Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Tetsuya Ishiu

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Eda

Linearly ordered topological spaces Theorem of Eda and Kamijo

A little flavor of logic

Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks Corollaries

# Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Why is it better to live in an Euledean space

Tetsuya Ishiu

Department of Mathematics Miami University

February 22, 2018

### Outline

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

### Question of Eda

- **■** ℝ2
- Linearly ordered topological spaces
- Theorem of Eda and Kamijo
- Question
- - $\mathbf{K}$ 
    - $\blacksquare$   $g: K \rightarrow L$

    - $\blacksquare f: K_0 \times K_1 \to L$
    - $f: K_0 \times K_1 \rightarrow L_0 \times L_1$
- - Corollaries
  - Open Problems

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Proof K

 $g: K \to L$   $f: K_0 \times K_1 - K_1 + K_0 \times K_1 - K_1 \times K_1 + K_1 \times K_1 \times K_$ 

Concluding remarks

### Consider $\mathbb{R}^2$ .

The function that swap the coordinates is a homemorphism from  $\mathbb{R}^2$  to  $\mathbb{R}^2$ .

$$f(x,y) = \langle y, x \rangle$$

If  $g_1:\mathbb{R}\to\mathbb{R}$  and  $g_2:\mathbb{R}\to\mathbb{R}$  are homeomorphisms, from  $\mathbb{R}$  to  $\mathbb{R}$ , then

$$g(x,y) = \langle g_1(x), g_2(y) \rangle$$

is a homemorphism from  $\mathbb{R}^2$  to  $\mathbb{R}^2$ . For example  $g(x,y)=\langle 2x+1,y^3+1\rangle$  is a homeomorphism.

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g: K -

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 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$   $L_0 \times L_1$ 

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 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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These two types of homeomorphisms and their compositions form a class of coordinate-wise homeomorphisms.

Namely, a function  $f: \mathbb{R}^2 \to \mathbb{R}^2$  is coordinate-wise if and only if there are functions  $g_1: \mathbb{R} \to \mathbb{R}$  and  $g_2: \mathbb{R} \to \mathbb{R}$  such that either

- 1 for every  $\langle x, y \rangle \in \mathbb{R}^2$ ,  $f(x, y) = \langle g_1(x), g_2(y) \rangle$  or
- 2 for every  $\langle x, y \rangle \in \mathbb{R}^2$ ,  $f(x, y) = \langle g_1(y), g_2(x) \rangle$ .

# Example : $\mathbb{R}^2$

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Proo

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It is very easy to construct homeomophisms from  $\mathbb{R}^2$  to  $\mathbb{R}^2$  that are NOT coordinate-wise.

$$f(x,y) = \langle x+y, x-y \rangle$$
  $g(x,y) = \langle x, e^x y \rangle$ 

Is it generally true for linearly ordered topological spaces

R<sup>2</sup>
Linearly ordered topological spaces
Theorem of Eda and Kamijo

A little flavo of logic

Proof K  $g: K \to L$   $f: K_0 \times K_1 - K_1 \times K_2 \times K_1 - K_2 \times K_1 \times K_2 \times K_2$ 

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Linearly ordered topological spaces

**Definition** 

Let L be a linearly ordered set without minimum or maximum elements.

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 $\underset{K}{\mathsf{Proof}}$ 

 $\begin{array}{l} \ddots \\ g: K \to L \\ \vdots: K_0 \times K_1 \to \\ \vdots: K_0 \times K_1 \to \end{array}$ 

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### **Definition**

Let *L* be a linearly ordered set without minimum or maximum elements.

Then, the set of all open intervals

 $(a,b) = \{x \in L : a < x < b\}$  is a basis for a topology. The topology generated by this basis is called the order topology. A space whose topology is an order topology is

called a linearly ordered topological space(LOTS).

For example,  $\mathbb{R}$  is a LOTS with ordinary order.

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Proof

 $f: K \to L$   $f: K_0 \times K_1 \to K_1 \times K_2 \times K_1 \to K_2 \times K_2 \times K_2 \to K_1 \to K_2 \times K_2 \times K_2 \to K_2 \times K_$ 

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Definition

Let L be a linearly ordered set.  $D \subseteq L$  is dense in L if and only if D is dense in the order topology of L, namely for every  $x, y \in L$  with  $(x, y) \neq \emptyset$ , there exists  $d \in D$  such that x < d < y.

L is separable if and only if L has a countable dense subset.

#### Fact

- $\blacksquare$   $\mathbb R$  is separable since  $\mathbb Q$  is a countable dense subset
- A linearly ordered set L embeds into  $\mathbb{R}$  if and only if L is separable and has only countably many jumps.

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### Characterization of $\mathbb{R}$

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#### **Theorem**

Every complete separable linearly ordered space L that is dense in itself without maximum or minimum elements is order-isomorphic to  $\mathbb{R}$ . Here,

- L is complete if and only if every subset of L bounded above has a least upper bound.
- L is dense in itself if and only if for every  $x, y \in L$  with x < y, there exists  $z \in L$  such that x < z < y.

### Characterization of $\mathbb{R}$

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# Topological and order-theoretic properties

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#### Theorem

Let L be a linearly ordered set, equipped with the order topology.

- L is Hausdorff.
- L is connected if and only if L is complete and dense in itself.
- If L is connected, every bounded closed subset of L is compact.

