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PRACTICAL PORTFOLIO PERFORMANCE MEASUREMENT AND ATTRIBUTION

CARL R. BACON

WILEY

Practical Portfolio Performance Measurement and Attribution

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Third Edition

Carl R. Bacon

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This book is dedicated to Alex.

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Acknowledgements

This book is based on a series of performance measurement training courses I have had the pleasure of running around the world over the past couple of decades. I have learnt so much and continue to learn from the questions and observations of the participants over the years, all of whom must be thanked.

Naturally from the practitioner's perspective I've favoured certain methodologies over others; my strong preferences are difficult to disguise. Nevertheless, I've attempted to present each methodology as fairly as possible; apologies to those who may feel their methods have been unfairly treated.

Of course, all errors and omissions are my own.

Carl R. Bacon, CIPM
Deeping St James
July 2022

1
Introduction

We learn more by looking for the answer to a question and not finding it than we do from learning the answer itself.

Lloyd Alexander (1924–2007)

Who questions much, shall learn much, and retain much.

Francis Bacon (1561–1626)

WHY MEASURE PORTFOLIO PERFORMANCE?

Whether we manage our own investment assets or choose to hire others to manage the assets on our behalf we are keen to know "how well" our collection or portfolio of assets is performing.

The process of adding value via benchmarking, asset allocation, manager selection, security analysis, portfolio construction and executing transactions is collectively described as the investment decision process. The measurement of portfolio performance should be part of the investment decision process, not external to it.

Clearly there are many stakeholders in the investment decision process; this book focuses on the investors or owners of capital (asset owners) and the firms managing their assets (asset managers[1] or individual portfolio managers). Other stakeholders in the investment decision process include asset consultants tasked with providing advice to asset owners, custodians, independent performance measurers and audit firms (asset servicing firms).

Portfolio performance measurement answers the three basic questions central to the relationship between asset managers and asset owners:

(1) *What* is the return on their assets?
(2) *Why* has the portfolio performed that way?
(3) *How* can we improve performance?

Portfolio performance measurement is the quality control of the investment decision process providing the necessary information to enable asset managers and asset owners to assess exactly how the money has been invested and the results of the process. The US Bank Administration Institute (BAI) laid down the foundations of the performance measurement process as early as 1968.[2] The main conclusions of their study hold true today:

(1) Performance measurement returns should be based on asset values measured at market value, not at cost.
(2) Returns should be "total" returns, that is, they should include both income and changes in market value (realised and unrealised capital appreciation).

[1] Typically, in this book I will refer to all types of asset management firms as asset managers and occasionally refer to individual asset managers as portfolio managers.

[2] Bank Administration Institute, "Measuring the Investment Performance of Pension Funds for the Purpose of Inter Fund Comparison" (October 1968).

(3) Returns should be time weighted.
(4) Measurement should include risk as well as return.

THE PERFORMANCE MEASUREMENT PROCESS

Performance measurement is essentially a three-stage process:

(1) Measurement
    Calculation of portfolio and benchmark returns
    Distribution of information
(2) Analysis
    Return attribution
    Risk attribution (ex-post and ex-ante)
(3) Evaluation
    Feedback
    Control

Note

Performance is at the very least two dimensional – the combination of both risk and return.

THE PURPOSE OF THIS BOOK

The writing of any book is inevitably a selfish activity, taking precious time not only from family who suffer in silence but also work colleagues and friends, driven by the belief that you have something to contribute in your chosen subject.

The vocabulary and methodologies used by performance analysts worldwide are extremely varied and complex. Despite the development and global success of performance measurement standards there are considerable differences in terminology, methodology and attitude to performance measurement throughout the world.

The main aims of the first edition were to:

(1) Provide a reference of the available methodologies and to hopefully provide some consistency in their definition.
(2) Promote the role of performance measurers.
(3) Provide some insights into the tools available to performance measurers.
(4) Share my practical experience.

The motivation to write the first edition was simply to provide the book I most wanted to read as a performance analyst, which did not exist at the time. The motivation to write the second edition came from the readers of the first edition. Their praise, comments, corrections and requests encouraged me to expand and improve the material in the second edition. The gap between the first and second editions was a mere 4 years; 14 years later I write this third edition. The motivation for this third edition is more intense: I wish to share the invaluable experience gained from running performance measurement and risk control teams for an

entire career, chairing a performance analytics company for 17 years, setting standards with the CFA Institute for 27 years, and above all teaching performance analytics worldwide for 25 years. That experience shouldn't go to waste.

Since the first edition I'm pleased to say that the CFA Institute have launched the Certificate in Investment Performance (CIPM)[3] designation, which further reinforces the role of performance measurement and is a major step in developing performance measurement as a professional activity. I can certainly recommend the CIPM course of study, and I'm pleased to say I have successfully achieved the CIPM designation.

The CIPM curriculum has also to some degree influenced the content of this third edition.

With practical examples, this book should meet the needs of performance analysts, portfolio managers, senior management within asset management firms, custodians, verifiers and the ultimate asset owners. I'm particularly pleased that this third edition provides access to an Excel spreadsheet that includes all of the practical examples used throughout the book. Apologies to readers who believe there are slightly too many worked examples in this text, but I regard worked examples as an essential part of the learning and explanation process.

THE ROLE OF PERFORMANCE ANALYSTS

Performance measurement is a key function in an asset management firm; it deserves better than to be grouped with the back office. Performance analysts provide real added value, with feedback into the investment decision process and analysis of structural issues. Since their role is to understand in full and make transparent and communicate the sources of return and risk within portfolios, they are often the only independent source equipped to understand the performance of all the portfolios and strategies within the asset management firm.

Performance analysts are in effect alternative risk controllers able to protect the firm from rogue managers and the unfortunate impact of failing to meet client expectations.

My aim has been to present all approaches and methodologies as fairly as possible. It is not always possible to hide my preferences, prejudices and ignorance but in this edition, I've attempted to highlight my views as clearly identified notes, cautions and interpretations.

BOOK STRUCTURE

Chapter 2 is naturally a brief description of the asset management industry from the perspective of a performance analyst. The remaining chapters are structured in the same order as the performance measurement process itself, namely:

Chapter 3: Calculation of portfolio returns
Chapter 4: Comparison against an appropriate benchmark
Chapter 5: Proper assessment of the reward received for the risk taken
Chapter 6: Attribution of the sources of excess return
Chapter 7: Global Investment Performance Standards
Chapter 8: Bringing it all together, presentation and communication

[3] The Certificate in Investment Performance Measurement is a professional accreditation in the field of portfolio performance measurement and analysis. It is offered by the CIPM Association, a body associated with the CFA Institute.

In Chapter 3 the "what" of performance measurement is introduced, describing the many forms of return calculation including the relative merits of each method together with calculation examples. Performance gross and net of fees is discussed as well as performance fee structures.

Performance returns in isolation add little value; we must compare these returns against a suitable benchmark. Chapter 4 discusses the merits of good and bad benchmarks and examines the detailed calculation of commercial and customised indexes. Excess return is discussed in detail.

Chapter 5 is an attempt to catalogue and describe all the available ex-post risk measures used by performance analysts, including suggestions for consistent definitions where such definitions are lacking. The chapter concludes with an attempt to illustrate the relationship of risk measures to each other in terms of a periodic table of risk measures.

Together with Chapter 5, Chapter 6 is the core of this edition, attempting to answer the "why" of performance measurement. Attribution is a broad subject; the early development of return attribution is covered, leading to a fundamental examination of the Brinson model. With the foundations in place, security level attribution, off-benchmark attribution, attribution including derivative instruments, multi-currency attribution, fixed income attribution, multi-period attribution, multi-level attribution and balanced attribution are explained in detail with multiple worked examples.

Chapter 7 discusses the Global Investment Performance Standards (GIPS®) in detail. Focusing on GIPS for investment firms, the provisions are listed together with commentary providing the rationale and context for the provision (from my perspective), including tips and pitfalls to avoid.

Chapter 8 brings all the analytical tools together to address the "how" of performance measurement in the context of data visualisation, asset manager selection, communicating the four dimensions of performance and effective risk management.

2

The Asset Management Industry

The most treasured asset in investment management is a steady hand at the tiller.

Robert Arnott (1954–)

Active management is a zero-sum game before cost, and winners have to win at the expense of the losers.

Eugene Fama (1939–)

Asset management is an agency activity. Assets are managed by asset managers on behalf of asset owners. Asset owners include pension funds, endowments, insurance companies, sovereign wealth funds, family offices, high-net-worth individuals and retail investors. Assets under management are huge and growing: PwC[1] estimate global assets under management at the end of 2020 in excess of US$110 trillion, growing more than 40% in the past five years – see Tables 2.1, 2.2 and 2.3.

Assets are managed either separately in segregated accounts or collectively in various collective vehicles such as open-ended mutual funds, closed-end investment trusts, hedge funds or exchange-traded funds (ETFs). Private equity assets are typically managed collectively in limited partnerships. Some asset managers are managers of other asset managers (multi-fund managers).

Asset owners and asset managers are the major stakeholders in the asset management industry, which also include asset consultants (who identify and model asset owner liabilities, advise and suggest asset allocation, and help select asset managers) and assets service providers (including custodians, third-party administrators, index providers, auditors and verifiers, prime brokers, authorised corporate advisors, peer group providers and independent performance measurers).

Boundaries between these activities are blurred: many asset owners (large pension funds and endowments) manage their own assets internally, and in addition, occasionally manage assets for other asset owners externally, becoming simultaneously asset owners and asset managers. Some asset owners will also act as asset consultants, modelling their own liabilities, determining asset allocation and selecting specialist external asset managers. These asset owners, therefore, are also asset consultants and on occasion asset managers as well. On rare occasions, where rules permit, some organisations fulfil all four roles. Fiduciary managers might act as asset consultants as well as asset managers and many asset managers may provide some asset service roles. Performance measurement is relevant to all four types of stakeholders.

A fifth and important stakeholder is the regulator. Regulation fulfils two roles, protecting the ultimate investor and ensuring a fair and safe securities market.

Performance measurement methodologies, requirements and challenges will vary between stakeholders, segregated or collective accounts and asset type. Stakeholder relationships are illustrated in Figure 2.1.

[1] PwC, "Asset and Wealth Management Revolution" (2020).

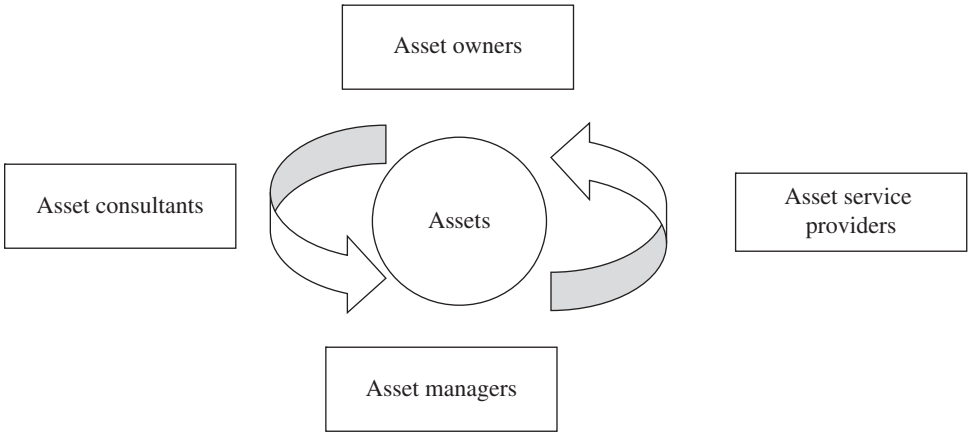


Figure 2.1 Asset management stakeholders

Table 2.1 Global assets under management (by product)

Product	2020 estimate (US$tn)	2025 estimate (US$tn)
Global	112.3	139.1
Collective funds	50.2	61.9
Active	37.8	45.2
Passive	12.4	16.7
ETFs	6.1	8.6
Segregated	48.5	58.3
Active	36.3	43.2
Passive	12.2	15.1
Alternatives	13.6	18.9

Table 2.2 Global assets under management (by region)

Region	2020 estimate (US$tn)	2025 estimate (US$tn)	Annual growth rate
North America	57.9	69.4	3.7%
Europe	32.3	37.1	2.8%
Asia-Pacific	18.3	26.2	7.4%
Latin America	2.8	4.8	11.8%
Middle East and Africa	1.1	1.6	8.4%

Table 2.3 Asset owner assets

	2020 estimate (US$tn)	2025 estimate (US$tn)	Annual growth rate
Pension funds	49.8	62.5	4.6%
Insurance funds	34.0	41.3	3.9%
Sovereign wealth funds	8.6	11.0	5.0%
High-net-worth individuals	88.2	108.3	4.2%
Mass affluent	77.3	94.6	4.1%
Total	**258.0**	**317.6**	**4.2%**
Assets under management	112.3	139.1	4.4%
Penetration	43.5%	43.8%	

ASSET CLASSES

An investment asset class should have distinct risk-return properties, be homogeneous, mutually exclusive and diversifying.[2]

Asset managers will manage various types of asset classes: public equities, bonds, real estate, private equity, private debt, infrastructure, cash, currency, derivatives and commodities. Balanced investment strategies will include a mix of asset classes seeking greater diversification and alignment to liabilities and/or the chosen strategic allocation. Different asset classes have different characteristics which may require different return and risk calculation methodologies.

PUBLIC EQUITIES

An equity is a share in a company. The company is owned by the shareholders, who have the right to income and capital of the company's business and usually have voting rights allowing them to influence the management of the company.

Equities may be defined as domestic or international (or global), usually by reference to countries in which a stock is listed. However, it should be noted that many companies of course are multi-national, some may not have any economic exposure to the country where they are listed, and a few stocks are dual-listed. Although a stock may be denominated in the currency of the market in which it is listed, underlying revenues and profits may be exposed to any number of other currencies. Countries may be categorised as developed (countries which are developed in terms of economy and industrialisation), emerging (emerging countries are typically countries with low to middle per-capita income that have undertaken economic development and reform programmes and have begun to "emerge" as significant players in the global economy[3]) or frontier (frontier countries are low-income in the beginning stages of economic growth and/or difficult to invest in because of investment infrastructure or regulation).

Definitions will vary but FTSE Russell's list of countries by category as of March 2019 is shown in Table 2.4.

Equity investing can also be further differentiated in terms of style (growth, value and momentum) and size (defined by market capitalisation: large-cap, mid-cap, small-cap and micro-cap).

[2] Cornelius, *International Investments in Private Equity* (2011).

[3] International Monetary Fund (IMF) definition.

Table 2.4 Countries by category

Developed	Advanced emerging	Secondary emerging	Frontier
Australia	Brazil	Chile	Argentina
Austria	Czech Republic	China	Bahrain
Belgium/Luxembourg	Greece	Colombia	Bangladesh
Canada	Hungary	Egypt	Botswana
Denmark	Malaysia	India	Bulgaria
Finland	Mexico	Indonesia	Côte d'Ivoire
France	South Africa	Kuwait	Croatia
Germany	Taiwan	Pakistan	Cyprus
Hong Kong	Thailand	Peru	Estonia
Ireland	Turkey	Philippines	Ghana
Israel		Qatar	Iceland
Italy		Russia	Jordan
Japan		Saudi Arabia	Kazakhstan
Netherlands		UAE	Kenya
New Zealand			Latvia
Poland			Lithuania
Norway			Macedonia
Portugal			Malta
Singapore			Mauritius
South Korea			Morocco
Spain			Nigeria
Sweden			Oman
Switzerland			Palestine
			Romania
			Serbia
			Slovakia
			Slovenia
			Sri Lanka
			Tunisia
			Vietnam

Value investing can be broadly defined as a fundamental long-term approach that seeks to invest in undervalued companies. In practice, value investing is often defined as investing in stocks with certain characteristics, such as low price–earnings ratios and high dividend yields. Asset managers seek to exploit valuation anomalies.

Growth investors seek to identify companies with good prospects for sales and earnings growth. The key to successful growth investing is identifying companies with superior long-term prospects which are not already reflected in the share price.

Momentum investors focus on companies that have a combination of stronger-than-average earning and/or price momentum, in the expectation that this trend will continue.

Definitions will vary, particularly in domestic markets of different size, but typically companies with a market capitalisation greater than US$10 billion would be considered large-cap, between US$2 billion and US$10 billion medium-cap, between US$250 million and US$2 billion small-cap, and below US$250 million micro-cap. Smaller-capitalisation

companies can often capture the early stages of new industries, are often less well researched and management may have a higher financial stake in the business. On the other hand, smaller-capitalisation companies can be more volatile and are typically less liquid and more expensive to trade, with variable standards of corporate governance and smaller, less experienced boards of directors.

The global equity market capitalisation by listed companies in 2018 was $74.7 trillion, of which 40.8% was US, 13.6% China and Hong Kong, 10.8% EU 27, 4.9% UK, 15.3% emerging markets and 14.6% other developed markets.[4]

BONDS (OR FIXED INCOME)

Bonds are securitised loans that may be bought or sold before they are repaid. For the most part they consist of predictable future cash flows, coupon (income) payments and a final redemption payment. Bonds are issued by governments (government issues), normally in their own currencies and by companies (corporates or non-government issues). The key feature that distinguishes non-government debt from government debt is a higher level of default or credit risk. Credit risk is a function of the credit quality of the issuer. The lower the credit quality of a bond, the greater the premium investors will require (yield spread) over the equivalent government bond.

Governments have the ability to raise cash to pay coupons and principal, either by taxation or printing money. Non-government issuers or governments issuing in currencies other than their own vary widely in their ability to make payments and this is recognised in credit ratings assessed by credit rating agencies, as shown in Table 2.5. The credit ratings measure the relative probability of an issuer defaulting. An issuer is in default if any payment is missed or delayed.

Table 2.5 Credit rating

S&P	Moody's	Fitch	
		Investment grade	
AAA	Aaa	AAA	Highest quality
AA	Aa	AA	High quality
A	A	A	Upper quality
BBB	Baa	BBB	Medium upper quality
		Non-investment grade	
BB	Ba	BB	Low quality
B	B1	B	Speculative
B-	B3	B–	Highly speculative
		High yield or in default	
CCC	Caa	CCC	High risk
CC	Ca	CC	Extremely speculative
C	C	C	
D		DDD	In default

[4] *Source:* World Federation of Exchanges.

Developed market bonds are issued by governments and corporates of developed markets; emerging market bonds are issued by governments or corporates of developing markets. The emerging market debt market totalled $2.5 trillion in 2018,[5] 2.4% of the total global bond markets. Over 50 countries are classified as emerging markets; many have liquid markets for bonds denominated in US dollars or other developed market currencies. Around 30 emerging markets also have local currency bond markets. Emerging markets typically offer higher yields to compensate investors for higher risk.

Conventional bond coupon and redemption payments are fixed, and inflation erodes the real value of these payments. Inflation risk can be reduced by investing in inflation-linked bonds which have their coupons and final redemption amounts determined by a specified measure of inflation. As of 2019 the inflation-linked bond market totalled $3.1 trillion,[6] mostly government issued with only a few corporate issuers.

As of August 2020, the overall size of the global bond market is estimated to be $128.3 trillion,[7] 68% of which is sovereign, supranational or agency (SSA) issuers and 32% corporate. Of the $87.5 trillion SSA bond markets, $22.4 trillion is US, $19.8 trillion is China and $12.4 trillion is Japan. Of the $40.9 trillion corporate issuers, $10.9 trillion is US and $7.4 trillion is China.

Cash (and near cash)

Cash, near cash and cash equivalents cover a range of instruments that basically take the form of short-term loans with a repayment period of less than one year. Cash includes certificates of deposit, promissory notes, company commercial paper and bonds with a maturity shorter than one year. Returns and, of course, risk are typically lower than for longer-duration bonds. Cash is unique in that as well as being an asset class in its own right it is an integral part of most investment portfolios and is typically present in most.

PRIVATE ASSETS

Private assets is a broad term and includes assets typically not available on public markets, including:

(1) Real estate
(2) Private equity
(3) Private debt
(4) Infrastructure
(5) Natural resources

As of July 2021, the size of globally managed private assets in closed-end funds was $9.8 trillion,[8] of which 53% is in North America, 22% Europe, 21% Asia and 4% the rest of the world; 65% is private equity, 12% is private debt, 13% is infrastructure and 11% is natural resources.

[5] *Source:* Bank of International Settlements (BIS).
[6] *Source:* Barclays Capital Research.
[7] *Source:* International Capital Markets Association.
[8] *Source:* McKinsey Global Private Markets Review 2022.

Real estate

Real estate refers to the commercial sectors of office, retail, industrial, and to a lesser degree leased residential and agricultural.

Real estate covers not only the physical asset and the rights that have been granted to the land on which that asset has been developed but also the right to the future income stream from that land and/or building. As a landlord, the right to these future income streams is governed by a lease with a tenant. The value of this lease will reflect:

(1) The duration of the lease
(2) Tenant default risk
(3) Management costs
(4) Vacancy rates
(5) Expected revenue growth
(6) Liquidity risk

Real estate is periodically valued (typically annually) by independent real estate valuers or appraisers.

The retail sector includes high street shops, shopping centres and retail warehouses.

As of 2019 the size of the globally managed real estate market was $9.6 trillion,[9] of which 35.8% is US, 21.3% is EU 27, 10.2% is China and Hong Kong, 9.2% is Japan and 7.8% is UK.

As of July 2021, the size of globally managed real estate assets in closed-end funds was $1.2 trillion,[10] of which 58% is in North America, 27% Europe, 11% Asia and 5% the rest of the world.

Private equity

Private equity is a broad term that refers to any type of equity investment in an asset in which the equity is not freely tradeable on a public stock exchange. Private equity is an important source of capital for start-up and young companies, for firms in financial distress and those seeking growth or buy-out financing. Private equity is illiquid and can be difficult to value.

Private equity funds pool capital from investors into strategy-specific funds. These are often organised as limited partnerships, which are the most common legal structures for private equity investments.

Private equity is typically illiquid but with lower volatility than public equities. Private equity is further differentiated from other assets classes by the J-curve.[11] The J-curve represents the tendency of private equity funds to generate negative returns in early years, due to high investment costs and fees, moving to increasing returns in later years as investments hopefully mature. Performance analysis of private equity therefore requires returns measured over longer time horizons supplemented by additional measures.[12]

Private equity strategies include:

(1) Buy-out

Buy-out refers to assets seeking to acquire controlling interests in established companies. Debt financing and leverage is often present.

[9] *Source:* MSCI Real Estate Market Size Report 2019/2020.
[10] *Source:* McKinsey Global Private Markets Review 2022.
[11] Fraser-Sampson, *Private Equity as an Asset Class* (2010).
[12] Multiples such as TVPI, DPI, PICC and RVPI; see Equations 7.3–7.6.

(2) Venture capital
Venture capital refers to investments in new and emerging companies, at the pre-product and/or pre-revenue stage, or just as companies start to generate revenue.
(3) Growth
Growth financing often follows venture capital in companies generating revenue but not necessarily profitable that need capital to expand as quickly as possible
(4) Other
Other includes distressed investments, turnaround investments and mezzanine capital.

As of July 2021, the size of globally managed private equity assets in closed-end funds was $6.3 trillion[13] of which 48% is buy-out, 29% is venture capital, 16% is growth and 7% is other.

Private debt

Private debt is typically investments that are not financed by banks and are not issued or traded in an open market. Private debt investments are used to finance business growth, provide working capital, or fund infrastructure or real estate development. Private debt tends to be very illiquid and typically is held to maturity. Just like public debt, regular income is received plus a final payment at maturity.

As of July 2021, the size of globally managed private debt assets in closed-end funds was $1.2 trillion,[14] of which 60% was in North America, 30% Europe, 7% Asia and 3% the rest of the world.

Infrastructure

Infrastructure is defined as the permanent facilities and structures that a society requires for the operation of its economy, including:

(1) Transportation (airports, toll roads, bridges, etc.)
(2) Utility and energy (wind and solar farms, water and waste services, fuel and energy storage, electricity interconnectors, gas networks, power generation, etc.)
(3) Communications (transmission towers, satellites, internet, etc.)
(4) Social (healthcare, recreation, education facilities, etc.)

Infrastructure is characterised by high barriers to entry, illiquidity, long time horizons, low correlation with other asset categories, high cash yields, regulation, and protection against inflation. Asset owners can use infrastructure to help match their liabilities with a reasonably predictable and partially inflation-protected income stream.

As of July 2021, the size of globally managed infrastructure and natural resources in closed-end funds was $1.1 trillion,[15] of which 49% was in North America, 30% Europe, 10% Asia and 10% the rest of the world.

[13] *Source:* McKinsey Global Private Markets Review 2022.
[14] *Source:* McKinsey Global Private Markets Review 2022.
[15] *Source:* McKinsey Global Private Markets Review 2022.

Natural resources

Natural resources are materials or substances that occur naturally. They may be mined or farmed and include:

(1) Agriculture
(2) Timberland
(3) Water
(4) Mining
(5) Energy

Total assets under management total $230 billion.[16] This asset category is really a blend of private equity and infrastructure with similar characteristics.

COMMODITIES

Commodities include gold, other precious and base metals, energy, agricultural products and I would also include cryptocurrencies. Commodities typically produce no direct income (in fact there is more likely to be a cost of storage and/or delivery) but they are liquid and are perceived to provide a high level of diversification and a hedge against inflation.

DERIVATIVES

Derivatives are not an asset class. They are financial instruments whose value is *derived* from the value of another underlying or reference asset. Derivatives are used in many different ways: they can be used to transfer risk and hedge, change asset allocations (quickly and efficiently), create synthetic instruments, and used purely for speculation. Derivatives are often seen as very high risk. This view arises from their use to speculate and create leverage. Individual derivative instruments taken in isolation may appear to be high risk but set in a context of the portfolio of assets of which they are a part, this need not be the case. Derivatives should be measured and reported in the context of the asset manager's intended investment decision process as a whole. The four basic types of derivatives are futures, forwards, swaps and options.

Futures

A futures contract is an agreement transacted through an organised exchange to deliver an asset at a fixed price in the future. Most futures contracts are equity index futures contracts.

An equity index future contract is an agreement to exchange cash compensation payments based on the movements in the level of an equity index. Physical assets are not delivered.

Equity index futures are the most basic form of derivative. They have no market value in a portfolio, rather two legs of economic exposure, each of equal value, one leg representing the equity index and the other leg economic or notional cash. Buying an equity index future will result in a long equity position and a short cash position and selling an equity index future, a short equity position and a long cash position.

[16] *Source:* Prequin Global Infrastructure Report (2022).

The price of the equity index future implicitly includes the underlying dividend stream before expiry and the cost of carry from interest rates, therefore the futures price will naturally differ from the underlying index discounting the differential between dividend and interest income. The contract may also trade at a discount or premium to fair value depending on the supply and demand of the futures contract relative to the underlying reference or cash index, converging only at the time of expiry.

Equity index futures never generate any currency exposure. Gains and losses in the futures contract are realised daily in the margin account. Opening an equity futures position (long or short) requires depositing an initial margin (collateral) with a broker who handles payments made to and received from the clearinghouse. If the position is not closed by the end of the day, it will be marked to market and profits and losses are added or deducted from the margin account. The margin account is an asset of the fund and should be included in all performance and attribution calculations generating currency exposure, just like any cash asset. Since the initial margin is a great deal smaller than the economic exposure, significant leverage can be achieved.

Forwards

A forward contract is an agreement made directly between two parties to buy and sell a commodity or financial asset on a specified date in the future at a fixed price agreed at the outset by both parties. Most forward contracts in asset management are currency forwards.

Currency forwards should be valued using the current forward rate at the point of valuation.

Unlike futures contracts there is no variation margin paid during the life of the contract; any gain or loss is unrealised.

The currency return between spot rates can be broken down to the forward currency return and the forward premium (or interest rate differential).

Swaps

A swap is a contract between two parties agreeing to make payments to each other on specified future dates (typically quarterly) over an agreed time period, where the amount that each has to pay is calculated on a different basis. Swaps can be considered a collection of forward contracts over multiple periods.

Just like with forward contracts, there is no variation margin and therefore the swap itself might be positively or negatively valued. Each leg of the swap can be represented by an appropriate notional economic asset, the difference between the two representing the unrealised gain or loss on the contract. In effect a swap is a cross between a futures contract and a forward contract; unrealised gain and losses are realised at each payment date. Different types of swaps include:

(1) Interest rate swap

The most common type of interest rate swap is the fixed/floating rate swap (or plain vanilla swap); the notional is normally fixed at a set amount. One party agrees to pay a fixed interest rate based on the notional amount and the other party agrees to pay a floating rate (Libor) at set times for an agreed time period.

Clearly the fixed interest leg behaves exactly as a bond and should be valued as such, with the notional asset in the correct category, and the other leg should be included as cash.

(2) Total return swap

Total return swaps transfer the entire economic performance of a reference asset or index, including all cash flows. They are often used to transfer the whole credit risk of an instrument, in return receiving interest-related payments.

(3) Credit default swap

A credit derivative is a contract whose pay-out depends on the creditworthiness of the reference asset. In the standard structure the buyer of protection pays a periodic premium to the seller of so many basis points per annum applied to the notional value of the reference asset. If during the term of the swap one of a set number of credit events occurs such as insolvency, failure to meet payments, credit ratings downgrade, etc., then the seller must take delivery of the assets at a set price or, alternatively, pay cash compensation.

The credit default premium is likely to be very close to the credit spread of the reference asset between government bonds.

The notional assets in this case are the appropriate corporate bond and government bond sectors, duration adjusted if that is appropriate to the investment decision process.

(4) Equity index swap

Equity index swaps are very similar to equity index futures contracts, the main difference being the lack of variation margin.

In a standard equity index swap one party agrees to make periodic payments based on the change in value of an equity index plus dividend yield applied to a notional value whilst receiving a fixed or floating rate of interest applied to the same notional value. The notional value can be set to remain constant or change in value in line with the equity index. Note that if the index falls in value the seller might receive additional payments for the negative change in value plus interest payments.

Equity index swaps can be represented by notional assets in a similar way to equity index futures contracts, with the notable exception that the notional values may not be identical, the difference representing unrealised losses and gains. These gains or losses are realised at each payment date.

Options

The buyer of a European-style option contract has the right, but not the obligation, to buy (call option) or sell (put option) an agreed amount of a specified asset (the underlying), at a specified price, called the exercise or strike price on a future date, called the expiry or expiration date.

European options can only be exercised at expiry, whereas American-style (the greater majority of options written) can be exercised on any business day up to and including expiry. A Bermudan option – halfway between America and Europe – is a halfway house allowing the option to be exercised on certain days up to expiry.

The purchaser of an option pays a premium to the seller or writer of the contract. Obviously, the writer of an option contract will receive a premium but retains a liability in the form of a negatively valued asset (for example, a short call or a short put).

The performance of option contracts is measured in exactly the same way as any other asset, but they also generate economic exposures. Unlike futures contracts, the economic exposure of option contracts is not linear, but convex, and will change depending on how close the price of the underlying asset is to the exercise or strike price.

The total value of an option is made up of intrinsic value and time value (see Figure 2.2). The intrinsic value of the option is the value that would be realised through immediately exercising

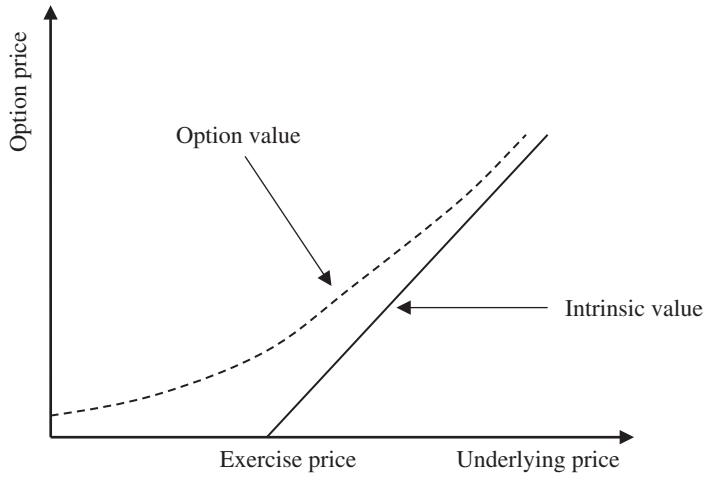


Figure 2.2 Option time value and intrinsic value

the option. If no value would be realised the intrinsic value would be zero (never negative; the option holder can never be compelled to exercise) and the option is described as "out of the money".

The time value reflects the possibility that the option may be "in the money" before expiry and is a function of time and volatility. Time value peaks around the exercise price; for a low underlying price well below the exercise price there is no intrinsic value and little time value. If the option is deeply in the money, it is virtually certain to be exercised and time value will also be low.

Option price sensitivity (the Greeks)

Options can be valued using the Black–Scholes model, which requires:

(1) The price of the underlying asset
(2) The strike or exercise price of the option
(3) Time to expiry
(4) The variability (or volatility) of the underlying asset
(5) The cost of carry

The price sensitivities of options are measured by the "Greeks": delta, gamma, theta, vega and rho:

- Delta (Δ or δ) is the change in value of an option for a small change in underlying asset price.
- Gamma (Γ or γ) measures the change in delta for a small change in underlying asset price.
- Theta (θ) is the change in value of an option as time elapses, all other factors remaining constant.
- Vega (sometimes called kappa (κ)) is the change in value of an option for a given change in volatility. Unlike the other Greeks, vega is not a Greek letter.
- Rho (ρ) is the change in value of an option for a given change in interest rates.

The impact of the change in price of an option for a positive change in each of the Greeks is summarised in Table 2.6.

Delta is the most important Greek from the point of view of performance measurement and attribution analysis. If a long call option is out of the money the delta will be less than 50% and converges to zero the more deeply out of the money it becomes. If the long call option is at the money the delta is approximately 50%; there is a 50:50 chance of the price moving either way. If the long call option is in the money the delta will be greater than 50% and converges to 100% if the option is deeply in the money, moving in line with the price of the underlying asset.

The delta is a first-order approximation of the change of price of the option; the actual relationship is convex, as shown in Figure 2.3. The option delta is the slope of the line at a tangent to the curve at the strike price.

Delta works well for a small change in value of the underlying price. Gamma measures the rate of change of this delta. For long calls the delta underestimates the gain in option price and overestimates the loss. For short calls and puts convexity is negative, overestimating gains and underestimating losses – clearly not a desirable situation.

Warrants

A warrant is a time-limited right to subscribe for shares, debentures, loan stock or government securities, and is exercisable against the original issuer of the underlying securities. Some other

Table 2.6 Impact of positive change in Greek sensitivity

Option type	Delta	Gamma	Vega	Rho	Theta
Long call	Positive	Positive	Positive	Positive	Positive
Long put	Negative	Positive	Positive	Negative	Positive
Short call	Negative	Negative	Negative	Negative	Negative
Short put	Positive	Negative	Negative	Positive	Negative

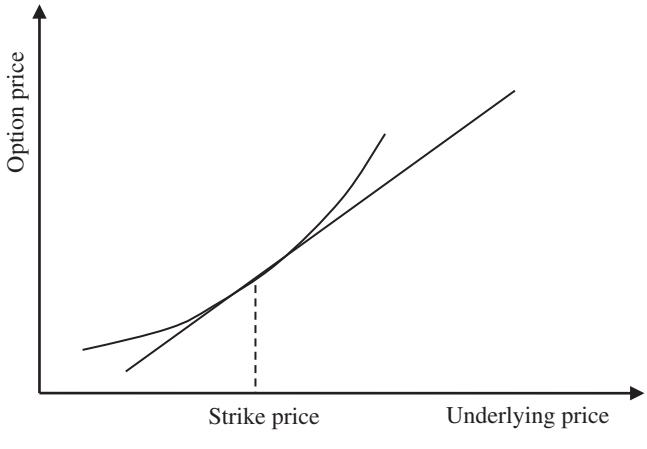


Figure 2.3 Option convexity

instruments are also called warrants but are actually options; for example, a right to acquire securities which is exercisable against someone other than the original issuer of the securities, is often called a covered warrant.

Convertible bonds

A convertible bond is a type of bond that can be converted into stock of the issuing company. In effect a convertible is a bond with an embedded long call option and from the perspective of performance measurement and attribution analysis it is appropriate to separate the two types of exposure if consistent with the investment decision process.

Buyers of convertible bonds might be seeking to arbitrage by selling the underlying security and to take advantage of pricing anomalies or in a similar way might be seeking to synthetically create the underlying security by buying the convertible but holding an appropriate amount of equitised cash to replicate the overall exposure of the desired asset at a cheaper price. This equitised cash, which may or may not be adjusted in line with the delta of the embedded option in the convertible, should be included as appropriate notional equity and notional (negative) cash assets for the purposes of attribution.

The participation rate of a convertible bond unlike delta describes the entire instrument's exposure to price changes in the underlying asset. Because much of the combined valuation is derived from the bond element it's likely that the upside participation will be greater than the downside participation, for example 75% upside capture but only 65% participation on the downside.

Contracts for difference (CFDs)

A contract for difference is an agreement between two parties, buyer and seller, in which the buyer will pay the seller the difference in value of a reference asset between the current value and its value at contract time (if the value is negative the seller will pay the buyer). CFDs avoid stamp duty, provide an opportunity for leverage and are less transparent, allowing the building of large virtual positions in an individual security.

Overlay strategies

An overlay strategy is carried out separately from the underlying (or underlaid) portfolio. Overlay strategies are typically designed, using derivative instruments, to limit the risk exposures of underlying portfolios or to add value through tactical changes to the underlying portfolio's asset allocation.

CURRENCY

Currency is not strictly an asset class in its own right. International portfolios are naturally exposed to currency; this exposure can be hedged, or currency asset managers might seek to add value. The ability to add value from currency exposures independent of the management of the underlying assets exists thanks to the depth, liquidity and flexibility of the forward currency market. Forward currency contracts may be managed independently in conjunction with the underlying portfolio or separately in a currency overlay account.

HEDGE FUNDS

Hedge funds are not an asset class; rather they are an investment vehicle that has the ability to use leverage and to take both long and short positions, with the aim of achieving an absolute return. Ineichen defines a hedge fund as follows:

> *A hedge fund constitutes an investment program whereby the managers or partners seek absolute returns by exploiting investment opportunities while protecting principal from potential financial loss.*[17]

This definition captures the asymmetric nature of hedge funds, seeking the upside whilst attempting to avoid the downside.

As of 2021, the size of globally managed hedge funds was $4.8 trillion,[18] plus an additional $0.4 trillion in hedge funds of funds.

ASSET ALLOCATION

Strategic asset allocation

Strategic asset allocation is the process in which asset owners periodically determine the asset allocation of various asset classes. These allocations will be based on liabilities, time horizons, risk tolerance, liquidity requirements and investment objectives. Strategies that include different asset classes might be described as balanced.[19]

Risk, return and liquidity characteristics are crudely illustrated in Figure 2.4.

