wise* if and only if there exists $\pi: m \to n$ such that for every j < m, the value of f(x)(j) only depends on $x(\pi(j))$, i.e. there exists $h_j: X_{\pi(j)} \to Y_j$ such that for every $x \in \prod_{i < n} X_i$, $f(x)(j) = h_j(x(\pi(j)))$.

Note that there are so many homeomorphisms $f : \mathbb{R}^2 \to \mathbb{R}^2$ that are not coordinate-wise, for example, f(x, y) = (x + y, x - y).

The theorem of Eda and Kamijo

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda and Kamijo

Proof of New

Do you remember Calc I? Notations n-dim to 1-dim

No analogue of Peano curve Byproduct Let K be a linearly ordered set and $x \in K$. Then, let cf(x) and ci(x) denote the cofinality and coinitiality of x.

Theorem (K. Eda and R. Kamijo)

Let L be a connected linearly ordered set. If either cf(x) or ci(x) is uncountable for a dense set of x in L, then every homeomorphism $f: L^n \to L^n$ is coordinate-wise for every natural number n.

The theorem of Eda and Kamijo

Some results on the products of connected linearly ordered sets

Tetsuya Ish

Introduction
The Theorem of Eda
and Kamijo
Aronszajn lines?

Proof of New Theorem

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct Let K be a linearly ordered set and $x \in K$. Then, let cf(x) and ci(x) denote the cofinality and coinitiality of x.

Theorem (K. Eda and R. Kamijo)

Let L be a connected linearly ordered set. If either cf(x) or ci(x) is uncountable for a dense set of x in L, then every homeomorphism $f: L^n \to L^n$ is coordinate-wise for every natural number n.

Outline

Some results on the products of connected linearly ordered sets

Γetsuya Ishiι

The Theorem of Ec and Kamijo Aronszajn lines?

Proof of New Theorem

Do you remember Calc I? Notations

n-dim to n-dim

No analogue of Peano curve Byproduct

- 1 Introduction
 - The Theorem of Eda and Kamijo
 - Aronszajn lines?
- 2 Proof of New Theorem
 - Do you remember Calc I?
 - Notations
 - n-dim to 1-dim
 - n-dim to n-dim
- 3 No analogue of Peano curve
 - Byproduct
 - Proof

What about Aronszajn lines?

Some results on the products of connected linearly ordered sets

Tetsuva Ish

Introduction
The Theorem of Ediand Kamijo

Aronszajn lines?
Proof of New Theorem

Do you remember Calc I? Notations

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct

Question (Eda and Kamijo)

What if every nonempty convex set has an Aronszajn suborder?

In case of the product of two linearly ordered sets, we already gave the following answer.

What about Aronszajn lines?

Some results on the products of connected linearly ordered sets

Tetsuya Ish

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc I? Notations n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve Byproduct

Question (Eda and Kamijo)

What if every nonempty convex set has an Aronszajn suborder?

In case of the product of two linearly ordered sets, we already gave the following answer.

What about Aronszajn lines?

Some results on the products of connected linearly ordered sets

Tetsuya Ishir

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc !?

Calc I?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct

Question (Eda and Kamijo)

What if every nonempty convex set has an Aronszajn suborder?

In case of the product of two linearly ordered sets, we already gave the following answer.

Nowhere real linearly ordered sets

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember
Calc !?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct Proof

Definition

Let K be a linearly ordered set. We say that K is *nowhere real* if and only if no uncountable convex set of K is separable.

In other words, the closure of any countable set is nowhere dense.

For example, if a linearly ordered set satisfies the assumption of the Theorem of Eda and Kamijo, then it is nowhere real.

Nowhere real linearly ordered sets

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember
Calc I?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct Proof

Definition

Let K be a linearly ordered set. We say that K is *nowhere real* if and only if no uncountable convex set of K is separable.

In other words, the closure of any countable set is nowhere dense.

For example, if a linearly ordered set satisfies the assumption of the Theorem of Eda and Kamijo, then it is nowhere real.

Nowhere real linearly ordered sets

Some results on the products of connected linearly ordered sets

letsuya Ishii

Introduction
The Theorem of Ed
and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember
Calc I?
Notations
n-dim to 1-dim
n-dim to n-dim

No analogue of Peano curve Byproduct Proof

Definition

Let K be a linearly ordered set. We say that K is *nowhere real* if and only if no uncountable convex set of K is separable.

In other words, the closure of any countable set is nowhere dense.

For example, if a linearly ordered set satisfies the assumption of the Theorem of Eda and Kamijo, then it is nowhere real.

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of Nev Theorem

Do you remember Calc I? Notations n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve

Theorem (I.)

Let K_0 , K_1 , L_0 , L_1 be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: K_0 \times K_1 \to L_0 \times L_1$ is coordinate-wise.

So, while it is restricted to dimension 2, this theorem improves the result of Eda and Kamijo in the following ways

- Just being continuous and injective is sufficient.
- These four linearly ordered sets may be different (and they may be the same).
- Now, the completion of an Aronszajn line is covered.

Some results on the products of connected linearly ordered sets

Tetsuya Is

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Notations

n-dim to 1-dim
n-dim to n-dim

No analogue of Peano curve Byproduct

Theorem (I.)

Let K_0 , K_1 , L_0 , L_1 be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: K_0 \times K_1 \to L_0 \times L_1$ is coordinate-wise.

So, while it is restricted to dimension 2, this theorem improves the result of Eda and Kamijo in the following ways.

- Just being continuous and injective is sufficient.
- These four linearly ordered sets may be different (and they may be the same).
- Now, the completion of an Aronszajn line is covered.

Some results on the products of connected linearly ordered sets

Tetsuya Is

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc I?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct

Theorem (I.)

Let K_0 , K_1 , L_0 , L_1 be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: K_0 \times K_1 \to L_0 \times L_1$ is coordinate-wise.

So, while it is restricted to dimension 2, this theorem improves the result of Eda and Kamijo in the following ways.

- Just being continuous and injective is sufficient.
- These four linearly ordered sets may be different (and they may be the same).
- Now, the completion of an Aronszajn line is covered.