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# Theorem of Eda and Kamijo

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### Theorem (K. Eda and R. Kamijo)

Let L be a connected linearly ordered space such that for every non-empty open interval I, there exists an uncountable monotone subset of I. Then every homeomorphism  $f: L^n \to L^n$  is coordinate-wise for every natural number n.

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### Example

Recall that  $\omega_1$  is the uncountable well-ordered set whose all proper initial segments are countable.

Let L be the set of all functions s from  $\omega_1$  into  $\{0,1\}$  that is not eventually 1, i.e. for unboundedly many  $\alpha \in \omega_1$ ,  $s(\alpha) = 0$ . Let L be ordered lexicographically. Namely, s < 0 if and only if  $s(\alpha) < t(\alpha)$  when  $\alpha$  is the least element of  $\omega_1$  such that  $s(\alpha) \neq t(\alpha)$ .

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 $g: K \to L$   $f: K_0 \times K_1 - K_1 \times K_2 \times K_3 = K_1 + K_2 \times K_3 = K_3 \times K_4 + K_4 \times K_$ 

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Proof

K  $g: K \to L$   $f: K_0 \times K_1 \to K_1 \to K_2 \times K_1 \to K_2 \times K_1 \to K_2 \times K_1 \to K_2 \times K_2 \to K_2 \to K_1 \to K_2 \times K_2 \to K_2$ 

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  - $\blacksquare K$ 
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    - $f: K_0 \times K_1 \to L$
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- 4 Concluding remarks
  - Corollaries
  - Open Problems

### Question of Eda

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Tetsuya Ishiu

Question of Eda

Linearly ordered topological spaces Theorem of Eda and Kamijo

A little flavor of logic

Proof  $\kappa$ 

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$   $L_0 \times L_1$ 

### Definition

A linearly ordered space *L* is *nowhere real* if and only if no nonempty open interval is separable.

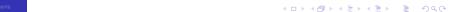
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Eda asked the following question.

#### Question

Can we replace "every non-empty open interval contains an uncountable monotone sequence" by "nowhere real"?

Note that connectedness is necessary because we may just rearrange the connected components.



### Question of Eda

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

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Proo

K  $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks Corollaries Open Problems

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Eda

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Theorem of Eda and Kamijo

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Proof K  $g: K \to L$  $f: K_0 \times K_1 \to f: K_0 \times K_1 \to K_1$ 

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Proo

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks Corollaries Open Problems

### We know that it is not vacuous question.

(In ZFC), there also exists a linearly ordered set that do not contain

- any uncountable separable suborder, or
- any uncountable monotone sequence.

Such a linearly ordered set is called an Aronszajn line. It is easy to construct an Aronszajn line that is dense in itself.

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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Proo

 $g: K \to L$   $f: K_0 \times K_1 \to K_1 \to K_1 \times K_1 \to K_$ 

Concluding remarks
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Question o Eda

Linearly ordered topological spaces Theorem of Eda and Kamijo

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K  $g: K \to L$   $f: K_0 \times K_1$ 

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks

Corollaries

Open Problems

Let L be an Aronszajn line that is dense in itself. Consider the cut-completion  $\hat{L}$  of L, namely the set  $\hat{L}$  of all Dedekind cuts of L.

Then, L is a connected nowhere real linearly ordered space that has no uncountable monotone sequence. ( $\hat{L}$  has both minimum and maximum elements, but it is easy to remove them).

So, Eda's question is not vacuous

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 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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Proof K

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks
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Tetsuya Ishiu

Question o

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

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 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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Corollaries
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Tetsuya Ishiu

Eda

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Linearly ordered topological spaces
Theorem of Eda ar

Theorem of Eda and Kamijo Question

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Proo

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks
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Answer: Yes, Theorem of Eda and Kamijo can be extended to every connected nowhere real linearly ordered spaces.

#### Theorem

Let  $K_0, K_1, ..., K_{n-1}, L_0, L_1, ..., L_{n-1}$  be connected nowhere real linearly ordered spaces. Let  $f : \prod_{i < n} K_i \to \prod_{j < n} L_j$  be a continuous injective function. Then, f is coordinate-wise.

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Tetsuya Ishiu

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Linearly ordered topological space

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

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Proo

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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Tetsuya Ishiu

Eda

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Linearly ordered topological spaces

Theorem of Eda an

topological spaces Theorem of Eda and Kamijo Question

A little flavor of logic

Proc

 $g: K \to L$   $f: K_0 \times K_1 \to K_1 \to K_2 \times K_1 \to K_2 \times K_1 \to K_2 \to K_2 \to K_1 \to K_2 \to K_$ 

Concluding remarks
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Question Eda

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Proc

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks
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Question of Eda

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks

Corollaries

Main Theorem improves the Theorem of Eda and Kamijo in the following sense.

- 11 The domain and codomain do not have to be the same
- Neither the domain nor codomain has to be the power of a single linearly ordered space, but the product of different linearly ordered spaces.
- The function does not have to be a homeomorphism but a continuous injection.

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Tetsuya Ishiu

Question of Eda

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Linearly ordered

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

A little flavo

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 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks
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Question of Eda

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

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Proof

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Tetsuya Ishiu

Questi Eda R<sup>2</sup>

> Linearly ordered topological spaces Theorem of Eda and Kamijo

A little flavo of logic

Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks
Corollaries

First, let me formally define coordinate-wise functions.

