Answer Key

Q1 (1)	**Q2** (2)	**Q3** (1)	**Q4** (9)
Q5 (2)	**Q6** (4)	**Q7** (1)	**Q8** (3)
Q9 (2)	**Q10** (3)	**Q11** (1)	**Q12** (2)
Q13 (2)	**Q14** (2)	**Q15** (4)	

Q1 (1)

Let, the consecutive terms T_r and T_{r+1} having equal coefficient.

$\therefore$ Coefficient of T_r^{th} term $=$ coefficient of T_{r+1}^{th} term

$\therefore\ {}^{74}C_{r-1}3^{74}\left(\frac{2}{3}\right)^{r-1} = {}^{74}C_r 3^{74}\left(\frac{2}{3}\right)^r$

$\Rightarrow\ {}^{74}C_{r-1} \times 3 = {}^{74}C_r \times 2$

$\Rightarrow \frac{3\times 74!}{(r-1)!(75-r)!} = \frac{2\times 74!}{r!(74-r)!}$

$\Rightarrow\ \frac{3}{75-r} = \frac{2}{r}$

$\Rightarrow\ 150 - 2r = 3r \Rightarrow r = 30$

$\therefore T_{30}$ and T_{31} are two consecutive terms whose coefficients are same.

Q2 (2)

Since there is no constant term the coefficient of 8^{th} and 19^{th} term are same as the binomial coefficients of 8^{th} and 19^{th} term.

${}^nC_7 = {}^n C_{18} \Rightarrow n = 7 + 18 = 25$

$T_{n+1} = {}^{25} C_r\left(x^{\frac{4}{5}}\right)^{25-r}\left(r^{-\frac{1}{5}}\right)^r$

${}^{25}C_r\ x^{\frac{4}{5}(25-r)-\frac{r}{5}}$

To be independent of x, $\frac{100-5r}{5} = 0 \Rightarrow r = 20$

Hence, the required term is ${}^{25}C_{20}$

Q3 (1)

Coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in $(1+x)^{2n}$ are ${}^{2n}\mathrm{C}_1$, ${}^{2n}\mathrm{C}_2$ and ${}^{2n}C_3$ respectively..

Since, coefficients of 2^{nd}, 3^{rd} and 4^{th} terms are in A.P.

$\Rightarrow 2\,{}^{2n}\mathrm{C}_2 = {}^{2n}\mathrm{C}_1 + {}^{2n}\mathrm{C}_3 \quad \ldots [2b = a + c]$

$\Rightarrow 2\left(\frac{2n!}{2!(2n-2)!}\right) = \frac{2n!}{1!(2n-1)!} + \frac{2n!}{3!(2n-3)!}$

$\Rightarrow 2\left(\frac{2n(2n-1)}{2}\right) = 2n + \frac{2n(2n-1)(2n-2)}{6}$

$\Rightarrow 6n(2n-1) = 6n + 4n^3 - 6n^2 + 2n$

$\Rightarrow 6n(2n-1) = 2n(2n^2 - 3n + 4)$

$\Rightarrow 3(2n-1) = 2n^2 - 3n + 4$

$\Rightarrow 2n^2 - 9n + 7 = 0$

$\Rightarrow 2n^2 - 9n + 8 = 1$

Q4 (9)

$P = {}^nC_6 \cdot \left(3^{\frac{1}{3}}\right)^{n-6} \cdot \left(4^{-\frac{1}{3}}\right)^6$

$Q = {}^nC_{n-6} \cdot \left(3^{\frac{1}{3}}\right)^6 \cdot \left(4^{-\frac{1}{3}}\right)^{n-6}$

$\therefore \frac{Q}{P} = 12 \Rightarrow (12)^{\frac{n-6}{3}} = (12)^1$

$\Rightarrow \frac{n-6}{3} = 1 \Rightarrow n = 9$

Q5 (2)

The general term of the expansion $(2^{1/4} + 3^{1/10})^{55}$ is given by

$$T_{r+1} = {}^{55}C_r (2)^{\left(\frac{55-r}{4}\right)} (3)^{\left(\frac{r}{10}\right)}$$

If a term is rational, then both of $\left(\frac{55-r}{4}\right)$ and $\left(\frac{r}{10}\right)$ should be equal to integers.

$\because$ $\frac{55-r}{4}$ is an integer.

$\therefore$ $55 - r = 4k$

$\Rightarrow$ $r = 55 - 4k = 4k' + 3$

$\because$ $\frac{r}{10}$ is an integer.

$\therefore$ $r = 10\,k = 4k' + 2$

There is not a single possible integral value of r such that $\frac{55-r}{4}$ as well as $\frac{r}{10}$ is an integer.

Number of rational terms in the given expansion is equal to zero.

Number of irrational terms in the given expansion is equal to 56.

Q6 (4)

The term independent of x is

$2 \cdot {}^9C_3 \left(\frac{3}{2}\right)^3 \left(\frac{-1}{3}\right)^6 + a \cdot {}^9C_2 \left(\frac{3}{2}\right)^2 \left(\frac{-1}{3}\right)^7$

$= \frac{7}{9} - \frac{a}{27} = 1$, giving $a = -6$.

Q7 (1)

Last digit of $(2017)^{2018}$ is 9, last digit of $(2018)^{2019}$ is 2, last digit of $(2019)^{2020}$ is 1
So remainder on division with 5 will be 4,2 and 1 respectively.