Some results on the products of connected linearly ordered sets

Tetsuya Is

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc I?

Notations

notations n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve Byproduct Proof

Theorem (I.)

Let K_0 , K_1 , L_0 , L_1 be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: K_0 \times K_1 \to L_0 \times L_1$ is coordinate-wise.

So, while it is restricted to dimension 2, this theorem improves the result of Eda and Kamijo in the following ways.

- Just being continuous and injective is sufficient.
- These four linearly ordered sets may be different (and they may be the same).
- Now, the completion of an Aronszajn line is covered.

Some results on the products of connected linearly ordered sets

Tetsuya Is

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remembe
Calc I?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct

Theorem (I.)

Let K_0 , K_1 , L_0 , L_1 be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: K_0 \times K_1 \to L_0 \times L_1$ is coordinate-wise.

So, while it is restricted to dimension 2, this theorem improves the result of Eda and Kamijo in the following ways.

- Just being continuous and injective is sufficient.
- These four linearly ordered sets may be different (and they may be the same).
- Now, the completion of an Aronszajn line is covered.

Some results on the products of connected linearly ordered sets

Tetsuya Is

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of Nev
Theorem

Do you remembe
Calc I?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct

Theorem (I.)

Let K_0 , K_1 , L_0 , L_1 be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: K_0 \times K_1 \to L_0 \times L_1$ is coordinate-wise.

So, while it is restricted to dimension 2, this theorem improves the result of Eda and Kamijo in the following ways.

- Just being continuous and injective is sufficient.
- These four linearly ordered sets may be different (and they may be the same).
- Now, the completion of an Aronszajn line is covered.

Some results on the products of connected linearly ordered sets

Tetsuya Is

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of Nev
Theorem
Do you remembe
Calc !?
Notations
n-dim to 1-dim

No analogue of Peano curve Byproduct

Theorem (I.)

Let K_0 , K_1 , L_0 , L_1 be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: K_0 \times K_1 \to L_0 \times L_1$ is coordinate-wise.

So, while it is restricted to dimension 2, this theorem improves the result of Eda and Kamijo in the following ways.

- Just being continuous and injective is sufficient.
- These four linearly ordered sets may be different (and they may be the same).
- Now, the completion of an Aronszajn line is covered.

Some results on the products of connected linearly ordered sets

Tetsuva Ishir

Introduction
The Theorem of Eda and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remembe Calc I? Notations n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve Byproduct Proof We extended this result to the product of any finite number of nowhere real connected linearly ordered sets. Namely,

Theorem (I.)

Let $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1}$ be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: \prod_{i < n} K_i \to \prod_{j < n} L_j$ is coordinate-wise.

I will present the outline of the proof of this therem

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Editand Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc I? Notations n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve Byproduct Proof We extended this result to the product of any finite number of nowhere real connected linearly ordered sets. Namely,

Theorem (I.)

Let $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1}$ be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: \prod_{i < n} K_i \to \prod_{j < n} L_j$ is coordinate-wise.

I will present the outline of the proof of this therem

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct Proof We extended this result to the product of any finite number of nowhere real connected linearly ordered sets. Namely,

Theorem (I.)

Let $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1}$ be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: \prod_{i < n} K_i \to \prod_{j < n} L_j$ is coordinate-wise.

I will present the outline of the proof of this therem.

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember
Calc !?

Notations

n-dim to 1-dim
n-dim to n-dim

No analogue of Peano curve Byproduct Proof We extended this result to the product of any finite number of nowhere real connected linearly ordered sets. Namely,

Theorem (I.)

Let $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1}$ be nowhere real connected linearly ordered sets. Then, every injective continuous function $f: \prod_{i < n} K_i \to \prod_{j < n} L_j$ is coordinate-wise.

I will present the outline of the proof of this therem.

Outline

Some results on the products of connected linearly ordered sets

Tetsuya Ishiι

Introduction The Theorem of Ed and Kamijo

Proof of New Theorem

Do you remember Calc I?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct

- 1 Introduction
 - The Theorem of Eda and Kamijo
 - Aronszajn lines?
- 2 Proof of New Theorem
 - Do you remember Calc I?
 - Notations
 - n-dim to 1-dim
 - *n*-dim to *n*-dim
- 3 No analogue of Peano curve
 - Byproduct
 - Proof

Intermediate Value Theorem

Some results on the products of connected linearly ordered sets

Tetsuya Ishit

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem Do you remember Calc I? Notations

Notations

n-dim to 1-din

n-dim to n-din

No analogue of Peano curve Byproduct

Lemma

Let K and L be connected linearly ordered sets, and $g: K \to L$ a continuous function. Let a < b be both in K. If $z \in L$ is between g(a) and g(b), then there exists a $c \in (a,b)$ such that g(c) = z.

Though it is simple, it plays a huge role in the proof of the theorem.

Intermediate Value Theorem

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct

Lemma

Let K and L be connected linearly ordered sets, and $g: K \to L$ a continuous function. Let a < b be both in K. If $z \in L$ is between g(a) and g(b), then there exists a $c \in (a,b)$ such that g(c) = z.

Though it is simple, it plays a huge role in the proof of the theorem.

Outline

Some results on the products of connected linearly ordered sets

Notations

- - The Theorem of Eda and Kamijo
 - Aronszajn lines?
- **Proof of New Theorem**
 - Do you remember Calc !?
 - Notations
 - = *n*-dim to 1-dim
 - = *n*-dim to *n*-dim
- - Byproduct
 - Proof

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

The Theorem of Eda and Kamijo

Aronszain lines?

Proof of New Theorem

Do you remembe

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve

Definition

Let K be a connected linearly ordered set, and M a set (typically a countable elementary submodel of $H(\theta)$).

Let J(K, M) be the set of all $x \in K \setminus cl(K \cap M)$ with $\inf(K \cap M) \le x \le \sup(K \cap M)$.

$$\eta(x) = \eta(K, M, x) = \sup \{ y \in cl(K \cap M) : y \le x \}
\zeta(x) = \zeta(K, M, x) = \inf \{ y \in cl(K \cap M) : y \ge x \}
I(x) = I(K, M, x) = [\eta(K, M, x), \zeta(K, M, x)]
C(x) = C(K, M, x) = { \eta(K, M, x), \zeta(K, M, x) }$$

if they exist. When it is clear from the context, we may omi K and M.