#### Definition

Let  $\prod_{i < n} K_i$  and  $\prod_{j < n} L_j$  be the products of sets. We say that a function  $f: \prod_{i < n} K_i \to \prod_{j < n} L_j$  is coordinate-wise if and only if for every j < n, there exists i < n such that the value of the j-th coordinate of f(x) depends only on the i-th coordinate of x.

i.e. there exist a function  $h: \{j: j < n\} \rightarrow \{i: i < n\}$ , and  $g_j: K_{h(j)} \rightarrow L_j$  for each j < n such that for every  $x \in \prod_{i < n} K_i$ , the j-th coordinate of f(x) equals to  $g_j(x(h(j)))$ .

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Questic Eda R<sup>2</sup>

Linearly ordered topological spaces Theorem of Eda and Kamijo

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to K_1 \times K_2 \times K_1 \times K_1 \times K_2 \times K_2 \times K_1 \times K_2 \times K_$ 

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Proo

 $g: K \to L$   $f: K_0 \times K_1 \to K_1 \to K_1 \times K_1 \to K_1 \to K_1 \times K_1 \to K_$ 

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Tetsuya Ishiı

Eda

R²

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Theorem of Eda an Kamijo

A little flavor of logic

Proof  $K
g: K \to L
f: K_0 \times K_1 \to G
f: K_0 \times K_1 \to G
L_0 \times L_1$ 

We shall outline the proof in case n = 2. The case n > 2 is done by the same idea with a little more tricks and notational difficulties.

Fortunately, it has nothing to do with forcing, large cardinals relative consitency, and so on.

However, we need "elementary submodels"

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Proof K  $g: K \to L$  $f: K_0 \times K_1 \to f: K_0 \times K_1 \to K_1$ 

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Proof  $K
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Finte Products of Connected Nowhere Real Linearly Ordered Spaces

A little flavor of logic

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Proof K  $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks

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(Technical remark) By Gödel's Incompleteness Theorem, there may not exist a set that satisfies ZFC.

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g: K \to L
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## Downward Löwenheim-Skolem Theorem

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Linearly ordered

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K  $g: K \to L$   $f: K_0 \times K_1 \to K_1$ 

Concluding remarks

Corollaries

By Löwenheim-Skolem Theorem, there exists a countable elementary submodel M of such an  $H(\theta)$ .

Namely, for every first-order formula  $\varphi$  and  $a_1, a_2, \ldots, a_n \in M$ ,  $H(\theta)$  satisfies  $\varphi(a_1, a_2, \ldots, a_n)$  if and only if M satisfies  $\varphi(a_1, a_2, \ldots, a_n)$ .

More practically, it means that M has all "definable" objects as elements, and is closed under every function in M.

## Downward Löwenheim-Skolem Theorem

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Linearly ordered topological space

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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Corollaries
Open Problems

By Löwenheim-Skolem Theorem, there exists a countable elementary submodel M of such an  $H(\theta)$ . Namely, for every first-order formula  $\varphi$  and  $A = \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, A_2, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only } \{A \in M, H(\theta) \text{ satisfies } \varphi(A_1, \dots, A_n) \text{ if and only$ 

Namely, for every first-order formula  $\varphi$  and  $a_1, a_2, \ldots, a_n \in M$ ,  $H(\theta)$  satisfies  $\varphi(a_1, a_2, \ldots, a_n)$  if and only if M satisfies  $\varphi(a_1, a_2, \ldots, a_n)$ .

More practically, it means that M has all "definable" objects as elements, and is closed under every function in M.

## Downward Löwenheim-Skolem Theorem

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Tetsuya Ishiu

Eda

R²

Linearly ordered topological spaces

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

A little flavor of logic

Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks
Corollaries
Open Problems

By Löwenheim-Skolem Theorem, there exists a countable elementary submodel M of such an  $H(\theta)$ .

Namely, for every first-order formula  $\boldsymbol{\varphi}$  and

 $a_1, a_2, \ldots, a_n \in M$ ,  $H(\theta)$  satisfies  $\varphi(a_1, a_2, \ldots, a_n)$  if and only if M satisfies  $\varphi(a_1, a_2, \ldots, a_n)$ .

More practically, it means that M has all "definable" objects as elements, and is closed under every function in M.

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Tetsuya Ishiu

Question of Eda

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

A little flavor of logic

Proof K  $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$  $L_0 \times L_1$ 

Concluding remarks
Corollaries
Open Problems

Let M be a countable elementary submodel of a sufficiently large  $H(\theta)$ .

- $\mathbb{I}$   $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ , and all definable objects are in M.
- 2 All definable fuctions are in M.
- Since  $\mathbb{R}$  is complete in  $H(\theta)$ ,  $\mathbb{R}$  is complete in M. Namely, if A is a subset of  $\mathbb{R}$  that is bounded above and  $A \in M$ , then A has the least upper bound in M.
- The supremum of  $\omega_1 \cap M$  is NOT in M. It is OK since  $\omega_1 \cap M$  is not an element of M.

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Tetsuya Ishiu

Question Eda

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A little flavor of logic

Proof K  $g: K \to L$  $f: K_0 \times K_1 \to K_2 \times K_1 \to K_2 \times K_1 \to K_2 \times K_2 \to K_1 \to K_2 \times K_2 \to K$ 

Concluding remarks Corollaries Open Problems Let M be a countable elementary submodel of a sufficiently large  $H(\theta)$ .