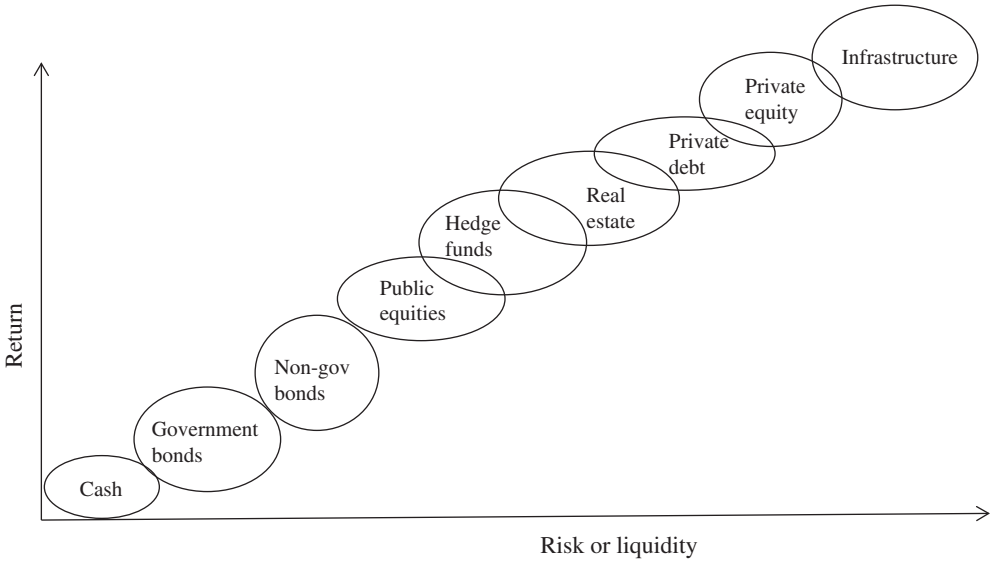


Figure 2.4 Asset classes (risk, return and liquidity)

[17] Ineichen, *Asymmetric Returns* (2003).
[18] *Source:* Barclayhedge.
[19] A balanced strategy or balanced portfolio contains two or more (not counting cash) asset classes.

Tactical asset allocation

Tactical asset allocation is the process by which asset managers seek to add value for asset owners by moving away from the strategic asset allocation (policy- or liability-driven benchmark) for periods of time to take advantage of valuation anomalies across the different asset classes, within limits defined by the asset owner.

3

The Mathematics of Portfolio Return

Time is your friend; impulse is your enemy. Take advantage of compound interest and don't be captivated by the siren song of the market.

Warren Buffett (1930–)

Mathematics is the gate and key of the sciences. … Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world.

Roger Bacon, "Doctor Mirabilis" (1214–1294), *Opus Majus*

The foundations of portfolio return calculation were laid in the 1960s and early 1970s by Dietz,[1] the Bank Administration Institute (BAI)[2] and the UK's Society of Investment Analysts (SIA).[3] All three of these important papers relate to the performance of pension funds. The calculation of portfolio return is the first step in the performance measurement process. We calculate portfolio returns because, fundamentally, the calculation of gain and loss in cash terms, in isolation, is insufficient, it doesn't allow for a fair comparison, and it doesn't allow for the assessment of the quality of performance. A $10,000 profit on a capital base of $100,000 is quite different from a $10,000 profit on a capital base of $1,000,000. Return calculations adjust for the capital base, and thus allow for a fair comparison against:

(1) Expectations
(2) History
(3) Benchmarks
(4) Other asset managers

Over the decades a variety of different return methodologies have evolved; the methodology employed will depend on data availability, data quality, accuracy required, operational constraints, investment objectives, asset type and asset owner requirements. Typically, there will always be an element of compromise.

Naturally returns are calculated for whole portfolios, components of portfolios, individual securities and groups of portfolios. This chapter will cover each in turn.

SIMPLE RETURN

In measuring the return of a "portfolio" or collection of investment assets we are concerned with the increase or decrease in the value of those assets over a specific time period; in other words, the change in "wealth".

This change in wealth can be expressed either as a "wealth ratio" or a "rate of return".

[1] Dietz, *Pension Funds: Measuring Investment Performance* (1966).
[2] BAI, "Measuring the Investment Performance of Pension Funds for the Purpose of Inter-Fund Comparison" (1968).
[3] SIA, "The Measurement of Portfolio Performance for Pension Funds" (1972).

The wealth ratio describes the ratio of the end value of the portfolio relative to the start value, mathematically:

$$\frac{V_E}{V_S} \tag{3.1}$$

where: V_E = the end value of the portfolio
V_S = the start value of the portfolio

A wealth ratio greater than 1.0 indicates an increase in value; a wealth ratio less than 1.0 a decrease in value.

Starting with a simple example, take a portfolio valued at £100 million initially and valued at £112 million at the end of the period. The wealth ratio is calculated in Exhibit 3.1.

Exhibit 3.1 Wealth ratio

$$\frac{112}{100} = 1.12$$

The value of a portfolio of assets is not always easy to obtain but should represent a reasonable estimate of the current economic value of the assets. Firms should ensure that internal valuation policies are in place, well documented and consistently applied over time. A change in valuation policy may generate spurious return differences over a specific time period.

Economic value implies that the traded market value, rather than the settled market value, of the portfolio should be used. For example, if an individual security has been bought but the trade has not been settled (i.e., paid for) then the portfolio is economically exposed to any change in price of that security. Similarly, any dividend declared and not yet paid, or interest accrued on a fixed income asset, is an entitlement of the portfolio and should be included in the valuation. Since it is a potential asset of the portfolio any reclaimable withholding tax should also be accrued in the market value. Although it may take some time before any withholding tax is recovered, it should remain in the market value until either it is recovered or written off thus capturing return in the appropriate period.

The rate of return, denoted r, describes the gain (or loss) in value of the portfolio relative to the starting value, mathematically:

$$r = \frac{V_E - V_S}{V_S} \tag{3.2}$$

Rewriting Equation 3.2:

$$r = \frac{V_E}{V_S} - \frac{V_S}{V_S} - \frac{V_E}{V_S} - 1 \tag{3.3}$$

Using the previous example, the rate of return is calculated in Exhibit 3.2.

Exhibit 3.2 Rate of return

$$\frac{112}{100} - 1 = 12\%$$

Equation 3.3 can be conveniently rewritten as:

$$1 + r = \frac{V_E}{V_S} \tag{3.4}$$

Hence, the wealth ratio is actually the rate of return plus 1.

If there are no "external cash flows" it is easy to show that the rate of return for the entire period is the "compounded return" over multiple sub-periods.

Let V_t equal the value of the portfolio after the end of period t then:

$$\frac{V_1}{V_S} \times \frac{V_2}{V_1} \times \frac{V_3}{V_2} \times \cdots \frac{V_{n-1}}{V_{n-2}} \times \frac{V_E}{V_{n-1}} = \frac{V_E}{V_S} = 1 + r \tag{3.5}$$

External cash flow is defined as any new money added to or taken from the portfolio, whether in the form of cash or other assets. Dividend and coupon payments, purchases and sales, and corporate transactions funded from within the portfolio are not considered external cash flows; rather, they are described as internal cash flows. Income from a security or stock lending programme initiated by the asset manager is not considered to be external cash flow. If initiated by the asset owner such income should be treated as external cash flow; the return (or the risk for that matter) does not belong to the asset manager.

Substituting Equation 3.4 into Equation 3.5 we establish Equation 3.6:

$$(1 + r_1) \times (1 + r_2) \times (1 + r_3) \times \cdots (1 + r_{n-1}) \times (1 + r_n) = (1 + r) \tag{3.6}$$

This process, demonstrated in Exhibit 3.3, of compounding a series of sub-period returns to calculate the entire period return is called "geometric" or "chain-linking".

Exhibit 3.3 Chain linking

		Market value (£m)	Return (%)
Start value	V_S	100	
End of period 1	V_1	112	12.0
End of period 2	V_2	95	−15.18
End of period 3	V_3	99	4.21
End of period 4	V_4	107	8.08
End value	V_E	115	7.48

$$\frac{112}{100} \times \frac{95}{112} \times \frac{99}{95} \times \frac{107}{99} \times \frac{115}{107} = 1.15$$

$$1.12 \times 0.8482 \times 1.0421 \times 1.0808 \times 1.0748 = 1.15$$

Therefore $r = 15.0\%$

CONTINUOUSLY COMPOUNDED (OR LOGARITHMIC) RETURNS

Simple or discrete returns are not symmetrical; positive simple returns are simply not equivalent in absolute impact to negative simple returns of the same absolute size. For example, if a positive return of 10% is followed by a negative return of 10% the combined return over both periods is $(1.1 \times 0.9) - 1 = -1.0\%$, not 0.0%. In other words, and somewhat counter-intuitively, if a portfolio increases by a given return and then decreases by that same return, it does *not* return to its starting value. This is because the second "equivalent" return starts from a higher absolute base and therefore does not result in the same absolute change.

We observe from the operation of our bank accounts that interest paid into our account compounds over time; in other words, we receive interest on our interest payments. The more frequent the payments the higher the compounded return at the end of the year.

For example, to achieve an equivalent rate of 12% in one year we need only obtain a rate of return in each half-year period of 5.83% (1.0583 × 1.0583 = 1.12). The nominal rate of return in each half-year period required to achieve an effective rate of return of 12% is therefore 5.83% × 2 = 11.66%.

For n periods in the year, we can calculate the effective return r as follows:

$$r = \left(1 + \frac{\dot{r}}{n}\right)^n - 1 \tag{3.7}$$

where $\dot{r}$ = nominal return.

The nominal rate of return in each monthly period required to achieve an effective rate of return of 12% is 11.387%.

If we continue to break down the periods into smaller and smaller periods, eventually we find the continuously compounded return or, in effect, the "force of return":

$$1 + r = \lim_{n \to \infty} \left(1 + \frac{\dot{r}}{n}\right)^n = e^{\dot{r}} \tag{3.8}$$

where e is the base of the natural logarithm, a constant approximately equal to 2.718.[4] Extending Equation 3.8:

$$1 + r = e^{\dot{r}} = \frac{V_E}{V_S} \tag{3.9}$$

Therefore, the end value can be expressed as an exponential function of return and the start value of the portfolio. The exponential function has several desirable mathematical properties that make it a logical choice for quants.[5]

$$V_E = V_S \times e^{\dot{r}} \tag{3.10}$$

and

$$\dot{r} = \ln(1 + r) \tag{3.11}$$

The continuously compounded return required to achieve an effective return of 12% is therefore the natural logarithm of the wealth ratio: ln(1.12) = 11.333%.

Nominal rates at different compounding frequencies that give the same effective rate of 12.0% are shown in Table 3.1.

The main advantage of continuously compounded returns is that they are additive. The total return can be calculated as follows:

$$\ln(1 + r) = \ln(1 + r_1) + \ln(1 + r_2) + \cdots + (1 + r_n) \tag{3.12}$$

Continuously compounded returns have the advantage of being:

(1) Additive (over multiple periods)
(2) Symmetric
(3) Normally distributed

[4] Euler's number discovered in 1683 by Bernoulli while studying compound interest.
[5] An expert in the use of mathematical models and statistics.

Table 3.1 Compounding frequencies

Compounding frequency	Nominal rate (%)
Annual	12.000
Semi-annual	11.660
Quarterly	11.495
Monthly	11.387
Weekly	11.345
Daily	11.335
Continuously compounded	11.333

Simple or discrete returns have the advantage of:

(1) Being simple to calculate
(2) Allowing aggregation across the portfolio
(3) Being widely understood and used by both asset owners and asset managers

Hudert, Schmitt and von Thanden usefully explore the methodical, practical and regulatory issues of using continuously compounded returns in some detail.[6]

MONEY-WEIGHTED RETURNS (MWRs)

Unfortunately, in the event of external cash flows we cannot continue to use the ratio of market values to calculate wealth ratios and hence rates of return. The cash flow itself will contribute to the valuation. Therefore, we must develop alternative methodologies that adjust for external cash flow.

Note

External cash flow represents capital (cash or assets) that enters or exits a portfolio. Internal cash flow includes dividends and coupon payments retained within the portfolio, corporate actions and normal transactions within the portfolio.

Internal rate of return (IRR)

To allow for external cash flow we can borrow a methodology used throughout finance, the "internal rate of return" or IRR.

The internal rate of return has been used for many decades to assess the value of capital investment or other business ventures over the future lifetime of a project. John Maynard Keynes,[7] the famous economist, first suggested the use of IRR to justify an investment in the context of market comparison.[8]

[6] Hudert, Schmitt and von Thanden, "Portfolio Performance Evaluation: What Difference Do Logarithmic Returns Make?" (2018/2019).

[7] Keynes, *The General Theory of Employment, Interest and Money* (1936).

[8] Magni, "Average Internal Rate of Return and Investment Decisions: A New Perspective" (2010).

Note

It is perhaps less well known that Keynes was a hugely successful institutional investor, managing the Kings College Cambridge Endowment fund between 1921 and his death in 1946, achieving an annualised outperformance of 8% and influencing many investment management giants such as Warren Buffett, George Soros and David Swensen.[9]

Normally the initial outlay, estimated costs and expected gains of a project are well known and the internal rate of return can be calculated to determine whether the investment is worth undertaking. IRR is also used to calculate the future rate of return on a bond, which is called the yield to redemption.

Ex-ante internal rate of return

The formal definition of the IRR is the discount rate that makes the net present value equal to zero in a discounted cash flow analysis. Used in this context, this return calculation methodology is best described as an ex-ante IRR, primarily used to aid imminent investment decisions. It's not difficult to understand why actuaries based at insurance companies and pension funds concerned about forecast actuarial return might turn to IRR to measure portfolio returns ex-post.

Caution

If you trawl through published financial literature on the internal rate of return it will undoubtedly reference the ex-ante IRR.

There are many criticisms of IRR as a method, including:

(1) It is difficult to calculate, requiring an iterative approach
(2) Its multiple potential solutions
(3) Its constant rate of return assumption
(4) Its assumption that future cash flows will be reinvested in the continuing rate of return
(5) The opportunity for the return calculation to be manipulated

Today, spreadsheets offer easy solutions to IRR calculations. However, historically solutions were more difficult to obtain; there were no direct solutions and time-consuming iterative methods had to be employed. Controversy surrounds the authorship of early iterative methods, in particular the Newton[10]–Raphson method, which perhaps should be attributed to Thomas

[9] Chambers and Dimson , "John Maynard Keynes, Investment Innovator" (2013).

[10] Named after the famous physicist Sir Isaac Newton. Newton may have been a great physicist and Master of the Royal Mint, but he was not such a great investor. In 1720, Newton, carried away with profits made by colleagues, invested much of his personal fortune in the South Sea Company. This company was responsible for perhaps the world's first financial crash, in the Autumn of 1720. Newton lost close to £40 million in today's money. Newton forbade anyone to ever utter the words "South Sea" in his presence and famously complained he "could calculate the motions of the heavenly bodies, but not the madness of people".

Simpson.[11] Of course, ultimately all iterative methods will achieve the same answer, just with more or less efficiency.

Note

Excel® offers two IRR functions, IRR and XIRR (extended IRR). The IRR function assumes regular periods; for most performance return calculations the XIRR function will be more appropriate.

Note

Performance analysts analyse historic, ex-post performance and are therefore less concerned about ex-ante IRR and more concerned about the calculation of ex-post IRR.

Multiple potential solutions are indeed possible,[12] but you need to work very hard, with quite extreme data, to generate suitable examples. Potential solutions are technically possible for each change in sign of cash flow data over a period of analysis but are extremely unlikely to be experienced in practice.

The assumption of a constant force of return throughout the period of measurement is a real problem. We know markets are volatile, therefore we know this assumption is not true. For the ex-ante assessment of a future project or investment, a constant force of return assumption is appropriate, but for ex-post analysis of asset performance it is much more problematic. This assumption also means we cannot disaggregate the IRR into different asset categories since we cannot continue to use a single constant rate for each asset category. This is a major problem for attribution analysis.

The reinvesting of future cash flows in the continuing rate of return is a problem specifically for ex-ante IRR and not ex-post IRR. Ex-post analysis of a self-contained portfolio of assets or a single asset need not be concerned about how ex-post cash flows have been invested. Even the reinvestment assumption in ex-ante IRR can be challenged – see the work of Magni and Martin.[13]

The manipulation of IRRs is a controversial area. Clearly delaying capital calls and returning capital to asset owners earlier will enhance the IRR (some might describe controlling the timing of cash flow in this manner good management, not manipulation). However, to the extent that the asset manager has control over the timing of cash flows, the opportunity to manipulate returns exists. Phalippou suggests:

> IRR is probably the worst performance metric one may use in an investment context. It exaggerates the variation across funds, exaggerates the performance of the best funds, can be readily inflated, and provide perverse incentives to fund managers.[14]

[11] Kollerstrom, "Thomas Simpson and Newton's Method of Approximation: An Enduring Myth" (1992).
[12] Fischer and Wermers, *Performance Evaluation and Attribution of Security Portfolios* (2013), p. 340.
[13] Magni and Martin, "The Reinvestment Assumption Fallacy for IRR and NPV: A Pedagogical Note" (December 2017).
[14] Phalippou, "The Hazards of Using IRR to Measure Performance" (2008).

Simple internal rate of return

For a single period, the IRR method in its most simple form requires that a return r be found that satisfies the following equation:

$$V_E = V_S \times (1 + r) + C \times (1 + r)^{0.5} \tag{3.13}$$

where C = external cash flow.

In this form we are assuming that all cash flows are received at the mid-point of the period under analysis. To calculate the simple IRR, we need only the start and end market values and the total external cash flow, as shown in Exhibit 3.4.

Exhibit 3.4 Simple IRR

Market start value	\$74.2m
Market end value	\$104.4m
External cash flow	\$37.1m

$$104.4 = 74.2 \times (1 + r) + 37.1 \times (1 + r)^{0.5}$$

We can see that $r = -7.41\%$ satisfies the above equation.

$$74.2 \times (0.9259) + 37.1 \times (0.9259)^{0.5} = 104.4$$

Ex-post internal rate of return

Assuming that all cash flows are received midway through the period of analysis is a fairly crude estimate. The mid-point assumption can be modified for all cash flows to adjust for the fraction of the period of measurement that the cash flow is available for investment as follows:

$$V_E = V_S \times (1 + r) + \sum_{t=1}^{t=T} C_t \times (1 + r)^{W_t} \tag{3.14}$$

where: C_t = the external cash flow on day t
W_t = weighting ratio to be applied on day t

Obviously, there will be no external cash flow for most days:

$$W_t = \frac{TD - D_t}{TD} \tag{3.15}$$

where: TD = total number of days within the period of measurement
D_t = number of days since the beginning of the period, including weekends and public holidays

In addition to the information in Exhibit 3.4 to calculate the ex-post internal rate of return shown in Exhibit 3.5, we need to know the date of the cash flow and the length of the period of analysis.

IRR is an example of a money-weighted return methodology; each amount or dollar invested is assumed to achieve the same effective rate of return irrespective of when it was invested.

Exhibit 3.5 Ex-post IRR

Market start value	31 December	\$74.2m
Market end value	31 January	\$104.4m
External cash flow	14 January	\$37.1m

Assuming the cash flow is at the end of day 14:

$$104.4 = 74.2 \times (1 + r) + 37.1 \times (1 + r)^{\frac{17}{31}}$$

We can see that $r = -7.27\%$ satisfies the above equation.

$$74.2 \times (0.9273) + 37.1 \times (0.9273)^{\frac{17}{31}} = 104.4$$

In the US the term "dollar-weighted" rather than "money-weighted" may be used; however, the term "money-weighted" is increasingly common in the US and is used worldwide.

Note

The weight of money invested at any point of time will ultimately impact the final return calculation. Therefore, if using this methodology, it is important to perform well when the amount of money invested is largest.

For the simple internal rate of return only, the solution of the quadratic equation can be used:

$$\text{Quadratic formula } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{3.16}$$

where $ax^2 + bx + c = 0$

Let $x = (1 + r)^{0.5}$
Using data from Exhibit 3.4 in Exhibit 3.6:

Exhibit 3.6 Simple IRR – using quadratic formula

Market start value	\$74.2m
Market end value	\$104.4m
External cash flow	\$37.1m

$$74.2 \times (1 + r) + 37.1 \times (1 + r)^{0.5} - 104.4 = 0$$

$$(1 + r)^{0.5} = \frac{-37.1 \pm \sqrt{37.1^2 - 4 \times 74.2 \times -104.4}}{2 \times 74.2} = \frac{-37.1 + 179.895}{148.4} = 0.96223$$

Therefore $r = -7.41\%$, the same as in Exhibit 3.4.

Simple Dietz

Even in its simple form the internal rate of return is not a particularly practical calculation, especially over longer periods with multiple cash flows. Peter Dietz[15] suggested as an alternative the following simple adaptation of Equation 3.2 to adjust for external cash flow:

$$r = \frac{V_E - V_S - C}{V_S + {}^{C}/_{2}} \tag{3.17}$$

This is now known as the simple (or original) Dietz method.

The numerator of Equation 3.17 represents the investment gain (or loss) in the portfolio. In the denominator, replacing the initial market value we now use the average capital invested represented by the initial market value plus half the external cash flow. An assumption has been made that the external cash flow is invested midway through the period of analysis and has been weighted accordingly.

⚠ Caution

The average capital invested is absolutely not the average of the start and end values which would factor in an element of portfolio return into the denominator.

To calculate a simple Dietz return, like the simple IRR, only the start market value, end market value and total external cash flow are required, as shown in Exhibit 3.7.

Exhibit 3.7 Simple Dietz

Using the existing example data:

Market start value	\$74.2m
Market end value	\$104.4m
External cash flow	\$37.1m

The simple Dietz rate of return is:

$$\frac{104.4 - 74.2 - 37.1}{74.2 + {}^{37.1}/_{2}} = \frac{-6.9}{92.75} = -7.44\%$$

Dietz originally described his method as assuming that one half of the net contributions are made at the beginning of the time interval and one half at the end of the time interval:

$$r = \frac{V_E - {}^{C}/_{2}}{V_S + {}^{C}/_{2}} - 1 \tag{3.18}$$

This simplifies to the more common description:

$$r = \frac{V_E - {}^{C}/_{2}}{V_S + {}^{C}/_{2}} - \frac{V_S + {}^{C}/_{2}}{V_S + {}^{C}/_{2}} = \frac{V_E - V_S - C}{V_S + {}^{C}/_{2}} \tag{3.19}$$

[15] Dietz, *Pension Funds: Measuring Investment Performance* (1966).

The Dietz method is easier to calculate and easier to visualise than the IRR method, and it can also be disaggregated, that is to say, the total return is the sum of the individual parts.

ICAA method

The Investment Counsel Association of America (ICAA) proposed a straightforward extension of the simple Dietz method[16] as follows:

$$r = \frac{V'_E - V_S - C' + I}{V_S + \dfrac{C'}{2}} \tag{3.20}$$

where: I = total portfolio income
C' = external cash flow including any reinvested income
V'_E = market end value including any reinvested income

The standard example is extended to the ICAA method in Exhibit 3.8.

Exhibit 3.8 ICAA method

Market start value	\$74.2m
Market end value	\$104.2m
External cash flow	\$37.1m
Total income	\$0.4m
Income reinvested	\$0.2m

$$\frac{104.2 - 74.2 - (37.1 + 0.2) + 0.4}{74.2 + {}^{(37.1+0.2)}/_{2}} = \frac{-6.9}{92.85} = -7.43\%$$

In this method, income (equity dividends, interest or coupon payments) is not automatically assumed to be available for reinvestment. The gain in the numerator is appropriately adjusted for any reinvested income included in the final value by including reinvested income in the definition of external cash flow.

Interestingly, although the average capital is increased for any reinvested income in the denominator there is no negative adjustment for any income not reinvested. This is perhaps not unreasonable from the perspective of the client if the income is retained and not paid until the end of period.

However, from the asset manager's viewpoint, if this income is not available for reinvestment it should be treated as a negative cash flow as follows:

$$r = \frac{V_E - V_S - C + I}{V_S + \dfrac{(C - I)}{2}} \tag{3.21}$$

[16] Investment Counsel Association of America, "The Standards of Measurement and Use for Investment Performance Data" (1971).

Extending our previous example again in Exhibit 3.9:

Exhibit 3.9 Income unavailable

Market start value	\$74.2m
Market end value	\$104.0m
External cash flow	\$37.1m
Total income	\$0.4m

$$\frac{104.0 - 74.2 - 37.1 + 0.4}{74.2 + {}^{(37.1-0.4)}/_{2}} = \frac{-6.9}{92.55} = -7.46\%$$

In Equation 3.21 any income received by the portfolio is assumed to be unavailable for investment by the asset manager and transferred to a separate income account for later payment or alternatively paid directly to the client.

Obviously, income paid or transferred is no longer included in the final value, V_E, of the portfolio. In effect, in this methodology income is treated as negative cash flow. Since income is normally always positive, this method has the effect of reducing the average capital employed, decreasing the size of the denominator, and thus leveraging (or gearing) the final rate of return.

Consequently, this method should only be used if portfolio income is genuinely unavailable to the asset manager for further investment. Typically, this method is used to calculate the return of an asset category (sector or component) within a portfolio.

Modified Dietz

Assuming that all cash flows are received midway through the period of analysis is a fairly crude estimate. The simple Dietz method can be further modified by day-weighting each cash flow by the following formula to establish a more accurate average capital employed.

$$r = \frac{V_E - V_S - C}{V_S + \sum C_t \times W_t} \tag{3.22}$$

where: C = total external cash flow within period
C_t = external cash flow on day t
W_t = weighting ratio to be applied to external cash flow on day t

$$W_t = \frac{TD - D_t}{TD} \text{ (Equation 3.15, from earlier)}$$

where: TD = total number of days within the period of measurement
D_t = number of days since the beginning of the period, including weekends and public holidays

In determining D_t the performance analyst must establish if the cash flow is received at the beginning or end of the day. If the cash flow is received at the start of the day, then it is reasonable to assume that the portfolio manager is aware of the cash flow and is able to respond to it; therefore it is reasonable to include this day in the weighting calculation. On the other hand, if the cash flow is received at the end of the day the portfolio manager is unable to take any action at that point and therefore it is unreasonable to include the current day in the weighting calculation.

For example, take a cash flow received on the 14th day of a 31-day month. If the cash flow is at the start of the day, then there are 18 full days including the 14th day available for investment and the weighting factor for this cash flow should be (31 – 13)/31. Alternatively, if the cash flow is at the end of the day, then there are 17 full days remaining and the weighting factors should be (31 – 14)/31. Day counts include weekends and public holidays.

Performance analysts should determine a company policy and apply this consistently to all cash flows.

Fisher and Wermers[17] demonstrate that the modified Dietz method is, in fact, a first-order linear approximation to the IRR. A second-order approximation is available, but is more complex to calculate, less practical to use, and the modified Dietz method is already a good approximation. The Dietz methods are therefore both money-weighted methodologies.

⚠ Caution

The modified Dietz method is sometimes erroneously described as an approximation of the time-weighted rate of return. The modified Dietz method is a money-weighted rate of return and can be accurately described as an approximation of the internal rate of return.

The modified Dietz return is calculated using our standard example data in Exhibit 3.10.

Exhibit 3.10 Modified Dietz

Market start value	31 December	\$74.2m
Market end value	31 January	\$104.4m
External cash flow	14 January	\$37.1m

Assuming the cash flow is at the end of day 14:

$$\frac{104.4 - 74.2 - 37.1}{74.2 + \frac{(31-14)}{31} \times 37.1} = \frac{-6.9}{94.55} = -7.30\%$$

Assuming the cash flow is at the beginning of day 14 with 18 full days in the month left:

$$\frac{104.4 - 74.2 - 37.1}{74.2 + \frac{(31-13)}{31} \times 37.1} = \frac{-6.9}{95.74} = -7.21\%$$

TIME-WEIGHTED RETURNS (TWRs)

True time-weighted

Time-weighted rate of returns provide a popular alternative to money-weighted returns in which each time period is given equal weight regardless of the amount invested.

[17] Fischer and Wermers, "Performance Evaluation and Attribution of Security Portfolios" (2013).

In the "true or classical time-weighted" methodology, performance is calculated for each sub-period between cash flows using simple wealth ratios. The sub-period returns are then chain-linked as follows:

$$\frac{V_1 - C_1}{V_S} \times \frac{V_2 - C_2}{V_1} \times \frac{V_3 - C_3}{V_2} \times \cdots \frac{V_{n-1} - C_{n-1}}{V_{n-2}} \times \frac{V_E - C_n}{V_{n-1}} = 1 + r \tag{3.23}$$

where: V_t is the valuation immediately after the cash flow C_t at the end of period t.

Since $\frac{V_t - C_t}{V_{t-1}} = 1 + r_t$ is the wealth ratio immediately prior to receiving the external cash flow, Equation 3.23 simplifies to the familiar Equation 3.6 from before:

$$(1 + r_1) \times (1 + r_2) \times (1 + r_3) \times \cdots (1 + r_{n-1}) \times (1 + r_n) = (1 + r)$$

In Equation 3.23 we have assumed that any cash flow is only available for the asset manager to invest at the end of the day. If we assume that the cash flow is available from the beginning of the day, we must change Equation 3.23 to:

$$\frac{V_1}{V_S + C_1} \times \frac{V_2}{V_1 + C_2} \times \frac{V_3}{V_2 + C_3} \times \cdots \frac{V_{n-1}}{V_{n-2} + C_{n-1}} \times \frac{V_E}{V_{n-1} + C_n} = 1 + r \tag{3.24}$$

Alternatively, we may wish to assume that the cash flow is available for investment midday and use a half-weight assumption as follows:

$$\frac{V_1 - \frac{C_1}{2}}{V_S + \frac{C_1}{2}} \times \frac{V_2 - \frac{C_2}{2}}{V_1 + \frac{C_2}{2}} \times \cdots \times \frac{V_E - \frac{C_n}{2}}{V_{n-1} + \frac{C_n}{2}} = 1 + r \tag{3.25}$$

Note from Equation 3.19 that:

$$r_t = \frac{V_t - V_{t-1} - C_t}{V_{t-1} + \frac{C_t}{2}} = \frac{V_t - \frac{C_t}{2}}{V_{t-1} + \frac{C_t}{2}} - 1$$

Equation 3.25 is really a hybrid methodology, combining both time-weighting and a money-weighted return for each individual day, and therefore ceases to be a true time-weighted rate of return.

Note

The terms time-weighted and money-weighted are used in the original BAI paper from 1968 and were probably in common use prior to that. Money-weighted is easy to understand, but the term time-weighted is more difficult – what exactly is time-weighted? In modern use the term *time*-weighted is used to describe weighting each time period equally, but historically time-weighted most likely refers to annualised continuously compounded or logarithmic returns.

For example, using Equation 3.10, $V_E = V_S \times e^{\dot{r}}$.

Assume, over a 10-year period, annualised continuously compounded returns of 4.5% for 2 years, 5.5% for 3 years and 6.5% for 5 years.

The 10-year continuously compounded return is derived from:

$$V_E = V_S \times e^{4.5\% \times \frac{2}{10}} \times e^{5.5\% \times \frac{3}{10}} \times e^{6.5\% \times \frac{5}{10}}$$

$$V_E = V_S \times e^{\left(\frac{4.5\% \times 2 + 5.5\% \times 3 + 6.5\% \times 5}{10}\right)}$$

The 10-year annualised continuously compounded return is the *time-weighted* average of:

$$\frac{(4.5\% \times 2 + 5.5\% \times 3 + 6.5\% \times 5)}{10} = 5.8\%$$

The continuously compounded returns have in effect been time-weighted.

Using our standard example data, we now need to know the value of the portfolio immediately after the cash flow as shown in Exhibits 3.11, 3.12 and 3.13.

The difference in return between Exhibits 3.11, 3.12 and 3.13 are quite shocking: the timing assumption of start, end or midday cash flow appears to have made a return difference of close to 0.5%. The reason for this difference is quite simple to explain – it depends entirely on the size of the denominator in each period. Obviously, the loss in the portfolio is the same for all exhibits; the return difference results entirely from the allocation of this loss into each return

Exhibit 3.11 True time-weighted (end of day cash flow)

End of day cash flow assumption:

Market start value	31 December	\$74.2m
Market end value	31 January	\$104.4m
External cash flow	14 January	\$37.1m
Market value end of	14 January	\$103.1m

$$\frac{103.1 - 37.1}{74.2} \times \frac{104.4}{103.1} - 1 = 0.8895 \times 1.0126 - 1 = -9.93\%$$

Exhibit 3.12 True time-weighted (start of day cash flow)

Start of day cash flow assumption:

Market start value	31 December	\$74.2m
Market end value	31 January	\$104.4m
External cash flow	14 January	\$37.1m
Market value start of	14 January	\$67.0m

$$\frac{67.0}{74.2} \times \frac{104.4}{67.0 + 37.1} - 1 = 0.9030 \times 1.0029 - 1 = -9.44\%$$

Exhibit 3.13 Time-weighted (midday cash flow)

Midday cash flow assumption:

Market start value	31 December	\$74.2m
Market end value	31 January	\$104.4m
External cash flow	14 January	\$37.1m
Market value start of	14 January	\$67.0m
Market value end of	14 January	\$103.1m

$$\frac{67.0}{74.2} \times \frac{103.1 - \frac{37.1}{2}}{67.0 + \frac{37.1}{2}} \times \frac{104.4}{103.1} - 1 = 0.9030 \times 0.9883 \times 1.0126 - 1 = -9.63\%$$

period, subsequently divided by a denominator. The end of day assumption in Exhibit 3.11 results in more of the loss being allocated to the first sub-period.

Unit price method

The "unit price" or "unitised" method is a useful variant of the true time-weighted methodology. Rather than use the ratio of market values between cash flows, a standardised unit price or "net asset value" price is calculated immediately before each external cash flow by dividing the market value by the number of units previously allocated. Units are then added or subtracted (bought or sold) in the portfolio at the unit price corresponding to the time of the cash flow – the unit price is in effect a normalised market value.

Unit prices can be calculated for any type of portfolio but are typically calculated for "pooled funds". Pooled funds are investment vehicles that have multiple investors, including unit trusts, mutual funds, pooled pension funds, collective funds, hedge funds and ETFs. Clearly investors will want to enter and exit these funds at different times and unit prices offer a mechanism to allocate the value of the assets of the portfolio fairly.

The starting value of the portfolio is also allocated to units, often using a notional starting unit price of, say, 1 or 100.

The main advantage of the unit price method is that the ratio of the end of period unit price with the start of period unit price always provides the rate of return irrespective of the change of value in the portfolio due to external cash flows. Therefore, to calculate the rate of return between any two points the only information you need to know is the start and end unit prices.

Let NAV_i equal the net asst value unit price of the portfolio after the end of period i. Then:

$$\frac{NAV_1}{NAV_S} \times \frac{NAV_2}{NAV_1} \times \frac{NAV_3}{NAV_2} \times \cdots \frac{NAV_{n-1}}{NAV_{n-2}} \times \frac{NAV_E}{NAV_{n-1}} = \frac{NAV_E}{NAV_S} = 1 + r \tag{3.26}$$

Note

The unitised method is so convenient for quickly calculating performance that returns calculated using other methodologies are often converted to unit prices for ease of use, particularly over longer time periods.

The unitised method is a variant of the true or classical time-weighted return and will always give the same answer, as can be seen in Exhibit 3.14.

Exhibit 3.14 Unit price method

		$m
Market value of portfolio at start of period	(31/12)	74.2
Market value of portfolio at end of period	(31/01)	104.4
Cash flow at end of day 14	(14/01)	37.1
Market value of portfolio immediately prior to cash flow (14/01)		66.0
Emerging market index return in January		−7.92%
Index return 31/12 to 14/01		−10.68%
Index return 14/01 to 31/01		+3.09%

	Valuation	Unit price	Units allocated	Total units
Start value	74.2	1.0000	74.20	74.2
Valuation 14/01	66.0	0.8895		74.2
Cash flow 14/01	37.1	0.8895	41.71	115.9
End value	104.4	0.9007		115.9

$$\frac{90.07}{100.00} - 1 = -9.93\%$$

Typically, the performance of mutual funds is calculated using net asset value prices for external presentation and with a true time-weighted methodology for internal analysis. It can be a challenge to reconcile both sets of returns; the performance analyst must ensure that the internal valuations are aligned with valuations produced by the unit pricing administrator including the timing of external cash flows. Unless there are no external cash flows it will be impossible to reconcile the time-weighted unit price method with a money-weighted return. Mutual fund unit prices include various fees and expenses, not only expenses associated with asset management including transaction fees and investment management fees but also custodial and other administrations fees not necessarily under the control of the asset manager but associated with the costs of the investment vehicle.

Unit price method with distributions

The unit price method shown in Equation 3.26 works provided there are no distributions of income, and that income is retained within the pooled fund. If income is distributed, then the return calculation must allow for the assumption of reinvested distributions using an adjusted NAV unit price:

$$\text{Adjusted unit price } NAV'_n = NAV_n \times \prod_{i=1}^{i=n} \left(1 + \frac{D_i}{NAV_i}\right) \tag{3.27}$$

Pooled fund returns with distributions are shown in Exhibit 3.15.

Exhibit 3.15 Pooled fund return with distributions

Date	Distribution D_i	Unit price NAV_i	Adjustment factor $\left(1+\frac{D_i}{NAV_i}\right)$	Cumulative adjustment factor $\prod_{i=1}^{i=n}\left(1+\frac{D_i}{NAV_i}\right)$	Adjusted unit price $NAV_i \times \prod_{i=1}^{i=n}\left(1+\frac{D_i}{NAV_i}\right)$
30/06/2010		100.00			
31/03/2011	3.98	120.73	1.033	1.033	124.71
31/03/2012	4.34	117.27	1.037	1.071	125.62
31/03/2013	4.55	130.07	1.035	1.109	144.20
31/03/2014	4.69	134.13	1.035	1.147	153.91
31/03/2015	4.94	137.18	1.036	1.189	163.07
31/03/2016	5.04	126.00	1.040	1.236	155.77
31/03/2017	5.49	148.35	1.037	1.282	190.19
31/03/2018	5.92	144.29	1.041	1.335	192.58
31/03/2019	6.68	148.35	1.045	1.395	206.91
31/03/2020	4.75	115.83	1.041	1.452	168.18
31/03/2021	4.22	136.15	1.031	1.497	203.81
31/07/2021		142.25			

Cumulative return 30/06/2010 to 31/07/2021:

$$\frac{142.25 \times 1.497}{100} - 1 = \frac{212.95}{100} - 1 = 112.95\%$$

Three-year annualised return 31/03/2018 to 31/03/2021:

$$\left(\frac{136.15 \times 1.497}{144.29 \times 1.335}\right)^{(1/3)} - 1 = \left(\frac{203.81}{192.58}\right)^{(1/3)} - 1 = 1.91\%$$

Five-year annualised return 31/03/2013 to 31/03/2018

$$\left(\frac{144.29 \times 1.335}{130.07 \times 1.109}\right)^{(1/5)} - 1 = \left(\frac{192.58}{144.20}\right)^{(1/5)} - 1 = 5.96\%$$

TIME-WEIGHTED VERSUS MONEY-WEIGHTED RATES OF RETURN

Time-weighted returns measure the returns of the assets irrespective of the amount invested. This can generate counterintuitive results, as shown in Exhibit 3.16.

In Exhibit 3.16 the client has lost £400 over the entire period, yet the time-weighted return is calculated as a positive 16.67%. The money-weighted return reflects this loss, −67.7% of the average capital employed. It is important to perform well in the second period when the majority of client money is invested.

If the client had invested all the money at the beginning of the period of measurement, then a 16.67% return would have been achieved. The difference in return calculated is due to the timing of cash flow. Over a single period of measurement, the money-weighted rate of return will always reflect the cash gain and loss over the period.

With such large potential differences between methodologies, which method should be used and in what circumstances?