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Eda
and Kamijo
Aronezain lines?

Proof of New Theorem

Do you remembe

Notations
n-dim to 1-dim

No analogue of Peano curve

Definition

Let K be a connected linearly ordered set, and M a set (typically a countable elementary submodel of $H(\theta)$). Let J(K, M) be the set of all $x \in K \setminus cl(K \cap M)$ with $\inf(K \cap M) \le x \le \sup(K \cap M)$.

For every $x \in K$, let

$$\eta(x) = \eta(K, M, x) = \sup \{ y \in cl(K \cap M) : y \le x \}
\zeta(x) = \zeta(K, M, x) = \inf \{ y \in cl(K \cap M) : y \ge x \}
I(x) = I(K, M, x) = [\eta(K, M, x), \zeta(K, M, x)]
C(x) = C(K, M, x) = { \eta(K, M, x), \zeta(K, M, x) }$$

if they exist. When it is clear from the context, we may omin K and M.

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ediand Kamijo
Aronszajn lines?

Proof of New Theorem

Calc I?
Notations

n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve Byproduct

Definition

Let K be a connected linearly ordered set, and M a set (typically a countable elementary submodel of $H(\theta)$). Let J(K,M) be the set of all $x \in K \setminus cl(K \cap M)$ with $\inf(K \cap M) \le x \le \sup(K \cap M)$.

For every $x \in K$, let

$$\eta(x) = \eta(K, M, x) = \sup \{ y \in cl(K \cap M) : y \le x \}
\zeta(x) = \zeta(K, M, x) = \inf \{ y \in cl(K \cap M) : y \ge x \}
I(x) = I(K, M, x) = [\eta(K, M, x), \zeta(K, M, x)]
C(x) = C(K, M, x) = \{ \eta(K, M, x), \zeta(K, M, x) \}$$

if they exist. When it is clear from the context, we may omit K and M.

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ediand Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember
Calc 12

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct

Definition

Let K be a connected linearly ordered set, and M a set (typically a countable elementary submodel of $H(\theta)$). Let J(K,M) be the set of all $x \in K \setminus cl(K \cap M)$ with $\inf(K \cap M) \leq x \leq \sup(K \cap M)$.

For every $x \in K$, let

$$\eta(x) = \eta(K, M, x) = \sup \{ y \in cl(K \cap M) : y \le x \}
\zeta(x) = \zeta(K, M, x) = \inf \{ y \in cl(K \cap M) : y \ge x \}
I(x) = I(K, M, x) = [\eta(K, M, x), \zeta(K, M, x)]
C(x) = C(K, M, x) = \{ \eta(K, M, x), \zeta(K, M, x) \}$$

if they exist. When it is clear from the context, we may omit K and M.

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ediand Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct

Definition

Let K be a connected linearly ordered set, and M a set (typically a countable elementary submodel of $H(\theta)$). Let J(K, M) be the set of all $x \in K \setminus cl(K \cap M)$ with $\inf(K \cap M) \le x \le \sup(K \cap M)$.

For every $x \in K$, let

$$\eta(x) = \eta(K, M, x) = \sup \{ y \in cl(K \cap M) : y \le x \}
\zeta(x) = \zeta(K, M, x) = \inf \{ y \in cl(K \cap M) : y \ge x \}
I(x) = I(K, M, x) = [\eta(K, M, x), \zeta(K, M, x)]
C(x) = C(K, M, x) = \{ \eta(K, M, x), \zeta(K, M, x) \}$$

if they exist. When it is clear from the context, we may omit K and M.

Note that if $x \in J(K, M)$, then all of them exist.

Multidimensional notations

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction The Theorem of F

The Theorem of Ed and Kamijo Aronszajn lines?

Proof of New Theorem

Theorem

Notations 1

n-dim to 1-dim *n*-dim to *n*-dim

No analogue of Peano curve Byproduct

Definition

Let $\vec{K} = \prod_{i < n} K_i$ where each K_i is a connected linearly ordered sets.

$$\vec{J}(\vec{K}, M) = \prod_{i < n} J(K_i, M)$$

$$\vec{I}(x) = \vec{I}(\vec{K}, M, x) = \prod_{i < n} I(K_i, M, x(i))$$

$$\vec{C}(x) = \vec{C}(\vec{K}, M, x) = \prod_{i < n} C(K_i, M, x(i))$$

if they exist. Note that if $x \in \vec{J}(\vec{K}, M)$, then all of them exist.

Multidimensional notations

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ed and Kamijo

Proof of New Theorem

Ineorem

Notations

n-dim to 1-din n-dim to n-din

No analogu of Peano curve Byproduct

Definition

Let $\vec{K} = \prod_{i < n} K_i$ where each K_i is a connected linearly ordered sets.

$$\vec{J}(\vec{K}, M) = \prod_{i < n} J(K_i, M)$$
$$\vec{I}(x) = \vec{I}(\vec{K}, M, x) = \prod_{i < n} I(K_i, M, x(i))$$
$$\vec{C}(x) = \vec{C}(\vec{K}, M, x) = \prod_{i < n} C(K_i, M, x(i))$$

if they exist. Note that if $x \in \vec{J}(\vec{K}, M)$, then all of them exist.

Multidimensional notations

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ed and Kamijo

Proof of New Theorem

Ineorem

Calc I? Notations

n-dim to 1-din

No analogu of Peano curve Byproduct

Definition

Let $\vec{K} = \prod_{i < n} K_i$ where each K_i is a connected linearly ordered sets.

$$\vec{J}(\vec{K}, M) = \prod_{i < n} J(K_i, M)$$
$$\vec{I}(x) = \vec{I}(\vec{K}, M, x) = \prod_{i < n} I(K_i, M, x(i))$$
$$\vec{C}(x) = \vec{C}(\vec{K}, M, x) = \prod_{i < n} C(K_i, M, x(i))$$

if they exist. Note that if $x \in \vec{J}(\vec{K}, M)$, then all of them exist.