- **11**  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ , and all definable objects are in M.
- 2 All definable fuctions are in M.
- 3 Since  $\mathbb R$  is complete in  $H(\theta)$ ,  $\mathbb R$  is complete in M. Namely, if A is a subset of  $\mathbb R$  that is bounded above and  $A \in M$ , then A has the least upper bound in M.
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Finte Products of Connected Nowhere Real Linearly Ordered Spaces

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Question o

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

A little flavor of logic

Proof K  $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks
Corollaries
Open Problems

Let M be a countable elementary submodel of a sufficiently large  $H(\theta)$ .

- 1  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ , and all definable objects are in M.
- 2 All definable fuctions are in M.
- Since  $\mathbb{R}$  is complete in  $H(\theta)$ ,  $\mathbb{R}$  is complete in M. Namely, if A is a subset of  $\mathbb{R}$  that is bounded above and  $A \in M$ , then A has the least upper bound in M.
- 4 The supremum of  $\omega_1 \cap M$  is NOT in M. It is OK since  $\omega_1 \cap M$  is not an element of M.

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

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Eda

R²

Linearly ordered topological spaces

A little flavor

Proof K  $g: K \to L$   $f: K_0 \times K_1 \to G$   $f: K_0 \times K_1 \to G$   $f: K_0 \times K_1 \to G$ 

Concluding remarks
Corollaries
Open Problems

Let M be a countable elementary submodel of a sufficiently large  $H(\theta)$ .

- 1  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ , and all definable objects are in M.
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- 3 Since  $\mathbb{R}$  is complete in  $H(\theta)$ ,  $\mathbb{R}$  is complete in M. Namely, if A is a subset of  $\mathbb{R}$  that is bounded above and  $A \in M$ , then A has the least upper bound in M.
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Question of Eda  $\mathbb{R}^2$ Linearly ordered

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

A little flavor of logic

Proof K  $g: K \to L$   $f: K_0 \times K_1 \to K_1 \times K_2 \times K_1 \to K_2 \times K_1 \to K_2 \times K_2 \to K_2 \to K_2 \times K_2 \to K_2$ 

Concluding remarks
Corollaries
Open Problems

Let M be a countable elementary submodel of a sufficiently large  $H(\theta)$ .

- 1  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ , and all definable objects are in M.
- 2 All definable fuctions are in *M*.
- Since  $\mathbb R$  is complete in  $H(\theta)$ ,  $\mathbb R$  is complete in M. Namely, if A is a subset of  $\mathbb R$  that is bounded above and  $A \in M$ , then A has the least upper bound in M.
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## Example

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ielsuya isii

Question of Eda

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

A little flavor of logic

Proof  $K
g: K \to L
f: K_0 \times K_1 \to f: K_0 \times K_1 \to K_1$ 

 $f: \mathcal{K}_0 \times \mathcal{K}_1 - \mathcal{L}_0 \times \mathcal{L}_1$ Concluding

Let M be a countable elementary submodel of a sufficiently large  $H(\theta)$ .

Then,

- 1  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ , and all definable objects are in M.
- 2 All definable fuctions are in M.
- Since  $\mathbb R$  is complete in  $H(\theta)$ ,  $\mathbb R$  is complete in M. Namely, if A is a subset of  $\mathbb R$  that is bounded above and  $A \in M$ , then A has the least upper bound in M.
- **4** The supremum of  $\omega_1 \cap M$  is NOT in M. It is OK since  $\omega_1 \cap M$  is not an element of M.

## Outline

Finte Products
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Tetsuya Ishiu

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#### Proof

- $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$
- Concluding remarks

  Corollaries

- Question of Eda
  - $\mathbb{R}^2$
  - Linearly ordered topological spaces
  - Theorem of Eda and Kamijo
  - Question
- 2 A little flavor of logic
- 3 Proof
  - K
  - g : K → L
  - $f: K_0 \times K_1 \to L$
  - $\blacksquare f: K_0 \times K_1 \rightarrow L_0 \times L_1$
- 4 Concluding remarks
  - Corollaries
  - Open Problems

## Characterization of connected nowhere real LOTS

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Fix a connected nowhere real linearly ordered space K and a sufficiently good countable elementary submodel M with  $K \in M$ .

## Characterization of connected nowhere real LOTS

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Fix a connected nowhere real linearly ordered space K and a sufficiently good countable elementary submodel M with  $K \in M$ .

Let a < b both in K. Since K is nowhere real,  $(a, b) \cap M$  is not dense in (a, b). So,  $Cl((a, b) \cap M)$  does not contain (a,b).

## Characterization of connected nowhere real LOTS

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

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We will play with this gap. By the way, this gap does not

# Characterization of connected nowhere real LOTS

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to K_$ 

Fix a connected nowhere real linearly ordered space K and a sufficiently good countable elementary submodel M with  $K \in M$ .

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We will play with this gap. By the way, this gap does not exists in case of  $\mathbb R$  and it explains why a similar argument does not work for  $\mathbb R$ .

### **Notations**

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Tetsuya Ishiu

R<sup>2</sup>
Linearly ordered topological space

Linearly ordered topological spaces Theorem of Eda and Kamijo Question

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#### Proof

- $g: K \to L$   $f: K_0 \times K_1 \to K_2 \times K_3 \to K_4 \times K_4 \to K_5 \times K_5 \to K_6 \times K_6 \times K_6 \to K_6 \times K_6 \times K_6 \to K_6 \times K_$
- Concluding remarks

  Corollaries

#### Definition

Let J(K, M) be the set of all  $x \in K$  such that  $x \notin Cl(K \cap M)$ , and there exist  $a, b \in K \cap M$  such that  $a \le x \le b$ .

By the previous slide, for every  $a, b \in K$  with a < b, there exists  $x \in J(K, M)$  such that a < x < b.