Exhibit 3.16 TWRs versus MWRs

Start period 1 Market value	£100
End period 1 Market value	£200
Cash flow	£1,000
Start period 2 Market value	£1,200
End period 2 Market value	£700

Time-weighted return:

$$\frac{1{,}200 - 1{,}000}{100} \times \frac{700}{1{,}200} - 1 = 16.67\%$$

Money-weighted return:

$$\frac{700 - 100 - 1{,}000}{100 + \frac{1{,}000}{2}} = -66.7\%$$

The time-weighted rate of return adjusts for cash flow and weights each time period equally, measuring the return that would have been achieved had there been no cash flows. Clearly this return is most appropriate for comparing the performance of different asset managers with different patterns of cash flows and with benchmark indexes, which, for the most part, are calculated using a time-weighted approach.

The "received wisdom" within the asset management industry is that the time-weighted rate of return measures the asset manager's performance adjusting for cash flows and the money-weighted rate of return measures the performance of the asset owner's invested assets including the impact of external cash flow. Put another way, if the asset manager does not control the timing of external cash flow, then a time-weighted return must be used; if the asset manager does control the timing of external cash flow, then a money-weighted return should be used.

This received wisdom is at least partially correct; I agree for the purpose of fair comparability that time-weighted returns should be used if the asset manager does not control the timing of external cash flow. However, if the asset manager does control the timing of external cash flow, the situation is more complex. I would suggest that it is the liquidity of the asset class that is more important in determining the choice of return methodology, rather than the control of the timing of external cash flow. The whole rationale for calculating return is to aid comparability; unfortunately, to the extent that money-weighted returns are used, comparability is damaged.

Note

Comparability is the whole point of calculating portfolio return; if comparability is not essential, then use profit and loss. If comparability is important, use time-weighted returns, if possible.

Most performance analysts would prefer time-weighted returns. By definition, time-weighted returns weight each time period equally, irrespective of the amount invested; therefore the timing of external cash flows or the amount of money invested does not affect the calculation of return. In most cases asset managers do not determine the timing of external cash flows, nor does the amount of money invested normally change the investment decision process; therefore it is desirable to use a methodology that is not impacted by the timing of cash flow.

A few performance analysts argue that from a presentational perspective, particularly when dealing with private clients, money-weighted returns are preferred since over a single period a loss always results in a negative return and a gain in a positive return. Although this is indeed true it may not be a true reflection of the asset manager's performance, and we are certain that the central assumption of money-weighted returns, that of a constant force of return, is most unlikely to be correct.

A major drawback of true time-weighted returns is that accurate valuations are required at the date of each cash flow. This is an onerous and expensive requirement for some asset managers. The asset manager must assess the benefits of increased accuracy against the costs of frequent valuations for each external cash flow and the potential for error. Asset management firms must have a daily valuation mindset to succeed with daily performance calculations. Exhibit 3.17 demonstrates the impact of a valuation error on the return calculation.

Exhibit 3.17 Valuation error

Market start value	31 December	\$74.2m
Market end value	31 January	\$104.4m
External cash flow	14 January	\$37.1m
Erroneous market value	14 January	\$101.1m

$$\frac{101.1 - 37.1}{74.2} \times \frac{104.4}{101.1} - 1 = 0.8625 \times 1.0326 - 1 = -10.93\%$$

A significant and permanent difference from the accurate time-weighted return of −9.93% calculated in Exhibit 3.11.

Not unreasonably, institutional clients such as large pension funds paying significant fees might expect that the asset manager has sufficient quality information on a daily basis to manage their portfolio accurately. Most large asset managers will also have mutual or other pooled funds in their stable which in most cases will already require daily valuations (not just at the date of each external cash flow). The industry, driven by performance presentation standards and the demand for more accurate analysis, is gradually moving to daily calculations as standard.

In terms of statistical analysis, daily calculation adds more noise than information; however, in terms of return analysis, daily calculation (or at the least valuation at each external cash flow, which practically amounts to the same thing) is essential to ensure the accuracy of long-term returns.

Note

I do not believe in the daily analysis of performance, which is far too short term for long-term investment portfolios, but I do believe in accurate returns, which require daily calculation. It is also useful for the portfolio manager or performance measurer to analyse performance between any two dates other than the standard calendar period ends.

APPROXIMATIONS TO THE TIME-WEIGHTED RETURN

Asset managers without the capability or willingness to pay the cost of achieving accurate valuations on the date of each cash flow may still wish to use a time-weighted methodology and can use methodologies that approximate to the "true" time-weighted return by estimating portfolio values on the date of cash flow, such as the following methodologies.

Index substitution

Assuming an accurate valuation is not available, an index return may be used to estimate the valuation on the date of the cash flow, thus approximating the "true" time-weighted return as demonstrated in Exhibit 3.18.

Exhibit 3.18 Index substitution

Given an assigned benchmark performance of −10.68% up to the point of cash flow and using the data from Exhibit 3.11, the estimated valuation at the date of the cash flow is:

$$74.2 \times (1 - 10.68\%) = 66.28$$

Therefore, the approximate time-weighted return is:

$$\frac{66.28}{74.2} \times \frac{104.4}{66.28 + 37.1} - 1 = 0.8932 \times 1.0099 - 1 = -9.79\%$$

In Exhibit 3.18 the index is a good estimate of the portfolio value and therefore the resultant return is a good estimate of the true time-weighted rate of return. However, if the index is a poor estimate of the portfolio value (see Exhibit 3.19), then the resultant return may be inaccurate, although in this case may be a better estimate of underlying return than, say, the modified Dietz or IRR.

Exhibit 3.19 Index substitution (poor estimate)

Using an index return of −7.90% to estimate the portfolio value at the point of cash flow:

$$74.2 \times (1 - 7.9\%) = 68.34$$

Therefore, the approximate time-weighted return is:

$$\frac{68.34}{74.2} \times \frac{104.4}{68.34 + 37.1} - 1 = 0.9210 \times 0.9901 - 1 = -8.81\%$$

Regression method (*or* β method)

The regression method is an extension of the index substitution method. A theoretically more accurate estimation of portfolio value can be calculated adjusting for the systematic risk (as represented by the portfolio's beta) shown in Exhibit 3.20.

Exhibit 3.20 Regression method

Assuming a portfolio beta of 1.05 in comparison with the benchmark, the revised estimated valuation at the time of cash flow is:

$$74.2 \times (1 - 10.68\% \times 1.05) = 65.88$$

Therefore, the approximate time-weighted return is:

$$\frac{65.88}{74.2} \times \frac{104.4}{65.88 + 37.1} - 1 = 0.8879 \times 1.3789 - 1 = -9.99\%$$

The index substitution method is only as good as the resultant estimate of portfolio value; making further assumptions about portfolio beta need not improve accuracy.

Analyst's test

A more accurate approximation was proposed by a working group of the UK's Society of Investment Analysts.[18] They demonstrated that the ratio of the money-weighted return of the portfolio against the money-weighted return of the notional fund (portfolio market values and cash flows invested in the benchmark) approximates to the ratio of the time-weighted return of the portfolio with the time-weighted return of the notional fund, mathematically:

$$\frac{(1 + MWA)}{(1 + MWN)} = \frac{V_A - (C_T - C_W)}{V_N - (C_T - C_W)} \cong \frac{(1 + TWA)}{(1 + TWN)} \tag{3.28}$$

where: MWA = money-weighted return of actual portfolio
MWN = money-weighted return of notional fund
V_A = value of portfolio at end of period
V_N = value of notional fund at end of period
C_T = total external cash flow in period
C_W = weighted external cash flow in period
TWA = time-weighted return of actual portfolio
TWN = time-weighted return of notional fund

Rearranging Equation 3.28,

$$TWA \cong \frac{(1 + MWA)}{(1 + MWN)} \times (1 + TWN) - 1 \tag{3.29}$$

[18] SIA, "The Measurement of Portfolio Performance for Pension Funds: A Report Prepared by the Working Group Set Up by the Society of Investment Analysts" (1972).

or

$$TWA \cong \frac{V_A - (C_T - C_W)}{V_N - (C_T - C_W)} \times (1 + TWN) - 1 \tag{3.30}$$

In other words, the time-weighted return of the portfolio can be approximated by the ratio of the money-weighted return of the portfolio divided by the money-weighted return of notional fund and then multiplied by the notional fund time-weighted rate of return. Since all commercial indexes are time-weighted (they don't suffer cash flows and are therefore useful for comparative purposes) we can use an index return for the time-weighted notional fund. The analyst's test method, using standard example data, is shown in Exhibit 3.21.

Exhibit 3.21 Analyst's test

Market start value	31 December	\$74.2m
Market end value	31 January	\$104.4m
External cash flow	14 January	\$37.1m
Index return in January		−7.92%
Index return 31/12 to 14/01		−10.68%
Index return 14/01 to 31/01		+3.09%

Final value of notional fund:

$$V_N = (74.2 \times (1 - 0.1068) + 37.1) \times 1.0309 = 106.57$$

$$C_T = 37.1$$

$$C_W = 37.1 \times \left(\frac{31 - 14}{31}\right) = 20.35$$

$$TWA = \frac{104.4 - (37.1 - 20.35)}{106.57 - (37.1 - 20.35)} \times (1 - 0.0792) - 1$$

$$TWA = \frac{82.86}{85.03} \times 0.9208 - 1 = -10.14\%$$

The advantage of these three approximation methods is that a time-weighted return may be estimated even without sufficient data to calculate an accurate valuation and hence an accurate time-weighted return. The disadvantages are clear: if the index, regression and notional fund assumptions respectively are incorrect or inappropriate the resultant return calculated will also be incorrect. Additionally, the actual portfolio return appears to change if a different index is applied which is counterintuitive (surely the portfolio return ought to be unique) and is very difficult to explain to asset owners.

Note

The index substitution, regression and analyst's test method are all dated and unlikely to be encountered today. They represent approximations to the time-weighted return.

HYBRID METHODOLOGIES

In practice, many asset managers use neither true time-weighted nor money-weighted calculations exclusively but rather a hybrid combination of both.

If the standard period of measurement is monthly, it is far easier and quicker to calculate the modified (or even simple) Dietz return for the month and then chain-link the resulting monthly returns. This approach treats each monthly return with equal weight and is therefore a version of time-weighting. All of the methods mentioned previously can be calculated for a specific time period and then chain-linked to create a time-weighted type of return for that time period.

Linked modified Dietz

Currently the standard approach for institutional asset managers is to chain-link monthly modified Dietz returns. Often described as a time-weighted methodology, in fact it is a hybrid chain-linked combination of monthly money-weighted returns. Each monthly time period is given equal weight and therefore time-weighted, but within the month the return is money-weighted.

BAI method (or linked IRR)

The US Bank Administration Institute[19] proposed an alternative hybrid approach,[20] which essentially links simple internal rates of return rather than linking modified Dietz returns.

Because of the difficulties in calculating internal rates of return this is not a popular method and is virtually unknown outside the US.

For clarification, both the BAI method and the linked modified Dietz methods can be described as a type of time-weighted methodology because each standard period (normally monthly) is given equal weight. True time-weighting requires the calculation of performance between each cash flow.

The index substitution, regression and analyst's test methods are approximations to the true time-weighted rate of return. The simple Dietz, modified Dietz and ICAA methods are approximations of the internal rate of return and are therefore money-weighted. Perhaps the best way to illustrate the relationship of these methodologies is the spectrum shown in Figure 3.1.

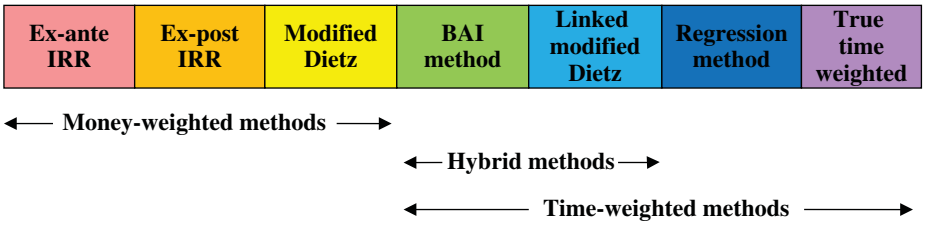


Figure 3.1 Spectrum of return methodologies

[19] Bank Administration Institute, "Measuring the Investment Performance of Pension Funds for the Purpose of Inter Fund Comparison" (1968).

[20] First proposed by Fisher, "An Algorithm for Finding Exact Rates of Return" (1966).

WHICH METHOD TO USE?

Determining which methodology to use will ultimately depend on the requirements of the asset owner, the degree of accuracy required, the type and liquidity of assets, availability of accurate valuations, operational constraints and capabilities, cost, and convenience factors.

Time-weighted returns neutralise the impact of cash flow. If the purpose of the return calculation is to measure and compare the asset manager's return against those of other managers and commercially published indexes, then time-weighting is the most appropriate. On the other hand, if there is no requirement for comparison and only the return of the asset owner's assets is to be analysed then money-weighting may be more appropriate.

As demonstrated in Exhibit 3.16 a time-weighted return, which does not depend on the amount of money invested, may lead to a positive rate of return over the period in which the asset owner may have lost money. This may be difficult to present to the ultimate asset owner, although in truth the absolute loss of money in this example is due to the asset owner giving the asset manager more money to manage prior to a period of poor performance in the markets. If there had been no cash flows the asset owner would have made money.

Confidence in the accuracy of asset valuation is crucial in determining which method to use. If accurate valuations are available only on a monthly basis, then a linked monthly modified Dietz methodology may well be the most appropriate. The liquidity of assets is also a key determinant of methodology. If securities are illiquid, it may be difficult to establish an accurate valuation at the point of cash flow. For illiquid assets such as private assets any perceived accuracy in the true time-weighted return could be quite spurious.

Internal rates of return are traditionally used for private asset categories for a number of reasons:

(1) The initial investment appraisal for non-quoted investments often uses an ex-ante IRR approach.
(2) Private assets are difficult to value accurately and are illiquid.
(3) Private asset managers often control the timing of cash flow.

It is interesting to note that, like private equity, real estate performance has been traditionally calculated using a money-weighted methodology; however, with increased frequency of property valuations (albeit not independent) real estate has strived to be treated with equivalence to other assets and more recently time-weighted approaches have become more common. In essence there is also a spectrum of liquidity; see Figure 3.2.

It is often said that private equity asset managers use a money-weighted methodology because the asset manager has more control of the timing of cash flows; that may well be true, but I believe the more likely reason historically is that accurate valuations at the time of cash flows are simply not available, and therefore a time-weighted methodology cannot be applied.

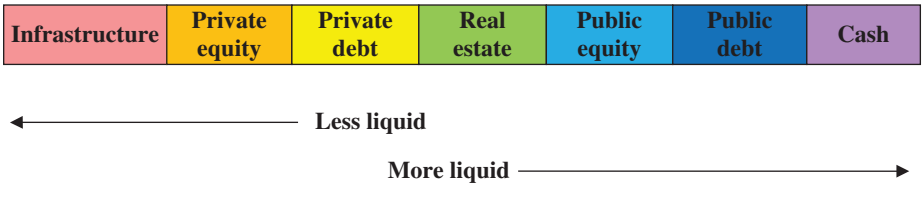


Figure 3.2 Spectrum of liquidity

Money-weighted rates of return are often used for private clients to avoid the difficulty of explaining why a loss could possibly lead to a positive time-weighted rate of return. Perhaps the most significant advantage of money-weighted returns is that if the portfolio generates a profit, the rate of return will be positive (and vice versa) regardless of the pattern of cash flows.

Because the recognised advantage of time-weighted returns is that they neutralise the impacts of cash flows they are clearly favoured when the asset manager does not influence the timing of cash flows. This does not imply that if the asset manager influences the timing of cash flows, then money-weighted returns should be preferred; the resultant return is simply the constant force of return required when applied to the pattern of cash flow that results in the end market value of the portfolio. This return is unique and not really comparable with other portfolios enjoying a different pattern of cash flow or indeed benchmark indexes; there are other ways of measuring the impact of timing, notably attribution analysis.

⚠ Caution

Control over the timing of external cash flow is not a sufficient reason to use a money-weighted return. Return calculation is about comparison, and time-weighted returns are better for comparative analysis than money-weighted rates of return. The management impact of the timing of cash flow can be measured in other ways.

Although it is often suggested that asset owners should use money-weighted returns, I argue that it is not automatically appropriate or desirable because:

(1) Comparability is damaged. With benchmarks, other asset owners and asset managers.
(2) Profit and loss is an alternative measure.
(3) Asset owners are not fully in control of their own cash flow (contributions and benefit payments). In any event, cash flows at the total fund level are unlikely to be large enough to cause a significant difference between money-weighted and time-weighted returns.
(4) Attribution and risk analysis are more difficult. Total fund attribution requires the use of a consistent methodology. Risk analysis requires more frequent return calculations.
(5) Increased cost and complexity. Asset managers will continue to use time-weighted returns and there will be continual discussion and confusion comparing results using different methodologies.

In effect money-weighted returns are appropriate only when time-weighted returns are not appropriate. Being an asset owner or in control of timing of external cash flows are not sufficient conditions. Time-weighted returns are potentially not appropriate when the assets being measured are largely illiquid (infrastructure, private assets and potentially real estate) because:

(1) Accurate valuations are not available at the time of cash flow.
(2) Benchmarks are typically not time-weighted for these categories.
(3) Asset managers are more likely to control the timing of cash flow (drawdown or return of capital).

To allow more detailed analysis, balanced strategies, including liquid and illiquid assets, are best measured consistently using a time-weighted methodology.

Late trading and market timing

Mutual funds suffer a particular performance problem caused by using backdating unit prices, as illustrated in Exhibit 3.22.

Exhibit 3.22 Late timing

Start portfolio value	$5,000,000
Units in issue	10,000,000
Start unit price	0.50
End portfolio value	$5,250,000
Units in issue	10,000,000
End period unit price	0.525

Assume because of administrative error that $500,000 should have been allocated at the start of the period. The administrator determines the client should not suffer and allocates the $500,000 for 1,000,000 units at 0.50.

The final price is now	$5,750,000
Units in issue	11,000,000
Erroneous unit price	0.5227

In effect existing unit holders have been diluted by 0.44%.
Units should only be issued at the current price, 1,000,000 at 0.525 = $525,000.
The administrator should inject $25,000 to correct the error.

This is in effect what happened in the "late trading and market timing" scandal in US mutual funds revealed in 2003.[21] Privileged investors were allowed to buy or sell units in international funds at slightly out-of-date prices with the knowledge that overseas markets had risen or fallen significantly already, resulting in small but persistent dilution of performance for existing unit holders.

SELF-SELECTION

With the choice of so many different acceptable calculation methodologies, asset managers should establish an internal policy to avoid both intentional and unintentional abuse. Table 3.2 illustrates the range of different returns calculated for our standard example in just the one period.

The fundamental reason for the difference in all these returns is the assumptions relating to external cash flow. Without cash flow, all these methods – money-weighted, time-weighted and approximations to time-weighted – will give the same rate of return.

The reason for the difference effectively lies in the denominator (or average capital invested) of the return calculation. Each of these methods makes different assumptions about the impact of external cash flow on the denominator: the greater the cash flow the greater the impact.

The differences in the simple Dietz and the modified Dietz returns in Table 3.2 are so significant in this example because the cash flow is large relative to the starting value. If the cash flow is not large, then the assumptions used to weight the cash flow will not have a measurable effect.

[21] Braceras, "Late Trading and Market Timing" (2004).

Table 3.2 Return variations due to methodology

Method	Return
Simple IRR	−7.41%
IRR	−7.27%
Simple Dietz	−7.44%
ICAA	−7.43%
ICAA (income unavailable)	−7.46%
Modified Dietz (end of day)	−7.30%
Modified Dietz (beginning of day)	−7.21%
True time-weighted (end of day)	−9.93%
True time-weighted (beginning of day)	−9.44%
True-time-weighted (midday cash flow)	−9.63%
Index substitution	−9.79%
Regression	−9.99%
Analyst's test	−10.14%

Large Cash Flow

Because this effect is often not significant it is not always worth revaluing the portfolio for each cash flow. Many institutional asset managers employ a standard modified Dietz method and only revalue for a large external cash flow above a set percentage limit (10% is common). Asset management firms should set a limit and apply it rigorously. This large cash flow limit may be defined to apply to a single cash flow during the period or the total cash flow during the period.

Note

In effect, large cash flows are cash flows large enough to distort the calculation of return.

Self-selection of methodologies

If multiple returns are routinely calculated for each methodology and the best return chosen for each period, even poor-performing portfolios could appear to be performing quite well. Clearly it is unethical to calculate returns using multiple methods and then choose the best return.

Caution

Intentional self-selection of the best methodology is easy to avoid but unintentional abuse can occur. Portfolio managers are well aware that cash flow can impact return calculations and often they have a good feel for the return of their own portfolios. If they have under-performed their expectations by say 0.2%, they may require the performance analyst to investigate the return. The analyst, recognising that a cash flow has occurred (but less than

the normal limit), may conclude that the return has been adversely impacted by the cash flow. It would be entirely inappropriate for the performance analyst to adjust the return (even though it is theoretically more accurate) because the portfolio manager is unlikely to require the same analysis if the return is 0.2% above expectations, resulting in only positive adjustments taking place.

Table 3.3 lists the advantages and disadvantages of each return methodology available to the performance measurer together with my personal preference from the asset manager's

Table 3.3 Calculation methodologies

Method	Advantages	Disadvantages	Author's ranking (highest number preferred)
Simple Dietz	(i) Simple to calculate (ii) Not sensitive to valuation errors (iii) Can be disaggregated (iv) Net gain = positive return (v) Net loss = negative return	(i) Crude estimate for timing of cash flows (ii) Not suitable for comparison against other funds or published indexes	**4**
Modified Dietz	(i) Simple to calculate (ii) Not sensitive to valuation errors (iii) Can be disaggregated (iv) Net gain = positive return (v) Net loss = negative return	(i) Not suitable for comparison against other funds or published indexes	**8**
ICCA	(i) Simple to calculate (ii) Not sensitive to valuation errors (iii) Can be disaggregated (iv) Net gain = positive return (v) Net loss = negative return	(i) Not suitable for comparison against other funds or published indexes (ii) Potential gearing effect because of treatment of income	**7**
Simple IRR	(i) Reflects the value added from the client's perspective (ii) Not sensitive to valuation errors (iii) Net gain = positive return (iv) Net loss = negative return	(i) Crude estimate for timing of cash flows (ii) Cannot be disaggregated (iii) Not suitable for comparison against other funds or published indexes	**3**
Internal rate of return	(i) Reflects the value added from the client's perspective. (ii) Not sensitive to valuation errors (iii) Common measure in finance. Frequently used in private equity/ venture capital	(i) Relatively difficult to calculate (ii) Cannot be disaggregated (iii) Not suitable for comparison against other funds or published indexes	**5**

Table 3.3 (*Continued*)

Method	Advantages	Disadvantages	Author's ranking (highest number preferred)
BAI	(i) Hybrid time-weighted, money-weighted return (ii) Not sensitive to valuation errors	(i) Relatively difficult to calculate (ii) Cannot be disaggregated	**6**
Linked modified Dietz	(i) Hybrid time-weighted, money-weighted return (ii) Easier to calculate than BAI method (iii) Can be disaggregated within single periods (iv) Well established and in common usage	(i) Can be impacted by large cash flows	**9**
True time-weighted	(i) Measures the true performance of the portfolio manager adjusting for cash flows (ii) Suitable for comparison with other asset managers and benchmarks	(i) Sensitive to incorrect valuations	**10**
Analyst's test	(i) Good estimate of true time-weighted rate of return (ii) Does not require accurate daily valuations	(i) Change in benchmark appears to change portfolio return	**2**
Index substitution	(i) Time-weighted methodology (ii) Does not require accurate daily valuations	(i) Approximation only as good as index assumption. Less accurate than analyst's test (ii) Change in benchmark appears to change portfolio return	**1**
Regression	(i) Time-weighted methodology (ii) Does not require accurate daily valuations (iii) Use of systematic risk may provide a more accurate valuation approximation than index substitution method	(i) Approximation only as good as index and systematic risk assumption. Low R^2 implies volatile systematic risk (ii) Change in benchmark or systematic risk calculation appears to change portfolio return	**0**

perspective. My preferences are consistent with the evolution of performance returns methodologies shown in Figure 3.3.

Different asset categories and different product types have progressed through this evolution of return methodologies at different rates – I would suggest driven by the availability of accurate valuations. Mutual funds evolved to true time-weighted return first because an accurate valuation must be derived to allow investors to enter and exit the mutual fund at a fair price. Pension funds are perhaps one step behind in evolutionary development, seeking time-weighted returns to neutralise the impact of cash flows and allow fair comparison of their managers with each other and benchmarks. In fact, pension funds are still in the process of

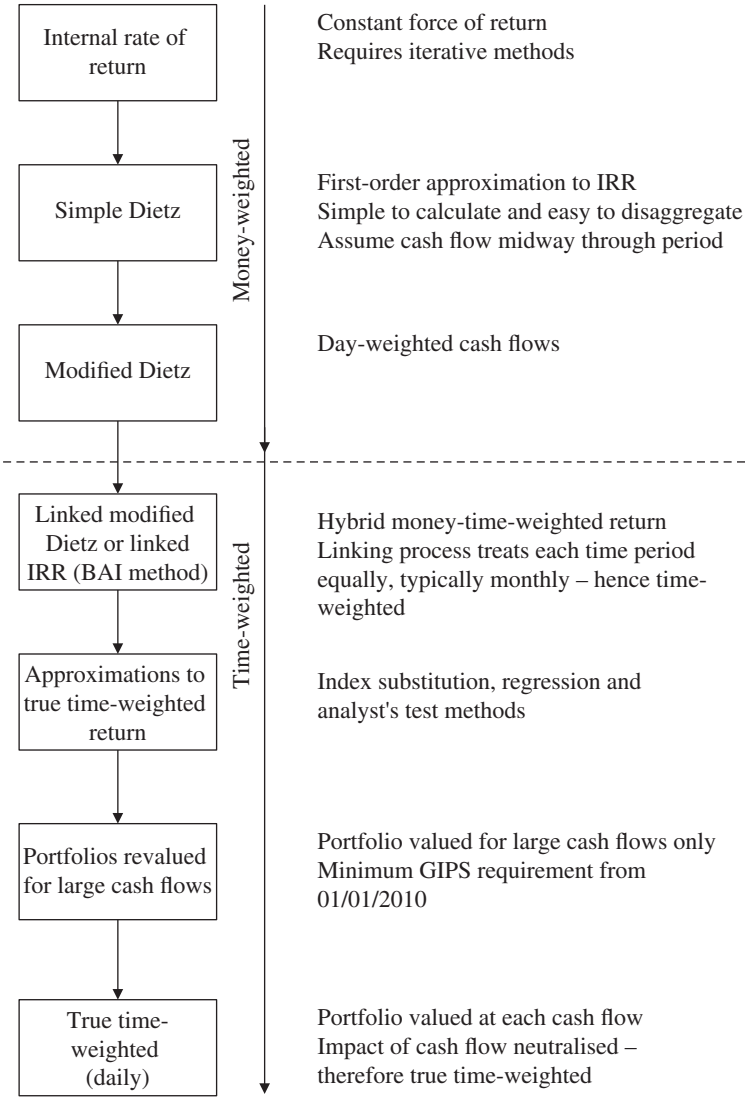


Figure 3.3 Evolution of performance returns

evolving to the final stage with many institutional asset managers still using linked modified Dietz; they are not able, or are unwilling because of cost, to calculate daily valuations. In fact, the Global Investment Performance Standards (GIPS)[22] mandate from 2010 a halfway house between linked modified Dietz and true time-weighting, which requires asset managers to value the portfolio for large cash flows only.

Internal rate of return is clearly the starting point, followed by the simple and modified Dietz approximations to the internal rate of return. Private equity because of the difficulty of obtaining accurate valuations is unable to evolve further. Interestingly, real estate has evolved from money-weighted to time-weighted more recently facilitated by more frequent property valuations.

ANNUALISED RETURNS

When comparing returns over long periods it is easier to think in terms of standardised periods, annual returns being the most convenient. The average (or mean) annual return over a number of years can be calculated arithmetically or geometrically as follows:

Arithmetic average or average return

$$\bar{r}_A = \frac{f}{n} \times \sum_{i=1}^{i=n} r_i \tag{3.31}$$

Geometric average or annualised return

$$\tilde{r} = \left(\prod_{i=1}^{n} (1 + r_i) \right)^{\frac{f}{n}} - 1 \tag{3.32}$$

where: n = the number of periods under analysis
f = the number of periods within the year (monthly $f = 12$, quarterly $f = 4$)

The geometric average or annualised return is the return which, if compounded with itself for the cumulative period, will result in the cumulative return.

Average and annualised returns are calculated in Exhibit 3.23.

Caution

It is poor performance practice to annualise returns for short periods (less than one year). It is inappropriate to assume the rate of return achieved in the year to date will continue for the remainder of the year.

Note

The terms "arithmetic" and "geometric" are common in the field of performance measurement; arithmetic reflects additive relationships and geometric reflects multiplicative or compounding relationships.

[22] See Chapter 7, Provision 2.A.23.c.

Exhibit 3.23 Average and annualised returns

Annual returns

2003	10.5%
2002	−5.6%
2001	23.4%
2000	−15.7%
1999	8.9%

Cumulative return:

$$(1.105 \times 0.944 \times 1.234 \times 0.843 \times 1.089) - 1 = 18.2\%$$

Arithmetic average:

$$\frac{10.5\% - 5.6\% + 23.4\% - 15.7\% + 8.9\%}{5} = 4.3\%$$

If the annual average return is compounded for five years the cumulative return is considerably greater than 18.2%:

$$(1.043 \times 1.043 \times 1.043 \times 1.043 \times 1.043) - 1 = 23.4\%$$

Geometric average or annualised return:

$$(1.105 \times 0.944 \times 1.234 \times 0.843 \times 1.089)^{\frac{1}{5}} - 1 = 3.4\%$$

As expected, if the annualised return is compounded for five years the cumulative return remains the same:

$$(1.034 \times 1.034 \times 1.034 \times 1.034 \times 1.034) - 1 = 18.2\%$$

Investment returns compound. When assessing historic performance, it is essential to use the constant rate of return that will compound to the same value as the historic series of returns, as shown in Exhibit 3.24.

Exhibit 3.24 Positive bias

Assume the rate of return is +20% in the first period and −20% in the second period

Assuming a start value of £100

The value at the end of the first period is £120

The value at the end of the second period is £96

The arithmetic average is $\frac{20.0\% - 20.0\%}{2} = 0\%$

The annualised return is $(1.2 \times 0.8)^{\frac{1}{2}} - 1 = -2.02\%$

−2.02% compounded over the two periods will generate the accurate end value of £96.

Note

Arithmetic averages are positively biased; if returns are not constant the annualised return will always be less than the arithmetic average. Note that the annualised return when compounded with itself over the entire period will reconcile to the original cumulative return, whereas the arithmetic average return will not. Hence the annualised return provides a better indicator of wealth at the end of the period than the arithmetic average. Performance analysts should use annualised rather than average returns.

The average return is always greater than or equal to the annualised return:

$$\bar{r}_A \geq \tilde{r} \tag{3.33}$$

A good approximation of the difference can be calculated from:

$$\tilde{r} \cong \bar{r}_A - \frac{\sigma^2}{2} \tag{3.34}$$

where: σ = the standard deviation or variability of returns[23]

This is an important relationship; the amount by which the annualised return is less than the average return depends on variability. The greater the variability of returns, the greater the difference between the arithmetic average and the geometric average.

If the average return is 12.0% and the variability of returns is 10% then the annualised return is approximately $12.0\% - (10.0\%)^2/2 = 11.5\%$.

Since-inception internal rate of return (SI-IRR)

To calculate the "annual" internal rate of return rather than the "cumulative" rate of return for the entire period we need to solve for r using the following formula:

$$V_E = V_S \times (1+r)^Y + \sum_{t=1}^{t=T} C_t \times (1+r)^{W_t^y} \tag{3.35}$$

where: Y = length of time period to be measured in years
W_t^y = factor to be applied to external cash flow on day t

$$W_t^y = Y - Y_t \tag{3.36}$$

where: Y_t = number of years since the beginning of the period of measurement

For example, assume cash flow occurs on the 236th day of the third year for a total measurement period of five years; then:

$$W_t^y = 5 - 2\frac{236}{365} = 2\frac{129}{365}$$

[23] See Equation 5.5.

For presentational purposes, internal rates of return are often calculated since inception and annualised.

Modified IRR (MIRR)

The modified IRR or MIRR is an ex-ante IRR approach that addresses two of the criticisms of the ex-ante IRR: the reinvestment of future cash flows at the IRR rate and the potential for multiple correct solutions. The modified IRR assumes that positive cash flows are reinvested at an assumed reinvestment rate and negative cash flows are financed at an assumed cost of capital. Clearly the reinvestment rate and cost of capital are of more concern for project appraisal than the measurement of asset performance. The MIRR is the ratio of the sum of assumed end value equivalent of positive cash flows (including the end value) divided by the present value of negative cash flows including the start value:

$$MIRR = \sqrt[n]{\frac{\sum_{t=1}^{t=t} V_E + C_t^+ \times (1+rr)^{W_t^y}}{\sum_{t=1}^{t=T} V_S + C_t^- \times (1+coc)^{-W_t^y}}} - 1 \tag{3.37}$$

where: coc = cost of capital
rr = reinvestment rate
C_t^+ = positive cash flow on day t
C_t^- = negative cash flow on day t

Performance analysts, focusing on ex-post analysis, are unlikely to use an ex-ante statistic using assumed reinvestment rates such as MIRR but for completeness an example calculation is shown in Exhibit 3.25.

Exhibit 3.25 Modified IRR

Initial outlay	Cash flow Year 1	Cash flow Year 2	Cash flow Year 3	Cash flow Year 4	Final value Year 5
£150m	£50m	£22m	£35m	–£45m	£330m

Assume a cost of capital of 8% and a reinvestment rate of 5%.

Final cash equivalent of outflows:

$$330 + 45 \times (1.05) = 377.25$$

Present value of inflows:

$$150 + \frac{50}{1.08} + \frac{22}{(1.08)^2} + \frac{35}{(1.08)^3} = 242.94$$

$$MIRR = \sqrt[5]{\frac{377.25}{242.94}} - 1 = 9.2\%$$

Note

Excel® offers a modified IRR function, MIRR.

Return hiatus

If there is any hiatus or gap in the performance track record, however short, it is impossible to bridge the gap and link performance returns to produce cumulative and hence annualised returns.

Caution

Some performance analysts might argue that it is possible to substitute an index return for a gap in performance and therefore calculate cumulative and annualised return. This is clearly not good practice and should be avoided.

GROSS- AND NET-OF-FEE CALCULATIONS

A key component in long-term investment performance is the fee charged by the asset manager. Fees are charged in many different ways by several different parties; in evaluating and comparing the performance of a portfolio manager it is essential that the impact of fees be appropriately assessed.

There are three basic types of fees or costs incurred in the management of an investment portfolio:

(1) Transaction fees – the costs directly related to buying and selling assets including broker's commission, bid/offer spread, and transaction-related regulatory charges and taxes (stamp duty, etc.), but excluding transaction-related custody charges.
(2) Portfolio management fee – the fees charged by the asset manager for the management of the account.
(3) Custody and other administrative fees including audit fees, performance measurement fees, legal fees and any other fee.

Asset managers should be evaluated against those factors, which are within their control. Clearly asset managers have a choice whether or not to buy or sell securities; therefore, performance should always be calculated after (or net of) transaction costs. This is naturally reflected in the valuations used in the methods described previously and no more action need be taken.

Portfolio management fees are traditionally taken direct from the account but need not be; the asset manager may invoice the client directly receiving payment from another source.

If payments were not taken directly from the portfolio, then any return calculated would be before or "gross" of fees.

The gross-of-fee effect can be replicated, if the fee is deducted from the account by treating the management fee as a negative external cash flow. If the fee is not treated as an external cash flow, then the return calculated is after, or "net", of fees.

The gross return is the investment return achieved by the asset manager and normally the most appropriate return to use for comparison purposes since asset owners are normally able to negotiate fees.

Custody and other administration fees are not normally in the control of the asset manager and hence should not be reflected in the calculation of return for evaluation purposes. It should be noted, however, that the "client return" after administration fees is the real return delivered to the asset owner or client.

Asset managers may bundle all of their services together and charge a "bundled fee". If the bundled fee includes transaction costs that cannot be separated, then the entire fee must be subtracted to obtain the investment return.

In most countries local regulators will require mutual funds to report and advertise their performance net of all fees.

If calculating performance net of fees then to reflect the correct economic value of the portfolio, fees (including performance fees) should be accrued negatively in the valuation.

Estimating gross- and net-of-fee returns

The most accurate way to calculate the gross and net series of returns for a portfolio would be to calculate each set of returns separately, treating the fee as an external cash flow for the gross return but making no adjustment for the net return.

Alternatively, if only one series is calculated (either gross or net) the other can be estimated using the fee rate as follows:

(1) "Grossing up" net returns:

$$r_g = (1 + r_n) \times (1 + f) - 1 \tag{3.38}$$

(2) "Netting down" gross returns:

$$r_n = \frac{(1 + r_g)}{(1 + f)} - 1 \tag{3.39}$$

where: r_g = return gross of portfolio management fee
r_n = return net of portfolio management fee
f = nominal period portfolio management fee rate

The grossing up of return calculation is normally applied to mutual fund returns, which are normally calculated net of all fees (including custody and other administrative fees). The total cost in fees and expenses of a mutual fund is normally expressed as the total expense ratio, or TER. The total net return can be grossed up for custody and administration fees only and/or the portfolio management fees depending on the requirement, taking care that regulatory requirements to advertise net of all expenses (or client return) are met. The netting down of return calculation is normally applied to institutional funds, although it is probably far easier and certainly more accurate to calculate net returns directly.

In Table 3.4 actual and estimated grossed-up returns are shown over a period of six months. Note that the underlying growth in the portfolio has appeared to exaggerate the arithmetic difference between gross and net returns: 35.8% – 35.0% = 0.8%. The expected fee difference for a 1.2% per annum fee over six months would be 0.6%. In fact, the geometric difference of $\frac{1.358}{1.35} = 0.6\%$ between gross and net returns will provide a better representation of the fees

Table 3.4 Gross- and net-of-fees calculations

Fee 1.2% per annum calculated monthly in arrears (assume paid at the midpoint of next month)

	Market value	Fee	Net-of-fees return	Gross-of-fees return	Estimated gross-of-fees return
Start value	£100m	£0.1m			
End of month 1	£112m	£0.112m	$\frac{112}{100} - 1 = 12.0\%$	$\frac{112 - 100 + 0.1}{100 - \frac{0.1}{2}} = 12.106\%$	$1.12 \times 1.001 - 1 = 12.112\%$
End of month 2	£95m	£0.095m	$\frac{95}{112} - 1 = -15.178\%$	$\frac{95 - 112 + 0.112}{112 - \frac{0.112}{2}} = -15.086\%$	$0.8482 \times 1.001 - 1 = -15.094\%$
End of month 3	£99m	£0.099m	$\frac{99}{95} - 1 = 4.211\%$	$\frac{99 - 95 + 0.095}{95 - \frac{0.095}{2}} = 4.313\%$	$1.04211 \times 1.001 - 1 = 4.315\%$
End of month 4	£107m	£0.107m	$\frac{107}{99} - 1 = 8.081\%$	$\frac{107 - 99 + 0.099}{99 - \frac{0.099}{2}} = 8.185\%$	$1.08081 \times 1.001 - 1 = 8.189\%$
End of month 5	£115m	£0.115m	$\frac{115}{107} - 1 = 7.477\%$	$\frac{115 - 107 + 0.107}{107 - \frac{0.107}{2}} = 7.580\%$	$1.07477 \times 1.001 - 1 = 7.584\%$
End of month 6	£135m		$\frac{135}{115} - 1 = 17.391\%$	$\frac{135 - 115 + 0.115}{115 - \frac{0.115}{2}} = 17.500\%$	$1.17391 \times 1.001 - 1 = 17.509\%$

Six-month return net of fees $\frac{135}{100} - 1 = 35.0\%$

Actual gross-of-fees return: $1.12106 \times 0.84914 \times 1.04313 \times 1.08185 \times 1.0758 \times 1.175 - 1 = 35.795\%$

Estimated gross-of-fees return: $1.12112 \times 0.84905 \times 1.04315 \times 1.08189 \times 1.07584 \times 1.17508 - 1 = 35.812\%$

charged. In this example the estimated gross return is a good estimate of the actual gross return. Timing of fee cash flows, payment in advance or in arrears and the frequency of calculation (i.e., monthly or quarterly) will all have minor impacts on the gross return calculation.