Outline

Some results on the products of connected linearly ordered sets

Tetsuva Ishiu

Introduction The Theorem of Ec

The Theorem of Eda and Kamijo Aronszajn lines?

Proof of New Theorem

Calc I?

Notations

n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve

- 1 Introduction
 - The Theorem of Eda and Kamijo
 - Aronszajn lines?
- 2 Proof of New Theorem
 - Do you remember Calc I?
 - Notations
 - n-dim to 1-dim
 - n-dim to n-dim
- 3 No analogue of Peano curve
 - Byproduct
 - Proof

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda and Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember Calc I?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct Proof Now suppose K_0, \ldots, K_{n-1}, L are nowhere real connected linearly ordered sets. Let $\vec{K} = \prod_{i < n} K_i$. We shall consider a continuous function $g : \vec{K} \to L$.

Let M be a countable elementary submodel of $H(\theta)$ with $K_0, \ldots, K_{n-1}, L, g \in M$.

We will assume that all linearly ordered sets are pairwise disjoint.

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda
and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct Proof Now suppose K_0, \ldots, K_{n-1}, L are nowhere real connected linearly ordered sets. Let $\vec{K} = \prod_{i < n} K_i$. We shall consider a continuous function $g : \vec{K} \to L$.

Let M be a countable elementary submodel of $H(\theta)$ with $K_0, \ldots, K_{n-1}, L, g \in M$.

We will assume that all linearly ordered sets are pairwise disjoint.

Some results on the products of connected linearly ordered sets

letsuya Ishi

Introduction
The Theorem of Ediand Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc 1?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct Proof Now suppose K_0, \ldots, K_{n-1}, L are nowhere real connected linearly ordered sets. Let $\vec{K} = \prod_{i < n} K_i$. We shall consider a continuous function $g : \vec{K} \to L$.

Let M be a countable elementary submodel of $H(\theta)$ with $K_0, \ldots, K_{n-1}, L, g \in M$.

We will assume that all linearly ordered sets are pairwise disjoint.

Max and min at corners

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Eda and Kamijo

Proof of New Theorem

Do you remember Calc I? Notations n-dim to 1-dim

No analogue of Peano curve Byproduct

Lemma

Let $x \in \vec{K}$ be so that for every i < n, inf $(K_i \cap M) \le x(i) \le \sup(K_i \cap M)$. Then,

$$\max g^{\rightarrow}(\vec{I}(x)) = \max g^{\rightarrow}(\vec{C}(x))$$

 $\min g^{\rightarrow}(\vec{I}(x)) = \min g^{\rightarrow}(\vec{C}(x))$

In particular, if g is constant on $\vec{C}(x)$, then g is constant on $\vec{I}(x)$.

Note that for example, if $x(i) \in J(K_i, M)$ for every i < k, and $x(i) \in cl(K_i \cap M)$ for every $k \le i < n$, then

$$\vec{I}(x) = \prod_{i < k} I(x(i)) \times \prod_{k < i < n} \{ x(i) \}$$

Max and min at corners

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Eda and Kamijo
Aronszain lines?

Theorem

Do you remember
Calc I?

Notations

n-dim to 1-dim

No analogu of Peano curve Byproduct

Lemma

Let $x \in \vec{K}$ be so that for every i < n, $\inf(K_i \cap M) \le x(i) \le \sup(K_i \cap M)$. Then,

$$\max g^{
ightarrow}(\vec{l}(x)) = \max g^{
ightarrow}(\vec{C}(x))$$
 $\min g^{
ightarrow}(\vec{l}(x)) = \min g^{
ightarrow}(\vec{C}(x))$

In particular, if g is constant on $\vec{C}(x)$, then g is constant on $\vec{I}(x)$.

Note that for example, if $x(i) \in J(K_i, M)$ for every i < k, and $x(i) \in cl(K_i \cap M)$ for every $k \le i < n$, then

$$\vec{l}(x) = \prod_{i < k} l(x(i)) \times \prod_{k < i < n} \{ x(i) \}$$

Some results on the products of connected linearly ordered sets

reisuya isni

Introduction

The Theorem of Eda and Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember Calc I?

Notations

n-dim to 1-dim

No analogue of Peano

of Peano curve Byproduct By using the previous lemma and its proof, we can show the following lemma.

Lemma

- If $g(x) \in M$, then g is constant on $\vec{l}(x)$.
- 2 If $g(x) \in J(L, M)$, then
 - $g \rightarrow (\vec{l}(x)) = l(g(x)), and$
 - for every $c \in C(x)$, $g(c) \in C(g(x))$.

Some results on the products of connected linearly ordered sets

ieisuya isiii

Introduction The Theorem of Ede and Kamijo Aronszain lines?

Proof of New Theorem

Do you remember Calc I? Notations

n-dim to 1-dim n-dim to n-dim

No analogu of Peano curve Byproduct By using the previous lemma and its proof, we can show the following lemma.

Lemma

- If $g(x) \in M$, then g is constant on $\vec{l}(x)$
- 2 If $g(x) \in J(L, M)$, then
 - $g \rightarrow (\vec{l}(x)) = l(g(x)), and$
 - for every $c \in \vec{C}(x)$, $g(c) \in C(g(x))$.

Some results on the products of connected linearly ordered sets

letsuya Ishil

Introduction
The Theorem of Eda
and Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember Calc I?

Notations

a-dim to 1-dim

n-dim to n-dim
No analogue

No analogue of Peano curve Byproduct By using the previous lemma and its proof, we can show the following lemma.

Lemma

- If $g(x) \in M$, then g is constant on $\vec{l}(x)$.
- If $g(x) \in J(L, M)$, then
 - $g \rightarrow (\vec{l}(x)) = l(g(x)), and$
 - for every $c \in \vec{C}(x)$, $g(c) \in C(g(x))$.

Some results on the products of connected linearly ordered sets

letsuya Ishil

Introduction
The Theorem of Educand Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember Calc I? Notations

n-dim to 1-dim
n-dim to n-dim

No analogue of Peano curve Byproduct Proof By using the previous lemma and its proof, we can show the following lemma.

