. Dr. Manfred Dürschner

. NWMA: improving recursive moving averages using the Nyquist criterion

This method was published by Dr. Manfred Dürschner in the 2012 issue of the IFTA Journal [1] in an article named “Moving Averages 3.0”.

In 1994, Patrick Mulloy introduced an approach to reduce the lag of EMA [Patrick Mulloy: Stocks & Commodities Magazine (February 1994)].

Figure . *Dr. Manfred Dürschner.*

$$TEMA_\alpha(x) = 3 \cdot EMA_\alpha(x) - 3 \cdot EMA_\alpha(EMA_\alpha(x)) + EMA_\alpha(EMA_\alpha(EMA_\alpha(x)))$$

where α is the smoothing factor of the EMA. He applied an EMA once and twice to itself and combined the results with the original EMA.

In 2001, John Ehlers introduced a moving average with reduced lag in [2], pp 167-175. He used a moving average (SMA, EMA or WMA) and applied this moving average a second time to itself. The resulting MA of MA is subtracted from the MA multiplied by the factor 2:

$$ZWMA_\ell(x) = 2 \cdot MA_\ell(x) - MA_\ell(MA_\ell(x))$$

where ℓ is the length of a moving average window.

Stating that the application of an moving average to itself can be seen as a sampling procedure, Manfred Dürschner concludes that it can be improved using the Nyquist criterion.

The sampled signal is a moving average (referred to as MA_1) and the sampling signal is a moving average as well (referred to as MA_2). If we want to avoid the aliasing (additional periodic cycles which are not included in the time series), the sampling must obey the Nyquist criterion.

With the cycle period as parameter, the Nyquist criterion reads as $\frac{\ell_1}{\ell_2} = \lambda \geq 2$. Here ℓ_1 is the cycle period of the sampled signal to which another sampling signal with cycle period ℓ_2 is applied; ℓ_1 must be at least twice as large as ℓ_2. In Mulloy's and Ehlers' approaches both cycle periods are equal.

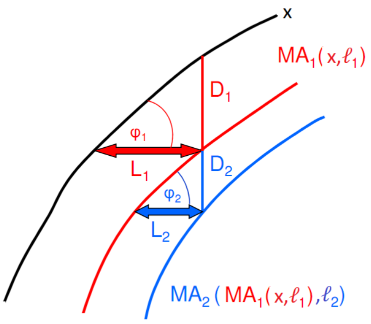

Figure . *In a fairly good approximation holds* $\varphi_1 = \varphi_2$. *From this* $D_1/L_1 = D_2/L_2$, $D_1 = x - MA_1$ *and* $D_2 = MA_1 - MA_2$.

In xxx, we see a price series x (black line), the first moving average MA_1 (red line) with a lag L_1 to the x and the second moving average MA_1 (blue line) with a lag L_2 to the MA_1. Based on fairly good approximation $\varphi_1 = \varphi_2$, $D_1/L_1 = D_2/L_2$, where $D_1 = x - MA_1$ and $D_2 = MA_1 - MA_2$. From this, the following equation holds:

$$\frac{D_1}{D_2} = \frac{x - MA_1}{MA_1 - MA_2} = \frac{L_1}{L_2}.$$

Denoting $\alpha \equiv \frac{L_1}{L_2}$ and replacing x with the approximation term NWMA (the notation for the New Weighted Moving Average), we can re-write the expression above as

$$NWMA = (1+\alpha)MA_1 - \alpha MA_2.$$

Let us recall now values of the lag for different moving averages with a window length ℓ in [2].

$$\begin{aligned} L_{SMA} &= \frac{1}{2}(\ell - 1) \\ L_{WMA} &= \frac{1}{3}(\ell - 1) \\ L_{EMA} &= \frac{1}{\alpha} - 1 \end{aligned}$$

Note that with $\alpha = \frac{2}{\ell - 1}$ we get the same EMA lag as for the SMA. According to these equations, $\frac{L_1}{L_2}$ can be written as

$$\alpha \equiv \frac{L_1}{L_2} = \frac{\ell_1 - 1}{\ell_2 - 1}.$$

In this expression, concludes Dr. Dürschner, denominator 2 for the SMA and EMA as well as denominator 3 for the WMA are missing; α is therefore valid for all three moving averages. Using the Nyquist criterion $\ell_2 = \ell_1/\lambda$, we get

$$\alpha = \frac{\ell_1 - 1}{\ell_2 - 1} = \frac{\ell_1 - 1}{\frac{\ell_1}{\lambda} - 1} = \lambda\frac{\ell_1 - 1}{\ell_1 - \lambda}, \quad \lambda \geq 2.$$

Combining both expressions for NWMA and α, we can finally write

$$NWMA_{\ell_1,\ell_2}(x) = (1+\alpha)MA_{1,\ell_1}(x) - \alpha MA_{2,\ell_2}(MA_{1,\ell_1}(x)), \quad \alpha = \frac{\ell_1 - 1}{\ell_2 - 1}, \ell_2 = \ell_1/\lambda, \quad \lambda \geq 2$$

using ℓ_1 and ℓ_2 parameters. Alternatively, we can use λ parameter instead of the ℓ_2:

$$NWMA_{\ell_1,\lambda}(x) = (1+\alpha)MA_{1,\ell_1}(x) - \alpha MA_{2,\lambda\ell_1}(MA_{1,\ell_1}(x)), \ \ \alpha = \lambda\frac{\ell_1 - 1}{\ell_1 - \lambda}, \ \ \lambda \geq 2$$

These equation, continues Dr. Dürschner, are independent of the choice of an moving average (he means probably only SMA, WMA, and EMA). As the WMA shows the smallest lag (($\frac{\ell-1}{3}$), it should generally be the first choice for the NWMA.

When $\ell_1 = \ell_1$, $\alpha = 1$ and $\lambda = 1$, respectively. Then the NWMA equation passes into Ehlers´ formula. Thus Ehlers´ formula from [2] is included in the NWMA as a limiting value.

It follows from a short calculation that the lag for NMA results in a theoretical value zero.

. References

[1] Dürschner, Manfred G. (2012). *Moving Averages 3.0*. IFTA Journal, vol. 12 pp. 27-31. ISSN:2409-0271 online offline

[2] Ehlers, John F. (2001). *Rocket Science for Traders: Digital Signal Processing Applications*. Wiley, 2001. ISBN:9780471405672 online offline