Initial fees

Initial fees (or front-load fees) are normally associated with investment product wrappers around a portfolio of assets. While initial fees will certainly impact the ultimate client return in terms of making judgements about the portfolio manager's performance, they can normally be ignored for both gross- and net-of-fees calculation. Their impact will vary with the time period measured and, in many cases, initial fees are wholly or partially rebated. Appropriate disclosure should be sufficient.

Performance fees

A performance fee is a variable fee based on the performance of the portfolio.

Supporters of performance fees would suggest that they are desirable because they align the interests of the asset manager with that of the asset owner.[24] If the asset manager performs well, then the owner's assets rise in value and for the most part asset owners appear happy to pay performance fees for good performance. If the asset manager performs poorly, only a smaller base fee and no incentive fee is paid.

Those not in favour claim that rather than aligning interests, performance fees create a conflict of interest, and for the most part performance fees are biased when designed in favour of the asset manager.[25] Without doubt, performance fees have had a beneficial impact on asset management revenues. In my early career the pressure on fees was always down while later average fees increased across the entire industry responding to incentive fees for alternative strategies. More recently low-fee passive strategies have again exerted downward pressure. Hedge funds demonstrated to the entire industry that asset owners, including institutions like endowments, pension funds and sovereign wealth funds, are prepared to accept high management fees if they are "dressed up" in the form of a performance or incentive fee.

Asymmetric or symmetric

Whether or not performance fees are beneficial for asset owners depends on how the performance fees are structured. There are many flavours but essentially there are two main types: asymmetric and symmetric. Asymmetric fees shown in Figure 3.4 are by far the most common and are very seductive to asset owners.

Typically, asset managers will present this type of fee to clients at a base fee rate considerably lower (perhaps less than half) than their normal management fee with an incentive fee for performance greater than an agreed hurdle rate.[26] This is extremely attractive to asset owners; they pay a lower fee and will only pay higher fees if the asset manager performs well. There is considerable opportunity to negotiate the hurdle rate, angle of the participation rate, time

[24] Davanzo and Nesbitt, "Performance Fees for Investment Managers" (1987).

[25] Goetzmann, Ingersoll and Ross, "High-water Marks and Hedge Fund Management Contracts" (2003).

[26] The minimum absolute return (risk-free rate or target return, perhaps) or relative return against an agreed benchmark that has to be reached before a performance fee is earned.

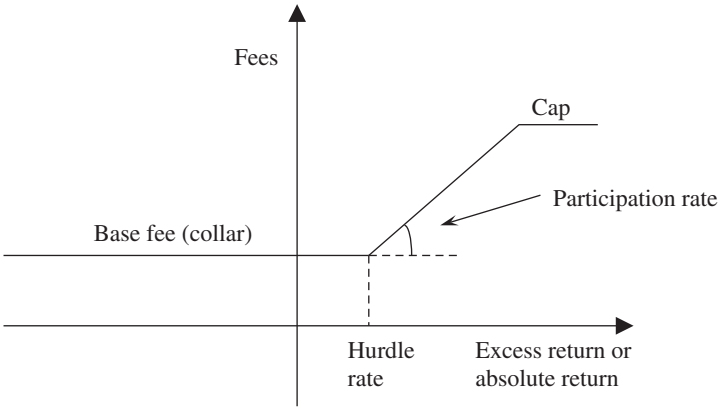


Figure 3.4 Asymmetric performance fees

period, excess or absolute return, caps and collars and perhaps a high-water mark,[27] but the basic structure is the same.

A rare alternative would be a symmetric structure.[28] In this type of structure, the base fee would probably not be lower and may even be higher than the normal management fee. Incentive fees are paid for outperformance, but crucially fees are rebated for underperformance to the extent that the asset manager may ultimately make a net payment to the client if performance is sufficiently bad. Though unpopular with asset managers, these types of fees genuinely align the interest of both parties.[29]

Some critics of performance fees make the claim that asymmetric fees encourage asset managers to take more risk if performance is poor because they have little further downside and considerable upside. This is not my personal experience: often when in a hole I've observed that portfolio managers in effect stop digging and take less risk when performing badly. On the flip side I have observed portfolio managers "lock in" outperformance when they have achieved the cap and take much less risk. This obviously reduces the business risk of the asset manager but presumably the asset owner would prefer the asset manager to continue taking risk if they are obtaining good rewards. Risk-adjusted returns such as M^2,[30] or, even better, adjusted M^2,[31] would help alleviate this problem. Muralidhar[32] reviews the available literature on performance fees and in conclusion supports the use of risk-adjusted returns such as M^2.

Schliemann and Stanzel[33] emphasise, among other issues, the option-like character of performance fees. Chevalier and Ellison[34] demonstrate that asymmetrical performance fees create call option–like components in the incentives of the asset manager. The price of the call option

[27] The highest peak in value. In subsequent periods asset managers must exceed the high-water mark before receiving performance incentives.

[28] In the US, under the Investment Advisers Act (1940) any performance fee charged to a mutual fund can only utilise symmetrical performance fee structures. Consequently, performance fees are rare in US mutual funds; asset managers do not like rebating fees to clients. In the US symmetrical fee structures are often called fulcrum fees.

[29] Starks, "Performance Incentive Fees: An Agency Theoretic Approach" (1987).

[30] See Equation 5.136.

[31] See Equation 5.142.

[32] Muralidhar, "Risk and Skill-adjusted Investment Compensation" (2009).

[33] Schliemann and Stanzel, "Performance-based Compensation Contracts in the Asset Management Industry" (2008).

[34] Chevalier Ellison, "Risk Taking by Mutual Funds as a Response to Incentives" (1997).

is implicitly borne by the asset owner, but the asset manager receives the benefits. The higher the risk, the more valuable the call option, thus incentivising the asset manager to take more risk, particularly when close to hurdle rate boundaries.

Sadly, performance fee arrangements often end in acrimony. For long-term agreements the original authors on both sides may have moved on, and, if the agreements were badly written, at the point when both partners should be celebrating good performance, relationships can be damaged by a dispute about the size of fees to be paid. Performance fee agreements should be clear, concise and as simple as possible. I would always recommend including a few worked examples to ensure that both parties fully understand the agreement. There is a tendency to complicate performance fees with high watermarks and variable hurdle rates; frankly, the more complex the agreement the more likely interests will be misaligned. Investor interests are best protected by structuring performance fees over longer time periods, three or even five years.

Endowments, pension funds and sovereign wealth funds typically select a portfolio of asset managers to diversify their selection risk. Asset owners should think through the implications before establishing a performance fee structure for their asset managers. Theoretically the existence of a performance fee should not alter the actual performance enjoyed. A portfolio manager is either skilled or unskilled; those extra hours staying behind in the office will not necessarily add to performance. Sufficient constraints and incentives already exist such that managers are always encouraged to deliver good performance with or without performance fees. The evidence on the impact of performance fees is mixed. In a recent study of Dutch pension funds, Broeders, van Oord and Rijsbergen find no statistical evidence that the returns of pension funds that pay performance fees to asset managers for active investing are significantly higher or lower than the returns of pension funds that do not pay performance fees.[35] They found that this is true for most asset classes and is more robust if corrected for risk. To attract new investors, asset managers must demonstrate a superior, consistent track record. The asset manager cannot favour performance fee clients over and above other clients without a performance fee; they have a fiduciary duty to treat all clients equally. Why pay a performance fee for performance you might expect to receive anyway? Obviously, the asset owner chooses the asset manager. Is it then not rather perverse that the asset owner is rewarded by a lower fee (i.e., no performance fee) for choosing an underperforming asset manager and penalised (by paying a performing fee) for choosing an outperforming asset manager? Consider for a moment that the asset owner is an average selector of asset managers; the overperformance of the good-performing asset managers will offset the underperformance of the poor-performing asset managers, leading to average performance overall. However, if the performance fee arrangement is asymmetric the performance fees paid will more than exceed the advantage gained from the lower base fees of the poor-performing asset managers, leading to above-average fees paid for average performance overall! Asset owners should treat performance fees as a necessarily evil, only accepting performance fees to access demonstrably superior asset managers where a simple base fee is not available.

Crystallisation

Performance fees by their very nature are variable and paid on an infrequent basis; entitlements can build up over a period of three years or longer. Because base fees are inevitably lower it

[35] Broeders, van Oord and Rijsbergen, "Does It Pay to Pay Performance Fees? Empirical Evidence from Dutch Pension Funds" (2019).

would be inappropriate not to accrue for any performance fee element earned in a net-of-fee calculation. The performance fee accrual should reflect the performance of the portfolio at that point in time and unlike other accruals can be quite volatile. A reducing performance fee accrual may over shorter time periods give the impression that the performance net of fees is greater than gross of fees performance.

The eventual payment of the performance fee accrual is described as crystallisation.

Crystallisation may be smoothed over a rolling three- or longer-year period – one third of the entitlement paid in the first year with subsequent payments deferred based on performance.

Some performance fees may only be crystallised when an investor leaves a fund in terms of an exit fee.

Performance fees in practice

If used, performance fees should be:

(1) Unambiguous and fair to both parties

The performance fee structure should be fair and equitable to both asset owners and asset managers. The performance fee structure should be consistent with the portfolio's objective and should provide reward commensurate with the skill exercised in achieving the performance.

The agreement should be clear and unambiguous, preferably including a worked example. Calculation methodologies should be agreed, and mechanisms for adjusting for cash flows, changing benchmarks and arbitration processes should be approved at the start of the relationship. The performance fee calculation should be verifiable. A worked example of the performance fee calculation should be included in the performance fee agreement.

(2) Symmetrical

Symmetrical fees are more likely to align the interests of the asset owner and asset manager on both the upside and the downside.

(3) Simple

The more complex the performance fee structure the more likely the interests of the asset manager and asset owner will be misaligned.

(4) Risk-adjusted

The performance fee structure should not influence the risk strategy of the asset manager. Ideally, performance calculations should be risk-adjusted.

Unfortunately, many performance fee structures are unfair and ambiguous, asymmetrical rather than symmetrical, frequently complex and rarely risk-adjusted.

Note

In terms of my own preference, if asked to rank fee arrangements, of any type, from an asset owner's perspective, I would simply prefer a fixed fee: a simple fixed amount, for example, $100,000 per annum. Clearly, asset managers need to cover costs, reward staff as appropriate and deliver a return on capital (mostly human capital). The ideal structure is an annual fee (paid quarterly) over a fixed period, say three

or five years (immediately cancellable if the investment mandate is breached), with or without an inflation uplift. Reward, based on good performance, would be renewing the mandate for an extended period at perhaps an increased cost. Admittedly this type of arrangement is rare in the asset management industry. More common is my second preference: an "ad valorem" fee, a fee rate based on the assets under management, for example, 40 basis points per annum. Some would say the asset manager is automatically rewarded with a higher fee for growing the asset base. My third preference is a symmetrical performance fee, for example, a 50-basis-points base fee, with 10% profit participation above, say, 1% outperformance and a 10% performance rebate below 1% underperformance. Least favoured, but common, is an asymmetrical performance fee, for example, a 2% base fee plus 20% profit participation above a 5% hurdle rate.

⚠ Caution

The parameters of a performance fee structure should clearly specify:

(1) The return calculation methodology
(2) The agreed benchmark (one only)
(3) Hurdle rate (soft or hard)
(4) High-water mark
(5) Resetting arrangements[36]
(6) Profit participation rate and rebate rate
(7) Time period (one, three or five years, for example)
(8) Fee accrual and payment schedule
(9) Capital basis for determination of base performance fee (a key area of ambiguity in practice)
(10) Clawback arrangements
(11) Arbitration procedures

Equalisation

Equalisation is designed to be fair to all investors in a collective mutual or hedge fund who inevitably invest at different times and are therefore subject to different performance returns. An investor who joins in the middle of a good period of performance may be charged a full performance fee even if they only participated for part of that performance; alternatively, an investor may enjoy a free ride if they invest at a low point after a high-water mark and avoid paying a performance fee as the fund recovers to previous highs.

One obvious solution would be to issue a different class of share for investors investing at different times, each with its own performance fee calculation. Clearly several different classes

[36] The periodic resetting of a contractually defined high-water mark. If high-water marks are not reset asset managers may find they are unable to generate performance fees and are thus disincentivised; alternatively, if high-water marks are reset too early asset managers may be rewarded for partially recovered performance below the previous high-water mark.

of shares can soon become a heavy administrative burden; alternatively, equalisation can be used. Any performance fee accrued but not yet crystallised will be included in the net asset value price of the fund; any investor buying units at this price in effect buys an equalisation credit equal to the accrual for the performance fee ultimately to be paid at the next crystallisation. The investor should not pay for performance not received. At crystallisation if any performance fee is due an amount is rebated either directly or in the form of additional units up to the amount of the equalisation credit.

If the unit price is below a high-water mark, an equalisation debit (or depreciation deposit) is established equal to the equivalent performance fee entitlement between the current net asset value price and the high-water mark level. If the high-water mark is achieved a performance fee is charged, normally by redeeming units in the fund. Example calculations are shown in Exhibit 3.26.

Exhibit 3.26 Equalisation

Assume a performance fee of 25%

	Gross assets	Net assets	Performance fee accrual
Unit price 31/12/2006	100	100	0
Unit price 31/03/2007	90	90	0
Unit price 30/06/2007	120	115	5
Unit price 30/09/2007	140	130	10
Unit price 31/12/2007	130	122.5	7.5

First investor invests 31/12/2006

Performance return	$130 - 100 = 30$
Performance fee due	$30 \times 25\% = 7.5$ (distributed from the fund)

Second investor invests 31/03/2007

Performance return	$130 - 90 = 40$
Performance fee due	$40 \times 25\% = 10$

Distributed from the fund = 7.5

Depreciation deposit	$(100 - 90) \times 25\% = 2.5$ (paid by redeemed units)
Total distribution	$7.5 + 2.5 = 10$

Third investor invests 30/06/2007

Performance return	$130 - 120 = 10$
Performance fee due	$10 \times 25\% = 2.5$

Distributed from the fund = 7.5

Equalisation credit	$20 \times 25\% = 5$ (rebated to client)
Total distribution	$7.5 - 5 = 2.5$

Exhibit 3.26 (*Continued*)

Fourth investor invests 30/09/2007	
Performance return	$130 - 140 = -10$
Performance fee due	nil
Distributed from the fund = 7.5	
Equalisation credit	$40 \times 25\% = 10$ (7.5 rebated to client)
Total distribution	$7.5 - 7.5 = 0$
Equalisation credit retained	$10 - 7.5 = 2.5$

Reporting hierarchy

Peterson lists the hierarchy of performance returns with or without fees available for presentation.[37] This range is shown in Figure 3.5. Pure gross return represents the return before transaction costs and management fees and of course is not a realistic number to present, but it should be noted that commercial indexes are typically calculated and presented pure gross. The gross return represents a more realistic return after transaction costs but before management fees. The semi-net return (sometimes erroneously described as a net return) represents the return after transaction fees and base management fees but before performance fees. The net return (sometimes described as net-net) represents the return after transaction fees, base management fees and performance fees (net-net in the sense of net of both base fees and performance fees). The pure net return (the actual return enjoyed by the client) represents the return after transaction fees, base management fees, performance fees and other administrative fees. Regulation will often demand the presentation of the pure net return, the actual return experienced by the client; although this is clearly appropriate for presenting to retail clients (who have no or little fee negotiating power) it is not necessarily appropriate for analysing the skill of the asset manager. For internal analysis and for institutional asset owners who have some fee-negotiating power, gross return is the most appropriate measure for analysing skill.

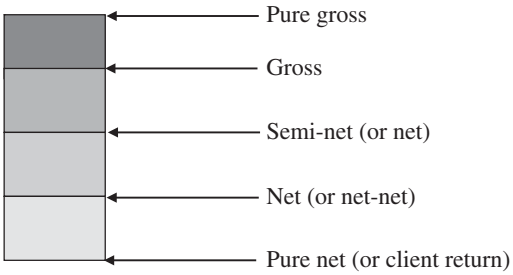


Figure 3.5 Fee reporting hierarchy

[37] Peterson, "Performance Measurement for Alternative Investments" (2015).

If management fees must be taken into account, then the net (or net-net) return is more appropriate than semi-net fees. In most cases base fees will be lower if there is also a performance fee and hence it would be misleading to present returns net of base fees only.

OVERLAY STRATEGIES

In traditional investment strategies the client will provide funds to be physically invested across a universe of assets. In addition, clients may wish to enhance returns or manage risk of these core physically invested assets by using other independent asset managers.

Overlay strategies utilise derivative instruments with reference to the underlaid or core assets. The overlay manager may have few or no physical assets to invest, resulting in small, zero or negative asset values. Traditional measurement methodologies using negative asset values are clearly inappropriate. The value of the assets underlaid or notional value should be introduced into the denominator of the return calculation. Thus, the denominator of the return calculation for overlaid assets will include both the notional value of the overlaid assets plus the value, including unrealised gains and losses of the overlaid assets themselves. Naturally the numerator is derived from the overlaid assets exclusively.

Overlay performance return calculations

Notional values for overlay portfolio return calculations are typically calculated by using one of the following methods:

(1) The notional exposure of the overlay strategy as of the beginning of the period
(2) The value of the underlying portfolio being overlaid as of the beginning of the period
(3) The specified target exposure as of the beginning of the period, which can be defined as either a target exposure or determined by a formula used to calculate the target exposure for each period.

Let's assume we have an overlay portfolio (shown in Table 3.5) consisting of forward and swap contracts with a net zero value of £0, £500 million long economic exposure and −£500 million short economic exposure (for the purposes of this example it does not matter what those exposures are) plus £10m million of cash to help implement the strategy. Let's also assume we have an underlay portfolio of total value £990 million. The aggregate portfolio value is therefore £1,000 million. For ease of calculation let's assume no external cash flow. The value of the portfolio over the quarter develops as follows:

Table 3.5 Overlay portfolio plus underlaid assets

	Start market value (£m)	End of month 1 (£m)	End of month 2 (£m)	End of month 3 (£m)
Long economic exposure	500	550	480	520
Short economic exposure	−500	−520	−530	−480
Cash	10	10	10	10
Overlay total	**10**	**40**	**−40**	**50**
Underlaid assets	990	970	1,040	1,050
Aggregate portfolio	**1,000**	**1,010**	**1,000**	**1,100**

Returns for the long and short economic exposures, the underlaid assets and aggregated portfolio are shown in Table 3.6. For convenience let's assume a zero return on cash.

Table 3.6 Portfolio returns

	Month 1	Month 2	Month 3	Quarter
Long economic exposure	10.0%	−12.7%	8.3%	4.0%
Short economic exposure	4.0%	1.9%	−9.4%	−4.0%
Cash	0.0%	0.0%	0.0%	0.0%
Overlay total	**300.0%**	**−200.0%**	**−225.0%**	**400.0%**
Underlaid assets	−2.0	7.2%	1.0%	6.1%
Aggregate portfolio	**1.0%**	**−1.0%**	**10.0%**	**10.0%**

The total aggregate portfolio return over the quarter and each month is calculated in Exhibit 3.27.

Exhibit 3.27 Aggregate portfolio return

Quarter return:	$\frac{1{,}100 - 1{,}000}{1{,}000} = 10.0\%$
Month 1:	$\frac{1{,}010 - 1{,}000}{1{,}000} = 1.0\%$
Month 2:	$\frac{1{,}000 - 1{,}010}{1{,}010} = -1.0\%$
Month 3:	$\frac{1{,}100 - 1{,}000}{1{,}000} = 10.0\%$

The overlay portfolio returns calculated in isolation in Exhibit 3.28, without introducing notional values into the denominator, look less meaningful since the capital base is only £10m million.

Exhibit 3.28 Overlay portfolio return

Quarter return:	$\frac{50 - 10}{10} = 400.0\%$
Month 1:	$\frac{40 - 10}{10} = 300.0\%$
Month 2:	$\frac{-40 - 40}{40} = -200.0\%$
Month 3:	$\frac{50 + 40}{-40} = -225.0\%$

The return of −225.0% in month 3 of Exhibit 3.28 looks strange, as a profit has been made, but don't forget that the starting position is an unrealised loss of −£40 million. A profit of £90 million on a capital base of −£40 million will generate a negative return of −225.0%.

It might seem counterintuitive but these apparently meaningless returns can be geometrically linked to calculate the quarter return:

$$(1 + 300\%) \times (1 - 200\%) \times (1 - 225\%) - 1 = 400\%$$

The added value of the overlay portfolio is calculated in Exhibit 3.29 using the aggregated portfolio size for the denominator. Note that the cash and unrealised gains and losses of the overlay portfolio are included in this denominator.

Exhibit 3.29 Added value of overlay portfolio

Quarter contribution:	$\frac{40}{1{,}000} = 4.0\%$
Month 1:	$\frac{30}{1{,}000} = 3.0\%$
Month 2:	$\frac{-80}{1{,}010} = -7.9\%$
Month 3:	$\frac{90}{1{,}000} = 9.0\%$

The contributions for all the assets are shown in Table 3.7.

Table 3.7 Portfolio contributions

	Month 1	Month 2	Month 3	Quarter
Long economic exposure	5.0%	−6.9%	4.0%	2.0%
Short economic exposure	−2.0%	−1.0%	5.0%	2.0%
Cash	0.0%	0.0%	0.0%	0.0%
Overlay total	**3.0%**	**−7.9%**	**9.0%**	**4.0%**
Underlaid assets	−2.0	6.9%	1.0%	6.0%
Aggregate portfolio	**1.0%**	**−1.0%**	**10.0%**	**10.0%**

In terms of independent overlay portfolios, it should be noted that the overlay returns are notional and not actual returns, therefore profit and losses are not automatically reinvested in previous notional returns and geometrically linked. If the notional asset is a fixed target exposure, then in one sense overlay returns are compounded with an underlying return of 0%. Therefore, unlike all other types of return, periodic overlay returns can be arithmetically linked.

Note

I don't think compounding overlay notional returns is an unreasonable assumption. Overlay portfolios create real profit and losses and ultimately, they compound with something, therefore why not make the assumption that they compound with previous notional returns?

BASE CURRENCY AND LOCAL RETURNS

A domestic asset is an asset that trades in the asset owner's domestic currency. Foreign assets (overseas or non-domestic assets) are assets denominated in currencies other than the asset owner's domestic currency. The domestic currency or base currency is the currency in which the portfolio is valued and returns reported. The portfolio return excluding currency is normally called the local currency (or local) return.

Clearly, international portfolios will include assets denominated in foreign currencies. The methods of calculation previously described are still appropriate to calculate returns, provided the securities denominated in foreign currencies are converted at appropriate exchange rates.

The impact of currency on a single asset portfolio is shown in Exhibit 3.30.

Exhibit 3.30 Currency returns

	Portfolio value in \$	Exchange rate €/\$	Portfolio value in €
Start value	\$100*	1.0	\$100 × 1.0 = €100
End value	\$110#	1.1	\$110 × 1.1 = €121

* 100 units of stock priced at \$1.0
# 100 units of stock priced at \$1.1

Note that the dollar buys 1.1 euros at the end of the period compared to 1.0 euro at the start; the dollar has increased in value.

The return of the portfolios in dollars (or local return) is:

$$\frac{110}{100} = 1.1 \text{ or a return of } 10\%$$

The currency return is:

$$\frac{1.1}{1.0} = 1.1 \text{ or a return of } 10\%$$

The return of the portfolio in euros is:

$$\frac{110 \times 1.1}{100 \times 1.0} = \frac{121}{100} = 1.21 \text{ or a return of } 21\%$$

Note that the portfolio return in Exhibit 3.30 is not the addition of the local return and currency return but the compound of returns:

$$(1 + r_L) \times (1 + r_C) = (1 + r) \tag{3.40}$$

where: r_L = return in local currency
r_C = currency return

r is the return of the portfolio in the base currency, the currency denomination or reference currency of the portfolio. Figure 3.6 illustrates the currency impact on portfolio valuation. The lightly shaded bottom right-hand area of €10 represents the underlying return of the portfolio in dollars or local return. The lightly shaded top left-hand area of €10 represents the currency return. The shaded top right-hand area of €1 represents the combined impact of local market returns with currency return. The initial value of the portfolio is the bottom left-hand area of €100 and the final value is the sum of all four areas, which is €121.

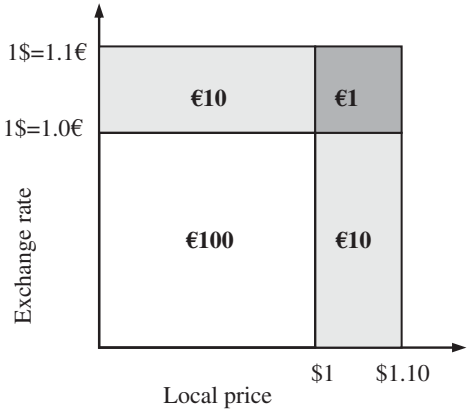


Figure 3.6 Components of base currency return

The example illustrated in Exhibit 3.30 and Figure 3.6 shows a positive contribution from the combined effect of local market returns with currency return. Exhibits 3.31 and 3.32 and Figures 3.7 and 3.8 illustrate a negative contribution from the combined effect of local market return with currency return.

Exhibit 3.31 Currency returns (positive local return, negative currency return)

	Portfolio value in $	Exchange rate €/$	Portfolio value in €
Start value	$100*	1.0	$100 × 1.0 = €100
End value	$110#	0.9	$110 × 0.9 = €99

* 100 units of stock priced at $1.0
# 100 units of stock priced at $1.1

Note that the dollar buys 0.9 euros at the end of the period compared to 1.0 euro at the start; the dollar has decreased in value.

The return of the portfolios in dollars (or local return) is:

$$\frac{110}{100} = 1.1 \text{ or a return of } 10\%$$

The currency return is:

$$\frac{0.9}{1.0} = 0.9 \text{ or a return of } -10\%$$

The return of the portfolio in euros:

$$\frac{110 \times 0.9}{100 \times 1.0} = \frac{99}{100} = 0.99 \text{ or a return of } -1.0\%$$

The shaded top right-hand area of €1 in Figure 3.7 still represents the combined impact of market return with currency return but, in this instance, it overlaps the bottom right-hand area, thus resulting in a negative contribution of €1. The final value of the portfolio is represented by the bottom left-hand quadrant of €90 (€100 initial value minus the overlapping loss from currency return of €10) plus the bottom right-hand area of €9 (€10 market return minus the overlapping loss from currency of €1). The total value is €99 ($90 × 1.1 = €99).

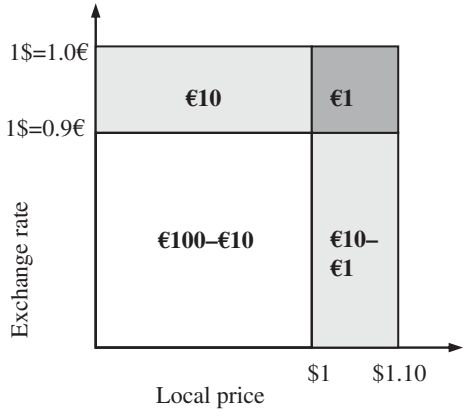


Figure 3.7 Components of base currency return (positive local return, negative currency return)

Exhibit 3.32 Currency returns (negative local return, positive currency return)

	Portfolio value in $	Exchange rate €/$	Portfolio value in €
Start value	$100*	1.0	$100 × 1.0 = €100
End value	$90#	1.1	$90 × 1.1 = €99

* 100 units of stock priced at $1.0
# 100 units of stock priced at $0.9

Note that the dollar buys 1.1 euros at the end of the period compared to 1.0 euro at the start; the dollar has increased in value.

The return of the portfolios in dollars (or local return) is:

$$\frac{90}{100} = 0.9 \text{ or a return of } -10\%$$

The currency return is:

$$\frac{1.1}{1.0} = 1.1 \text{ or a return of } +10\%$$

The return of the portfolio in euros is:

$$\frac{90 \times 1.1}{100 \times 1.0} = \frac{99}{100} = 0.99 \text{ or a return of } -1.0\%$$

The shaded top right-hand area of €1 in Figure 3.8 still represents the combined impact of market return with currency return but, in this instance, it overlaps the top left-hand area, thus resulting in a negative contribution of €1. The final value of the portfolio is represented by the bottom left-hand quadrant of €90 (€100 initial value minus the overlapping loss from portfolio return of €10) plus the top left-hand area of €9 (€10 market return minus the overlapping loss from currency of €1). The total value is €99 ($110 × 0.9 = €99).

Exhibit 3.33 and Figure 3.9 illustrate a positive contribution from the combined effect of a negative local market return with a negative currency return.

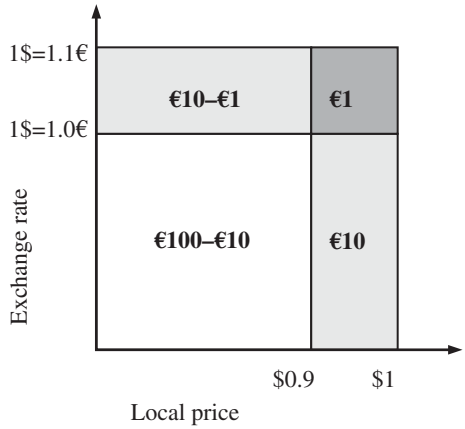


Figure 3.8 Components of base currency return (negative local return, positive currency return)

Exhibit 3.33 Currency returns (negative local return, negative currency return)

	Portfolio value in $	Exchange rate €/$	Portfolio value in €
Start value	$100*	1.0	$100 × 1.0 = €100
End value	$90#	0.9	$90 × 0.9 = €81

* 100 units of stock priced at $1.0
# 100 units of stock priced at $0.9

Note that the dollar buys 0.9 euros at the end of the period compared to 1.0 euro at the start; the dollar has decreased in value.

The return of the portfolios in dollars is:

$$\frac{90}{100} = 0.9 \text{ or a return of } -10\%$$

The currency return is:

$$\frac{0.9}{1.0} = 0.9 \text{ or a return of } -10\%$$

The return of the portfolio in euros is:

$$\frac{90 \times 0.9}{100 \times 1.0} = \frac{81}{100} = 0.81 \text{ or a return of } -19.0\%$$

The shaded top right-hand area of €1 in Figure 3.9 still represents the combined impact of market return with currency return but, in this instance, it overlaps both the top left-hand area and bottom right-hand area, resulting in a positive contribution since the value can't be lost twice. The final value of the portfolio is represented by the bottom left-hand quadrant of €81 (€100 initial value minus the overlapping loss from portfolio return of €10), minus the overlapping loss from currency return of €10, plus €1 from the double overlapping combination of local return and currency return. The total value is €81 ($90 × 0.9 = €81).

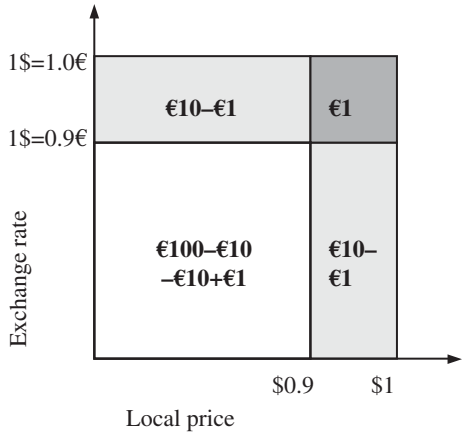


Figure 3.9 Components of base currency return (negative local return, negative currency return)

Note

The interaction of local market returns and currency returns is neither currency nor local market return, rather a combination of both.

Note

The interaction of local market returns and currency returns will be positive if returns are both positive or both negative and will be negative if either local market returns or currency returns is negative and the other is positive.

Currency conversions

The base currency returns of the portfolio can be readily converted to any other currency return for presentation purposes as follows:

$$(1 + r) \times (1 + c) - 1 = r_C \tag{3.41}$$

where: c = currency return relative to the base currency
r_C = portfolio return expressed in currency c

Local currency returns are calculated ignoring the impact of changes in the currency exchange rate. Although in reality the local currency return of a portfolio consisting of assets in multiple currencies does not exist, it is useful to make an intermediate calculation. The local return of a multi-currency portfolio is defined as the weighted average local return for assets in each currency as follows:

$$r_L = \sum_{j=1}^{j=n} w_i \times r_{Li} \tag{3.42}$$

where: w_i = weight of sector i
r_{Li} = local return of sector i

The ratio of the base currency return with the local return of the portfolio will calculate the implicit return due to currency in the portfolio.

Local, base currency (or just base) and currency returns for the portfolio data in Table 3.8 are calculated in Exhibits 3.34, 3.35 and 3.36 over a period of six months. Local returns are calculated in Exhibit 3.34.

Table 3.8 Local, base and currency returns

Period	Local market value in £m	$/£ spot rate	Base market value in $m
Start	100	1.5	150
End of month 1	100	1.6	160
End of month 2	110	1.55	170.5
End of month 3	115	1.4	161
End of month 4	105	1.3	136.5
End of month 5	95	1.4	133
End of month 6	90	1.4	126

Exhibit 3.34 Local returns

Local returns for each month:

Month 1 local return $\frac{100}{100} - 1 = 0\%$

Month 2 local return $\frac{110}{100} - 1 = 10\%$

Month 3 local return $\frac{115}{110} - 1 = 4.55\%$

Month 4 local return $\frac{105}{115} - 1 = -8.7\%$

Month 5 local return $\frac{95}{105} - 1 = -9.52\%$

Month 6 local return $\frac{90}{95} - 1 = -5.26\%$

The total local return over the six-month period is:

$$(1 + 0.0\%) \times (1 + 10.0\%) \times (1 + 4.55\%) \times (1 - 8.7\%) \times (1 - 9.52\%) \times (1 - 5.26\%) - 1 = -10.0\%$$

Alternatively, the local return over the six-month period is:

$$\frac{90}{100} - 1 = -10.0\%$$

Exhibit 3.35 Currency returns

Spot currency return for each month:

Month 1 currency spot return £ versus \$ $\frac{1.6}{1.5} - 1 = 6.67\%$

Month 2 currency spot return £ versus \$ $\frac{1.55}{1.6} - 1 = -3.13\%$

Month 3 currency spot return £ versus \$ $\frac{1.4}{1.55} - 1 = -9.68\%$

Month 4 currency spot return £ versus \$ $\frac{1.3}{1.4} - 1 = -7.14\%$

Month 5 currency spot return £ versus \$ $\frac{1.4}{1.3} - 1 = 7.69\%$

Month 6 currency spot return £ versus \$ $\frac{1.4}{1.4} - 1 = 0.00\%$

The total spot currency return over the six-month period is:

$$(1 + 6.67\%) \times (1 - 3.13\%) \times (1 - 9.68\%) \times (1 - 7.14\%) \times (1 + 7.69\%) \times (1 + 0.00\%) - 1 = -6.67\%$$

Alternatively, the currency return over the six-month period is:

$$\frac{1.4}{1.5} - 1 = -6.67\%$$

Spot currency returns, or simply currency returns, are calculated in Exhibit 3.35. Focusing on month 1 the dollar has fallen against the pound; the pound bought more dollars at the end of month 1 than at the start.

Note

There are always two ways to express currency movements – in the example of Exhibit 3.35, pound against the dollar or dollar against the pound. These are not the opposite of each other; for example, in month 1 of Exhibit 3.35 the dollar depreciated 6.25% but the pound appreciated 6.67% against the dollar, $\frac{1.6}{1.5} - 1 = 6.67\%$.

Currency pairs[38] are reciprocal or inverse relationships:

$$-6.25\% = 1 - \frac{1}{1 + 6.67\%} = \frac{1.5}{1.6} - 1 = \frac{0.62500}{0.66667} - 1$$

Base currency returns are calculated in Exhibit 3.36. The base currency for the portfolio in Table 3.8 is pounds.

Local, spot currency and base returns for each month and the total period are summarised in Table 3.9.

[38] A currency pair is the quotation of two different currencies, with the value of one currency being quoted against the other.

Exhibit 3.36 Base currency returns

Base currency return for each month:

Month 1 base currency return $\frac{100 \times 1.6}{100 \times 1.5} - 1 = \frac{160}{150} - 1 = 6.67\%$ or

$$(1 + 0\%) \times (1 + 6.67\%) - 1 = 6.67\%$$

Month 2 base currency return $\frac{110 \times 1.55}{100 \times 1.6} - 1 = \frac{170.5}{160} - 1 = 6.56\%$ or

$$(1 + 10\%) \times (1 - 3.13\%) - 1 = 6.56\%$$

Month 3 base currency return $\frac{115 \times 1.4}{110 \times 1.55} - 1 = \frac{161}{170.5} - 1 = -5.57\%$ or

$$(1 + 4.55\%) \times (1 - 9.68\%) - 1 = -5.57\%$$

Month 4 base currency return $\frac{105 \times 1.3}{115 \times 1.4} - 1 = \frac{136.5}{161} - 1 = -15.22\%$ or

$$(1 - 8.7\%) \times (1 - 7.14\%) - 1 = -15.22\%$$

Month 5 base currency return $\frac{95 \times 1.4}{105 \times 1.3} - 1 = \frac{133}{136.5} - 1 = -2.56\%$ or

$$(1 - 9.52\%) \times (1 + 7.69\%) - 1 = -2.56\%$$

Month 6 base currency return $\frac{90 \times 1.4}{95 \times 1.4} - 1 = \frac{126}{133} - 1 = -5.26\%$

$$(1 - 5.26\%) \times (1 + 0.00\%) - 1 = -5.26\%$$

Base currency return for six months:
The total base currency return over the six-month period is:

$$(1 + 6.67\%) \times (1 + 6.56\%) \times (1 - 5.57\%) \times (1 - 15.22\%) \times (1 - 2.56\%) \times (1 - 5.26\%) - 1 = -16.00\%$$

Alternatively, the total base currency return over the six-month period is:

$$(1 - 10.0\%) \times (1 - 6.67\%) - 1 = -16.00\%$$

Or, the total base currency return over the six-month period is:

$$\frac{90 \times 1.4}{100 \times 1.5} - 1 = \frac{126}{150} - 1 = -16.00\%$$

Table 3.9 Local, currency and base currency portfolio returns

Period	Market value in £m	£/$ spot rate	Local return	Currency return £ vs. $	Base return
Start	100	1.5			
End of month 1	100	1.6	0.00%	6.67%	6.67%
End of month 2	110	1.55	10.0%	−3.13%	6.56%
End of month 3	115	1.4	4.55%	−9.68%	−5.57%
End of month 4	105	1.3	−8.70%	−7.14%	−15.22%
End of month 5	95	1.4	−9.52%	7.69%	−2.56%
End of month 6	90	1.4	−5.26%	0.00%	−5.26%
			−10.0%	**−6.67%**	**−16.00%**

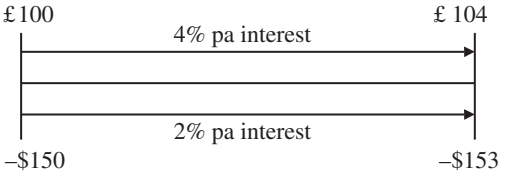
HEDGED RETURNS

Asset owners may decide that although they are content with the risk of holding the underlying asset, they are not content with the additional uncertainty of foreign currency exposure. Consequently, they may decide to hedge their currency risk. The purpose of currency hedging is to reduce or eliminate currency exposure. Asset owners can hedge their currency risk using forward currency contracts, currency swaps, currency futures contracts and currency options.