Lemma

- If $g(x) \in M$, then g is constant on $\vec{l}(x)$.
- 2 If $g(x) \in J(L, M)$, then
 - lacksquare $g^{\rightarrow}(I(x)) = I(g(x))$, and
 - for every $c \in C(x)$, $g(c) \in C(g(x))$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishit

Introduction
The Theorem of Ediand Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember Calc I? Notations

No analogu of Peano curve Byproduct By using the previous lemma and its proof, we can show the following lemma.

Lemma

- If $g(x) \in M$, then g is constant on $\vec{l}(x)$.
- 2 If $g(x) \in J(L, M)$, then
 - $g \rightarrow (\vec{l}(x)) = l(g(x))$, and
 - for every $c \in C(x)$, $g(c) \in C(g(x))$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishii

Introduction
The Theorem of Eda
and Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember Calc I? Notations n-dim to 1-dim

No analogu of Peano curve Byproduct By using the previous lemma and its proof, we can show the following lemma.

Lemma

- If $g(x) \in M$, then g is constant on $\vec{l}(x)$.
- 2 If $g(x) \in J(L, M)$, then
 - $g \rightarrow (\vec{I}(x)) = I(g(x))$, and
 - for every $c \in \vec{C}(x)$, $g(c) \in C(g(x))$.

Outline

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction The Theorem of Education and Kamijo

and Kamijo Aronszajn lines?

Proof of New Theorem

Do you remember Calc I?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct

- 1 Introduction
 - The Theorem of Eda and Kamijo
 - Aronszajn lines?
- 2 Proof of New Theorem
 - Do you remember Calc I?
 - Notations
 - n-dim to 1-dim
 - n-dim to n-dim
- 3 No analogue of Peano curve
 - Byproduct
 - Proof

Some results on the products of connected linearly ordered sets

n-dim to n-dim

Now, let $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1}$ be nowhere real connected linearly ordered sets. Let $\bar{K} = \prod_{i \geq n} K_i$ and

Let $f: \vec{K} \to \vec{L}$ is an injective continuous function. For every j < n, let $g_i : \vec{K} \to L_i$ be given by $g_i(x) = f(x)(j)$ (i.e. the j-th

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda and Kamijo

Proof of New Theorem

Do you remember Calc I? Notations n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve Byproduct Proof Now, let $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1}$ be nowhere real connected linearly ordered sets. Let $\vec{K} = \prod_{i < n} K_i$ and $\vec{L} = \prod_{j < n} L_j$.

Let M be a countable elementary submodel of $H(\theta)$ with $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1} \in M$.

Let $f : \vec{K} \to \vec{L}$ is an injective continuous function. For every j < n, let $g_j : \vec{K} \to L_j$ be given by $g_j(x) = f(x)(j)$ (i.e. the *j*-th component function).

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Ediand Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc I?

Notations

n-dim to n-dim

No analogue
of Peano
curve

Now, let $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1}$ be nowhere real connected linearly ordered sets. Let $\vec{K} = \prod_{i < n} K_i$ and $\vec{L} = \prod_{j < n} L_j$.

Let M be a countable elementary submodel of $H(\theta)$ with $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1} \in M$.

Let $f : \vec{K} \to \vec{L}$ is an injective continuous function. For every j < n, let $g_j : \vec{K} \to L_j$ be given by $g_j(x) = f(x)(j)$ (i.e. the *j*-th component function).

Some results on the products of connected linearly ordered sets

reisuya is

Introduction
The Theorem of Ediand Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

a-dim to a-dim

No analogue of Peano curve Byproduct Proof Now, let $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1}$ be nowhere real connected linearly ordered sets. Let $\vec{K} = \prod_{i < n} K_i$ and $\vec{L} = \prod_{j < n} L_j$.

Let M be a countable elementary submodel of $H(\theta)$ with $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1} \in M$.

Let $f : \vec{K} \to \vec{L}$ is an injective continuous function. For every j < n, let $g_j : \vec{K} \to L_j$ be given by $g_j(x) = f(x)(j)$ (i.e. the j-th component function).

Some results on the products of connected linearly ordered sets

iciouya io

Introduction
The Theorem of Eda and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember
Calc !?

Notations

n-dim to 1-dim
edim to p-dim

No analogue of Peano curve Byproduct Proof Now, let $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1}$ be nowhere real connected linearly ordered sets. Let $\vec{K} = \prod_{i < n} K_i$ and $\vec{L} = \prod_{j < n} L_j$.

Let M be a countable elementary submodel of $H(\theta)$ with $K_0, \ldots, K_{n-1}, L_0, \ldots, L_{n-1} \in M$.

Let $f : \vec{K} \to \vec{L}$ is an injective continuous function. For every j < n, let $g_j : \vec{K} \to L_j$ be given by $g_j(x) = f(x)(j)$ (i.e. the *j*-th component function).

Some results on the products of connected linearly ordered sets

Tetsuva Ishiu

Introduction
The Theorem of Ed and Kamijo

Proof of New Theorem

Do you remember Calc I?
Notations
n-dim to 1-dim

No analogue of Peano curve Byproduct

Lemma

Let $\hat{x} \in \vec{K}$ be so that $\hat{x}(0)$ is a limit point of $K_0 \cap M$, and for every $1 \le i < n$, $\hat{x}(i) \in J(K_i, M)$.

Then, there exists a unique j < n such that for every $x \in \vec{I}(\hat{x})$, $g_{\hat{i}}$ is constant on $\vec{I}(\hat{x})$.

To prove this, take $p \in K_0$ that is sufficiently close to $\hat{x}(0)$, and for every $c \in \vec{C}(\hat{x})$, compare f(c) and $f(p \cap c \upharpoonright [1, n))$. We can show that for every j < n, if g_j is not constant on $\vec{I}(\hat{x})$, then $g_j(c) = g_j(p \cap c \upharpoonright [1, n))$. Since f is injective, by using the inductive hypothesis, we can find a unique \hat{j} .