### **Notations**

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Tetsuya Ishii

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#### Proof

- $g: K \to L$   $f: K_0 \times K_1 \to K_1 \times K_2 \times K_1 \to K_2 \times K_2 \times K_2 \to K_3 \times K_4 \to K_4 \times K_$
- Concluding remarks

  Corollaries

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### **Notations**

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#### Proof

- $g: K \to L$   $f: K_0 \times K_1 \to K_1 \times K_2 \times K_1 \to K_2 \times K_2 \times K_2 \to K_3 \times K_4 \to K_4 \times K_$
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  Corollaries

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Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Tetsuya Ishiu

Question of

Linearly ordered topological spaces Theorem of Eda and Kamijo

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Proof

- $g: K \to L$   $f: K_0 \times K_1 f: K_0 \times K_1 -$
- $f: \mathcal{K}_0 \times \mathcal{K}_1 \to \mathcal{L}_0 \times \mathcal{L}_1$

Concluding remarks
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#### Definition

$$\eta(K, M, x) = \sup \{ y \in Cl(K \cap M) : y \le x \} 
\zeta(K, M, x) = \inf \{ y \in Cl(K \cap M) : y \ge x \} 
I(K, M, x) = [\eta(K, M, x), \zeta(K, M, x)] 
C(K, M, x) = \{ \eta(K, M, x), \zeta(K, M, x) \}$$

if they exist.

Note that if  $x \in J(K, M)$ , then all of them exist. *I* is for the "interval", and *C* is for the "corner". When *K* or *M* are clear from the context, we omit them.

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Tetsuya Ishit

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#### Proof

```
K
g: K \to L
f: K_0 \times K_1 \to K_1 \to K_2 \times K_1 \to K_2 \times K_1 \to K_2 \times K_2 \to K_2 \times K_1 \to K_2 \times K_2 \times K_2 K_2
```

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Question of Eda

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#### Proof

- K  $g: K \to L$   $f: K_0 \times K_1 \to K_1 \to K_2 \times K_1 \to K_2 \times K_1 \to K_2 \times K_2 \to K_2 \times K_1 \to K_2 \times K_2 \times K_2 K_2$
- Concluding remarks

  Corollaries

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$$\eta(K, M, x) = \sup \{ y \in Cl(K \cap M) : y \le x \} 
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#### Proof

- $g: K \to L$   $f: K_0 \times K_1 K_1 \times K_2 \times K_1 = K_1 \times K_2 \times K_1 = K_2 \times K_1 \times K_2 \times K_2 \times K_1 = K_1 \times K_2 \times K_2 \times K_2 \times K_1 = K_2 \times K_2 \times K_2 \times K_2 \times K_2 \times K_1 = K_1 \times K_2 \times K_$
- Concluding remarks Corollaries Open Problems

#### Definition

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if they exist.

Note that if  $x \in J(K, M)$ , then all of them exist. *I* is for the "interval", and *C* is for the "corner". When K or M are clear from the context, we omit them.

#### A little lemma

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

#### Lemma

Let  $\hat{x} \in J(K, M)$ . Then, either  $\eta(\hat{x}) \notin M$  or  $\zeta(\hat{x}) \notin M$ .

#### Proof.

Suppose that both  $\eta(\hat{x}) \in M$  and  $\zeta(\hat{x}) \in M$ . Since K is dense in itself and M is an elementary submodel, there exists  $x \in M$  such that  $\eta(\hat{x}) < x < \zeta(\hat{x})$ . If  $x < \hat{x}$ , then by the definition  $\eta(\hat{x}) \ge x$ . This is a contradiction. Similarly if  $x > \hat{x}$ .

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Proof

K  $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$   $L_0 \times L_1$ 

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## Outline

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Proof

- $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$   $L_0 \times L_1$
- Concluding
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- 1 Question of Eda
  - $\mathbb{R}^2$
  - Linearly ordered topological spaces
  - Theorem of Eda and Kamijo
  - Question
- 2 A little flavor of logic
- 3 Proof
  - $\blacksquare K$
  - $\blacksquare g: K \rightarrow L$
  - $f: K_0 \times K_1 \to L$
  - $\blacksquare f: K_0 \times K_1 \rightarrow L_0 \times L_1$
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### Max and min at the corners

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> Linearly ordered topological spaces Theorem of Eda and Kamijo

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to L$   $f: K_0 \times K_1 \to L$ 

Concluding remarks

Corollaries

Open Problems

Now consider two connected nowhere real linearly ordered spaces K and L, a continuous function  $g: K \to L$ , and a suffienctly good countable elementsry suubmodel M with  $K, L, g \in M$ .

#### Theorem

Let  $\hat{x} \in J(K, M)$ . Then,  $g \upharpoonright I(\hat{x})$  has maximum and minimum at either  $\eta(\hat{x})$  or  $\zeta(\hat{x})$ . In particular, if  $g(\eta(\hat{x})) = g(\zeta(\hat{x}))$ , then g is constant on  $I(\hat{x})$ .

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Concluding remarks
Corollaries
Open Problems

Now consider two connected nowhere real linearly ordered spaces K and L, a continuous function  $g: K \to L$ , and a suffienctly good countable elementsry suubmodel M with  $K, L, g \in M$ .

#### **Theorem**

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### Max and min at the corners

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Proo

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Now consider two connected nowhere real linearly ordered spaces K and L, a continuous function  $g:K\to L$ , and a sufficiently good countable elementary submodel M with  $K,L,g\in M$ .