Forward currency contracts are relatively simple derivative instruments. They are contracts between two parties for the exchange of an agreed amount of currency at a fixed exchange rate on a fixed date in the future. The two currencies are not actually exchanged until the value date is reached but the rate is agreed on the trade date. The fair rate for a forward currency contract can be established using the spot exchange rate and the interest rates of the two currencies involved as shown in Exhibit 3.37.

Exhibit 3.37 Forward currency contracts

Assume a current spot rate of \$/£1.5
Assume an annual rate of interest in £ of 4%
Assume an annual rate of interest in \$ of 2%



The one-year forward rate of £ against the \$ is:
At today's spot rate £100 would buy \$150. Investing £100 at an annualised interest rate of 4% for one year would yield $100 \times (1.04) = 104$. Investing \$150 at an annualised rate of 2% for one year would yield $150 \times (1.02) = 153$. The value of £ against the \$ in one year's time is therefore:

$$£104 = \$153$$

$$£1 = \frac{153}{104} = \$1.47115$$

The one-year forward rate (the currency exchange rate today for conversion in one year's time) is therefore £/\$ 1.47115.

If the forward rate does not reflect interest rates, then an arbitrageur could borrow in one currency, invest in another, and cover the future repayment of the loan without currency risk using a currency forward contract. The potential for this covered interest arbitrage results in a relationship described as "covered interest rate parity", which states that the currency forward rate will be a function of the currency spot rate and the respective interest rates of the two currencies as follows:

$$F_0 = S_0 \times \frac{1 + x_L}{1 + x_B} \tag{3.43}$$

where: F_0 = forward exchange rate at time $t = 0$
S_0 = spot exchange rate at time $t = 0$
x_L = interest rate in local currency at time $t = 0$
x_B = interest rate in base currency at time $t = 0$

Exhibit 3.38 applies Equation 3.43 to the forward contract data in Exhibit 3.37.

Exhibit 3.38 Forward currency exchange rate

$$F_0 = S_0 \times \frac{1 + x_L}{1 + x_B} = 1.5 \times \frac{1.02}{1.04} = 1.47115$$

The one-year forward rate is £/$ 1.47115.

The spot currency return for period t is:

$$= \frac{S_{t+1}}{S_t} - 1 \tag{3.44}$$

where: S_{t+1} = spot exchange rate at time $t + 1$

The return on a forward currency contract is not the same as the spot currency return (unless the interest rates of both currencies are the same). The return on the forward currency contract is known as the forward currency return (or simply the forward return or currency surprise).

The currency forward return for period t is:

$$= \frac{S_{t+1}}{F_t} - 1 \tag{3.45}$$

where: F_t = forward exchange rate at time t for conversion at time $t + 1$

The forward premium or interest rate differential for period t is:

$$= \frac{F_t}{S_t} - 1 \tag{3.46}$$

Therefore, the spot currency return is the compound of the interest rate differential with the currency forward return:

$$= \frac{S_{t+1}}{S_t} - 1 = \frac{S_{t+1}}{F_t} \times \frac{F_t}{S_t} - 1 \tag{3.47}$$

Assuming a spot exchange rate of $/£ of 1.6 in one year's time, spot currency returns, forward returns and forward premiums are calculated in Exhibit 3.39 for the forward contract in Exhibit 3.40. From the perspective of the pound as base currency the forward return in this example is greater than the spot currency return because interest rates in the pound are greater than the dollar. Notionally to gain exposure to the pound and hedging dollars, cash must be borrowed in dollars at a lower rate of interest and reinvested in the pound at a greater rate of interest. This is called the benefit of hedging. Traders might recognise this as the carry trade,[39] traditionally borrowing in a low-interest-rate currency such as the Japanese yen or Swiss franc and reinvesting in a high-interest currency such as the Australian or New Zealand dollar.

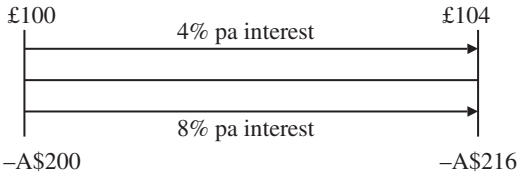
[39] The carry trade involves borrowing at a low rate of interest and reinvesting at a high rate of interest.

Exhibit 3.39 Spot currency return, forward return and forward premium

Spot currency return	$= \frac{S_{t+1}}{S_t} - 1 = \frac{1.6}{1.5} - 1 = 6.67\%$
Forward currency return	$= \frac{S_{t+1}}{F_t} - 1 = \frac{1.6}{1.47115} = 8.76\%$
Forward premium	$= \frac{F_t}{S_t} - 1 = \frac{1.47115}{1.5} - 1 = \frac{1.02}{1.04} - 1 = -1.92\%$
Spot currency	$= \frac{S_{t+1}}{F_t} \times \frac{F_t}{S_t} - 1 = (1 + 8.76\%) \times (1 - 0.192\%) - 1 = 6.67\%$

Exhibit 3.40 Pound–Australian dollar forward contract

Assume a current spot rate of £/A$2.0
Assume an annual rate of interest in £ of 4%
Assume an annual rate of interest in A$ of 8%



The one-year forward rate of £ against the A$ is:
At today's spot rate £100 would buy A$200.
Investing £100 at an annualised interest rate of 4% for one year would yield $100 \times (1.04) = 104$.
Investing A$200 at an annualised rate of 8% for one year would yield $200 \times (1.08) = 216$.
The value of £ against the A$ in one year's time is therefore:

$$£104 = A\$216$$

$$£1 = \frac{216}{104} = \$2.077$$

The one-year forward rate (the currency exchange rate today for conversion in one year's time) is therefore £/A$ 2.077.

Or using Equation 3.43:

$$F_0 = S_0 \times \frac{1 + x_L}{1 + x_B} = 2.0 \times \frac{1.08}{1.04} = 2.077$$

Exhibit 3.40 illustrates a forward currency contract in a high-interest-rate currency: Australian dollars.

Assuming a spot exchange rate of £/A$ of 1.8 in one year's time, spot currency returns and forward returns are calculated in Exhibit 3.41 for the forward contract in Exhibit 3.40. From the perspective of the pound as base currency the forward return in this example is worse that the spot currency return because interest rates in the pound are less than the Australian dollar. Notionally to gain exposure to the pound and hedging Australian dollars, cash must be

Exhibit 3.41 Currency return, forward return – high-interest-rate currency

Pound against Australian dollar

Spot currency return $= \frac{S_{t+1}}{S_t} - 1 = \frac{1.8}{2.0} - 1 = -10.0\%$

Forward currency return $= \frac{S_{t+1}}{F_t} - 1 = \frac{1.8}{2.077} = -13.33\%$

Australian dollar against the pound (the reciprocal or inverse relationship)

Spot currency return $= \frac{S_t}{S_{t+1}} - 1 = \frac{0.55556}{0.5} - 1 = \frac{2.0}{1.8} - 1 = 11.11\%$

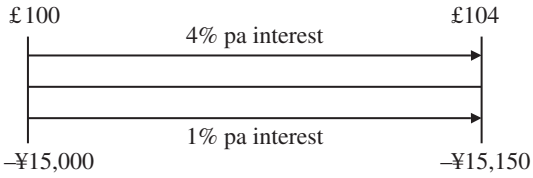
Forward currency return $= \frac{F_t}{S_{t+1}} - 1 = \frac{0.55556}{0.48146} - 1 = \frac{2.077}{1.8} = 15.39\%$

borrowed in Australian dollars at a higher rate of interest and reinvested in the pound at a lower rate of interest. This is called the cost of hedging. From the perspective of the Australian dollar, borrowing at lower interest rates and investing in higher interest rates is a benefit of hedging.

Exhibit 3.42 illustrates a forward currency contract in a low-interest-rate currency – the Japanese yen.

Exhibit 3.42 Pound–Yen forward contract

Assume a current spot rate of £/¥150
Assume an annual rate of interest in £ of 4%
Assume an annual rate of interest in ¥ of 1%



The one-year forward rate of £ against the ¥ is:
At today's spot rate £100 would buy ¥15,000.
Investing £100 at an annualised interest rate of 4% for one year would yield $100 \times (1.04) = 104$.
Investing ¥15,000 at an annualised rate of 1% for one year would yield $15{,}000 \times (1.01) = 15{,}150$.
The value of £ against the ¥ in one year's time is therefore:

$$£104 = ¥15{,}150$$

$$£1 = \frac{15{,}150}{104} = ¥145.67$$

The one-year forward rate (the currency exchange rate today for conversion in one year's time) is therefore £/¥ 145.67.

Or using Equation 3.43:

$$F_0 = S_0 \times \frac{1 + x_L}{1 + x_B} = 150 \times \frac{1.01}{1.04} = 145.67$$

Assuming a spot exchange rate of £/¥ of 135 in one year's time, spot currency returns and forward returns are calculated in Exhibit 3.43 for the forward contract in Exhibit 3.42. The forward return in this example is better than the spot currency return because interest rates in the pound are greater than the yen. Notionally to gain exposure to the pound and hedging yen, cash must be borrowed in yen at a lower rate of interest and reinvested in the pound at a higher rate of interest.

Exhibit 3.43 Currency return, forward return – low-interest-rate currency

Pound against Japanese yen

Spot currency return $= \frac{S_{t+1}}{S_t} - 1 = \frac{135}{150} - 1 = -10.0\%$

Forward currency return $= \frac{S_{t+1}}{F_t} - 1 = \frac{135}{145.67} = -7.32\%$

Japanese yen against the pound (the reciprocal or inverse relationship)

Spot currency return $= \frac{S_t}{S_{t+1}} - 1 = \frac{150}{135} - 1 = 11.11\%$

Forward currency return $= \frac{F_t}{S_{S+1}} - 1 = \frac{145.67}{135} = 7.90\%$

Currency overlay returns

The local returns in Table 3.9 are unobtainable to a dollar-based investor; the only returns that are obtainable are either the local returns converted to dollars or returns hedged back into dollars. Unless interest rates in both currencies are the same there will be a cost or benefit associated with hedging.

To hedge the pound exposure in Table 3.8 we can use forward currency contracts, in effect shorting the pound and gaining exposure to dollars. We can either trade these forward contracts within the existing portfolio or hire a currency overlay asset manager to trade these forward contracts in a separate currency overlay portfolio. Exhibit 3.44 demonstrates hedged returns using a separate currency overlay portfolio.

Using the same spot currency exchange rates in Table 3.9 and assuming constant interest rates of 4.0% per annum in pounds and 2.0% per annum in dollars throughout the six-month period, one-month forward rates are shown in Table 3.10.

Hedged portfolio returns are summarised in Table 3.11.

Perfectly hedged returns

The month 1 hedged return in Exhibit 3.44 is slightly negative even though the local return is 0.0% and the initial currency exposure is fully hedged. This is caused by the cost of borrowing pounds at a high rate of interest and investing in dollars at a lower rate of interest. This cost can be derived directly from the interest rates of 4% per year in pounds and 2% per year in dollars as follows:

$$\frac{(1.02)^{1/12}}{(1.04)^{1/12}} - 1 = -0.16\%$$

Exhibit 3.44 Currency overlay

Hedging pound exposure in month 1 requires the sale of £100 million at a forward rate of 1.49757. At the end of month 1, the loss in dollars on this contract is:

$$100 \times 1.49757 - 100 \times 1.6 = -10.24$$

This results in a base currency return of $\frac{-10.24}{150} = -6.83\%$

Adding the currency overlay return in month 1 to the base currency of the portfolio in Table 3.9 provides the fully hedged return:

$$6.67\% - 6.83\% = -0.16\%$$

Because there is a 0% underlying local market return, −0.16% represents the cost of hedging pounds into dollars.

Hedging pound exposure in month 2 also requires the sale of £100 million but at a forward rate of 1.59741.

At the end of month 2, the profit in dollars on this contract is:

$$100 \times 1.59741 - 100 \times 1.55 = 4.74$$

This results in a base currency return of $\frac{4.74}{160} = 2.96\%$.

Therefore, the hedged return in month 2 is 6.56% + 2.96% = 9.53%.

Hedging pound exposure in month 3 requires the sale of £110 million at a forward rate of 1.54749.

At the end of month 3, the profit in dollars on this contract is:

$$110 \times 1.54749 - 110 \times 1.4 = 16.22$$

This results in a base currency return of $\frac{16.22}{170.5} = 9.52\%$.

Therefore, the hedged return in month 3 is −5.57% + 9.52% = 3.94%.

Hedging pound exposure in month 4 requires the sale of £115 million at a forward rate of 1.39774. At the end of month 4, the profit in dollars on this contract is:

$$115 \times 1.39774 - 115 \times 1.3 = 11.24$$

This results in a base currency return of $\frac{11.24}{161} = 6.98\%$.

Therefore, the hedged return in month 4 is −15.22% + 6.98% = −8.24%.

Hedging pound exposure in month 5 requires the sale of £105 million at a forward rate of 1.29790. At the end of month 5, the loss in dollars on this contract is:

$$105 \times 1.29790 - 105 \times 1.4 = -10.72$$

This results in a base currency return of $\frac{-10.72}{136.5} = -7.85\%$.

Therefore, the hedged return in month 5 is −2.56% − 7.85% = −10.42%.

Hedging pound exposure in month 6 requires the sale of £95 million at a forward rate of 1.39774. At the end of month 6, the loss in dollars on this contract is:

$$95 \times 1.39774 - 95 \times 1.4 = -0.22$$

This results in a base currency return of $\frac{-0.22}{133.0} = -0.16\%$.

Exhibit 3.44 (*Continued*)

Therefore, the hedged return in month 6 is $-5.26\% - 0.16\% = -5.42\%$.
Because there is no change in spot rates, -0.16% is simply the cost of hedging. Because interest rates have remained constant for both currencies throughout, the cost of hedging is identical for each month.
The total hedged return over the six-month period is:

$$(1 - 0.16\%) \times (1 + 9.53\%) \times (1 + 3.54\%) \times (1 - 8.24\%) \times (1 - 10.42\%) \times (1 - 5.42\%) - 1$$
$$= -11.63\%$$

Table 3.10 One-month forward rates

Period	\$/£ spot rate	\$/£ one-month forward rate
Start	1.5	1.49757
End of month 1	1.6	1.59741
End of month 2	1.55	1.54749
End of month 3	1.4	1.39774
End of month 4	1.3	1.29790
End of month 5	1.4	1.39744
End of month 6	1.4	1.39744

Table 3.11 Hedged portfolio returns

Period	Base return	Currency overlay return	Hedged return
Month 1	6.67%	−6.83%	−0.16%
Month 2	6.65%	2.96%	9.53%
Month 3	−5.57%	9.52%	3.94%
Month 4	−15.22%	6.98%	−8.24%
Month 5	−2.56%	−7.85%	−10.24%
Month 6	−5.26%	−0.16%	−5.24
Six-month	**−16.00%**	**n/a**	**−11.63%**

However, currency exposures will change with underlying market movements. Hedges taken out at the start of the measurement period will not capture changing exposures caused by market movements. From the perspective of a currency overlay manager, unless informed differently, they will be unaware of residual currency exposures caused by underlying local market movements. If these residual positions are perfectly hedged, then the only difference between the local return and the hedged return will be due to interest rate differentials between the respective currencies.

The theoretically perfect hedge returns for the portfolio in Table 3.8 are calculated in Exhibit 3.45 and compared to the hedged returns calculated for the currency overlay portfolio in Table 3.12.

Exhibit 3.45 Perfectly hedged returns

Month 1 $(1 + 0.0\%) \times (1 - 0.16\%) - 1 = -0.16\%$
Month 2 $(1 + 10.0\%) \times (1 - 0.16\%) - 1 = 9.82\%$
Month 3 $(1 + 4.55\%) \times (1 - 0.16\%) - 1 = 4.38\%$
Month 4 $(1 - 8.70\%) \times (1 - 0.16\%) - 1 = -8.84\%$
Month 5 $(1 - 9.52\%) \times (1 - 0.16\%) - 1 = -9.67\%$
Month 6 $(1 - 5.26\%) \times (1 - 0.16\%) - 1 = -5.42\%$

Table 3.12 Hedged versus perfectly hedged returns

Period	Hedged return	Perfectly hedged return
Month 1	−0.16%	−0.16%
Month 2	9.53%	9.82%
Month 3	3.94%	4.38%
Month 4	−8.24%	−8.84%
Month 5	−10.24%	−9.67%
Month 6	−5.24%	−5.42%
Six-month	**−11.63%**	**−10.87%**

The hedged return in month 2 of 9.53% is less than the perfectly hedged return of 9.82% because there is a residual unhedged exposure to the falling pound from the underlying market return. The hedged return in month 4 of −8.24% is greater than the perfectly hedged return of −8.84% because there is less exposure to the falling pound due to a negative underlying local market return.

⚠ Caution

It is virtually impossible to perfectly hedge portfolios; underlying market values are constantly changing. Currency overlay managers are best advised to manage against the currency exposures they are aware of, rather than assumptions of currency exposures based on the impact of market movements. This will cause problems for multi-currency attribution as described in Chapter 6.

⚠ Caution

Illiquid assets with currency exposure also cause problems for currency overlay managers. Illiquid assets are typically revalued at the end of a measurement period; overlay managers are simply unaware of the residual currency exposure prior to updated valuations.

PORTFOLIO COMPONENT RETURNS

Calculating the performance of the total portfolio is only part of the analytical process. If we are to understand all the sources of return in a portfolio, we must calculate the returns of subsets, or components, of the portfolio (asset classes, categories, regions, countries, sectors, etc.), including individual securities and instruments that contribute to the total return of a portfolio.

The calculation methodologies for component returns are the same as for the total portfolio; however, internal cash flows between components or sectors should be treated as external cash flows within that component. Dividend and coupon payments should be treated as cash flow out of the relevant sector into an appropriate cash sector, if, that is, these sectors are defined separately.

Provided return calculation methodologies are consistent, the sum of component returns should equal the total portfolio return. This is a key requirement for both attribution and contribution analysis. Because internal rates of return assume a constant rate of return for all assets within the portfolio it is not appropriate to use internal rates of return to calculate component returns.

Note

I cannot understate the importance of the total portfolio return being the sum of its parts. This not only facilitates the good analysis of the sources of return, consistent with the investment decision process, but also allows diagnostic processes that improve operational performance.

Money-weighted component returns

Both simple and modified Dietz total returns can be broken down or disaggregated into component returns.

Let r_i be the return of the portfolio in the ith component, sector or asset category.

Then, using modified Dietz:

$$r_i = \frac{{}^iV_E - {}^iV_S - {}^iC}{{}^iV_S + \sum {}^iC_t \times {}^iW_t} \tag{3.48}$$

where: ${}^iV_E$ = end value of sector i
${}^iV_S$ = start value of sector i
${}^iC$ = total cash flow in sector i
${}^iC_t$ = cash flow in sector i on day t
${}^iW_t$ = weighting ratio in sector i on day t

Then the total portfolio return

$$r = \sum_{i=1}^{i=n} w_i \times r_i \tag{3.49}$$

where: w_i = weight of the portfolio in the ith asset class

$$w_i = \frac{{}^iV_S + \sum {}^iC_t \times {}^iW_t}{V_S + \sum C_t \times W_t} \tag{3.50}$$

Note that $\sum_{i=1}^{i=n} w_i = 1$ since $\sum_{i=1}^{i=n} {}^iV_S = V_S$ and $\sum_{i=1}^{i=n} \sum {}^iC_t \times {}^iW_t = \sum C_t \times W_t$

Time-weighted component returns

Time-weighted returns can be disaggregated as well. The weight allocated to transactions within the time-weighted period must be the same as the return methodology associated with the overall portfolio (i.e., beginning of day, middle of day or end of day). Because cash flows will exist between categories within a portfolio it is no longer sufficient to revalue only at the date of an external cash flow. To calculate the time-weighted return for a component or sector, valuations are required for internal cash flows, which in effect require daily valuations.

Note

Equation 3.49 applies equally to time-weighted as well as money-weighted returns. This equation is fundamental to much of the detailed attribution and contribution analysis that follows.

In rare circumstances, due to transaction activity, the effective weight in a sector may total zero but still contribute a gain or loss to the overall portfolio. The most common instance of this is a buy transaction (an inflow) in a sector with no current holding but using the end of day cash flow assumption. In these circumstances it is acceptable to use the size of cash flow as the weight for that sector, ensuring there is an offsetting cash flow in the cash sector.

It is this problem of apparent zero weights for individual sectors and most particularly for individual securities that has led some firms to use a midday weighting assumption, which of course guarantees that the cash flow will be represented in the denominator of the return calculation.

Although both midday weighting and inflow-start of day and outflow-end of day approaches both remove the zero-weighting problem, they are systems-inspired solutions. I prefer end of day as a logical timing for all cash flows; if this method results in a zero weight in an individual category it is appropriate to default to start of day for that sector only with a balancing adjustment in the cash sector. Therefore, the overall return is unaffected by this adjustment and the sum of the parts still equals the total because of the equalising adjustment in the cash sector.

The timing of external cash flow assumptions within the day impacts true time-weighted return calculations as well as money-weighted return calculations. The assumptions asset managers make include:

(1) End of day
(2) Beginning of day

(3) Intra-day weighted
(4) Differentiated
(5) Actual time
(6) Rule-based

End of day

The end of day cash flow assumption is perhaps the most common assumption applied by asset managers. Using an end of day cash flow assumption requires that the return for that day is calculated using an end of day valuation excluding the value of the cash flow (see Equation 3.23). The economic rationale for the end of day assumption is clear: asset managers need time to decide how to respond to any cash flow. Assuming that cash flow is only available for investment at the end of the day is reasonable, given the time required to respond.

Of course, the reality is that cash flows have occurred during the day: assets may have been bought or sold and profits made or lost. These profits (or losses) will be applied to a smaller capital employed for inflows and a larger capital employed for outflows. For small cash flows relative to the initial capital employed this will normally have little impact; for larger cash flows on occasion the impact might be more significant.

If there is no initial capital in the sector then for an inflow, an end of day cash flow assumption will generate a zero denominator, resulting in the failure to calculate any contribution to return for that period.

Beginning of day

The beginning of day assumption is an obvious alternative to the end of day assumption. Using this assumption, the cash flow is added to the previous period end valuation resulting in profits and losses being applied to a larger capital employed for inflows and a smaller capital employed for outflows (see Equation 3.24). The economic rationale of this assumption presupposes that the asset manager has been alerted to the possibility of a cash flow and is therefore ready to respond. Clearly this method results in a non-zero denominator in an empty sector for cash inflows. But of course, zero, near zero or even negative[40] denominators may result from a cash outflow of the entire holding within the day.

Intra-day weighted

The intra-day method is a type of Dietz method calculation within the day. The most common intra-day method, making a midday assumption, is to simply weight the cash flow by 50%, see Equation 3.25. This has the advantage of eliminating most zero denominator issues of beginning and end of day methods. The main disadvantage is that the capital employed, within the day, is arbitrarily 50% of the cash flow, not 100%, a real compromise. More extreme versions of this method may choose to weight by the hour; for example, given an 8:30 a.m. to 4:30 p.m. working day, a 10:30 a.m. cash flow may be given a 75% weight of capital employed.

[40] Negative denominators are caused by the sale transaction during the day being greater in value than the start of period value.

Differentiated

The differentiated method applies a different approach to each cash flow depending on whether they are inflows or outflows.

Either:

(1) Beginning of day for a cash inflow and end of day for a cash outflow
or
(2) End of day for a cash inflow and beginning of day for a cash outflow

The first of these differentiated methods has the advantage of eliminating zero denominators for both inflows and outflows at the expense of a structurally higher capital employed. Higher capital employed naturally results from assuming that all inflows occur at the beginning of the day and all outflows occur at the end of the day. If we are to assume, certainly over the long term, that investment returns are positive, then higher capital employed will tend to dilute these positive returns – no asset manager would wish to employ a method that structurally dilutes positive returns.

Note

Although both the intra-day-weighted and differentiated approaches both remove the zero-weighting problem, they are systems-inspired solutions. I prefer end of day as a logical timing for all cash flows; if this method results in a zero weight in an individual category it is appropriate to default to start of day for that sector only with a balancing adjustment in the cash sector. Therefore, the overall return is unaffected by this adjustment and the sum of the parts still equals the total because of the equalising adjustment in the cash sector.

The impact of the first differentiated method on the overall return for our standard example is shown in Exhibit 3.46. Unsurprisingly given that the net external cash flow is predominately inflow, the resultant return of −9.42% is close to the start of day assumption return of −9.44%.

Exhibit 3.46 Time-weighted (inflow start of day, outflow end of day)

Inflow start of day, outflow end of day		
Market start value	31 December	\$74.2m
Market end value	31 January	\$104.4m
External inflow	14 January	\$39.5m
External outflow	14 January	\$2.4m
Net external cash flow		\$37.1m
Market value start of	14 January	\$67.0m (before inflow)
Market value end of	14 January	\$103.1m (after outflow)

$$\frac{67.0}{74.2} \times \frac{103.1 + 2.4}{67.0 + 39.5} \times \frac{104.4}{103.1} - 1 = 0.9030 \times 0.9906 \times 1.0126 - 1 = -9.42\%$$

The second differentiated method suffers the disadvantage of potential zero denominators for both inflows and outflows but employs a lower capital base, thus potentially leveraging returns.

The impact of the second differentiated method on the overall return for our standard example is shown in Exhibit 3.47. This differentiated approach leverages the loss within the day of the cash flow on a smaller capital base, resulting in a more negative return over the entire period.

Exhibit 3.47 Time-weighted (inflow end of day, outflow start of day)

Inflow start of day, outflow end of day		
Market start value	31 December	\$74.2m
Market end value	31 January	\$104.4m
External inflow	14 January	\$39.5m
External outflow	14 January	\$2.4m
Net external cash flow		\$37.1m
Market value start of	14 January	\$67.0m (before inflow)
Market value end of	14 January	\$103.1m (after outflow)

$$\frac{67.0}{74.2} \times \frac{103.1 - 39.5}{67.0 - 2.4} \times \frac{104.4}{103.1} - 1 = 0.9030 \times 0.9845 \times 1.0126 - 1 = -9.98\%$$

Exhibits 3.46 and 3.47 add two further return variations due to methodology to Table 3.2, all based on the same sample data. The differences in all these methodologies are caused by assumptions addressing the timing of cash flows resulting in differing effective capital employed.

Note

The very wide range of return methodologies offers significant opportunities for self-selection. Ex-post it is often quite easy to construct a rationale for a particular methodology in a particular circumstance. This should be avoided; consistency and transparency are key – methodologies are only changed ex-post if there is a return advantage!

Actual time

The actual time method is the most accurate but least practical method. It requires an accurate valuation at the specific time of each cash flow. If there are multiple cash flows in the day at different times, then the portfolio must be valued at each time and returns between each cash flow chain-linked – a pure, true time-weighted return if you like. The difficulties of achieving accurate valuations at any time and the cost effectively eliminate this method as a practical alternative.

Rule-based

The rule-based method is not a method in its own right but an ex-ante rule that allows switching between certain methodologies in certain circumstances.

Extremely large cash flows

Extremely large cash flows are cash flows extreme enough to dominate the return calculation for that period. Extremely large cash flows are policy determined but are typically greater than 100% of initial capital. Note that if there is no initial capital than any cash flow is extreme. For example, an asset manager with an end of day standard assumption might require a beginning of day assumption if the cash flow qualifies as an extreme cash flow. This clearly works on the first day of a portfolio's history but is most likely to be applied on sub-sectors or individual securities within the portfolio.

Which timing assumption to use for time-weighted returns?

Exhibit 3.48 demonstrates, with a simple example, the difference in return for each of the timing assumptions typically used in true time-weighted returns. In this example we have a small portfolio of \$100 with a cash inflow of \$10 on day 1. The value at the time of this cash inflow is \$100.50. The value of the portfolio at the end of day 1 including the cash inflow is \$111, a profit of \$1 during day 1. On day 2 there is a cash outflow of \$10. The value of the portfolio at the time of this outflow is \$111.50. The value of the portfolio at the end of day 2 excluding the cash outflow is \$102, a profit of \$1 during day 2.

Exhibit 3.48 Timing of cash flow – true time-weighted

	Day 1	Day 2
Beginning market value	\$100	\$111
Cash flow	+\$10	−\$10
Actual value	\$100.50	\$111.50
End market value	\$111	\$102
End of day	$\frac{101}{100} \times \frac{112}{111} - 1 = 1.91\%$	
Beginning of day	$\frac{111}{110} \times \frac{102}{101} - 1 = 1.908\%$	
Intra-day	$\left(1 + \frac{111 - 100 - 10}{100 + 10/2}\right) \times \left(1 + \frac{102 - 111 + 10}{111 - 10/2}\right) = 1.905\%$	
Differentiated	Inflow (beginning of day), outflow (end of day)	$\frac{111}{110} \times \frac{112}{111} - 1 = 1.818\%$
	Inflow (end of day), outflow (beginning of day)	$\frac{101}{100} \times \frac{102}{101} - 1 = 2.0\%$
Actual	$\frac{100.5}{100} \times \frac{111.5}{110.5} \times \frac{102}{101.5} - 1 = 1.909\%$	

The end of day method assumes the cash inflow is not available to the asset manager until the end of day so to calculate the return on day 1 the cash inflow is excluded from the end of day valuation, \$101. The start value is \$100: a profit of \$1 (\$101 – \$100) on a capital base of \$100 generates a day 1 return of 1.0%. To calculate the return on day 2 we must assume the cash outflow has not yet occurred; therefore, we have a profit of \$1 (\$112 – \$111) applied to a capital base of \$111 = 0.901%.

Compounding day 1 with day 2 we get a return of 1.91%.

The beginning of day method assumes the cash inflow is available to the asset manager from the beginning of the day so the capital base on day 1 is \$110. The \$1 profit on day 1 (\$111 – \$100) generates a return of 0.909% rather than the 1.0% from the end of day method – a significant return difference caused by the capital available assumption. In this example much of this return difference is recovered on day 2 because the capital base assumption is now \$101, resulting from assuming the cash outflow occurred at the beginning of day. The day 2 return is 0.990%. Compounding day 1 with day 2 we get a return of 1.908%, admittedly only 0.2 of a basis point different, but different nevertheless.

The midday, intra-day assumption results in a capital base on day 1 of \$105 and hence a return of 0.952%, and a capital base on day 2 of \$106 and hence a return of 0.943%. Compounding day 1 with day 2 we get 1.905%, 0.3 of a basis point less than the beginning of day method and 0.5 of a basis point less than the end of day method.

The differentiated method produces the most interesting return difference. Assuming beginning of day for a cash inflow and end of day for a cash outflow results in a higher capital base for both days. Day 1 is a cash inflow hence a return of 0.909%, day 2 is a cash outflow hence a return of 0.901%. Compounding day 1 with day 2 we get a two-day return of 1.818% – an astonishing nine-basis-point-plus difference from the end of day methodology.

The second differentiated method assumes beginning of day for a cash outflow and end of day for a cash inflow, resulting in a lower capital base for both days. Day 1 is a cash inflow hence a return of 1.0%, day 2 is a cash outflow hence a return of 0.990%. Compounding day 1 with day 2 we get a two day return of 2.0%. Again an astonishing difference from the end of day methodology, this time in the other direction.

The most accurate, and least practical, method is to value the portfolio accurately at the time of each cash flow. In this example we have two cash flows and hence three sub-period returns. The return immediately before the cash flow on day 1 is 0.5%, the return between cash flows is 0.905% and the return after the second cash flow is 0.493%. Compounding the three sub-period returns results in a two-day return of 1.909%.

This of course is a contrived example with symmetrical profits and symmetrical, offsetting cash flows. As a result, the actual return falls midway between the end of day and beginning of day methodologies; this is atypical. Different return and cash flow profiles will generate different impacts. The main takeaway from this example, however, is that the differentiated method diverges the most from the actual return.

My timing methodology preferences are listed in Table 3.13.

Carve-outs

Sub-sector returns of larger portfolios are often calculated and presented separately by asset managers to demonstrate competence in managing assets of that type, particularly if the

Table 3.13 Time-weighted timing assumption methodologies

Method	Advantages	Disadvantages	Author's ranking (highest number preferred)
End of day	(i) Most common (ii) Simple to understand (iii) Good economic rationale	(i) Potential zero denominator for inflows	6
Beginning of day	(i) Simple to calculate (ii) Avoids potential zero denominator for inflows	(i) Potential zero, near zero or negative denominator for outflows (ii) Assumes advance notice of cash flows	4
Intra-day weighting	(i) Simple to calculate (ii) Avoids zero, near zero and negative denominators in most circumstances	(i) Weak economic rationale	3
Differentiated inflow (beginning of day) Outflow (end of day)	(i) Avoids zero, near zero and negative denominators for inflows and outflows	(i) Structural higher capital base, potentially diluting returns (ii) Less economic rationale – more systems driven	2
Differentiated inflow (end of day) Outflow (beginning of day)	(i) Structural lower capital base potentially leveraging returns. Might be described as unethical	(i) Potential zero, near zero or negative denominator for outflows (ii) Potential zero denominator for inflows	1
Actual time	(i) Technically the most accurate	(i) Not practicable	5
Rule-based	(i) Can combine preferred method in specific circumstances based on ex-ante rules	(i) Complex	7

manager does not manage that type of asset in stand-alone portfolios. These sub-sector returns are often called "carve-outs".

Sub-portfolios

Carve-outs with their own dedicated cash and managed just like a stand-alone portfolio are called sub-portfolios. Carve-outs and sub-portfolios are discussed in more detail in Chapter 7.

Cash sectors

Cash and cash equivalents[41] in a portfolio should be measured using the same methodology as any other asset within the portfolio. Because the cash sector naturally suffers large and frequent cash flows, the calculation assumptions with regard to cash flow may result in a return which on face value is most unlike a cash return. Crucially, the combination of average weight and return will replicate the contribution of the cash component to total return.

Individual security returns

Accounting policies, valuation procedures and return methodologies flow directly to individual securities. To ensure that the total portfolio return is the sum of its parts it is essential that policies, procedures and methodologies are consistent throughout all levels of analysis. Total return, of course, includes income. For equities, income is in the form of discretionary payments known as dividends; for bonds, income (interest or coupon payments) is a contractual obligation, the timing and amount of which are normally known in advance.

When an equity dividend is announced, a date is set – the ex-dividend date – from which point any purchaser of that equity is not entitled to that dividend. The dividend is actually paid on the pay date, which may be several weeks later. The price of the equity before the ex-dividend date will implicitly include the value of the dividend entitlement and exclude its value after the ex-dividend date. Thus, there is a period after the ex-dividend date but before the pay date in which the price of the equity does not reflect the economic value of the dividend yet to be paid. Policies will vary between asset management firms, but if the correct return is to be included in the appropriate period, then outstanding dividend payments should be accrued after the ex-dividend date.

Note

To facilitate the correct comparison of return and to avoid unnecessary volatility I strongly recommend that equity dividends are accrued.

Accruals are then reversed when the cash dividend is paid. Reclaimable withholding taxes are sometimes deducted from dividend payments. If these withholding taxes are recoverable, they should also be accrued – although it may take many years before tax is eventually recovered. Asset owners, at some point, may choose to write off accrued taxes they believe are not recoverable, although of course the reversal of an accrual without a corresponding cash payment will generate a negative return impact.

In terms of return methodology, a dividend payment is an external cash flow for an individual security and should be calculated as such. Whether a dividend is treated as an external cash flow for an investment category depends on whether the resulting cash is retained for investment by the category portfolio manager or transferred to a separate pot. Return calculations for a security with a dividend payment are shown in Exhibit 3.49.

[41] Assets that are so liquid that they can be converted into cash easily and readily and are considered cash for practical purposes, including very-short-term bonds, commercial paper, Treasury bills, certificates of deposit and other money market instruments.

Exhibit 3.49 Individual equity returns

Company A announced a dividend of \$2.50 per share on 6 July 2022 with an ex-dividend date of 13 July and a pay date of 27 July.

Closing prices for the month of July for company A are as follows:

30 June	\$100
6 July	\$101
13 July	\$99
27 July	\$98
29 July	\$99.50

29 July is the last business day of the month.

The money-weighted return for the month of July using modified Dietz, Equation 3.22, is:

$$r = \frac{V_E - V_S - C}{V_S + \sum C_t \times W_t} = \frac{99.5 - 100 + 2.5}{100 - 2.5 \times \frac{31 - 27}{31}} = \frac{2}{99.68} = 2.01\%$$

Using an end of day timing assumption there are only four calendar days left in the month after the 27 July pay date.

The time-weighted return for the month of July, using Equation 3.23, is:

30 June to 6 July $\frac{101 - 100}{100} = 1.0\%$

6 July to 13 July $\frac{99 + 2.5 - 101}{101} = 0.495\%$

Note that the dividend entitlement of \$2.50 has been included in the end value of the return calculation for the security.

13 July to 27 July $\frac{98 - (99 + 2.5) + 2.5}{99 + 2.5} = -0.99\%$

Note that the accrual of \$2.50 has been reversed but \$2.50 of income has also been received. Note also that the accrual is included in the start of period value.

27 July to 29 July $\frac{99.5 - 98}{98} = 1.53\%$

Note that the start value excludes the dividend payment which in effect was a negative cash flow at the end of 27 July.

Time-weighted return for July is therefore:

$$\frac{101}{100} \times \frac{101.5}{101} \times \frac{100.5}{101.5} \times \frac{99.5}{98} - 1 = 2.04\%$$

Note

Separation of income and capital matter in terms of tax and liquidity, but from a performance analysis perspective total return is key. Historically, there may have been some demand for separate income and capital returns, but these calculations are rarely required today. Multi-period capital and income returns will require an underlying assumption on the future reinvestment of capital and income.

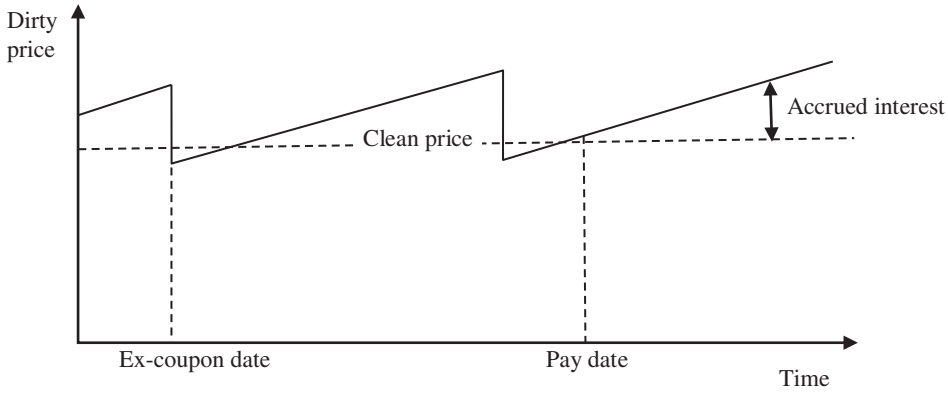


Figure 3.10 Clean price, accrued interest, dirty price and pay dates

Bonds are different: income payments are contractual. Interest payments are accrued constantly as a fraction of the days between interest or coupon payments, as shown in Figure 3.10. The price paid by an asset owner to buy a bond, or received to sell a bond, will include accrued interest. That said, accrued interest is not typically included in the quoted price. Prices excluding accrued interest are known as clean prices, and those including accrued interest, dirty prices. Practice varies between markets, but bonds also have ex-dates (ex-coupon dates), although the time between the ex-coupon date and the payment date is typically short. Interest will continue to be accrued after the ex-coupon date but before the pay date, leading to a subtraction of accrued interest in the valuation after ex-coupon dates in some markets.