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Educand Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc 12

Calc I?
Notations
n-dim to 1-dim
n-dim to n-dim

No analogue of Peano curve Byproduct

Lemma

Let $\hat{x} \in \vec{K}$ be so that $\hat{x}(0)$ is a limit point of $K_0 \cap M$, and for every $1 \le i < n$, $\hat{x}(i) \in J(K_i, M)$.

Then, there exists a unique $\hat{j} < n$ such that for every $x \in \vec{l}(\hat{x})$, $g_{\hat{j}}$ is constant on $\vec{l}(\hat{x})$.

To prove this, take $p \in K_0$ that is sufficiently close to $\hat{x}(0)$, and for every $c \in \vec{C}(\hat{x})$, compare f(c) and $f(p \cap c \upharpoonright [1, n))$. We can show that for every j < n, if g_j is not constant on $\vec{I}(\hat{x})$, then $g_j(c) = g_j(p \cap c \upharpoonright [1, n))$. Since f is injective, by using the inductive hypothesis, we can find a unique \hat{j} .

Some results on the products of connected linearly ordered sets

Tetsuya Ishio

Introduction
The Theorem of Editand Kamijo
Aronszajn lines?

Proof of New Theorem Do you remember Calc 1?

Calc I?
Notations
n-dim to 1-dim
n-dim to n-dim

No analogue of Peano curve Byproduct Proof

Lemma

Let $\hat{x} \in \vec{K}$ be so that $\hat{x}(0)$ is a limit point of $K_0 \cap M$, and for every $1 \le i < n$, $\hat{x}(i) \in J(K_i, M)$.

Then, there exists a unique $\hat{j} < n$ such that for every $x \in \vec{l}(\hat{x})$, $g_{\hat{i}}$ is constant on $\vec{l}(\hat{x})$.

To prove this, take $p \in K_0$ that is sufficiently close to $\hat{x}(0)$, and for every $c \in \bar{C}(\hat{x})$, compare f(c) and $f(p \cap c \upharpoonright [1, n])$. We can show that for every j < n, if g_j is not constant on $\bar{I}(\hat{x})$, then $g_j(c) = g_j(p \cap c \upharpoonright [1, n])$. Since f is injective, by using the inductive hypothesis, we can find a unique \hat{I} .

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda and Kamijo
Aronszajn lines?

Proof of New Theorem Do you remember Calc I? Notations

No analogue of Peano curve Byproduct

n-dim to n-dim

Lemma

Let $\hat{x} \in \vec{K}$ be so that $\hat{x}(0)$ is a limit point of $K_0 \cap M$, and for every $1 \le i < n$, $\hat{x}(i) \in J(K_i, M)$.

Then, there exists a unique $\hat{j} < n$ such that for every $x \in \vec{l}(\hat{x})$, $g_{\hat{j}}$ is constant on $\vec{l}(\hat{x})$.

To prove this, take $p \in K_0$ that is sufficiently close to $\hat{x}(0)$, and for every $c \in \vec{C}(\hat{x})$, compare f(c) and $f(p \cap c \upharpoonright [1, n])$. We can show that for every j < n, if g_j is not constant on $\vec{l}(\hat{x})$, then $g_j(c) = g_j(p \cap c \upharpoonright [1, n])$. Since f is injective, by using the inductive hypothesis, we can find a unique \hat{j} .

Some results on the products of connected linearly ordered sets

letsuya Ishi

Introduction
The Theorem of Eda and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember
Calc !?

Notations

n-dim to 1-dim
edim to n-dim

No analogue of Peano curve Byproduct Proof

Lemma

Let $\hat{x} \in \vec{K}$ be so that $\hat{x}(0)$ is a limit point of $K_0 \cap M$, and for every $1 \le i < n$, $\hat{x}(i) \in J(K_i, M)$.

Then, there exists a unique $\hat{j} < n$ such that for every $x \in \vec{l}(\hat{x})$, $g_{\hat{j}}$ is constant on $\vec{l}(\hat{x})$.

To prove this, take $p \in K_0$ that is sufficiently close to $\hat{x}(0)$, and for every $c \in \vec{C}(\hat{x})$, compare f(c) and $f(p \cap c \upharpoonright [1, n))$. We can show that for every j < n, if g_j is not constant on $\vec{l}(\hat{x})$, then $g_j(c) = g_j(p \cap c \upharpoonright [1, n))$. Since f is injective, by using the inductive hypothesis, we can find a unique \hat{j} .

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda
and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember
Calc !?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct

Lemma

Let $\hat{x} \in \vec{K}$ be so that $\hat{x}(0)$ is a limit point of $K_0 \cap M$, and for every $1 \le i < n$, $\hat{x}(i) \in J(K_i, M)$.

Then, there exists a unique $\hat{j} < n$ such that for every $x \in \vec{l}(\hat{x})$, $g_{\hat{j}}$ is constant on $\vec{l}(\hat{x})$.

To prove this, take $p \in K_0$ that is sufficiently close to $\hat{x}(0)$, and for every $c \in \vec{C}(\hat{x})$, compare f(c) and $f(p \cap c \upharpoonright [1, n])$. We can show that for every j < n, if g_j is not constant on $\vec{l}(\hat{x})$, then $g_j(c) = g_j(p \cap c \upharpoonright [1, n])$. Since f is injective, by using the inductive hypothesis, we can find a unique \hat{j} .

Global Lemma

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Ed and Kamijo

Proof of New Theorem

Do you remembe Calc I?
Notations
n-dim to 1-dim
n-dim to n-dim

No analogu of Peano curve Byproduct By pasting this local result, we can prove the following lemma.

Lemma

Let p be a limit point of $K_0 \cap M$. Then, there exists j < n such that g_j is constant on $\{x \in \vec{J}(\vec{K}, M) \mid x(0) = p\}$.

By applying this lemma to all coordinates of K, we can finish proving the new theorem.

Global Lemma

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remembe Calc I? Notations n-dim to 1-dim n-dim to n-dim

No analogue of Peano curve Byproduct Proof By pasting this local result, we can prove the following lemma.

Lemma

Let p be a limit point of $K_0 \cap M$. Then, there exists $\hat{j} < n$ such that $g_{\hat{j}}$ is constant on $\{x \in \vec{J}(\vec{K}, M) \mid x(0) = p\}$.