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Let  $\hat{x} \in J(K, M)$ . Then,  $g \upharpoonright I(\hat{x})$  has maximum and minimum at either  $\eta(\hat{x})$  or  $\zeta(\hat{x})$ . In particular, if  $g(\eta(\hat{x})) = g(\zeta(\hat{x}))$ , then g is constant on  $I(\hat{x})$ .

## No elements of M in $g^{\rightarrow}l(\hat{x})$

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

#### Tetsuya Ishiu

Let  $\hat{x} \in J(K, M)$  with  $g(\hat{x}) \in M$ . Then, g is constant on  $I(\hat{x})$ .

#### **Theorem**

Theorem

Let  $\hat{x} \in J(K, M)$  with  $g(\hat{x}) \in J(L, M)$ . Then,

$$\{g(\eta(\hat{x})),g(\zeta(\hat{x}))\}=\{\eta(g(\hat{x})),\zeta(g(\hat{x}))\}$$

So, the behavior of g is very restricted by M.

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## No elements of M in $g^{\rightarrow}l(\hat{x})$

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Let  $\hat{x} \in J(K, M)$  with  $g(\hat{x}) \in M$ . Then, g is constant on  $I(\hat{x})$ .

#### Theorem

Theorem

Let  $\hat{x} \in J(K, M)$  with  $g(\hat{x}) \in J(L, M)$ . Then,

$$\{g(\eta(\hat{x})),g(\zeta(\hat{x}))\}=\{\eta(g(\hat{x})),\zeta(g(\hat{x}))\}$$

$$g: K \to L$$

$$f: K_0 \times K_1 \to$$

$$f: K_0 \times K_1 \to$$

$$L_0 \times L_1$$

## No elements of M in $g^{\rightarrow}l(\hat{x})$

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Let  $\hat{x} \in J(K, M)$  with  $g(\hat{x}) \in M$ . Then, g is constant on  $I(\hat{x})$ .

#### **Theorem**

Theorem

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$$\{g(\eta(\hat{x})),g(\zeta(\hat{x}))\}=\{\eta(g(\hat{x})),\zeta(g(\hat{x}))\}$$

So, the behavior of g is very restricted by M.

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Proof

$$\begin{split} \mathbf{g} &: \mathbf{K} \to \mathbf{L} \\ \mathbf{f} &: \mathbf{K}_0 \times \mathbf{K}_1 \to \\ \mathbf{f} &: \mathbf{K}_0 \times \mathbf{K}_1 \to \\ \mathbf{L}_0 \times \mathbf{L}_1 \end{split}$$

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Proof

- $g: K \to L$   $f: K_0 \times K_1 \to I$   $f: K_0 \times K_1 \to I$   $L_0 \times L_1$
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### **Product**

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Proof K  $g: K \to L$   $f: K_0 \times K_1 \to K_1 \to K_0 \times K_1 \to K_1 \to K_0 \times K_1 \to K$ 

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- $K_0, K_1, L$  are connected nowhere real linearly ordered spaces,
- $f: K_0 \times K_1 \to L$  is a continuous function, and
- M is a sufficiently good countable elementary submodl with  $K_0, K_1, L, f \in M$ .

## **Notation**

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 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_2 \to$ 

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#### Definition

$$I(K_0 \times K_1, M, \langle x, y \rangle) = I(\hat{x}) \times I(\hat{y})$$

$$C(K_0 \times K_1, M, \langle x, y \rangle) = C(\hat{x}) \times C(\hat{y})$$

## When $\hat{y} \in M, ...$

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 $g: K \to L$   $f: K_0 \times K_1 \to K_1 \to K_1 \times K_1 \to K_$ 

#### Lemma

Let  $\hat{x} \in J(K_0, M)$  and  $\hat{y} \in M$ . Then,

- 1  $f \upharpoonright (I(\hat{x}) \times \{ \hat{y} \})$  has maximum and minimum at either  $\langle \eta(\hat{x}), \hat{y} \rangle$  or  $\langle \zeta(\hat{x}), \hat{y} \rangle$ ,
- **2** *if*  $f(\hat{x}, \hat{y}) \in M$ , then  $f \upharpoonright (I(\hat{x}) \times \{ \hat{y} \})$  is constant, and
- $\exists$  if  $f(\hat{x}, \hat{y}) \in J(L, M)$ , then

$$f^{\rightarrow}(I(\hat{x}) \times \{ \hat{y} \}) = I(f(\hat{x}, \hat{y}))$$
$$\{ f(\eta(\hat{x}), \hat{y}), f(\zeta(\hat{x}), \hat{y}) \} = \{ \eta(f(\hat{x}, \hat{y})), \zeta(f(\hat{x}, \hat{y})) \}$$

#### Proof.

Note that  $f \upharpoonright (K_0 \times \{\hat{y}\})$  is in M. Apply the result in the previous section.