Multi-period component returns

Portfolio managers will make asset allocation decisions in different portfolio components over time. The timing of these decisions will impact the overall return of the portfolio. It is entirely possible that the overall portfolio return is less or more than all of the component returns (see Exhibit 3.50).

Exhibit 3.50 Multi-period component returns

Period	Equity weight	Fixed income weight	Equity return	Fixed income return	Total return
Q1	20%	80%	10.4%	2.3%	**3.9%**
Q2	60%	40%	3.5%	1.3%	**2.6%**
Q3	90%	10%	−15.7%	1.4%	**−14.0%**
Q4	30%	70%	12.7%	8.7%	**9.9%**
Year	**50%**	**50%**	**8.6%**	**14.2%**	**0.8%**

In Exhibit 3.50 both the equity and fixed income returns exceed the total return for the full year by a considerable margin. The total return is so low because of the high weight in equities in the third quarter. The timing of asset allocation decisions can make a significant difference.

Abnormal returns

Bacon, Thompson and van der Westhuizen (2016) define an abnormal return as a return that appears to misrepresent the true economic performance of the asset being measured.[42]

Short positions

All the return formulae previously described, time-weighted and money-weighted, are equally useable if market values are negative; in other words, if the return denominator is negative $\left(V_S < 0 \text{ or } V_s + C < 0 \text{ or } V_s + \sum C_t \times W_t < 0\right)$. For individual security level, as Menchero,[43] Laker,[44] and Bacon, Thompson and van der Westhuizen[45] have pointed out, the return has the desirable property of being independent of whether the security is held long or short. So, for example, if the price changes from \$100 to \$110 over the period, the return of the stock is simply +10% regardless of whether it is held long or short. This is a fundamental and crucial observation: the return of the asset is independent of the holding; the return experience of that asset is the same for all. Arbitrarily changing the sign of the return for presentational convenience, as well as being incorrect, will also lead to many potential calculation issues. However, the contribution, portfolio gains and losses will depend on the holding quantity and of course whether the holding is long or short.

Negative market values are caused by short positions (selling securities you do not own), but also derivative instruments and through time leverage. Derivative instruments might be relatively straightforward instruments, such as futures or contracts for difference, which create symmetric positive and negative economic exposures that offset. Gains and losses in these simple derivative instruments manifest as real cash in the margin account. More complex derivative instruments such as swaps and forwards in which the notional exposures are not symmetric might result in unrealised gains or losses during the lifetime of the instrument. As single-line instruments, swaps and forwards have the undesirable property of changing signs positive to negative and negative to positive as one leg of the contract changes value relative to the other. This may cause a discontinuity in the return series, which in isolation suggests that returns cannot be linked. Leverage (in which the total value of positive assets is greater than the total portfolio value) can also cause negative total portfolio values through time. If the portfolio of assets also includes liabilities (in effect negative assets) then the value of positive assets[46] can fall to a value smaller than the liabilities and create, just like a single-line swap, a portfolio that changes from positive to negative value.

But what if the total portfolio value is negative and this portfolio value itself becomes more negative? Take a portfolio with a start value of −£1,000 and an end value −£1,100; the return is calculated in Exhibit 3.51.

The positive return in Exhibit 3.51 seems quite counterintuitive. Starting with a negative value and ending with a more negative value means that we've *lost* money; most portfolio managers would expect a negative return in such circumstances. However, consider a portfolio with one security only, and this was held short. If the security's price increases, the individual security return would still be positive (even though it is held short). This is another important

[42] Bacon, Thompson and van der Westhuzien, "Abnormal Returns" (2016).

[43] Menchero, "Performance Attribution with Short Positions" (2002/2003).

[44] Laker, "Performance Measurement for Short Positions" (2000/2001).

[45] Bacon, Thompson and van der Westhuzien, "Abnormal Returns Part 2. Returns for Short Positions and Portfolios" (2018).

[46] *Assets* is a generic term that includes positive assets and negative assets. Long positions and assets might be used to describe positive assets. Liabilities and short positions might also (confusingly) be used to describe negative assets.

Exhibit 3.51 Negative portfolio values?

Using Equation 3.2:

$$\frac{-1{,}100-(-1{,}000)}{-1{,}000}=\frac{-100}{-1{,}000}=+10.0\%$$

observation: the return is meaningful only when viewed in the context of the market value. A positive return causes a negative contribution when the starting market value is negative and the total portfolio starting market value is positive. Most asset owners and indeed analysts are conditioned to expect positive contributions from positive returns, but that is only because most of the time we are dealing with positive market values.

The context is vital. If the portfolio in question was really part of an aggregate portfolio, then the fact that there was a positive return but a negative weight (due to its negative value) correctly generates a negative *contribution* to the aggregate portfolio – which is clearly what we would expect. In effect the negative portfolio is a short holding of the portfolio in the context of the aggregated portfolio.

It seems unlikely that any negative value portfolios would persist independently in the long term, as the portfolio would not remain solvent. However, there may be cases where a portfolio could be in overdraft position for a short period of time. In this situation, what return should be calculated? Moreover, if the portfolio fluctuates between negative and positive values over time, how should the period return calculation work? There are situations where a portfolio's total market value may be negative for a period of time. Should it be possible to calculate a return for the period where the portfolio market value is negative? Yes – as long as the return is presented along with the market value so that the total contribution is interpretable. A portfolio which starts the period as negative and ends the period as even more negative (but is negative throughout the period) actually generates a positive return, which, as discussed earlier, is initially counterintuitive. However, as we know, the portfolio market value is negative; we know therefore that the combination of negative assets and positive return would result in a negative contribution (in effect shorting an asset going up in value). If this total contribution were also reported, irrespective of whether the portfolio was stand-alone or part of an aggregate, then these results would be more meaningful.

However, should it be possible to calculate a compounded return across a period which included both negative and positive market values?

It could be argued that switching from a negative to positive portfolio (or an asset portfolio to a liability portfolio) over time creates a discontinuity and returns should not be reported under such circumstances. It can certainly be argued that reporting returns without knowledge of the portfolio's asset values over the period would be misleading. The time-weighted return is independent of whether the portfolio value is positive or negative – but crucially, contribution, attribution and gain and losses obviously depend on the sign.

⚠ Caution

If asset values change sign (positive to negative, or negative to positive), this creates a discontinuity. Just like a gap or hiatus in return, returns cannot be linked across a discontinuity.

Why not just switch the sign of the return? Wouldn't this give the same result? Apart from the obvious issues in attribution and contribution calculations and arbitrarily altering a return calculation, a more fundamental issue is that a positive return is not equivalent to a negative return of the same size. To demonstrate this just take +10% compounded with −10%; the resulting return is not 0%, it is −1%:

$$(1 + 10\%) \times (1 - 10\%) - 1 = 0.99 - 1 = -1.0\%$$

Caution

It is simply not appropriate to naively switch positive and negative returns for presentational convenience. If portfolio managers make gains shorting stocks, they want to see positive returns – this is not appropriate, the return is negative. It is the combination of negative values and negative returns that generate positive contributions to overall returns.

CONTRIBUTION TO RETURN

From Equation 3.49 we know that the total return of the portfolio is the sum of its parts. The contribution to return for each investment category or individual security for a single period is straightforward to calculate:

$$\text{Contribution to return } k_i = w_i \times r_i \tag{3.51}$$

However, for multiple periods we must make assumptions about how contributions to return are compounded over time. For static portfolios we can make the assumption that each category or security is compounded with itself; see Exhibit 3.52 for a simple example of an equally weighted four-security portfolio.

Exhibit 3.52 Contribution to return – static portfolio

	Weight	Return	Contribution
Security 1	25%	9.18%	2.30%
Security 2	25%	2.23%	0.56%
Security 3	25%	−3.24%	−0.81%
Security 4	25%	4.47%	1.12%
Total	**100%**	**3.16%**	**3.16%**

In Exhibit 3.52 we can see the contributions for each security sum to the total return of the portfolio for the single period. For dynamic portfolios, category weights and individual securities will change in response to transactions and the assumption of securities continuing to compound with each other is not valid. Instead for dynamic portfolios we must assume the contribution from each investment category or security in each period is compounded with the total portfolio as follows:

$$k_i = \sum_{t=1}^{n} k_{i,t} \times \prod_{t=1}^{n} (1 + r_t) \tag{3.52}$$

Equation 3.52[47] ensures that the contributions for each category or security over multiple periods adds up to the total multi-period return. Exhibits 3.53 to 3.56 break down the static portfolio returns in Exhibit 3.52 into four periods. Exhibit 3.57 demonstrates the adjusted contributions assuming contributions are reinvested in the total portfolio rather than compounded with itself.

Exhibit 3.53 Contribution to return – static portfolio – period 1

	Weight	Return	Contribution
Security 1	25%	−1.3%	−0.33%
Security 2	25%	−0.8%	−0.20%
Security 3	25%	−2.0%	−0.50%
Security 4	25%	1.7%	0.43%
Total	**100%**	**−0.6%**	**−0.6%**

Exhibit 3.54 Contribution to return – static portfolio – period 2

	Weight	Return	Contribution
Security 1	24.82%	1.9%	0.47%
Security 2	24.95%	1.4%	0.35%
Security 3	24.65%	2.5%	0.62%
Security 4	25.58%	0.4%	0.10%
Total	**100%**	**1.54%**	**1.54%**

Exhibit 3.55 Contribution to return – static portfolio – period 3

	Weight	Return	Contribution
Security 1	24.91%	2.7%	0.67%
Security 2	24.92%	3.6%	0.90%
Security 3	24.88%	−1.2%	−0.30%
Security 4	25.29%	0.6%	0.15%
Total	**100%**	**1.42%**	**1.42%**

The sum of raw uncompounded contributions, contributions assuming reinvested in each security (static portfolio) and reinvested in total portfolio (adjusted) are summarised in Table 3.14. Although this is limited data for a short period, there are a few differences. Note that the uncompounded contributions do not add up. Both the static portfolio and adjusted contributions add up but with slightly different results according to reinvesting assumptions. Both results are acceptable, it is difficult to argue that the static portfolio is incorrect – but the assumption is invalid for dynamic portfolios which of course is the normal situation.

[47] This is an adaptation of the GRAP method, Equation 6.167, explained in Chapter 6.

Exhibit 3.56 Contribution to return – static portfolio – period 4

	Weight	Return	Contribution
Security 1	25.23%	5.7%	1.44%
Security 2	25.45%	−1.9%	−0.48%
Security 3	24.24%	−2.5%	−0.61%
Security 4	25.09%	1.7%	0.43%
Total	**100%**	**0.77%**	**0.77%**

Exhibit 3.57 Adjusted contributions

Security 1	
Period 1	$-0.33\% \times (1.0154) \times (1.0142) \times (1.0077) = -0.34\%$
Period 2	$0.47\% \times (1.0142) \times (1.0077) = 0.48\%$
Period 3	$0.67\% \times (1.0077) = 0.68\%$
Period 4	1.44% (last period of analysis)
Full period	2.26%
Security 2	
Period 1	$-0.20\% \times (1.0154) \times (1.0142) \times (1.0077) = -0.21\%$
Period 2	$0.35\% \times (1.0142) \times (1.0077) = 0.36\%$
Period 3	$0.90\% \times (1.0077) = 0.90\%$
Period 4	−0.48% (last period of analysis)
Full period	0.57%
Security 3	
Period 1	$-0.50\% \times (1.0154) \times (1.0142) \times (1.0077) = -0.52\%$
Period 2	$0.62\% \times (1.0142) \times (1.0077) = 0.63\%$
Period 3	$-0.30\% \times (1.0077) = -0.30\%$
Period 4	−0.61% (last period of analysis)
Full period	−0.8%
Security 4	
Period 1	$0.43\% \times (1.0154) \times (1.0142) \times (1.0077) = 0.44\%$
Period 2	$0.10\% \times (1.0142) \times (1.0077) = 0.10\%$
Period 3	$0.15\% \times (1.0077) = 0.15\%$
Period 4	0.43% (last period of analysis)
Full period	1.13%

Table 3.14 Multiple period contribution to return

Contribution	Security 1	Security 2	Security3	Security 4	Total
Uncompounded	2.26%	0.56%	−0.79%	1.11%	3.14%
Static portfolio	2.30%	0.56%	−0.81%	1.12%	3.16%
Adjusted	2.26%	0.57%	−0.80%	1.13%	3.16%
				Total	**3.16%**

COMPOSITE RETURNS

One of the ways asset managers demonstrate their competency to manage a particular asset class is to group together the past performance of a number of similar portfolios and sub-portfolios and calculate an average or composite return. Composite returns are typically calculated using one of the following three methods:

(1) Beginning weight
(2) Beginning weight plus weighted cash flows
(3) Aggregate

The beginning market weight method is the simplest approach because it requires the fewest calculations. The drawback of this method is that the weights of the individual portfolios do not take into account external cash flows throughout the period, which can be significant. Portfolios that experience cash inflows during the period will be underweighted in the composite return, and portfolios that experience cash outflows will be overweighted. This issue can be addressed to some extent by applying beginning weights plus weighted cash flows (in effect the denominator of the modified Dietz method) at the expense of a few more calculations. The third approach, the aggregate method, combines all the portfolios into one "super portfolio" and in effect treats the portfolio as one single portfolio in terms of return calculation.

Sample composite returns are calculated in Exhibit 3.58 for the composite data in Table 3.15. The standard return methodology for the portfolios in the composite are modified Dietz (end of day cash flow assumption) with a 10% large cash flow limit.

Note

Of course, these methodologies may generate different results, but there is no structural bias, and each method is equally valid. Because of simplicity and ease of calculation the beginning weight method is my preference.

Table 3.15 Composite data

Portfolio	Market value 30 June (£m)	Cash flow 5 July (£m)	Cash flow 9 July (£m)	Cash flow 14 July (£m)	Market value 31 July (£m)	Return
A	15.7				17.1	8.92%
B	29.8				32.3	8.39%
C	41.7	3.8			48.7	7.13%
D	57.0		−5.6		57.1	10.75%
E	205.2			49.9*	275.0	8.16%
Total	349.4	3.8	−5.6	49.9*	430.2	

*Note that the cash flow in Portfolio E on 14 July is defined as a large cash flow for both portfolio E and in the context of the aggregated composite.
Value of Portfolio E on 14 July is £208.7 million and the aggregated portfolio £350.2 million.

Exhibit 3.58 Composite returns

The return for portfolio E, taking into account the large cash flow greater than 10%, is:

$$= \frac{208.7}{205.2} \times \frac{275}{(208.7 + 49.9)} - 1 = 8.16\%$$

Using the beginning market value method, the composite return is:

$$\frac{15.7}{349.4} \times 8.92\% + \frac{29.8}{349.4} \times 8.39\% + \frac{41.7}{349.4} \times 7.13\% + \frac{57.0}{349.4} \times 10.75\% + \frac{205.2}{349.4} \times 8.16\% = 8.51\%$$

Using the beginning market value plus external cash flow method, the portfolio weight in the composite is:

Portfolio A	$15.7 + 0 = 15.7$	$\frac{15.7}{375.98} = 4.2\%$
Portfolio B	$29.8 + 0 = 29.8$	$\frac{29.8}{375.98} = 7.9\%$
Portfolio C	$41.7 + 3.8 \times \frac{26}{31} = 44.89$	$\frac{44.89}{375.98} = 11.9\%$
Portfolio D	$57.0 - 5.6 \times \frac{22}{31} = 53.03$	$\frac{53.03}{375.98} = 14.1\%$
Portfolio E	$205.2 + 49.9 \times \frac{17}{31} = 232.56$	$\frac{232.56}{375.98} = 61.9\%$
Composite	$349.4 + 3.8 \times \frac{26}{31} - 5.6 \times \frac{22}{31} + 49.9 \times \frac{17}{31} = 375.98$	

Therefore, using the beginning market value plus external cash flow method the composite return is:

$$8.92\% \times 4.2\% + 8.39\% \times 7.9\% + 7.13\% \times 11.9\% + 10.75\% \times 14.1\% + 8.16\% \times 61.9\% = 8.45\%$$

Using the aggregate method, the composite return is:

First 14-day sub-period return using modified Dietz:

$$\frac{350.2 - 3.8 + 5.6 - 349.4}{349.4 + 3.8 \times \frac{9}{14} - 5.6 \times \frac{5}{14}} = 0.74\%$$

Second sub-period return:

$$\frac{430.2}{350.2 + 49.9} - 1 = 7.52\%$$

Aggregate method total return: $(1 + 0.74\%) \times (1 + 7.52\%) - 1 = 8.32\%$

The large cash flow rule applies at the composite level as well as at the composite level. However, it is possible for the large cash flow rule to apply to multiple portfolios in the composite without triggering the rule at the composite level.

4

Benchmarks

No one can hit their target with their eyes closed.

Paulo Coelho (1947–)

Business is a good game – lots of competition and a minimum of rules. You keep score with money.

Atari founder Nolan Bushnell (1943–)

BENCHMARKS

Measuring the return of a portfolio in isolation provides only part of the story; we need to know if the return is good or bad. In other words, we need to evaluate performance (risk and return), against an appropriate benchmark.

Benchmark is defined in the *Oxford English Dictionary* as "a standard or point of reference against which things may be compared". Not readily translated into other languages, it derives from a surveyor's mark cut in a wall and used as a reference point in measuring altitudes.

Benchmark attributes

Good benchmarks should have the following attributes:

(1) *Appropriate*
The chosen benchmark must be relevant to the appropriate investment strategy. It is essential that the benchmark matches the asset owner's requirements.

(2) *Investable*
The asset manager should be able to invest in all the securities included in the benchmark. If not, there will always be an element of relative performance for which the asset manager has no control.

(3) *Accessible*
To allow the asset manager to construct the portfolio relative to the benchmark it is essential that there is access not only to the return information of the benchmark but the constituent elements and their weights at the start of the period of measurement.

(4) *Independent*
An independent third party should construct and calculate all benchmark returns to ensure a fair comparison.

Best benchmark practice

I draw a distinction between the design of a good benchmark and best practice in the use of that benchmark. Benchmarks should be:

(1) *Unambiguous*
The chosen benchmark should be clear, well documented and unambiguous. It is bad performance measurement practice to measure performance against more than one benchmark and to change benchmarks retrospectively.

(2) *Specified in advance*
The chosen benchmark should be determined in advance (ex-ante) of the measurement period.

(3) *Never retrospectively changed*
The chosen benchmark should not be retrospectively changed (ex-post). It is clearly inappropriate to deliver performance against an agreed benchmark, and if underperforming to search for an alternative benchmark against which the portfolio can demonstrate apparent outperformance.

Note

Bailey, Richards and Tierney[1] combined both good attributes and best practice in their description of "properties of a valid benchmark":

Unambiguous – The names and weights of securities comprising the benchmark are clearly delineated.
Investable – The option is available to forgo active management and simply hold the benchmark.
Measurable – The benchmark's return can be readily calculated on a reasonably frequent basis.
Appropriate – The benchmark is consistent with the manager's investment style.
Reflective of current investment opinions – The manager has current investment knowledge (be it positive, negative or neutral) regarding the securities that constitute the benchmark.
Specified in advance – The benchmark is constructed prior to the start of an evaluation period.
Accountable – The manager accepts ownership and accountability for the composition and performance of the benchmark.

THE ROLE OF BENCHMARKS

Benchmarks are crucial for the relationship between asset owners and asset managers. Even prior to considering this relationship, benchmarks will be needed to model liabilities,

[1] Bailey, Richards and Tierney, "Benchmark Portfolios and the Manager/Plan Sponsor Relationship" (1988).

establish diversification opportunities and determine asset allocations. Asset owners will carefully select benchmarks to articulate objectives, communicate risk tolerances and ultimately evaluate performance. Typically, benchmark relationships can be described as:

(1) *Passive*
The investment strategy is to replicate the benchmark precisely or as closely as possible. In effect the investment decision process has been delegated to the index provider.
(2) *Relative*
Investment decisions are made explicitly in relation to benchmark weights, exposures and risk. Most active management relationships will fall into this category.
(3) *Aware*
Investment decisions are not directly tied to benchmark weights or risk, but the benchmark description articulates the broad strategy and universe from which securities may be chosen.
(4) *Agnostic*
In this relationship the benchmark is background information, a market barometer potentially, often a target return, and perhaps used in reporting as a reference to provide some context and certainly not closely referenced by the asset manager.

TYPES OF BENCHMARKS

There are many types of benchmarks used in the evaluation of portfolio performance:

(1) Commercial indexes
(2) Customised indexes
(3) Peer groups
(4) Random portfolios
(5) Exchange-traded funds (ETFs)
(6) Target (or absolute) returns
(7) Blended benchmarks
(8) Spliced benchmarks
(9) Money-weighted benchmarks (or public market equivalents)
(10) Normal portfolio

COMMERCIAL INDEXES

Commercial indexes and averages[2] have been available for many decades and continue to be invented to this day. The first indexes were calculated to measure price inflation. Today index providers create indexes for a variety of reasons: to promote investment in certain markets (e.g., emerging markets), to provide added value services for trading clients, to provide a broad indicator of market movements, to promote the use of derivative instruments and to generate revenue.

Index providers create "intellectual property" in the construction of indexes in terms of the selection of securities to be included, calculation formulae, weighting schemes, categorisation

[2] The word *average* entered the English language in the twelfth century via the Italian "avaria", which is from the Arabic for "damaged goods". Mathematically it is derived from the fact that each party would pay half the value of the damage for goods damaged in transit.

(e.g., large-, mid- or small-cap, developed, emerging or frontier) and construction rules. The relevance of the index to the asset manager or ultimate asset owner will depend very much on the investment strategy and the original purpose of the index.

Note

Students of Latin may disagree but according to the *Oxford English Dictionary* both *indexes* and *indices* are acceptable plurals of *index*. I prefer to use indexes.

Calculation methodologies

For effective portfolio construction asset managers should be cognisant of the calculation methodologies and the rules for including and excluding various securities.

Indexes may be calculated excluding income (price-only) or including income (total return). Performance analysis should always be conducted using total return indexes.

Caution

Price-only indexes, even if there is no total return option, should never be used; they are always misleading. Naturally some indexes do not include income if the underlying assets do not generate income. In effect, these indexes are total return indexes but with a zero contribution from income.

Aggregate price index (price-weighted index or Carli[3] type)

The simplest index can be constructed by adding the share prices of the constituent securities together and dividing by a divisor.

The index at time t is:

$$I_t = \frac{I_0 \times \sum_{i=1}^{n} P_{t,i}}{D_t} \tag{4.1}$$

where: I_0 = the base index number (e.g., 100 or 1,000)
$P_{t,i}$ = the price of security i at time t
D_t = the value of the divisor at time t after adjusting for past capital and constituent changes

Note:

$$D_0 = \sum_{i=1}^{n} P_{0,i} \tag{4.2}$$

[3] Carli, "Del valore e della proporzione de'metalli monetati" (1804).

If there is a capital or constituent change at time t, the new divisor D'_t is calculated from the former divisor as follows:

$$D'_t = \frac{\sum_{i=1}^{i=n} P'_{t,i}}{\sum_{i=1}^{i=n} P_{t,i}} \times D_t \tag{4.3}$$

where: $P'_{t,i}$ = the price of constituent i immediately after the change.

Thus, the notional price of the security determines the weighting of the security in the index.[4] The Dow Jones Industrial Average, first published in 1928 with 30 securities, and the Nikkei Stock Average, first published in 1950 with 225 securities, are examples of aggregate price indexes.

Geometric (or Jevons[5] type) index

Geometric indexes are calculated by multiplying the wealth ratios of each security and taking the nth root of the product and multiplying by the base index value:

$$\text{Geometric index } I_t = I_0 \times \left(\frac{P_{t,1}}{P_{0,1}} \times \frac{P_{t,2}}{P_{0,2}} \times \cdots \times \frac{P_{t,n}}{P_{0,n}} \right)^{\left(1/n\right)} \tag{4.4}$$

A percentage change in the price of any constituent affects the index to the same extent. Although a form of equal weighting, geometric indexes are impossible for portfolio managers to replicate in real portfolios and therefore are not appropriate indexes. In theory, if one constituent falls to zero the whole index falls to zero. In practice, a failing security would be removed from the index before it reached zero. The FT 30 index established in 1935 is an example of a geometric index.

A pure Carli index is unweighted:

$$\text{Carli index } I_t = I_0 \times \frac{1}{n} \times \sum_{i=1}^{n} \frac{P_{t,i}}{P_{0,i}} \tag{4.5}$$

A Dutot[6] index divides the average price at the end of the period by the average price at the start:

$$\text{Dutot index } I_t = I_0 \times \frac{\frac{1}{n} \times \sum_{i=1}^{n} P_{t,i}}{\frac{1}{n} \times \sum_{i=1}^{n} P_{o,i}} = I_0 \times \frac{\sum_{i=1}^{n} P_{t,i}}{\sum_{i=1}^{n} P_{o,i}} \tag{4.6}$$

Jevons-type indexes, though, are favoured over Carli-type indexes in terms of calculating price inflation.[7,8]

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[4] Technically, price-weighted indexes are not pure Carli indexes, which are unweighted.
[5] Jevons, "The Variation of Prices and the Value of the Currency since 1782" (1865).
[6] Dutot, *Reflexions Politiques sur les Finances et le Commerce* (1738).
[7] Levell, "Is the Carli Index Flawed? Assessing the Case for the New Retail Price Index RPIJ" (2014).
[8] Carli-type indexes such as the UK's RPI systematically understate price inflation.

Market capitalisation index

Price and geometric indexes are neither investable nor accessible and hence most indexes used in the context of performance measurement are market capitalisation indexes in which the weight of each security in an index is normally determined by its market capitalisation (price multiplied by number of shares in issue); however, this may be adjusted by a free float factor if an index provider determines that not all the market capitalisation of a security is available to the general investing public.

For international indexes the total weight of respective countries is determined by the sum of securities in that country qualified for entry into the index. In some indexes entire country weights might be adjusted to better reflect the economic strength of that country by reweighting in proportion to countries according to GDP.

Laspeyres index

The majority of market capitalisation indexes are a form of Laspeyres index,[9] a methodology proposed by German economist Étienne Laspeyres (1834–1913) for measuring price inflation. In a Laspeyres index the quantity of each product, in this case the weight of a security, is fixed at the start of the period. The index return is calculated for each finite period, typically one month, and then monthly returns are chain-linked.

Therefore, a Laspeyres index is calculated for period t as follows:

$$\text{Laspeyres index } b = \sum_{i=1}^{i=n} W_i \times \frac{P_{t,i}}{P_{t-1,i}} \tag{4.7}$$

where: W_i = weight of security i at the beginning of period

Replacing the return performance of securities with the return of each component sector return within the benchmark we obtain the standard formula for most commercial indexes:

$$b = \sum_{i=1}^{i=n} W_i \times b_i \tag{4.8}$$

Paasche index

Paasche indexes,[10] developed by German economist Hermann Paasche (1851–1925), are also used for measuring price inflation but use the end of period rather than start of period weight for the quantity of each product. In the context of price inflation, the distinction between start and end of period is important since consumers will typically react to price changes by changing the quantities they buy; if prices go up then quantity should fall. Hence Laspeyres indexes systematically overstate inflation and Paasche indexes understate inflation. Therefore, Paasche indexes are not investable from a portfolio manager's point of view since the quantity

[9] Laspeyres, "Die Berechnung einer mittleren Waarenpreissteigerung" (1871).
[10] Paasche (1874).

is unknown until the end of the period; Laspeyres indexes are more relevant in the context of performance measurement.

$$\text{Paasche index } b = \sum_{i=1}^{i=n} W_i^E \times \frac{P_{t,i}}{P_{t-1,i}} \tag{4.9}$$

where: W_i^E = weight of security i at the end of period

Marshall–Edgeworth index

There are a few methods that attempt to negate the natural bias of Laspeyres and Paasche indexes. The Marshall–Edgeworth Index[11,12] (proposed by Alfred Marshall and Francis Ysidro Edgeworth) takes the arithmetic average between the start and end weights as follows:

$$\text{Marshall–Edgeworth index } b_{M-E} = \sum_{i=1}^{i=n} \frac{\left(W_i + W_i^E\right)}{2} \times \frac{P_{t,i}}{P_{t-1,i}} \tag{4.10}$$

Fisher index

Also wishing to negate the natural bias of Laspeyres and Paasche indexes, Irving Fisher[13] suggested the geometric mean of the Laspeyres and Paasche indexes as follows:

$$\text{Fisher index } b_F = \sqrt{b_P \times b} \tag{4.11}$$

Neither Marshall–Edgeworth nor Fisher indexes are investable and hence are rarely used in the context of portfolio performance measurement.

Equal-weighted indexes

Some professionals may argue that equal-weighted indexes, in which each security is given equal weight, represent a fair index for asset managers because they have an equal opportunity to buy each stock, arguing that capitalisation weights are arbitrary. Unfortunately, such indexes give equal weight to relatively illiquid smaller companies by definition and therefore are not practical for the entire industry to use.

Equal-weighted indexes are also difficult for passive managers to track; to maintain equal weight, positively performing stocks must be sold and negatively performing stocks must be bought, generating unnecessary transaction costs and of course bad news if stock returns are persistent; continuously selling a good-performing stock is not necessarily a good strategy if the stock continues to perform well.

[11] Marshall (1887).

[12] Edgeworth, "Measurement of Change in Value of Money" (1887, reprinted 1925).

[13] Irving Fisher (1867–1947), president of the American Statistical Association in 1932, not to be confused with Sir Ronald Aylmer Fisher (1890–1962), professor of genetics at University College London 1943–1957.

The periodicity of this rebalancing process must also be arbitrarily chosen. Should rebalancing to equal weight take place daily, weekly, monthly or annually? There is no correct answer, just compromise. Increased frequency increases turnover and hence cost and complexity; reduced frequency increases potential divergence.

Fundamental indexes

Many commentators argue that market capitalisation is a very poor choice of weight, tending to overweight overvalued stocks and underweight undervalued stocks.[14] Alternative indexes known as fundamental indexes use other, more stable, characteristics of stocks to establish appropriate weights including book value, sales, free cash flow, number of employees or dividends.

The question remains of which stock characteristic, or combination of characteristics should be used. Dividend-weighted indexes will overweight sectors of the market that pay large dividends (utilities and financial services); sales-weighted indexes will overweight high sales, low-margin stocks; free cash flow-weighted indexes will overweight mature, ex-growth businesses; book value weighted indexes will tend to overweight companies that are capital-intensive rather than those reliant on human capital.

Note

Because fundamental and equal-weighted indexes avoid the basic flaw of market capitalisation indexes, overweighting overpriced securities and underweighted underpriced securities, they tend to outperform.

Optimised indexes (efficient or minimum variance indexes)

Optimised weighted indexes attempt to find the optimum combination of securities to minimise variability or maximise a particular risk measure. Using quantitative information on expected returns, standard deviation and correlations, optimised indexes are intended to provide better risk–return trade-offs.

Caution

To a certain degree part of the investment decision process is embedded in the design of all indexes, to a greater degree in terms of optimised indexes. Asset owners should be cognisant of where they draw the line and take that into account in their asset manager selection process.

[14] Arnott, Hsu and West, *The Fundamental Index* (2008).

Style- and factor-based indexes

In the 1970s and 1980s researchers (Banz,[15] Reinganum,[16] Basu,[17,18] Fama and French,[19] Sharpe[20]) began to identify factors or styles that better explain market returns:

(1) Size – small-cap, mid-cap, large-cap
(2) Valuation – value, growth

Growth investors look for companies that are likely to have strong earnings growth. Strong earnings growth will lead to higher dividends, encouraging other investors to buy, leading to higher prices. Growth investors are less concerned about the current value and more concerned with future prospects.

On the other hand, value investors look for companies whose value is not reflected in the current price. Value investors will use valuation tools like discounted cash flow to calculate a current fair price – if the market price is less than the calculated fair price the value investor is likely to buy, assuming other investors will eventually realise the company is undervalued.

In the technology boom, growth investors fared best because they bought loss-making companies on the expectation of future profits, with valuations often based on multiples of revenue (real or manipulated) not profits. Value investors are unlikely to buy new loss-making companies.

Portfolios managed with a growth or value strategy should be measured against growth or value indexes. The index should be appropriate to the asset manager's style.

Fixed income indexes

Fixed income indexes are quite different from equity indexes. Fixed income indexes might be categorised by:

(1) Issuer (government, corporate, municipal, asset-backed and mortgage-backed securities)
(2) Country
(3) Currency
(4) Credit risk (specific credit rating, e.g., AAA, AA, A, BBB)
(5) Liquidity
(6) Maturity (1–3 years,[21] 3–5 years, 5–7 years, 7–10 years, over 10 years, undated)
(7) Sector classification (industrial, utility and financial sectors)

As well as weights and return, other data might be calculated such as average coupon, average life, modified duration, convexity, average redemption yield and average current yield.

Fixed income indexes have inherited the all-inclusive market capitalisation–weighted (perhaps debt-weighted is a better term) approach of equity indexes. Typically, there are more

[15] Banz, "The Relationship Between Return and Market Value of Common Stocks" (1981).
[16] Reinganum, "Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earning Yields and Market Values" (1981).
[17] Basu, "Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis" (1977).
[18] Basu, "The Relationship between Earning Yield, Market Value, and Return for NYSE Common Stocks: Further Evidence" (1983).
[19] Fama and French, "Common Risk Factors in Stock and Bond Returns" (1993).
[20] Sharpe, "Determining a Fund's Effective Asset Mix" (1988).
[21] Fixed income indexes typically excluded bonds with maturities shorter than one year.

bonds than stocks and because of liquidity issues some bonds are difficult to price accurately. Debt-weighted indexes of course tend to weight more indebted companies and more indebted countries more heavily, which of course is riskier (the "bums" problem described by Siegel[22]). Most of the return (roughly 95%) in a fixed income index can be explained by yield curve moves.[23] Perhaps changing yields and spreads are a better representation of relative performance for fixed income portfolios than excess return against a benchmark. The duration of the benchmark naturally reflects the duration preferences of issues of debt, not the preferences of investors in debt. Rarely is it the case that asset owners should hold the duration of the benchmark; the natural starting position is the duration of their liabilities (the duration problem, also described by Siegel.[24]

⚠ Caution

The value of public equity indexes is self-evident. Because of the duration problem, the "bums" problem, the number of securities, liquidity issues and pricing problems, the value of public fixed income indexes is far less obvious.

Index providers

In return for providing transparency, methodology, construction rules, weightings, categorisation, rebalancing information, returns, other characteristic data and importantly distribution of data, "index providers" receive a fee.

The concept of index provision is simple; all that is required is a methodology and data. Index providers will make significant investments in licencing financial data primarily from approved stock exchanges and supporting technology.

Global index provider revenues increased by 23.1% in 2021 to a breath-taking $5.0 billion.[25] In fact, global index provider revenues have almost doubled in the five years since 2017; see Table 4.1. The "Big Three" – MSCI, FTSE Russell and S&P Dow Jones Indices – hold a combined market share in excess of two thirds of the overall market. According to Petry, Fitchtner

Table 4.1 Global index provider revenues

Year	Revenue	Growth
2021	$5.0 billion	23.1%
2020	$4.1 billion	9.7%
2019	$3.7 billion	8.0%
2018	$3.5 billion	13.4%
2017	$2.7 billion	18.8%

Source: Burton-Taylor International Consulting.

[22] Siegel, "Benchmarks and Investment Management" (2003).
[23] Nemerever, "Overcoming Cap-weighted Bond Benchmark Deficiencies" (2007).
[24] Siegel, "Benchmarks and Investment Management" (2003).
[25] Burton-Taylor International Consulting.

and Heemskerk,[26] index providers operate in an oligopolistic industry with high barriers to entry.

Index construction is not as objective as it might initially seem; in fact, it is highly subjective. Index providers differentiate their indexes by applying specific rules related to:

(1) Sampling
(2) Filtering
(3) Buffering
(4) Free float

Given the task of constructing a 100-stock country-specific index, the three main index providers would no doubt produce indexes that were slightly different. Index providers might not construct such an index with the strictly largest 100 stocks; their ambition might be to reflect the entire market as best as possible, using sampling techniques.

Filters and screening introduce additional differences. Index providers aiming to ensure that their indexes are as easy to track as possible will attempt to exclude illiquid stocks. Political screening might also take place; a simple listing on a national stock exchange does not determine the inclusion of the stock in a national index. Stocks may be excluded because they do not offer shareholders sufficient voting rights (in the view of the index provider), do not generate sufficient revenue in that national market, or are domiciled in another market.

Note

These index provider decisions have real business consequences. Post Brexit, Unilever sought to consolidate to one holding company, headquartered in Rotterdam with one class of shares listed in Amsterdam, New York and London. Unfortunately, this simplification would have resulted in the shares being ejected from the FTSE 100 index requiring passive and, in many cases, active managers selling holdings. Despite extensive lobbying of the index provider, shareholders rebelled and eventually forced Unilever to consolidate its headquarters in London.

Over time, indexes might further diverge: index providers establish buffering at size boundaries (e.g., between large-, mid- and small-cap) to ensure that securities are not oscillating in and out of the index causing unnecessary turnover.

On 7 December 1999, Yahoo! was included in the S&P 500 index; the stock rose 24% on day one. From the announcement that Yahoo! would join the index the stock had already risen 32%. The cause of this extraordinary performance was primarily due to the design of the index. Yahoo! had been added to the index using its full market capitalisation, but because of shares held by employees, senior management and venture capital firms with extensive lock-ins, only about 10% of the market capitalisation was available to the general public.[27] Passive managers

[26] Petry, Fichtner and Heemskerk, "Steering Capital: The Growing Private Authority of Index Providers in the Age of Passive Asset Management" (2021).

[27] Siegel, "Benchmarks and Investment Management" (2003).

obviously need to buy stock, and even active managers who are not bullish might not wish to be fully underweight, and hence to buy some stock. A massive misbalance between supply and demand had been created; worse, it had been signalled in advance. As a result, passive managers failed to track the index and the massive majority of active managers in the period failed to outperform the index. The solution is a free-float adjusted index in which the market capitalisation of stocks are adjusted to the amount of stock available for the general public to buy. The problem with free-float indexes is that the free-float is another subjective assessment made by the index provider; it is entirely possible for a portfolio to be overweight a stock against one index and to be simultaneously underweight against an equivalent index provided by a different index provider.

Even the calculation of security returns might differ; price sources tend to be similar (at least for equity indexes), but the treatment of corporate actions and dividends can be different and the treatment of suspended stocks and particularly the timing of writing down the value of stock to zero (if appropriate) may be quite different.

Index providers are not necessarily concerned about producing indexes that are pure, objective reflections of the intended markets but rather indexes that might be easier and cheaper to replicate for passive managers with lower turnover. Some indexes are totally rules based, whereas others employ an index committee. An index committee, almost by definition, must be subjective, even if the purpose is to respond when a rigid rule book appears not to work in a particular instance.