By applying this lemma to all coordinates of $ec{K}$, we can finish proving the new theorem.

Global Lemma

Some results on the products of connected linearly ordered sets

Tetsuva Ish

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc I?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct Proof By pasting this local result, we can prove the following lemma.

Lemma

Let p be a limit point of $K_0 \cap M$. Then, there exists $\hat{j} < n$ such that $g_{\hat{j}}$ is constant on $\{x \in \vec{J}(\vec{K}, M) \mid x(0) = p\}$.

By applying this lemma to all coordinates of \vec{K} , we can finish proving the new theorem.

Outline

Some results on the products of connected linearly ordered sets

letsuya Ishi

Introduction

The Theorem of Eda and Kamijo Aronszajn lines?

Proof of New Theorem

Do you remember Calc I? Notations

n-dim to 1-dim n-dim to n-dim

of Peano
Curve

Byproduct

- 1 Introduction
 - The Theorem of Eda and Kamijo
 - Aronszajn lines?
- 2 Proof of New Theorem
 - Do you remember Calc I?
 - Notations
 - n-dim to 1-dim
 - n-dim to n-dim
- 3 No analogue of Peano curve
 - Byproduct
 - Proof

Some results on the products of connected linearly ordered sets

Tetsuya Ishii

Introduction
The Theorem of Ediand Kamijo

Proof of New Theorem

Do you remembe Calc !? Notations n-dim to 1-dim

No analogue of Peano curve Byproduct As a byproduct of this research, we showed the following theorem, too.

Theorem

Let K, L_0 , L_1 be nowhere real connected linearly ordered sets such that K has the minimum and maximum elements Then, there is no continuous surjective function from K to $L_0 \times L_1$.

So, there cannot be an analogue of Peano curve in this situation.

Some results on the products of connected linearly ordered sets

letsuya Is

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Calc I?
Notations
n-dim to 1-dim

No analogu of Peano curve Byproduct As a byproduct of this research, we showed the following theorem, too.

Theorem

Let K, L_0 , L_1 be nowhere real connected linearly ordered sets such that K has the minimum and maximum elements. Then, there is no continuous surjective function from K to $L_0 \times L_1$.

So, there cannot be an analogue of Peano curve in this situation.

Some results on the products of connected linearly ordered sets

Tetsuya Isl

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem Do you remember Calc I? Notations

No analogu of Peano curve Byproduct As a byproduct of this research, we showed the following theorem, too.

Theorem

Let K, L_0 , L_1 be nowhere real connected linearly ordered sets such that K has the minimum and maximum elements. Then, there is no continuous surjective function from K to $L_0 \times L_1$.

So, there cannot be an analogue of Peano curve in this situation.

Some results on the products of connected linearly ordered sets

letsuya Is

Introduction
The Theorem of Eda
and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember
Calc !?

Notations

No analogue of Peano curve Byproduct As a byproduct of this research, we showed the following theorem, too.

Theorem

Let K, L_0 , L_1 be nowhere real connected linearly ordered sets such that K has the minimum and maximum elements. Then, there is no continuous surjective function from K to $L_0 \times L_1$.

So, there cannot be an analogue of Peano curve in this situation.

Outline

Some results on the products of connected linearly ordered sets

.o.o.aya .o....

The Theorem of Ed

The Theorem of Eda and Kamijo Aronszajn lines?

Proof of New Theorem

Do you remember Calc I?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct Proof

- 1 Introduction
 - The Theorem of Eda and Kamijo
 - Aronszajn lines?
- 2 Proof of New Theorem
 - Do you remember Calc I?
 - Notations
 - n-dim to 1-dim
 - n-dim to n-dim
- 3 No analogue of Peano curve
 - Byproduct
 - Proof

Some results on the products of connected linearly ordered sets

etsuya Ishiu

Introduction
The Theorem of Eda
and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc I?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct Proof Let $f: K \to L_0 \times L_1$ be a continuous surjective function. Let M be a countable elementary substructure of $H(\theta)$ with $K, L_0, L_1, f \in M$ for a sufficiently large regular cardinal θ . Let $g_0: K \to L_0$ and $g_1: K \to L_1$ be the component functions of f.

Let $\langle y_i : i < \omega \rangle$ be an increasing sequence in $L_0 \cap M$ and $z \in J(L_1, M)$. Then, for every $i < \omega$, there exists an $x_i \in K$ such that $f(x_i) = \langle y_i, z \rangle$.

Since $z \in J(L_1, M)$, we have $x_i \notin \operatorname{cl}(K \cap M)$ for every $i < \omega$ Since K is compact, we have $x_i \in J(K, M)$. Then, g_0 is constant on $I(x_i)$, i.e. for every $x \in I(x_i)$, $g_0(x) = y_i$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc I?
Notations
n-dim to 1-dim
n-dim to n-dim

No analogue of Peano curve Byproduct Proof Let $f: K \to L_0 \times L_1$ be a continuous surjective function. Let M be a countable elementary substructure of $H(\theta)$ with $K, L_0, L_1, f \in M$ for a sufficiently large regular cardinal θ . Let $g_0: K \to L_0$ and $g_1: K \to L_1$ be the component functions of f.

Let $\langle y_i : i < \omega \rangle$ be an increasing sequence in $L_0 \cap M$ and $z \in J(L_1, M)$. Then, for every $i < \omega$, there exists an $x_i \in K$ such that $f(x_i) = \langle y_i, z \rangle$.

Since $z \in J(L_1, M)$, we have $x_i \notin \operatorname{cl}(K \cap M)$ for every $i < \omega$ Since K is compact, we have $x_i \in J(K, M)$. Then, g_0 is constant on $I(x_i)$, i.e. for every $x \in I(x_i)$, $g_0(x) = y_i$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ed and Kamijo
Aronszajn lines?

Proof of New Theorem

Calc I?
Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct Proof Let $f: K \to L_0 \times L_1$ be a continuous surjective function. Let M be a countable elementary substructure of $H(\theta)$ with $K, L_0, L_1, f \in M$ for a sufficiently large regular cardinal θ . Let $g_0: K \to L_0$ and $g_1: K \to L_1$ be the component functions of f.