## When $\hat{y} \in M, ...$

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## Proof

- $g: K \to L$   $f: K_0 \times K_1 \to K_1 \times K_2 \times K_3 \to K_4 \to K_1 \times K_4 \times K_5 \to K_$
- $f: K_0 \times K_1 \longrightarrow L_0 \times L_1$

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#### Lemma

Let  $\hat{x} \in J(K_0, M)$  and  $\hat{y} \in M$ . Then,

- 1  $f \upharpoonright (I(\hat{x}) \times \{ \hat{y} \})$  has maximum and minimum at either  $\langle \eta(\hat{x}), \hat{y} \rangle$  or  $\langle \zeta(\hat{x}), \hat{y} \rangle$ ,
- 2 if  $f(\hat{x}, \hat{y}) \in M$ , then  $f \upharpoonright (I(\hat{x}) \times \{ \hat{y} \})$  is constant, and
- $if f(\hat{x}, \hat{y}) \in J(L, M)$ , then

$$f^{\rightarrow}(I(\hat{x}) \times \{ \hat{y} \}) = I(f(\hat{x}, \hat{y}))$$
$$\{ f(\eta(\hat{x}), \hat{y}), f(\zeta(\hat{x}), \hat{y}) \} = \{ \eta(f(\hat{x}, \hat{y})), \zeta(f(\hat{x}, \hat{y})) \}$$

#### Proof.

Note that  $f \upharpoonright (K_0 \times \{\hat{y}\})$  is in M. Apply the result in the previous section.

## When $\hat{y}$ is a limit point, ...

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

$$g: K \to L$$

$$f: K_0 \times K_1 \to K_2 \times K_3 \to K_4 \to K_4 \times K_4 \to K_5 \to K_6 \times K_6 \to K_$$

#### Lemma

Let  $\hat{x} \in J(K_0, M)$  and  $\hat{y}$  is a limit point of  $K_1 \cap M$ . Suppose  $f(\hat{x}, \hat{y}) \in J(L, M)$ . Then,

$$f^{\rightarrow}(I(\hat{x}) \times \{ \hat{y} \}) = I(f(\hat{x}, \hat{y}))$$
$$\{ f(\eta(\hat{x}), \hat{y}), f(\zeta(\hat{x}), \hat{y}) \} = \{ \eta(f(\hat{x}, \hat{y})), \zeta(f(\hat{x}, \hat{y})) \}$$

$$\{ f(\eta(\hat{x}), y), f(\zeta(\hat{x}), y) \} = \{ \eta(f(\hat{x}, y)), \zeta(f(\hat{x}, y)) \}$$
  
= \{ \eta(f(\hat{x}, \hat{y})), \zeta(f(\hat{x}, \hat{y})) \}



## When $\hat{y}$ is a limit point, ...

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Proo

- $g: K \to L$   $f: K_0 \times K_1 \to L$
- $t: K_0 \times K_1 L_0 \times L_1$

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#### Lemma

Let  $\hat{x} \in J(K_0, M)$  and  $\hat{y}$  is a limit point of  $K_1 \cap M$ . Suppose  $f(\hat{x}, \hat{y}) \in J(L, M)$ . Then,

$$f^{\rightarrow}(I(\hat{x}) \times \{ \hat{y} \}) = I(f(\hat{x}, \hat{y}))$$
$$\{ f(\eta(\hat{x}), \hat{y}), f(\zeta(\hat{x}), \hat{y}) \} = \{ \eta(f(\hat{x}, \hat{y})), \zeta(f(\hat{x}, \hat{y})) \}$$

#### Proof.

If  $y \in K_1 \cap M$  and  $f(\hat{x}, y) \in I(f(\hat{X}, \hat{y}))$ , then

$$\{ f(\eta(\hat{x}), y), f(\zeta(\hat{x}), y) \} = \{ \eta(f(\hat{x}, y)), \zeta(f(\hat{x}, y)) \}$$
  
= \{ \eta(f(\hat{x}, \hat{y})), \zeta(f(\hat{x}, \hat{y})) \}

Note that  $\hat{y}$  is a limit point of such y's



## When $\hat{y}$ is a limit point, ...

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$$f: K_0 \times K_1 \to L$$

 $L_0 \times L_1$ Concluding

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#### Lemma

Let  $\hat{x} \in J(K_0, M)$  and  $\hat{y}$  is a limit point of  $K_1 \cap M$ . Suppose  $f(\hat{x}, \hat{y}) \in J(L, M)$ . Then,

$$f^{\rightarrow}(I(\hat{x}) \times \{ \hat{y} \}) = I(f(\hat{x}, \hat{y}))$$
$$\{ f(\eta(\hat{x}), \hat{y}), f(\zeta(\hat{x}), \hat{y}) \} = \{ \eta(f(\hat{x}, \hat{y})), \zeta(f(\hat{x}, \hat{y})) \}$$

#### Proof.

If  $y \in K_1 \cap M$  and  $f(\hat{x}, y) \in I(f(\hat{X}, \hat{y}))$ , then

$$\{ f(\eta(\hat{\mathbf{x}}), \mathbf{y}), f(\zeta(\hat{\mathbf{x}}), \mathbf{y}) \} = \{ \eta(f(\hat{\mathbf{x}}, \mathbf{y})), \zeta(f(\hat{\mathbf{x}}, \mathbf{y})) \}$$
$$= \{ \eta(f(\hat{\mathbf{x}}, \hat{\mathbf{y}})), \zeta(f(\hat{\mathbf{x}}, \hat{\mathbf{y}})) \}$$

Note that  $\hat{y}$  is a limit point of such y's.



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K  $g: K \to L$   $f: K_0 \times K$ 



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 $g: K \to L$   $f: K_0 \times K_1 =$   $f: K_0 \times K_1 =$ 

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- $K_0, K_1, L_0, L_1$  are connected nowhere real linearly ordered spaces,
- $f: K_0 \times K_1 \to L_0 \times L_1$  is a continuous injection, and
- M is a sufficiently good countable elementary submodl with  $K_0, K_1, L_0, L_1, f \in M$ .