Note

The purpose of the index will obviously determine the rigour of objectivity. Indexes that are intended to be barometers of the market will be more objective, indexes that are intended for use by passive managers will be more subjective. Asset owners should consider the difference between objective and subjective indexes as part of their selection process.

Caution

The starting point for asset owners should be a fully objective index including as many securities as possible, listed on a stock exchange, available to purchase, avoiding liquidity, free float, size, buffering and political filters.

Index providers take different approaches to partitioning the US equity market by size.[28] S&P, Dow Jones and Russell assign size definitions by a fixed number of stocks, while CRSP assigns size definitions by percentage of market-cap breakpoints. Table 4.2 lists these breakpoints while Table 4.3 illustrates the impact on coverage. Using this approach, the size in

[28] McCullough, "Untangling How Index Providers Break Down the Market by Size"(2019).

Table 4.2 Index provider size segment definitions

Size segment	S&P	Dow Jones	Russell	CRSP
Mega	Top 100	n/a	Top 200	Top 70%
Large	Top 500	Top 750	Top 1,000	Top 85%
Mid	501–900	401–1,100	201–100	70–85%
Small	901–1,500	751–2,500	1,001–3,000	85–98%
Micro	n/a	<2,501	3,001–4,000	98–100%

CRSP = Center for Research in Security Prices
Source: Morningstar.

Table 4.3 Coverage by size segment

Size segment	S&P	Dow Jones	Russell	CRSP
Mega	52.6%	n/a	66.0%	71.1%
Large	82.2%	88.8%	91.4%	85.6%
Mid	5.8%	10.6%	25.5%	15.4%
Small	2.5%	9.5%	7.0%	12.3%
Micro	n/a	n/a	1.3%	n/a

Source: Morningstar Direct. Data as of March 2019.

market cap to qualify in any segment will vary through time. An alternative approach might define market cap criteria by specific size.

International indexes introduce additional issues. Defining whether a country is a developed, emerging or frontier market has the potential to significantly change the complexion of any index. Controversy over the years has surrounded decisions involving China,[29] Russia,[30] South Korea,[31] Malaysia,[32] Saudi Arabia,[33] Israel,[34] Argentina[35] and Peru,[36] among others. In truth I could have listed many other examples. Index providers are not suppliers of information; they are important actors whose decisions steer capital to countries and companies.

These decisions impact not only performance analysis, but also the actual management of portfolios. Country inclusions and exclusions and other significant changes are often flagged months in advance. How should asset managers respond? Waiting until the change date may not be appropriate, if it means asset managers are trying to trade in crowded markets. Changing allocations in advance of the formal change may impact relative performance, as may changing positions after the formal change. The best solution might be to customise the index, in agreement with the asset owner, to smooth the change over a period of months.

[29] Senator Marco Rubio accused MSCI of enabling China to gain unprecedented access to US capital.
[30] Flood and Johnson, "MSCI and JP Morgan Under Pressure to Axe 'Uninvestable' Russia from Indices" (2022).
[31] "South Korea, Developed or Emerging," FTSE Russell, 19 February 2014.
[32] Siegel, "Benchmarks and Investment Management" (2003).
[33] Freeman, "Saudi Arabia Wins Emerging Market Status" (2018).
[34] Yago, "MSCI Needs to Add Israel to Its Europe Index" (2022).
[35] Andrade and DoRosario, "Argentina Shares Fall after MSCI Cuts Emerging Markets Status" (2021).
[36] Alloway, Burger and Evans, "Index Providers Rule the World – For Now, at Least" (2017).

Choice of index provider

Notwithstanding cost and the quality of the index, asset owners and asset managers will assess index providers using the following criteria:

(1) Brand awareness
(2) Provenance of data
(3) Longevity of track record
(4) Sustainability of business model
(5) Accessibility of data

⚠ Caution

Index construction is not just a technical performance measurement issue; it is an issue of asset management. In order to manage portfolios appropriately asset managers must understand how indexes are constructed and importantly anticipate future changes in the construction of indexes.

Self-indexing

Asset managers might try to side-step the high cost of index providers and establish their own index, in other words, self-index. Brand awareness will be an issue as will the appearance of independence, but the operational aspects of index construction can easily be outsourced to an independent service provider using a set of objective criteria in the design of the index.

⚠ Caution

Index providers provide a useful, independent service, but probably at a cost that is an order of 10 too much. Profit margins are in excess of 70%,[37] probably well in excess of that if more recent revenue growth is taken into account. Asset owners, and therefore the ultimate investor, do not necessarily pay the direct cost of high barriers to entry but they certainly do indirectly via higher asset manager fees. Asset owners and asset managers should work together to create a not-for-profit index provider for the benefit of the asset management industry as a whole.

Benchmark regulation

Following the Libor[38] scandal, unsurprisingly regulators across the world moved to regulate interest rate benchmarks. The EU moved further and regulated financial indexes in general.[39] This regulation seeks to address the provision of benchmarks, input data and the use of a benchmark by a supervised entity within the EU.

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[37] Fichtner, Heemskerk and Petry, "Index Funds Might Sound Boring. But Who Decides Which Countries and Companies to Include?" (2020).

[38] Vaughan and Finch, *The Fix* (2017).

[39] "Regulation (EU) 2016/1011 of the European Parliament and of the Council of 8 June 2016 on indices used as benchmarks in financial instruments and financial contracts or to measure the performance of investment funds and amending Directives 2008/48/EC and 2014/17/EU and Regulation (EU) No 596/2014."

The EU regulation defines a benchmark as any "index by reference to which the amount payable under a financial instrument is determined" or an "index that is used to measure the performance of an investment fund". The regulation provides for three categories of benchmark:

(1) *Critical benchmarks*
- Used for financial instruments, contracts and measuring performance of funds having a total value of at least €500bn and meeting various critical importance criteria.
- Very few benchmarks meet this criterion.
- Index administrators (index providers) must comply with all requirements of the regulation.

(2) Significant benchmarks
- Used for financial instruments, contracts and measuring performance of funds having a total value of at least €50bn and for which there is no reliable substitute and whose absence will lead to market disorder.
- Index providers must comply with some but not all the requirements of the regulation.

(3) Nonsignificant benchmarks
- Benchmarks that are neither critical or significant.
- Index providers are subject to lighter regulation but must comply with the code of conduct rules.

The regulation applies to index tracking, determining asset allocation and calculating performance fees, but not necessarily performance analysis.

To ensure the integrity of the benchmarks, index providers must:

(1) Implement adequate governance
(2) Establish an oversight function
(3) Adopt a code of conduct
(4) Provide adequate systems and controls, recordkeeping and complaints procedures.

Input data must:

(1) Accurately reflect the market
(2) Be obtained from a reliable panel of sources
(3) Be verifiable
(4) Not be obtained from providers that do not adhere to the code of conduct

Choice of index

Ultimately the choice of index will depend on the asset owner's requirements. Asset owners will be concerned about the coverage, concentration, turnover and costs of indexes.

Coverage is the percentage by market capitalisation of securities included in the index compared to the total market capitalisation of all securities in that market.

Concentration measures the percentage weight of the top few securities in the index. A highly concentrated index may well introduce significant specific risk for the asset owner.

Turnover provides an indication of the underlying cost of an index. Indexes are not always stable; securities will be added (typically new issues but also previously smaller companies that have grown sufficiently to be included in larger-capitalisation indexes) and securities will be removed (takeovers or company collapse). Typically, indexes do not suffer transaction costs

(broker commission, custody charges, taxes and bid/offer spread) so index turnover is "free". Asset managers, of course, suffer transaction costs; certainly passive managers but active managers are also impacted. The removal from the index of a previously mid-cap stock because it has been promoted to a large-cap index will change the relative "bets" of an asset manager benchmarked against the mid-cap index significantly.

Costs of indexes are both direct and indirect. Indirect costs include the underlying turnover of the index and the cost of processing the index (collection, maintenance, compliance checking, etc.).

Currency effects in benchmarks

Benchmark base currency and local returns are linked using a similar approach to Equation 3.40.

$$(1 + b_L) \times (1 + b_C) = (1 + b) \tag{4.12}$$

where: b_L = return in local currency
b_C = currency return

Hedged indexes

Many index providers calculate index returns in a specific base currency, in local currency terms and hedged back to the base currency.

Hedged calculation methods differ, one approach being to sell notional one-month forward contracts at the start of the period of measurement and calculating a return based on the gain or loss on those contracts in conjunction with the gain or loss in the underlying assets. This method does not hedge dynamically. Currency positions caused by the market gains of the underlying assets during the month are therefore not hedged, leading paradoxically to a residual currency element in the hedged return.

Alternatively, the local currency return can be compounded with the interest rate differential (between the base currency and the currency of the underlying asset) as in Equation 4.13.

$$\text{Hedged return } b_{Hi} = (1 + b_{Li}) \times (1 + d_i) - 1 \tag{4.13}$$

where: b_{Li} = local benchmark return for category i
b_{Hi} = benchmark return for category i hedged back to the base currency
d_i = interest rate differential

Since there is no currency exposure in a hedged return, the only contributing factor to the difference between the local and hedged returns is the interest rate differential. The hedged return will be greater than the local return if interest rates are lower in the underlying currency than the base return if d_i is positive.

Alternatively, the hedged return can be derived from the base return as follows:

$$\text{Hedged return } b_{Hi} = \frac{(1 + b_i)}{(1 + f_i)} - 1 \tag{4.14}$$

where: f_i = return on forward currency contract (forward currency return)

CUSTOMISED INDEXES

Generic indexes are irrelevant to asset owners who have specific requirements based on their liabilities. Increasingly, asset owners require benchmarks that are customised to their own requirements. Asset owners may choose to exclude certain "sin" stocks (alcohol, fossil fuels, gambling, smoking, armaments, etc.) from indexes or increase/decrease weights for securities, industries or countries.

Index providers in the past may have been more relaxed about the reuse of their data in customised form but are increasingly seeking to leverage their intellectual capital and will often charge for the extended use of their data. Asset managers should ensure that they are licensed to use the index data in the form they want to use it.

Index information is normally provided in a specified base currency with local and possibly hedged returns. Given base local and hedged returns it is relatively simple to convert these returns into any alternative base currency.

The sample index in Table 4.4 is customised to exclude Australia in Exhibit 4.1.

From Table 4.4 currency returns and hedge differentials can be calculated for each currency as shown in Table 4.5. Exhibit 4.2 demonstrates how to convert the base currency of an index.

Capped indexes

Due to regulatory requirements or the specific requirements of asset owners, customised indexes often include maximum limits for securities, countries or industrial sectors, etc. These limits should be reflected by fixed weights in the customised index, either at the maximum limit or to allow the portfolio manager an overweight allocation at a fixed level lower than the limit.

Table 4.4 Sample index data

	Weight	Base currency return (£)	Local return	Hedged return
UK	20%	15.00%	15.00%	15.00%
Norway	4%	−4.21%	−7.00%	−7.19%
Sweden	3%	−3.10%	−5.00%	−5.10%
France	15%	−10.75%	−15.00%	−14.75%
US	35%	6.70%	10.00%	10.55%
Japan	20%	26.50%	15.00%	15.81%
Australia	3%	20.75%	5.00%	4.48%
Total	100%	**9.39%**	**6.97%**	**7.34%**

Exhibit 4.1 Customised index

Contribution to total return excluding Australia

$$20\% \times 15.0\% + 4\% \times -4.21\% + 3\% \times -3.1\% + 15\% \times -10.75\%$$

$$35\% \times 6.7\% + 20\% \times 26.5\% = 9.04\%$$

Customised index excluding Australia $\dfrac{8.77}{1 - 3\%} = 9.04\%$

Table 4.5 Index currency returns

	Currency return $\frac{b_i}{b_{Li}} - 1$	Interest differential $\frac{b_{Hi}}{b_{Li}} - 1$
Norwegian krona	$\frac{0.9579}{0.93} - 1 = 3.0\%$	$\frac{0.9281}{0.93} - 1 = -0.2\%$
Swedish krone	$\frac{0.969}{0.95} - 1 = 2.0\%$	$\frac{0.949}{0.95} - 1 = -0.1\%$
Euro	$\frac{0.8925}{0.85} - 1 = 5.0\%$	$\frac{0.8525}{0.85} - 1 = 0.3\%$
US dollar	$\frac{1.067}{1.1} - 1 = -3.0\%$	$\frac{1.1055}{1.1} - 1 = 0.5\%$
Yen	$\frac{1.265}{1.15} - 1 = 10.0\%$	$\frac{1.1581}{1.15} - 1 = 0.7\%$
Australian dollar	$\frac{1.2075}{1.05} - 1 = 15.0\%$	$\frac{1.0448}{1.05} - 1 = -0.5\%$

Exhibit 4.2 Benchmark currency conversions

Converting the Sterling-based customised index in Exhibit 4.1 return to an Australian dollar return:

$$\frac{1.0904}{1.15} - 1 = -5.18\%$$

PEER GROUPS AND UNIVERSES

Peer groups are collections of competitor portfolios of similar strategies grouped together to provide both an average and range of competitor returns.

Some would argue that peer groups offer a more appropriate comparison than indexes for an asset manager because they represent a genuine alternative to the asset manager and because they consist of real portfolios, they suffer transaction costs; therefore comparisons are on a like-for-like basis. Asset managers are always up against an inbuilt disadvantage in that indexes never suffer transaction costs.

There are a number of disadvantages associated with peer groups; firstly, asset owners are reliant on the independent peer group complier to control the quality of the peer group. Comparisons are only relevant between similar strategies if the peer group complier adopts loose entry criteria; the peer group may be larger but may consist of widely divergent strategies.

Peer groups generate different challenges for asset managers: not only are asset managers required to make good investment decisions, but they must have a good understanding of what their competitors are doing. For example, if an asset manager likes IBM, against the index it is easy to ensure that the position is overweight, but in the peer group there is an element of guesswork involved: the asset manager must guess the average weight in the competitor portfolios and then determine the weight of IBM required. A peer group benchmark requires an additional competency or increased skillset from the asset manager.

Peer groups also suffer "survivorship bias": poor-performing portfolios are either closed, removed from the universe or merged with another fund within the asset manager's stable of

funds because the asset manager is unwilling to keep a poor-performing portfolio in the survey. The result is increasing the good long-term performance of the peer group as a poor-performing portfolio ceases to belong to the long-term track record.

Note

When funds are merged, seldom is the poor-performing track record retained. In virtually all cases, justified or not, the better-performing track record will be retained; otherwise the merger will not be willingly undertaken.

Percentile rank

One way of describing the relative performance of a portfolio or fund in a peer group is to provide the rank (or position) of the portfolio compared to the total number of portfolios in the universe.

Raw ranks, however, are very hard to compare against peer groups of different size; to allow comparison we can convert the raw rank to an equivalent rank based on a total peer group size of 100 using the following formula.

There are at least five different methods that might be used to calculate percentile rank:

$$\text{Percentile rank method 1} = \frac{n}{N} \tag{4.15}$$

$$\text{Percentile rank method 2} = \frac{n-1}{N} \tag{4.16}$$

$$\text{Percentile rank method 3} = \frac{n-1}{N-1} \tag{4.17}$$

$$\text{Percentile rank method 4} = \frac{n-0.5}{N} \tag{4.18}$$

$$\text{Percentile rank method 5} = \frac{n}{N+1} \tag{4.19}$$

where: n = rank of the observation (ranked best to worst)
N = total number of observations

Which percentile rank method to use really depends on the use to which it is put. For small samples the difference between methods can be significant (see Table 4.6), but for large samples the impact will be small. From the performance measurer's perspective, to rank portfolios in a peer group comparison I much prefer method 3; in effect, whatever the size of the peer group the percentile rank equates to the equivalent rank as if there were 100 portfolios in the peer group. The top-ranked fund would have a percentile rank of 0% and the bottom rank fund 100%. This method also ensures that the middle rank portfolio, the median, has a percentile rank of 50%. For example, the percentile rank of the portfolio ranked 8 out of a peer group size of 15 (the median portfolio) is calculated as follows: $\frac{8-1}{15-1} = \frac{7}{14} = 50\%$.

To test whether the percentile ranking methodology is sound, simply calculate the percentile rank of the middle rank fund in an odd sample size; if it is not exactly 50% it is a poor methodology.

Table 4.6 Percentile rank methodologies

Rank	Method 1 $\frac{n}{N}$	Method 2 $\frac{n-1}{N}$	Method 3 $\frac{n-1}{N-1}$	Method 4 $\frac{n-0.5}{N}$	Method 5 $\frac{n}{N+1}$
1	8.33%	0.00%	0.00%	4.17%	7.69%
2	16.67%	8.33%	9.09%	12.50%	15.38%
3	25.00%	16.67%	18.18%	20.83%	23.08%
4	33.33%	25.00%	27.27%	29.17%	30.77%
5	41.67%	33.33%	36.36%	37.50%	38.46%
6	50.00%	41.67%	45.45%	45.83%	46.15%
7	58.33%	50.00%	54.55%	54.17%	53.85%
8	66.67%	58.33%	63.64%	62.50%	61.54%
9	75.00%	66.67%	72.73%	70.83%	69.23%
10	83.33%	75.00%	81.82%	79.17%	76.92%
11	91.67%	83.33%	90.91%	87.50%	84.62%
12	100.00%	91.67%	100.00%	95.83%	92.31%

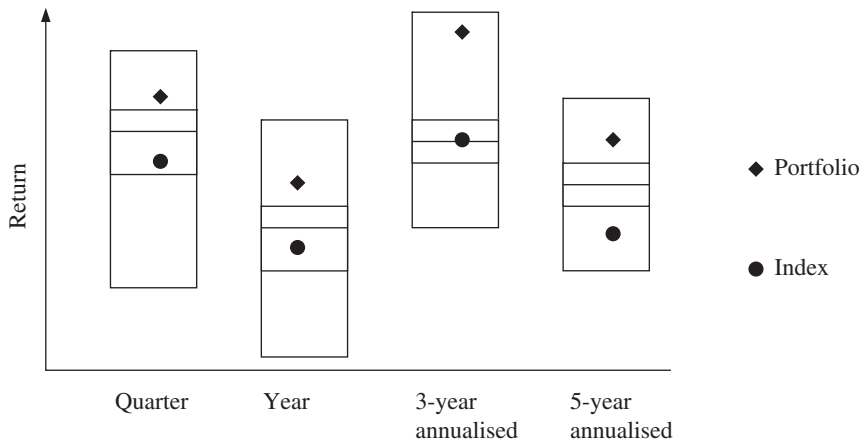

Figure 4.1 Floating bar chart

Percentile ranks are often banded as follows:

0–25%	First quartile
25–50%	Second quartile
50–75%	Third quartile
75–100%	Fourth quartile

Quintiles (20% bands) and deciles (10% bands) are also common.

An extremely useful way of showing peer group information is in the form of a floating bar chart as shown in Figure 4.1.

The bars represent the range of returns of the peer group, which are banded by quartiles. Note that the bandwidths for the second and third quartiles are much narrower than the first and fourth quartiles, indicating a normal distribution of returns. For convenience many peer group managers ignore the top and bottom 5% of returns to ensure that the distribution can fit onto the page.

In this example it is easy to see that the portfolio has performed well within the 1st quartile for all periods as well as outperforming the index.

RANDOM PORTFOLIOS

Peer groups and indexes both have their critics – peer groups because of survivorship bias, inefficient construction, investability and achievability issues and indexes because of the choice of stock weightings and the tendency for portfolio managers to be closet indexers, reducing the asset manager's business risk by taking only small deviations from index weights. Closet indexers will neither underperform nor outperform significantly since they are taking little relative risk; asset owners presumably pay asset managers to take risk on their behalf, and hence closet indexers are not acting in the best interests of their clients.

Random portfolios avoid most of these issues. Using Monte Carlo simulations, we can create all the possible outcomes given the restraints of the investment management mandate. If the actual performance of the portfolio manager is toward the top of all the possibilities the performance is good; if toward the bottom performance is bad. Since the construction of the median portfolio is unknown beforehand it is impossible to closely follow the index and of course there are no competitor construction or survivorship bias issues. Whether a random portfolio is investable or achievable is debatable: it is certainly impossible to construct the portfolio to match the median return of all possible outcomes at the outset of the period; the manager is required to take positions from the very start.

EXCHANGE-TRADED FUNDS (ETFs)

Exchange-traded funds (ETFs) are, in reality, a special case of index benchmark. ETFs are pooled investment vehicles intended to track a commercial index. They inherit the advantages and disadvantages of the underlying index but in addition they represent a genuine investable alternative to the asset owner, including transaction costs and management fees.

TARGET RETURNS

Target returns, such as a predetermined hurdle rate or arbitrary amount, inflation plus a specified amount, risk-free rate, minimum accepted return, and short-term interest rates are a relatively common type of benchmark. Target returns by their nature contain little information to aid performance analysis, typically do not reflect the investment mandate, and are not investable and are therefore very poor benchmarks.

BLENDED BENCHMARKS (OR BALANCED BENCHMARKS)

Blended benchmarks are a specific type of customised benchmark, consisting of a blend of multiple indexes, typically from different index providers and representing a range of asset classes, potentially using a range of methodologies. Blended benchmarks are most commonly used in conjunction with balanced investment strategies or policy benchmarks for asset owners.

Fixed-weight and dynamised benchmarks

Blended benchmarks are often defined using fixed weights for certain asset categories. The performance of assets categories will diverge over time, thus impacting the original strategic asset allocation. It is essential that the frequency of fixed-weight rebalancing is established

within the benchmark definition. If the initial fixed weight is allowed to float with the performance of the individual categories, the impact compared to a genuine fixed weight can be significant, as demonstrated in Table 4.7.

Exhibit 4.3 illustrates how the floating weights are calculated for each quarter or, in other words, dynamised. The quarterly balanced fixed-weight index outperforms the floating-weight index. This outperformance is for the most part generated in the third quarter; the fixed-weight index is required to reduce the exposure to equities at the beginning of the period immediately before a big fall in the market. Note that to replicate the annual return resulting from applying the 50%:50% fixed weight to the category annual returns, floating weights must be used each quarter.

Table 4.7 Fixed-weight and dynamised benchmarks

	First quarter		Second quarter		Third quarter		Fourth quarter		Year
	Weight	Return	Weight	Return	Weight	Return	Weight	Return	Return
Fixed weights									
Equities	50%	10.4	50%	3.5	50%	−15.7	50%	12.7	**8.56**
Bonds	50%	2.3	50%	1.3	50%	1.4	50%	8.7	**14.22**
Total		**6.35**		**2.4**		**−7.15**		**10.70**	**11.94**
Floating weights									
Equities	50%	10.4	51.90%	3.5	52.44%	−15.7	47.83%	12.7	**8.56**
Bonds	50%	2.3	48.10%	1.3	47.56%	1.4	52.17%	8.7	**14.22**
Total		**6.35**		**2.44**		**−7.57**		**10.61**	**11.39**

Exhibit 4.3 Dynamised benchmark

Appling the 50%:50% weight to the annual returns the total benchmark return is:

$$50\% \times 8.56 + 50\% \times 14.22 = 11.39$$

To achieve this return using quarterly data we need to reweight each quarter to reflect underlying market movements:

Second-quarter weights

$$50\% \times 10.4\% = 55.2\% \quad 50\% \times 2.3\% = 51.15\% \quad \text{Total } 106.35\%$$

Revised weights $\dfrac{55.2\%}{106.35\%} = 51.9\%$ $\quad \dfrac{51.15\%}{106.35\%} = 48.1\%$

Third-quarter weights

$$51.9\% \times 3.5\% = 53.72\% \quad 48.1\% \times 1.3\% = 48.72\% \quad \text{Total } 102.44\%$$

Revised weights $\dfrac{53.72\%}{102.44\%} = 52.44\%$ $\quad \dfrac{48.72\%}{102.44\%} = 47.56\%$

Fourth-quarter weights

$$52.44\% \times -15.7\% = 44.21\% 47.56\% \times 1.4\% = 48.23\% \quad \text{Total } 92.43\%$$

Revised weights $\dfrac{44.21\%}{92.43\%} = 47.83\%$ $\quad \dfrac{48.72\%}{92.43\%} = 52.17\%$

With the revised weights the quarterly benchmark returns compound to 11.39%; using fixed weights the benchmark return compounds to 11.94%, a significant difference.

SPLICED INDEXES

It is possible to change the index associated with the measurement of a specific portfolio over time, a good example being after a change in investment strategy. It is bad performance measurement practice to change the associated index retrospectively; therefore, a spliced index should be calculated to maintain the long-term return series of the associated benchmark. This can be achieved by chain-linking, or splicing, the respective indexes.

MONEY-WEIGHTED BENCHMARKS (OR PUBLIC MARKET EQUIVALENTS)

All commercially produced indexes are time-weighted; there is no need to adjust for cash flow between time periods (although from time to time indexes are subject to corporate actions of various types that in effect generate cash flow). Money-weighted portfolio returns are impacted by cash flow, to ensure a like-for-like comparison a commercial index return can be recalculated to reflect the cash flow experience of the portfolio.

A type of money-weighted benchmark, the index comparison method (ICM) was first suggested by Long and Nickels.[40] First used internally in 1992, this method gradually gained acceptance as the Long-Nickels-Coller (LNC) method, then the Bannock-Long-Nickels-Coller (BLNC) method, and finally as the public market equivalent (PME) method.[41]

The main disadvantage of money-weighted benchmarks is that the adjusted return is clearly different from the published index return and is unique to the cash flow experience of that portfolio. It makes the comparison of portfolio to public market equivalent return more accurate but does not aid the comparison against other asset managers with differing cash flow experience.

Note

In effect money-weighted benchmarks replicate the errors of money-weighted return calculations in the portfolio, but that does allow for an accurate one-on-one comparison.

NORMAL PORTFOLIO

Commercial indexes may not adequately reflect the investment options open to individual portfolio managers. Normal portfolios provide an alternative benchmark consisting of specific securities available for investment (for example, the recommended list of securities from the in-house research team).

Although normal portfolios are a useful comparison against the real alternatives of the portfolio manager, they lack the stamp of independence required in the definition of a good benchmark.

[40] Long and Nickels, "A Private Investment Benchmark" (1996).
[41] Long, "The Common Mathematical Foundation of ACG's ICM and AICM and the K&S PME" (2008).

BENCHMARK STATISTICS

Index turnover

Securities are constantly added and deleted from commercial indexes caused by takeover, business failure or simply major changes in market capitalisation. Most indexes suffer no transaction costs to effect these changes; however, asset managers will suffer costs.

$$\text{Index turnover} = \frac{\text{Market cap (Additions + Deletions)}}{\text{Average total market} \times 2} \tag{4.20}$$

Turnover will be greatest in mid-cap indexes due to traffic in both directions, including securities promoted into the large-cap index and previously large-cap securities falling into the mid-cap index.

All other things being equal, an index with high turnover will be more difficult for an asset manager to outperform because of potentially higher transaction costs. Asset managers start with a small structural disadvantage when compared to indexes.

Up-capture indicator

$$\text{Up-capture indicator} = \frac{\bar{r}^{+}}{\bar{b}^{+}} \tag{4.21}$$

where: $\bar{b}^{+}$ = average positive benchmark return
$\bar{r}^{+}$ = average portfolio return for each period in which the benchmark return is positive

The up-capture indicator divides the average portfolio return by the average benchmark return for each period in which the benchmark return is positive – the greater the value the better.

Down-capture indicator

$$\text{Down-capture indicator} = \frac{\bar{r}^{-}}{\bar{b}^{-}} \tag{4.22}$$

where: $\bar{b}^{-}$ = average negative benchmark return
$\bar{r}^{-}$ = average portfolio return for each period in which the benchmark return is negative

The down-capture indicator divides the average portfolio return by the average benchmark return for each period in which the benchmark return is negative. Lower values are preferred.

Up-number ratio

The up-number ratio measures the percentage of returns in each measurement period in which the portfolio returns are greater than zero when the benchmark returns are greater than zero.

Ideally the ratio should be 100% – the closer to 100% the better.

Down-number ratio

The down-number ratio measures the percentage of returns in each measurement period in which the portfolio returns are less than zero when the benchmark returns are less than zero.

Ideally, but very rarely, the ratio should be 0%. The lower the ratio the better, although for highly correlated returns, ratios close to 100% should be expected.

Up-percentage ratio

More interestingly the up-percentage ratio measures the percentage of periods in which the excess return of the portfolio against the benchmark is greater than zero in each measurement period when the benchmark return is greater than zero.

In other words, how often does the portfolio manager outperform a rising market?

Down-percentage ratio

The down-percentage ratio measures the percentage of periods in which the excess return is greater than zero in each measurement period when the benchmark return is less than zero.

In other words, how often does the portfolio manager outperform a falling market?

Percentage gain ratio

$$\text{Percentage gain ratio} = \frac{n_r^+}{n_b^+} \tag{4.23}$$

where: n_r^+ = number of portfolio returns greater than zero
n_b^+ = number of benchmark returns greater than zero

Clearly, higher-percentage gain ratios are better.

EXCESS RETURN

Given portfolio and benchmark returns, it is only natural to make a comparison and calculate the difference in return or "excess return".

There are two common measures of the excess return, "arithmetic" and "geometric".

Arithmetic excess return

Arithmetic excess return is the profit in excess of a notional or benchmark fund expressed as a percentage of the initial amount invested:

$$a = r - b \tag{4.24}$$

where: a = arithmetic excess return
b = benchmark return

Geometric excess return

Geometric excess return is the profit in excess of a notional or benchmark fund expressed as a percentage of the final value of the notional or benchmark fund:

$$g = \frac{1+r}{1+b} - 1 \tag{4.25}$$

where: g = geometric excess return

In both these definitions the added value (or profit) is identical. Both definitions attempt to explain the same added value in cash terms. The arithmetic excess return explains the added value relative to the initial amount invested and the geometric excess return explains the same added value but relative to the notional fund or the amount expected if the client had invested in the benchmark.

This simple difference is crucial, as demonstrated in Exhibit 4.4.

Both versions of excess return are used worldwide with neither dominating globally. Arithmetic is more common, but I would argue that the geometric method is more technically correct. The three main arguments for using geometric excess returns are quite persuasive:

(1) Proportionality
(2) Convertibility
(3) Compoundability

Most arguments in favour of using arithmetic excess returns centre on its ease of use, simplicity and intuitive feel. It is clearly easier to subtract two numbers than to calculate a

Exhibit 4.4 Arithmetic and geometric excess returns

Portfolio start value = \$1,000,000
Portfolio end value = \$1,070,000

$$\frac{1{,}070{,}000 - 1{,}000{,}000}{1{,}000{,}000} = 7\%$$

Notional fund start value = \$1,000,000 (i.e., the same as the portfolio start value)
For benchmark performance assume a 5% return
Therefore, notional fund end value = \$1,050,000
The portfolio's added value above the benchmark or notional fund is:

$$\$70{,}000 - \$50{,}000 = \$20{,}000$$

The arithmetic excess return $= \dfrac{\$20{,}000}{\$1{,}000{,}000} = 2\%$

or, alternatively, 7% – 5% = 2%
The geometric excess return or the increase in value of the portfolio compared to what would have been achieved if invested in the benchmark is:

$$\frac{\$1{,}070{,}000 - \$1{,}050{,}000}{\$1{,}050{,}000} = 1.9\%$$

Or

$$\$20{,}000/\$1{,}050{,}000 \quad \text{or} \quad \frac{1.07}{1.05} - 1 = 1.9\%$$

In other words, the portfolio is 1.9% larger than it would have been had it been invested in the benchmark.

wealth ratio. Even proponents of the use of geometric excess returns will occasionally resort to arithmetic excess returns when reporting to clients. They argue that it's simply not worth valuable time discussing why the ratio of returns is favoured over a simpler subtraction; the message of the report is more important.

To some extent I would agree, but the arithmetic or geometric debate is not a continuous discussion. Once understood, you need not have the same debate with the asset owner again.

In fact, the geometric excess return is *more*, not less, intuitive to the layperson.

The asset owner is most concerned about the value of the portfolio at the end of the period (not the start), and quite rightly should ask how much larger the portfolio is now than it would have been had the portfolio invested in the benchmark. This is the natural question of asset owners, particularly before calculating rates of return confuses the issue.

Rearranging Equation 4.23 we see that there is a relationship between arithmetic and geometric excess return:

$$\frac{1+r}{1+b} - 1 = \frac{1+r}{1+b} - \frac{1+b}{1+b} = \frac{r-b}{1+b} \tag{4.26}$$

This relationship is important: it demonstrates that in rising markets the arithmetic excess return is always greater than the geometric excess return and in falling markets the reverse is true. If I were a cynic, I would suggest that asset managers prefer arithmetic excess returns because they look better in most market conditions.

The geometric excess return represents a better measure of the relative added value of the asset manager's performance – it is proportionate, as demonstrated in Exhibit 4.5.

The most convincing argument for using geometric excess returns is their convertibility across different currencies. Exhibit 4.6 illustrates the impact of reporting excess returns in different currencies.

In Exhibit 4.6 we can see that simply by expressing the same performance in a different currency the arithmetic excess return appears to have increased from 2.0% in Exhibit 4.4 to 2.2%.

Asset managers cannot add more value simply by presenting returns in a different currency; the underlying increase in the benchmark, compounded with the currency return, has increased the added value relative to the initial amount invested. Crucially, because the portfolio and notional fund have been compounded by the same currency return, the added value relative to the final value of the notional fund remains the same regardless of the currency in which the report is denominated. The geometric excess return is always the same regardless of the currency used to calculate performance.

Exhibit 4.5 Proportionality

Portfolio start value \$1,000,000
Portfolio end value \$500,000
Portfolio return −50%
Notional fund start value \$1,000,000
Notional fund end value \$250,000
Notional or benchmark return −75%
Arithmetic excess return = −50% – (−75%) = +25%
Geometric excess return $\frac{0.5}{0.25} - 1 = +100\%$
The geometric excess return correctly demonstrates that the portfolio is double the size that would have been achieved by investing in the benchmark.

Exhibit 4.6 Convertibility

In an extension of Exhibit 4.4 let's look at the same portfolio from the perspective of a client whose base currency is euros (€).
Assume a beginning of period exchange rate of 1$ = 1€ and an end of period exchange rate of 1$ = 1.1€

Portfolio start value €1,000,000
Portfolio end value 1,070,000 × 1.1 = €1,177,000
Portfolio return in euros = 17.7%
Notional start value €1,000,000
Notional end value 1,050,000 × 1.1 = €1,155,000

The portfolio's added value above the notional fund is:

$$€177{,}000 - €155{,}000 = €22{,}000$$

The arithmetic excess return:

$$\frac{€22{,}000}{€1{,}000{,}000} = 2.2\%$$

alternatively, 17.7% – 15.5% = 2.2%
The geometric excess return or increase in value of the portfolio compared to what would have been achieved if invested in the benchmark is:

$$\frac{€1{,}177{,}000 - €1{,}155{,}000}{€1{,}1550{,}000} = 1.9\%$$

$$€22{,}000/€1{,}155{,}000 \text{ or } \frac{1.177}{1.155} - 1 = 1.9\%$$

This explains why geometric excess returns are more popular in the UK and Europe than, say, in the US and Australia. In Europe, particularly the UK, the asset management community has been more international in nature for some time. The problem of presenting the same portfolio or composite returns in many currencies was addressed many years ago.

This relationship holds since the currency return c is identical for both the portfolio and the benchmark.

Let r_L = the portfolio return in local currency
And b_L = the benchmark return in local currency

$$\text{Then the portfolio return in currency } c = (1 + r_L) \times (1 + c) - 1 = r_c \tag{4.27}$$

$$\text{and the benchmark return in currency } c = (1 + b_L) \times (1 + c) - 1 = b_c \tag{4.28}$$

It follows that:

$$\frac{(1 + r_c)}{(1 + b_c)} = \frac{(1 + r_L) \times (1 + c)}{(1 + b_L) \times (1 + c)} = \frac{(1 + r_L)}{(1 + b_L)} \tag{4.29}$$

Geometric excess returns are also compoundable over time. This is an extremely useful property in the measurement of portfolio performance.

Exhibit 4.7 Compoundability

Assume that the same returns in Exhibit 3.4 are repeated over four quarters.
The portfolio return is $(1.07) \times (1.07) \times (1.07) \times (1.07) - 1 = 31.1\%$
The benchmark return is $(1.05) \times (1.05) \times (1.05) \times (1.05) = 21.6\%$
The arithmetic excess return over the entire period is $31.1\% - 21.6\% = 9.5\%$
There is no apparent straightforward link between the individual arithmetic excess returns for each quarter and the total arithmetic excess return.

$$2\% + 2\% + 2\% + 2\% \neq 9.5\%$$

$$(1.02) \times (1.02) \times (1.02) \times (1.02) - 1 \neq 9.5\%$$

The geometric excess return over the entire period is $\frac{1.311}{1.216} - 1 = 7.8\%$

The geometric excess return for each finite period can be compounded to calculate the geometric excess return for the entire period.

$$(1.019) \times (1.019) \times (1.019) \times (1.019) - 1 = 7.8\%$$

In Chapter 3 we established by definition time-weighted rates of return are calculated by chain-linking each finite performance period within the overall period as follows:

$$(1 + r_1) \times (1 + r_2) \times (1 + r_3) \times \cdots (1 + r_{n-1}) \times (1 + r_n) = (1 + r) \tag{4.30}$$

The total benchmark return b can be derived in a similar calculation:

$$(1 + b) = (1 + b_1) \times (1 + b_2) \times \cdots (1 + b_{n-1}) \times (1 + b_n) \tag{4.31}$$

It can be seen that the geometric excess return for the total period g can be calculated by chain-linking the geometric excess returns of the sub-periods g_i as follows:

$$(1 + g) = \frac{(1 + r)}{(1 + b)} = \frac{(1 + r_1)}{(1 + b_1)} \times \frac{(1 + r_2)}{(1 + b_2)} \times \cdots \times \frac{(1 + r_n)}{(1 + b_n)} \tag{4.32}$$

or

$$(1 + g) = (1 + g_1) \times (1 + g_2) \times \cdots \times (1 + g_n) \tag{4.33}$$

Compoundability is demonstrated in Exhibit 4.7.

Note

Because of the properties of proportionality, convertibility, compoundability and others I much prefer geometric excess return to arithmetic excess return.

5
Risk

I am always doing that which I cannot do, in order that I might learn how to do it.
Pablo Picasso (1881–1973)

Do not worry about your difficulties in Mathematics. I can assure you mine are still greater.

As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.
Albert Einstein (1879–1955)

If you torture the data long enough, it will confess.
Ronald Coase (1910–2013)

DEFINITION OF RISK

Risk means very different things to different audiences at different times; risk is truly in the eye of the beholder. In the context of asset management, the *Oxford English Dictionary* provides a surprisingly good definition of risk:

> The potential impact of an event determined by combining the likelihood of the event occurring with the impact should it occur.

Risk is the combination of exposure and uncertainty. As Holton[1] so eloquently points out, it is not risky to jump out of an aircraft without a parachute because death is certain. Holton also points out that we can never operationally define risk; at best, we can operationally define our perception of risk.

Another common and effective, but broader definition of risk is *exposure to uncertainty*.

Risk types

Within asset management firms there are many types of risk that should concern asset managers and senior management. For convenience I've chosen to classify risk into five main categories:

(1) Compliance risk
(2) Operational risk
(3) Liquidity risk
(4) Counterparty risk
(5) Portfolio risk

[1] Holton, "Defining Risk" (2004).

These risks are ranked in my priority order of concern at the point in time I assumed the role of director of risk control at an asset management firm in the late 1990s.[2] What I didn't appreciate fully then, but did much later, is that priorities will vary through time; during the credit crisis I'm sure counterparty risk became the number-one priority for many firms.