Let $\langle y_i : i < \omega \rangle$ be an increasing sequence in $L_0 \cap M$ and $z \in J(L_1, M)$. Then, for every $i < \omega$, there exists an $x_i \in K$ such that $f(x_i) = \langle y_i, z \rangle$.

Since $z \in J(L_1, M)$, we have $x_i \notin \operatorname{cl}(K \cap M)$ for every $i < \omega$. Since K is compact, we have $x_i \in J(K, M)$. Then, g_0 is constant on $I(x_i)$, i.e. for every $x \in I(x_i)$, $g_0(x) = y_i$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Ediand Kamijo
Aronszajn lines?

Proof of New Theorem

Calc I?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct Proof Let $f: K \to L_0 \times L_1$ be a continuous surjective function. Let M be a countable elementary substructure of $H(\theta)$ with $K, L_0, L_1, f \in M$ for a sufficiently large regular cardinal θ . Let $g_0: K \to L_0$ and $g_1: K \to L_1$ be the component functions of f.

Let $\langle y_i : i < \omega \rangle$ be an increasing sequence in $L_0 \cap M$ and $z \in J(L_1, M)$. Then, for every $i < \omega$, there exists an $x_i \in K$ such that $f(x_i) = \langle y_i, z \rangle$.

Since $z \in J(L_1, M)$, we have $x_i \notin \operatorname{cl}(K \cap M)$ for every $i < \omega$. Since K is compact, we have $x_i \in J(K, M)$. Then, g_0 is constant on $I(x_i)$, i.e. for every $x \in I(x_i)$, $g_0(x) = y_i$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishiu

Introduction
The Theorem of Eda and Kamijo
Aronszajn lines?

Proof of New Theorem Do you remember Calc I? Notations

Notations
n-dim to 1-dim
n-dim to n-dim

No analogue of Peano curve Byproduct Proof Let $f: K \to L_0 \times L_1$ be a continuous surjective function. Let M be a countable elementary substructure of $H(\theta)$ with $K, L_0, L_1, f \in M$ for a sufficiently large regular cardinal θ . Let $g_0: K \to L_0$ and $g_1: K \to L_1$ be the component functions of f.

Let $\langle y_i : i < \omega \rangle$ be an increasing sequence in $L_0 \cap M$ and $z \in J(L_1, M)$. Then, for every $i < \omega$, there exists an $x_i \in K$ such that $f(x_i) = \langle y_i, z \rangle$.

Since $z \in J(L_1, M)$, we have $x_i \notin \operatorname{cl}(K \cap M)$ for every $i < \omega$. Since K is compact, we have $x_i \in J(K, M)$. Then, g_0 is constant on $I(x_i)$, i.e. for every $x \in I(x_i)$, $g_0(x) = y_i$.

Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct Proof Hence, for every $i < \omega$, $\{g_1(\eta(x_i)), g_1(\zeta(x_i))\} = C(z)$. Let $a_i, b_i \in C(x_i)$ be so that $g_1(a_i) = \eta(z)$ and $g_1(b_i) = \zeta(z)$.

Some results on the products of connected linearly ordered sets

letsuya Ish

Introduction
The Theorem of Eda and Kamijo

Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

No analogue of Peano curve Byproduct Proof Hence, for every $i < \omega$, $\{g_1(\eta(x_i)), g_1(\zeta(x_i))\} = C(z)$. Let $a_i, b_i \in C(x_i)$ be so that $g_1(a_i) = \eta(z)$ and $g_1(b_i) = \zeta(z)$.

Some results on the products of connected linearly ordered sets

letsuya Ishi

Introduction
The Theorem of Eda and Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct Hence, for every $i < \omega$, $\{g_1(\eta(x_i)), g_1(\zeta(x_i))\} = C(z)$. Let $a_i, b_i \in C(x_i)$ be so that $g_1(a_i) = \eta(z)$ and $g_1(b_i) = \zeta(z)$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda
and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct Hence, for every $i < \omega$, $\{g_1(\eta(x_i)), g_1(\zeta(x_i))\} = C(z)$. Let $a_i, b_i \in C(x_i)$ be so that $g_1(a_i) = \eta(z)$ and $g_1(b_i) = \zeta(z)$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda and Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember
Calc !?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct Hence, for every $i < \omega$, $\{g_1(\eta(x_i)), g_1(\zeta(x_i))\} = C(z)$. Let $a_i, b_i \in C(x_i)$ be so that $g_1(a_i) = \eta(z)$ and $g_1(b_i) = \zeta(z)$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda and Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember
Calc !?

Notations

n-dim to 1-dim
n-dim to n-dim

No analogue of Peano curve Byproduct Proof Hence, for every $i < \omega$, $\{g_1(\eta(x_i)), g_1(\zeta(x_i))\} = C(z)$. Let $a_i, b_i \in C(x_i)$ be so that $g_1(a_i) = \eta(z)$ and $g_1(b_i) = \zeta(z)$.

Some results on the products of connected linearly ordered sets

Tetsuya Ishi

Introduction
The Theorem of Eda and Kamijo
Aronszain lines?

Proof of New Theorem

Do you remember Calc !?

Notations

n-dim to 1-dim

n-dim to n-dim

No analogue of Peano curve Byproduct Proof Hence, for every $i < \omega$, $\{g_1(\eta(x_i)), g_1(\zeta(x_i))\} = C(z)$. Let $a_i, b_i \in C(x_i)$ be so that $g_1(a_i) = \eta(z)$ and $g_1(b_i) = \zeta(z)$.

Open problems

Some results on the products of connected linearly ordered sets

Tetsuva Ishir

Introduction
The Theorem of Eda and Kamijo
Aronszajn lines?

Proof of New Theorem

Do you remember Calc I? Notations n-dim to 1-dim n-dim to n-dim

No analoguof Peano
curve

Byproduct
Proof

- Can we weaken "linearly ordered"?
- Can there be an analogue of Peano curve if K has no maximum or minimum elements?
- How about other things that can be done for \mathbb{R} ?