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K  $g: K \to L$   $f: K_0 \times K_1 \to K_1$   $f: K_0 \times K_1 \to K_2$ 

Concluding remarks

Corollaries

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K  $g: K \to L$   $f: K_0 \times K_1 - K_0 \times K_1 = K_1 = K_1 \times K_1 = K_1$ 

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- $K_0, K_1, L_0, L_1$  are connected nowhere real linearly ordered spaces,
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## Yet another analogous lemma

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

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#### Lemma

Let  $\hat{x} \in J(K_0, M)$  and  $\hat{y} \in J(K_1, M)$ . Then,

$$f^{\rightarrow}(I(\hat{x},\hat{y}))=I(f(\hat{x},\hat{y}))$$

$$f^{\rightarrow}(C(\hat{x},\hat{y}))=C(f(\hat{x},\hat{y}))$$

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Here, we need the assumption that f is injective

## Yet another analogous lemma

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#### Lemma

Let  $\hat{x} \in J(K_0, M)$  and  $\hat{y} \in J(K_1, M)$ . Then,

$$f^{\rightarrow}(I(\hat{x},\hat{y})) = I(f(\hat{x},\hat{y}))$$
  
$$f^{\rightarrow}(C(\hat{x},\hat{y})) = C(f(\hat{x},\hat{y}))$$

Here, we need the assumption that *f* is injective.

## $f \upharpoonright (\eta(\hat{x}), K_1)$ is (almost) constant

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

#### Lemma

Let  $\hat{x} \in J(K_0, M)$ . Then, there exists  $i \in \{0, 1\}$  such that  $f \upharpoonright (\eta(\hat{x}) \times [\inf(K_1 \cap M), \sup(K_1 \cap M)])$  is constant.

$$g: K \to L$$

$$f: K_0 \times K_1 \to$$

$$f: K_0 \times K_1 =$$

# Wrapping all up

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By changing M and a little more argument, we can show that f is coordinate-wise.

#### Theorem

Let  $K_0, K_1, \ldots, K_{n-1}, L_0, L_1, \ldots, L_{n-1}$  be connected nowhere real linearly ordered spaces. Let  $f: \prod_{i < n} K_i \to \prod_{j < n} L_j$  be a continuous injective function. Then, f is coordinate-wise.

# Wrapping all up

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Proof

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# Corollary 1.

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Proof

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L<sub>0</sub> × L<sub>1</sub>
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### Corollary

Let  $K_0$ ,  $K_1$ ,  $L_0$ ,  $L_1$  be connected nowhere real linearly ordered spaces. Suppose that  $K_0 \times K_1$  and  $L_0 \times L_1$  are homeomorphic. Then, either

- $K_0$  and  $L_0$  are homeomorphic and  $K_1$  and  $L_1$  are homeomorphic, or
- $K_0$  and  $L_1$  are homeomorphic and  $K_1$  and  $L_0$  are homeomorphic.

# Corollary 1.

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

Corollaries

Corollary

Let  $K_0$ ,  $K_1$ ,  $L_0$ ,  $L_1$  be connected nowhere real linearly ordered spaces. Suppose that  $K_0 \times K_1$  and  $L_0 \times L_1$  are homeomorphic. Then, either

- $K_0$  and  $L_0$  are homeomorphic and  $K_1$  and  $L_1$  are homeomorphic, or
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# Corollary 1.

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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### Corollary

Let  $K_0$ ,  $K_1$ ,  $L_0$ ,  $L_1$  be connected nowhere real linearly ordered spaces. Suppose that  $K_0 \times K_1$  and  $L_0 \times L_1$  are homeomorphic. Then, either

- $K_0$  and  $L_0$  are homeomorphic and  $K_1$  and  $L_1$  are homeomorphic, or
- *K*<sub>0</sub> and *L*<sub>1</sub> are homeomorphic and *K*<sub>1</sub> and *L*<sub>0</sub> are homeomorphic.

## Corollary 2.

Finte Products of Connected Nowhere Real Linearly Ordered Spaces

Corollaries

### Corollary

Let K be a connected nowhere real linearly ordered spaces. Then, there is no group operation that makes K a topological group.

## Corollary 2.

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Proc

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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### Corollary

Let K be a connected nowhere real linearly ordered spaces. Then, there is no group operation that makes K a topological group.

#### Proof.

If K is a topological group, then  $\langle x, y \rangle \mapsto \langle x, x \cdot y \rangle$  is a homeomorphism that is not coordinate-wise. This is a contradiction.

#### Question

Is it known? More direct proof?

## Corollary 2.

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K  $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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### Corollary

Let K be a connected nowhere real linearly ordered spaces. Then, there is no group operation that makes K a topological group.

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#### Question

Is it known? More direct proof?

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# Open problems

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### Problem

Can we extend this to more spaces? Maybe one-dimensional in some sense?

#### Problem

Can other properties of  $\mathbb{R}$  be extended to other connected linearly ordered spaces?

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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# Open problems

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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### Problem

Can we extend this to more spaces? Maybe one-dimensional in some sense?

#### **Problem**

Can other properties of  $\mathbb{R}$  be extended to other connected linearly ordered spaces?

# Open problems

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Proof

 $g: K \to L$   $f: K_0 \times K_1 \to$   $f: K_0 \times K_1 \to$ 

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### Problem

Can we extend this to more spaces? Maybe one-dimensional in some sense?

#### **Problem**

Can other properties of  $\mathbb{R}$  be extended to other connected linearly ordered spaces?