Overall remainder will be $(4+2+1)\%5=2$

Q8 (3)

We have $2^{2000}=\left(2^4\right)^{500}=(17-1)^{500}$

$={ }^{500}C_0 17^{500}-{ }^{500}C_1 17^{499}+\cdots$

$=-{ }^{500}C_{499} 17+(-1)^{500}$

$=17m+1$, where m is some positive integer.

$\Rightarrow 2^{2003}=8\left(2^{2000}\right)=8(17 \mathrm{~m}+1)=17(8 \mathrm{~m})+8$

This show that the required remainder is 8.

Q9 (2)

Let $(k+1)^{\text{th}}$ term be independent of x

$T_{k+1}={ }^9 C_k\left(a x^{\frac{1}{6}}\right)^{9-k}\left(b x^{-\frac{1}{3}}\right)^k$

$={ }^9 C_k\, a^{9-k}\, b^k\, x^{\left(\frac{9-k}{6}-\frac{k}{3}\right)}$

For this to be independent of x,

$\frac{9-k}{6}-\frac{k}{3}=0 \Rightarrow \frac{9}{6}=\frac{k}{6}+\frac{k}{3}=\frac{k}{2} \Rightarrow k=3.$

$\Rightarrow T_{k+1}={ }^9 C_3 a^6 b^3=84\left(\sqrt{a^2 b}\right)^6$

Using $A.M. \geq G.M.$ we get $\frac{a^2+b}{2} \geq \sqrt{a^2 b}$

$\Rightarrow \sqrt{a^2 b} \leq 1$

$\Rightarrow T_{k+1} \leq 84$

Q10 (3)

$\because$ 5!, 6!, 7!, 8!, ... 175! are each multiple of 15 therefore, the required remainder is obtained when $(1!+2!+3!+4!=33)$ is divided by 15.

Hence, required remainder $=3$

$\Rightarrow \lambda=3$.

Q11 (1)

$\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$

term independent of x will be

$\frac{1}{60}\times$ term independent of x in $\left(2x^2-\frac{3}{x^2}\right)^6$

$-\frac{1}{8}\times$ term of z^{-8} in $\left(2x^2-\frac{3}{x^2}\right)^6$

$\mathrm{T}_{\mathrm{r}+1}$ in $\left(2\mathrm{x}^2-\frac{3}{\mathrm{x}^2}\right)^6$ will be

$1_{r+1}={}^6C_r\left(2x^2\right)^{6-r}\left(-\frac{3}{x^2}\right)^r$

$={}^6C_r 2^{6-r}(-1)^{\mathrm{r}}\times 3^{\mathrm{r}}\times \mathrm{x}^{12-2\mathrm{r}-2\mathrm{r}}$

Case-I :

For term independent of x is

$12-4r=0\Rightarrow r=3$

$T_4=-{}^6C_3\times 2^3\times 3^3x^6=-20\times 2^3\times 3^3$

Case-II :

For term of x^{-8} $\quad 12-4r=-8$

$\mathrm{T}_6={}^6C_5\cdot 2^1\cdot(-1)\cdot 3^5\cdot \mathrm{x}^{-8}$

$\Rightarrow 4\mathrm{r}=20\Rightarrow \mathrm{r}=5$

Required ans. $=\frac{1}{60}\times(-20)2^3\times 3^3$

$-\frac{1}{81}\times 6\times 2\times(-1)\times 3^5$

$=-72+36=-36$

Q12 (2)

Coefficient of x^{10} in the given expansion

$= {}^{15}C_{10} + {}^{16}C_{10} + {}^{17}C_{10} + \ldots {}^{30}C_{10}$

$= \left({}^{10}C_{10} + {}^{11}C_{10} + \ldots + {}^{14}C_{10} + {}^{15}C_{10} + \ldots + {}^{30}C_{10}\right) - \left({}^{10}C_{10} + {}^{11}C_{10} + \ldots + {}^{14}C_{10}\right)$

$= {}^{31}C_{11} - {}^{15}C_{11}$

Q13 (2)

$11^{1011} = \left(11^3\right)^{327} = (9k+8)^{327}$

when divided by 9 is equivalent to the division of

$8^{327} = (9-1)^{327}$

i.e. remainder$= -1 + 9 = 8$

Now, $(1011)^{11} = (9m+3)^{11}$

when divided by 9 is equivalent to the division of

$3^{11} = (9)^5 \times 3$

i.e. remainder$= 0$

$\therefore \quad 11^{1011} + 1011^{11}$ has same remainder as 8.

Q14 (2)

General term of $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$ is

$T_{r+1} = {}^{11}C_r \left(\frac{5}{2}x^3\right)^{11-r} \left(-\frac{1}{5x^2}\right)^r = {}^{11}C_r (-1)^r \cdot \frac{5^{11-2r}}{2^{11-r}} \cdot x^{33-5r}$

The term independent of x in the expansion of $\left(1 - x^2 + 3x^3\right)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$ will be the coefficient of x^0 in $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11} -$ coefficient of x^{-2} in $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11} + 3\times$ coefficient of x^{-3} in $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$

$= -{}^{11}C_7(-1)^7 \cdot \frac{5^{-3}}{2^4} = \frac{330}{5^3 \cdot 2^4} = \frac{33}{200}$

Q15 (4)

We can have $17^{256} = (290-1)^{128}$

$= 1000I + {}^{128}C_2(290)^2 - {}^{128}C_1\,(290) + 1,$ where I is an integer

$= 1000I + 128\,(290)\,(18415-1) + 1$

$= 1000m + 681$

$\therefore\ \ A_1 = 6, A_2 = 8, A_3 = 1$