Although a major concern of all asset managers, reputational risk does not warrant a separate category; a risk failure in any category can cause significant damage to a firm's reputation.

Compliance or regulatory risk is the risk of breaching a regulatory, client or internally imposed guideline, restriction or clear limit. I draw no distinction between internal or external limits; the breach of an internal limit indicates a control failure, which could just as easily have been a regulatory, or client mandated limit. Of course, the financial impact of breaching limits can be significant: in August 1996 Peter Young of Morgan Grenfell Asset Management allegedly cost Deutsche Bank £300–£400 million in compensation payments to investors in highly regulated authorised unit trusts.[3] Young used Luxembourg-listed shell companies to circumvent limits on unlisted and risky holdings.

Regulatory risk increasingly includes failure to live up to environmental, social and governance (ESG) expectations. In August 2021 shares in DWS fell more than 13% following reports that US and German regulators were investigating whistleblower claims that it misled clients and shareholders about its sustainable investing efforts,[4] in effect, "greenwashing".[5] It was alleged that the ESG risk-management system used by DWS was flawed because it relied on outdated technology. In 2022 Allianz pleaded guilty to fraud and agreed to pay $5.8 billion in compensation after misrepresenting risk in hedge funds that subsequently collapsed.[6] According to authorities, the portfolio managers "smoothed" performance data and altered documents detailing the fund's hedging strategies. Astonishingly, a 2017 internal audit identified red flags that if pursued might have identified major problems, but no meaningful follow-up was conducted. In fact, the follow-up was conducted by the structured-products group itself, whose compensation was directly tied to the quarterly performance of the funds.

⚠ Caution

Senior management failure to follow up on identified internal audit risks and lack of independence are two fundamental and relatively common failures that expose many companies to significant risk.

Operational risk, often defined as a residual catch-all category to include risks not defined elsewhere, actually includes the risk of human error, fraud, system failure, poor controls, management failure and failed trades. Risks of this type are more common but usually less severe. Nevertheless, it is important to continuously monitor errors and near-misses of all types, even those that do not result in financial loss. An increase in the frequency of errors regardless

[2] In truth I did not identify liquidity risk as a separate risk category at the time.

[3] "Fund Manager Faces Fraud Trial," BBC News Home, 30 April 1999.

[4] "DWS Shares Slide after BaFin Acts on Allegations of Greenwashing," *Financial Times*, 27 August 2001.

[5] Behaviour, activities or disinformation to make investors believe that a firm is doing more to protect the environment than it really is.

[6] B. Van Voris and S. Kahl, "Allianz Fund Collapse Ends in Guilty Plea," Bloomberg UK, 22 May 2022.

of size or sign may indicate a more serious problem that requires further investigation and corrective action. Although typically small in size, operational errors can lead to large losses. In December 2005 a trader at the Japanese brokerage firm Mizuho Securities[7] made a typing error and tried to sell 610,000 shares at 1 yen apiece in recruiting company J-Com Co., which was debuting on the exchange, instead of an intended sale of one share at 610,000 yen, an example of fat-finger syndrome.[8] Mizuho lost approximately 41 billion yen. In April 2007, a programmer at AXA Rosenberg incorrectly programmed a statistical model, leading to $217 million in losses for clients.[9] An SEC investigation found that senior management learned in June 2009 of a material error, but instead of disclosing and fixing the error a senior official directed others to keep quiet. The error was hidden from senior management until November 2009 and from clients until April 2010. According to the SEC, the firm failed to disclose the error and its impact on client performance, attributed the model's underperformance to market volatility and misrepresented the model's ability to control risk. AXA Rosenberg paid an additional penalty fine of $25 million.

Liquidity risk is the risk that assets cannot be traded quickly enough in a market to change asset and risk allocations, realise profits or prevent losses. Perhaps in the past liquidity risk received less attention than it should have: it is capable of causing significant damage. The demise of Long-Term Capital Management (LTCM) in 1998[10] was really a liquidity issue compounded by massive leverage. In less than 12 months LTCM lost $4.4 billion of its $4.7 billion capital. LTCM was a hedge fund based in Greenwich, Connecticut that used absolute return strategies combined with high leverage. LTCM had been making losses throughout the summer of 1998 which were further compounded by the Russian debt crisis in August and September, causing a flight to quality. This resulted in the bidding up of the price of the most liquid securities in which LTCM was short and depressing the price of less liquid securities of which LTCM was long. As rumours of LTCM's positions spread, market participants positioned themselves for forced liquidation; eventually LTCM was forced to liquidate at exactly the wrong time, increasing its losses. A more recent, and extremely relevant, example in the context of today's markets is the liquidity crisis at the LF Woodford Equity Income Fund (WEIF).[11] WEIF, an open-ended investment fund, was forced to suspend dealing in June 2019 to avoid a fire sale of unlisted assets, triggered by the attempted withdrawal of £250 million or 4% of the fund's assets by Kent County Council. In the preceding weeks, following a period of poor performance, investors had already withdrawn in excess of £500 million, increasing the already high percentage of unlisted, illiquid assets. The fund had previously circumvented a 10% limit on illiquid assets by bundling up the fund's unlisted assets and listing them on the Guernsey-headquartered International Stock Exchange which had barely any trading activity and was unable to provide sufficient liquidity.[12] Extraordinarily, the Governor of the Bank of England said, "These funds are built on a lie which is that you can have daily liquidity for assets that are fundamentally not liquid".[13] Understanding liquidity risk in both normal and

[7] David McNeill, "'Fat-finger' Trade Cost Tokyo Shares Boss His Job," *The Independent*, 21 December 2005.

[8] Fat-finger syndrome – accidently pressing the wrong button on a keyboard.

[9] "AXA Rosenberg Settles Coding-Error Case with SEC," *Morningstar Fund Times*, 3 February 2011.

[10] Lowenstein, *When Genius Failed* (2000).

[11] Mark Latham, "The Neil Woodford Crisis: An Accident Waiting to Happen?" Funds Europe, July–August 2019.

[12] Nick Corbishley, "Liquidity Crisis at Woodford Equity Fund Is Symptomatic of Systemic Problem, Bank of England Warns," *Wolf Street*, 12 July 2019.

[13] J. Booth, "Carney Warns That Woodford-style Funds Are 'Built on a Lie,'" *City A.M.*, 26 June 2019.

turbulent markets is a crucial element of effective risk control; the relatively recently identified phenomena of crowded exits is a characteristic of those turbulent markets.

Counterparty risk occurs when counterparties are unwilling or unable to fulfil their contractual obligations, at its most basic through corporate failure. Counterparty exposure could include profits on an OTC derivatives contract, unsettled transactions, cash management, administrators, custodians, prime brokers, and even with the comfort of appropriate collateral the failure to return stock that has been used for stock lending. Perhaps the most obvious example of counterparty risk is the failure of Lehman Brothers in September 2008.[14]

In the middle office of asset management firms, we are most concerned with portfolio risk, which I define as the uncertainty of meeting asset owner expectations. Is the portfolio of assets managed in line with the asset owner's investment objectives? The consequences of not meeting asset owner expectations can be quite severe. Early in 2001, the Unilever Superannuation Fund sued Merrill Lynch for damages of £130 million, claiming negligence in that Merrill Lynch had not sufficiently considered the risk of underperformance.[15] Ultimately, the case was settled out of court for an undisclosed sum, believed to be £70 million, the perception of many being that Unilever won.

Credit risk (or issuer risk), as opposed to counterparty risk, is a type of portfolio risk. Credit risk or default risk is the investor's risk of a borrower failing to meet their financial commitments in full. The higher the risk of default the higher the rate of interest investors will demand to lend their capital. Therefore, the reward or returns in terms of higher yields must offset the increased risk of default. Similarly, market, currency and interest rate risks taken by asset managers in the pursuit of asset owner objectives would constitute portfolio risks in this context.

I'm sure readers can quickly add to this brief list of risks and extend through various subdivisions, but I'm fairly certain any risk I've not mentioned so far can be allocated to one or more of the above categories.

Risk management versus risk control

It is useful to distinguish between the ways portfolio managers and risk professionals see risk. For this purpose, let us refer to portfolio managers as "risk managers" and to risk professionals as "risk controllers". Then there is a clear distinction between risk management and risk control. As risk managers, portfolio managers are paid to take risk; they need to take risk in order to achieve higher returns. For the risk manager, "Risk is good".

Risk controllers, on the other hand, are paid to monitor risk; their role is to measure risk and make transparent to the entire firm how much risk is being taken by the portfolio manager (and often, from their perspective, to reduce risk). The risk controller's objective is to reduce the probability or eliminate entirely a major loss event on their watch. For the risk controller, "Risk is bad".

Risk managers' and risk controllers' objectives are in conflict, leading to a natural tension between them. To resolve this conflict, we need measures that assess the quality of return and answer the question "Are we achieving sufficient return for the risk taken?"

[14] Macdonald, *A Colossal Failure of Common Sense* (2009).

[15] Perold and Alloway, *The Unilever Superannuation Fund vs. Merrill Lynch* (2003).

Risk aversion

It is helpful to assume that asset owners are risk averse, that given portfolios with equal rates of return they will prefer the portfolio with the lowest risk.

Asset owners will only accept additional risk if they are compensated by the prospect of higher returns.

Ex-post and ex-ante

Risk is calculated in two fundamentally different ways: ex-post and ex-ante. Ex-post, or historical, risk is the analysis of risk after the event; it answers the question of how risky the portfolio has been in the past.

On the other hand, ex-ante, or prospective, risk is forward-looking, based on a snapshot of the current securities and instruments within the portfolio and their historical relationship with each other; it is an estimate or forecast of the future risk of the portfolio. Obviously, the use of historical returns and correlations to forecast future risk is problematic, particularly for extreme, low-probability events. Increasing the length of the historical track record or increasing the frequency of observations does not always result in an improvement because of the changing nature of markets and underlying securities. Older returns may be less reliable for future predictions, but on the other hand more recent observations may not include the more extreme results.

Ex-post and ex-ante risk calculations are substantially different and therefore can lead to completely different results and conclusions. Differences between ex-post and ex-ante risk calculations provide significant additional information which should be monitored continuously.

DESCRIPTIVE STATISTICS

For the most part risk managers and risk controllers use dispersion measures of return as a proxy for their perception of risk.

There are several measures of return dispersion; they all measure some aspect of the range of portfolio returns experienced in a particular time period. They all report on what has *actually happened* during the time period of interest. Even from an ex-post perspective, one can ask whether the return variability truly represents how much risk the asset manager took during the time period or whether one needs to explore the range of possible returns that *might have (realistically) happened* during the time period.

Ex-post performance measurement is two-dimensional: we are concerned with both the return of the asset manager over a period of time and the risk of that return measured by the variability of return or another dispersion measure. Both the return and the shape of the return distribution are of interest to asset owners. We need descriptive statistics to help understand the underlying distribution of returns. The classic descriptive statistics are the mean, variance, skewness and kurtosis known as the first, second, third and fourth moments of the return distribution. These descriptive statistics are the basic components of many of the ex-post risk measures defined in this chapter.

Mean (or arithmetic mean)

The mean is the sum of returns divided by the total number of returns:

$$\text{Mean } \bar{r} = \frac{\sum_{i=1}^{i=n} r_i}{n} \qquad (5.1) \text{ or } (3.31)$$

where: n = total number of returns, and
r_i = return in period i

For example, one might calculate the average monthly return over a two-year period using 24 returns ($n = 24$), where each r_i is the monthly return.

Note that this mean (or average) return is calculated arithmetically and should not be confused with the annualised return, which is calculated geometrically. The average is a measure of central tendency; the median and the mode are also average measures. The mode is the most frequently occurring return and the median is the middle ranked when returns are ranked in order of size.

The annual arithmetic mean return ($\bar{r}_A$) or annual average return is simply the mean of annual returns over the time period being evaluated.

Mean absolute deviation (or mean deviation)

The mean of the distribution of returns provides useful information but as investors we are also interested in the deviation or dispersion of returns from the mean, as shown in Figure 5.1.

If one were to simply sum up all the positive and negative deviations from the mean, they would cancel each other out. By using the absolute difference (i.e., ignoring the sign), we are able to calculate the mean or average *absolute* deviation as follows:

$$\text{Mean absolute deviation } MAD = \frac{\sum_{i=1}^{i=n} |r_i - \bar{r}|}{n} \qquad (5.2)$$

The mean absolute deviation is the average *absolute* deviation of returns from the mean and penalises positive and negative deviations equally.

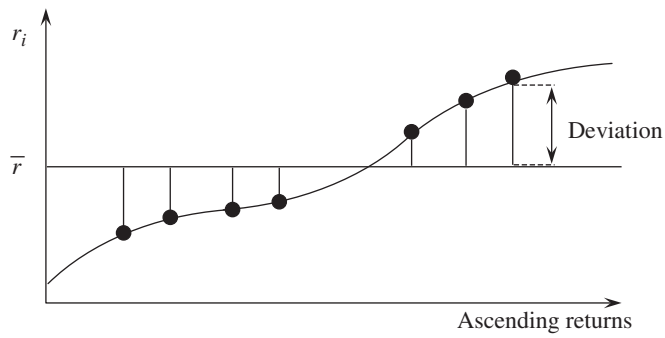

Figure 5.1 Deviation from the mean

Variance

The variance of returns is the average *squared* deviation of returns from the mean.

$$\text{Variance } \sigma^2 = \frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n} \tag{5.3}$$

Squaring the deviations from the mean avoids the problem of negative deviations cancelling with positive deviations and also penalises larger deviations from the mean. It provides a kind of weighted average deviation in which large deviations carry more weight (because the square of the difference is larger). It is a more accepted method for measuring deviation, hence the name "standard deviation", which is the square root of the variance.

Variance is a measure of variability (or dispersion) of returns from the average or mean return.

Table 5.1 contains 36 monthly portfolio returns. We will return to this standard portfolio data many times during the course of this chapter. The mean, annualised return, mean absolute deviation and variance for this portfolio are calculated in Exhibit 5.1.

Table 5.1 Portfolio variability

Portfolio monthly return (%) r_i	Unit price	Absolute deviation $\lvert r_i - \bar{r}\rvert$	Deviation squared $(r_i - \bar{r})^2$
	100.00		
0.3	100.30	0.8	0.64
2.6	102.91	1.5	2.25
1.1	104.04	0.0	0.00
−0.9	103.10	2.0	4.00
1.4	104.55	0.3	0.09
2.4	107.06	1.3	1.69
1.5	108.66	0.4	0.16
6.6	115.83	5.5	30.25
−1.4	114.21	2.5	6.25
3.9	118.67	2.8	7.84
−0.5	118.07	1.6	2.56
8.1	127.64	7.0	49.00
4.0	132.74	2.9	8.41
−3.7	127.83	4.8	23.04
−6.1	120.03	7.2	51.84
1.4	121.71	0.3	0.09
−4.9	115.75	6.0	36.00
−2.1	113.32	3.2	10.24
6.2	120.34	5.1	26.01
5.8	127.32	4.7	22.09
−6.4	119.18	7.5	56.25
1.7	121.20	0.6	0.36
−0.4	120.72	1.5	2.25

(Continued)

Table 5.1 (*Continued*)

Portfolio monthly return (%) r_i	Unit price	Absolute deviation $\lvert r_i - \bar{r}\rvert$	Deviation squared $(r_i - \bar{r})^2$
−0.2	120.48	1.3	1.69
−2.1	117.95	3.2	10.24
1.1	119.24	0.0	0.00
4.7	124.85	3.6	12.96
2.4	127.84	1.3	1.69
3.3	132.06	2.2	4.84
−0.7	131.14	1.8	3.24
4.7	137.30	3.6	12.96
0.6	138.13	0.5	0.25
1.0	139.51	0.1	0.01
−0.2	139.23	1.3	1.69
3.4	143.96	2.3	5.29
1.0	145.40	0.1	0.01
Total		**Total**	**Total**
$\sum_{i=1}^{i=n} r_i = 39.6$		$\sum_{i=1}^{i=n} \lvert r_i - \bar{r}\rvert = 90.8$	$\sum_{i=1}^{i=n} (r_i - \bar{r})^2 = 396.18$

Exhibit 5.1 Portfolio mean and variance

$$\text{Mean } \bar{r} = \frac{\sum_{i=1}^{i=n} r_i}{n} = \frac{39.6\%}{36} = 1.1\%$$

$$\text{Annualised return } \tilde{r} = \left(\frac{145.4}{100.0}\right)^{\frac{12}{36}} - 1 = 13.29\%$$

$$\text{Mean absolute deviation} = \frac{\sum_{i=1}^{i=n} \lvert r_i - \bar{r}\rvert}{n} = \frac{90.8\%}{36} = 2.52\%$$

$$\text{Variance } \sigma^2 = \frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n} = \frac{396.18}{36} = 11.0\%$$

Table 5.2 contains 36 months of benchmark returns associated with the portfolio in Table 5.1. The mean, annualised return, mean absolute deviation and variance for this benchmark are calculated in Exhibit 5.2.

Bessel's correction (population or sample, *n* or *n* – 1)

It might seem obvious that we should use n in the denominator of the calculation of variance; however, if we are using sample data to estimate the variance of the population, the sample mean will typically differ from the real mean of the population μ and as a consequence underestimate variance.

Table 5.2 Benchmark variability

Benchmark monthly return (%) b_i	Benchmark value	Absolute deviation $\lvert b_i - \overline{b}\rvert$	Deviation squared $(b_i - \overline{b})^2$
	100		
0.2	100.20	1.0	1.00
2.5	102.71	1.3	1.69
1.8	104.55	0.6	0.36
−1.1	103.40	2.3	5.29
1.4	104.85	0.2	0.04
2.3	107.26	1.1	1.21
1.4	108.76	0.2	0.04
6.5	115.83	5.3	28.09
−1.5	114.10	2.7	7.29
4.2	118.89	3.0	9.00
−0.3	118.53	1.5	2.25
8.3	128.37	7.1	50.41
3.9	133.38	2.7	7.29
−3.8	128.31	5.0	25.00
−6.2	120.35	7.4	54.76
1.5	122.16	0.3	0.09
−4.8	116.29	6.0	36.00
−2.0	113.97	3.2	10.24
6.0	120.81	4.8	23.04
5.6	127.57	4.4	19.36
−6.7	119.03	7.9	62.41
1.9	121.29	0.7	0.49
−0.3	120.92	1.5	2.25
−0.1	120.80	1.3	1.69
−2.6	117.66	3.8	14.44
0.7	118.48	0.5	0.25
4.3	123.58	3.1	9.61
2.9	127.16	1.7	2.89
3.8	132.00	2.6	6.76
−0.2	131.73	1.4	1.96
5.1	138.45	3.9	15.21
1.4	140.39	0.2	0.04
1.3	142.21	0.1	0.01
0.3	142.64	0.9	0.81
3.4	147.49	2.2	4.84
2.1	150.59	0.9	0.81
Total		**Total**	**Total**
$\sum_{i=1}^{i=n} b_i = 43.2$		$\sum_{i=1}^{i=n} \lvert b_i - \overline{b}\rvert = 92.8$	$\sum_{i=1}^{i=n} (b_i - \overline{b})^2 = 406.92$

Exhibit 5.2 Benchmark mean and variance

$$\text{Mean } \bar{b} = \frac{\sum_{i=1}^{i=n} b_i}{n} = \frac{43.2\%}{36} = 1.2\%$$

$$\text{Annualised return } \tilde{b} = \left(\frac{150.59}{100.0}\right)^{\frac{12}{36}} - 1 = 14.62\%$$

$$\text{Mean absolute deviation} = \frac{\sum_{i=1}^{i=n} |b_i - \bar{b}|}{n} = \frac{92.8\%}{36} = 2.58\%$$

$$\text{Variance } \sigma^2 = \frac{\sum_{i=1}^{i=n} (b_i - \bar{b})^2}{n} = \frac{406.92\%}{36} = 11.3\%$$

Bessel's correction helps correct this underestimation by multiplying by the term $\frac{n}{n-1}$. For a more detailed discussion on Bessel's correction see So.[16]

Note

It is a moot point whether the mean of the full period of 36 months is a sample of the asset manager's returns or is the true mean of the population being analysed – I incline to the full population. In any event for large n there is little practical difference, and the industry standard is n, not $(n - 1)$. This is sensible from the performance measurer's ex-post perspective; it is easy to appreciate from the risk controller's more conservative ex-ante perspective that $(n - 1)$ might be chosen.

Sample variance

Multiplying Equation 5.3 by Bessel's correction $\frac{n}{n-1}$ provides the formula for sample variance:

$$\text{Sample variance } \hat{\sigma}^2 = \frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n-1} \tag{5.4}$$

Standard deviation (variability or volatility)

For the analysis of portfolio variability, it is more convenient to use our original non-squared units of return; therefore, we take the square root of the variance to obtain the standard deviation:

$$\text{Standard deviation } \sigma = \sqrt{\frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n}} \tag{5.5}$$

[16] So, "Why Is the Sample Variance a Biased Estimator?" (2008).

The term "standard deviation" was first coined by the statistician Karl Pearson in 1894.[17] It is convenient to interpret standard deviation as a standardised measure of variance or even just a standard measure. A higher standard deviation would indicate greater uncertainty, variability or risk. Mean absolute deviation and standard deviation are related measures of variability. The standard deviation of portfolio returns is frequently (but less accurately) described as volatility; variability is a more appropriate term. Sample standard deviation is simply:

$$\text{Sample standard deviation } \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n-1}} \tag{5.6}$$

Interpretation

Many risk practitioners warn about interpreting a portfolio's standard deviation (also known as volatility) as a proper measure of "risk". Their reasoning is as follows: a portfolio's volatility is a measure of the portfolio's *usual* deviation from its average return and is therefore not a measure of how bad a performance can be generated by the portfolio. Most investors are concerned about large losses, not the usual deviation from the average return.

Note

The annualised return and the arithmetic average annual return are linked by the formula:

$$\tilde{r} \cong \bar{r}_A - \frac{\sigma^2}{2} \tag{3.34}$$

The annualised return is always less than the annual arithmetic average; the greater the variability of the returns the greater the difference.

Annualised risk (or time aggregation)

Equations 5.5 and 5.6 calculate standard deviation based on the periodicity of the data used, daily, monthly, quarterly, and so forth. For comparison of portfolio returns, standard deviation or variability is normally annualised for presentation purposes.

To annualise standard deviation we need to multiply by the square root of the number of observations in the year:

$$\text{Annualised standard deviation } \tilde{\sigma} = \sqrt{t} \times \sigma \tag{5.7}$$

[17] Pearson, "On the Dissection of Asymmetrical Frequency-Curves" (1894).

where: t = number of observations in year, quarterly = 4, monthly = 12, weekly = 52, etc.

For example, to annualise a monthly standard deviation, regardless of the number of monthly observations, multiply by $\sqrt{12}$; for quarterly standard deviation, multiply by $\sqrt{4}$ or 2.

Note

Because of weekends and public holidays, to annualise a daily standard deviation, a range of 250 to 260 (52 × 5 weekdays) observations in the year is typically used. The number itself is less important; the number chosen should be used consistently for comparison.

This calculation requires the assumption that each periodic return is independent and hence not correlated with other returns. Making this assumption the variance over one year is simply:

$$\sigma^2_{12\,\text{months}} = \sigma^2_{\text{Month 1}} + \sigma^2_{\text{Month 2}} + \ldots\ \sigma^2_{\text{Month 12}} \tag{5.8}$$

If we now assume that the variance of each month is the same, then the variance over the year is simply 12 times the variance over one month.

$$\sigma^2_{12\,\text{months}} = 12 \times \sigma^2 \tag{5.9}$$

Taking the square root leads to Equation 5.10, which ultimately leads back to Equation 5.7:

$$\sigma_{12\,\text{months}} = \sqrt{12} \times \sigma \tag{5.10}$$

Note

The typical annualised standard deviation of worldwide equities is about 16%, corresponding to a typical daily standard deviation of about 1% ($\sqrt{252} \times 1.0\% \approx 16\%$) since there are about 252 trading days per calendar year. This means that a 1% daily move in equities is normal: equities generate daily returns of ±1% about two-thirds (68%) of the time.

The central limit theorem

To annualise risk we must, in part, rely on the central limit theorem, which states that, if a number of observations of an independent random variable are taken:

(1) The mean of the resulting sample mean is equal to the mean of the underlying population.
(2) The standard deviation of the resulting sample mean is the standard deviation of the underlying population divided by the square root of the number of observations.
(3) Even if the underlying distribution is strongly non-normal the sampling distribution of means will increasingly approximate to a normal distribution as the sample size increases.

Frequency and number of data points

If variability is stable then clearly the more observations, the higher number of data points, the better, to maximise the accuracy of the estimation process. If variability is not stable, then we must find a balance between long measurement periods that are more accurate but slow to reflect structural changes and short recent measurement periods that reflect recent market conditions but are less accurate.

> ⚠ **Caution**
>
> The industry standard requires a minimum of 36 monthly periods and 20 quarterly periods. If absolutely pushed, I would provide risk statistics calculated using 24 months of monthly data but never less; the resulting information is less meaningful and potentially misleading.

Daily information is too noisy for long-term investment portfolios and should be ignored for standard risk measures (but not extreme risk measures for which there are few observations in the tail of the return distribution), although it is very tempting for short time periods for which there are an insufficient number of observations. On the other hand, the standard for ex-ante value-at-risk calculations is to use a 100-day (5-month) or 252-day (1-year) window of daily returns. Far better to refuse to calculate risk measures, for which there are insufficient data points, than to provide misleading information.

> ⚠ **Caution**
>
> Although it is easy to calculate annualised standard deviations, it is never appropriate to compare portfolios with risk statistics calculated using different frequencies. For daily valued mutual funds, you may wish to calculate the daily, weekly, monthly and quarterly annualised standard deviations, say over five years, and note the range of annualised standard deviations that result from the same data.

Using the data from Table 5.1 and Table 5.2 the standard deviation, sample standard deviation and annualised standard deviation are calculated in Exhibit 5.3 for both the portfolio and benchmark, respectively.

Normal (or Gaussian) distribution

A distribution is said to be normal if there is a high probability that an observation will be close to the average and a low probability that an observation is far away from the average, tailing away symmetrically. A normal distribution curve peaks at the average value. Returns are equally likely to lie above or below the mean. Normal distributions, sometimes called the "bell curve", are more formally known as Gaussian distributions, named after the German mathematician Carl Friedrich Gauss.

Exhibit 5.3 Standard deviation

Portfolio standard deviation $\sigma = \sqrt{\dfrac{\sum_{i=1}^{i=n}(r_i - \overline{r})^2}{n}} = \sqrt{\dfrac{396.18\%}{36}} = 3.32\%$

Sample standard deviation $\widehat{\sigma} = \sqrt{\dfrac{\sum_{i=1}^{i=n}(r_i - \overline{r})^2}{n-1}} = \sqrt{\dfrac{396.18\%}{35}} = 3.36\%$

Annualised standard deviation $\widetilde{\sigma} = \sqrt{t} \times \sigma = \sqrt{12} \times 3.32\% = 11.49\%$

Benchmark standard deviation $\sigma_b = \sqrt{\dfrac{\sum_{i=1}^{i=n}(b_i - \overline{b})^2}{n}} = \sqrt{\dfrac{406.92\%}{36}} = 3.36\%$

Sample standard deviation $\widehat{\sigma}_b = \sqrt{\dfrac{\sum_{i=1}^{i=n}(b_i - \overline{b})^2}{n-1}} = \sqrt{\dfrac{406.92\%}{35}} = 3.41\%$

Annualised standard deviation $\widetilde{\sigma}_b = \sqrt{t} \times \sigma_b = \sqrt{12} \times 3.36\% = 11.65\%$

A normal distribution (see Figure 5.2) has special properties that are useful to asset management. If returns are normally distributed, then we can use the average return and variability or standard deviation of returns to describe the distribution of returns, such that:

- Approximately 68% of returns will be within a range of one standard deviation above and below the average return.
- Approximately 95% of returns will be within a range of two standard deviations above and below the average return.
- Approximately 99.7% of returns will be within a range of three standard deviations above and below the average return.[18]

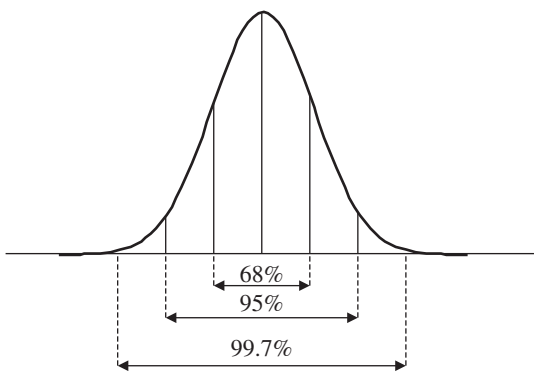

Figure 5.2 Normal distribution

[18] The "Empirical or 68-95-99.7" Rule; actually, more accurately 68.2689%, 95.4499% and 99.7300%.

This property is obviously especially useful for calculating the probability of an event occurring outside a specified range of returns. Normal distributions are popular because of these statistical properties and because many random events can be approximated by a normal distribution.

Caution

While it is tempting to use a mathematical function, like the normal curve, because it has convenient properties, care must be taken to ensure that it properly models the data. Equity returns are often shown to follow the normal curve within about two standard deviations but outside this range show a higher frequency of extreme returns than the normal curve implies. Equities exhibit so-called "fat tails" precisely for this reason: their return distributions have more weight (i.e., are fatter) for large returns. This means that large losses (and gains) occur more frequently that the normal curve implies.

Histograms

Histograms were first introduced by Pearson[19] to provide a graphical representation of the distribution of data and are ideal for illustrating the distribution of returns of a portfolio manager's track record. A histogram of the example data from the portfolio in Table 5.1 is shown in Figure 5.3. The horizontal axis is split into consecutive intervals of return or bins and the vertical axis represents the frequency in which periodic returns fall within the bin. A normal

Figure 5.3 Histogram of portfolio returns

[19] K. Pearson, "Contributions to the Mathematical Theory of Evolution" (1895).

distribution curve is imposed on the histogram to show that in this case the distribution of returns is quite normal. The term *histogram* is derived from the Greek *histos*, anything set upright such as the vertical bars on a graph, and *gramma*, drawing.

Skewness (Fisher's or moment skewness)

Not all distributions are normally distributed; if there are more extreme returns extending to the right tail of a distribution it is said to be positively skewed and if there are more returns extending to the left it is said to be negatively skewed.

We can measure the degree of skewness (or more accurately Fisher's skewness) in the following formula:

$$\text{Skewness } \zeta = \sum_{i=1}^{i=n} \left(\frac{r_i - \bar{r}}{\sigma} \right)^3 \times \frac{1}{n} \tag{5.11}$$

A normal distribution will have a skewness of 0. Note that in Equation 5.11, extreme values carry greater weight since they are cubed while maintaining their initial sign, positive or negative. Note also that the difference from the mean return is standardised by dividing by the standard deviation of returns. Skewness is a measure of asymmetry of the return distribution.

Positive and negative skew are illustrated in Figures 5.4 and 5.5, respectively.

We can use skewness in making a judgement about the possibility of large negative or positive outliers when comparing portfolio returns. Skewness provides more information about the shape of return distribution, where deviations from the mean are greater in one direction than the other; this measure will deviate from zero in the direction of the larger deviations.

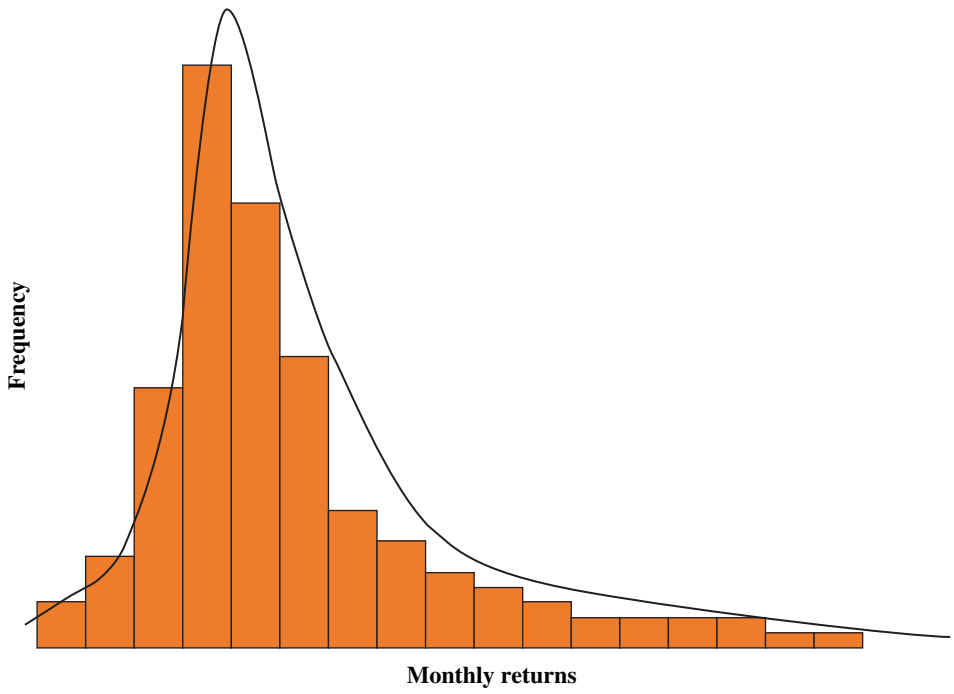

Figure 5.4 Positive skew

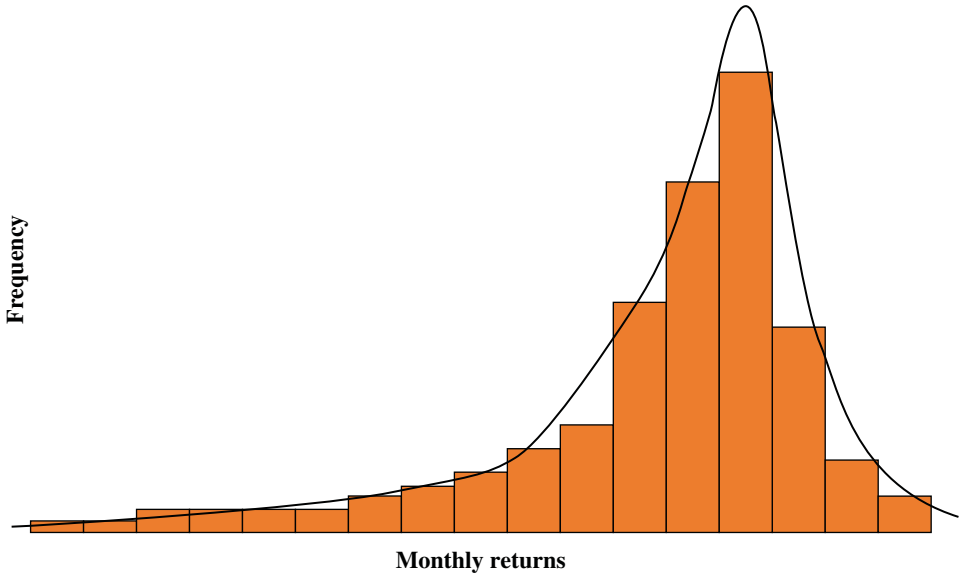

Figure 5.5 Negative skew

Sample skewness

Sample skewness is calculated using Bessel's correction as follows:

$$\text{Sample skewness } \hat{\zeta} = \sum_{i=1}^{i=n} \left(\frac{r_i - \bar{r}}{\hat{\sigma}} \right)^3 \times \frac{n}{(n-1) \times (n-2)} \tag{5.12}$$

Note

The standard Excel® function called skewness is in fact sample skewness.

Kurtosis (Pearson's kurtosis)

Kurtosis[20] (or more correctly Pearson's kurtosis) provides additional information about the shape of a return distribution; formally it measures the weight of returns in the tails of the distribution relative to standard deviation, but is more often associated as a measure of flatness or peakedness of the return distribution. In fact, DeCarlo[21] correctly points out that both tailedness and peakedness are components of kurtosis.

$$\text{Kurtosis } \kappa = \sum_{i=1}^{i=n} \left(\frac{r_i - \bar{r}}{\sigma} \right)^4 \times \frac{1}{n} \tag{5.13}$$

[20] Pearson, "Das Fehlergesetz und seine Verallgemeinerungen durch Fechner und Pearson. A Rejoinder" (1905).
[21] DeCarlo, "On the Meaning and Use of Kurtosis" (1997).

Note that in Equation 5.13 extreme results of either sign make exceptionally large positive contributions to kurtosis. Just like the formula for skewness the difference from the mean is again standardised by dividing by the standard deviation of returns.

The kurtosis of a normal distribution is 3 (called *mesokurtic*), greater than 3 may indicate a peaked distribution with fat tails (called *leptokurtic*) and less than 3 may indicate a less peaked distribution with thin tails (called *platykurtic*). The word *kurtosis* is derived from the Greek word for bulge, *kurtos. Platy* comes from the Greek word *platus* for broad or flat. *Leptos* is Greek for thin or slender describing the shape of the peak not the tails. For symmetric distributions with skewness close to zero, kurtosis greater than 3 could indicate an excess of returns either in the tails, or the centre, or both.

Note

Generally speaking, investors prefer platykurtic distributions with a kurtosis less than 3, less peaked but with fewer extreme returns.

A leptokurtic distribution is shown in Figure 5.6 and a platykurtic distribution in Figure 5.7.

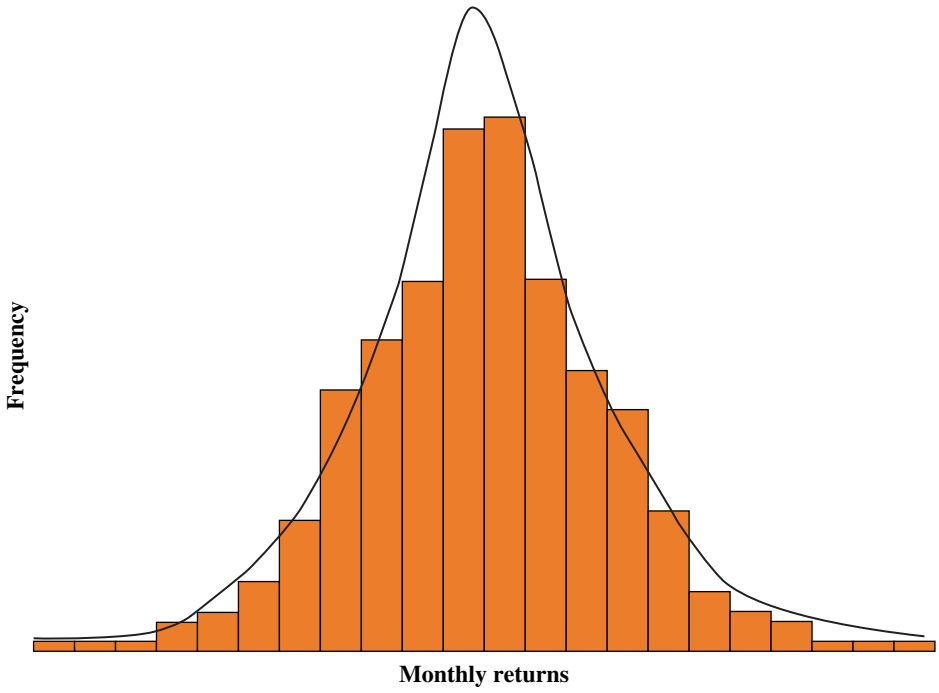

Figure 5.6 Kurtosis >3, thin peak with fat tails

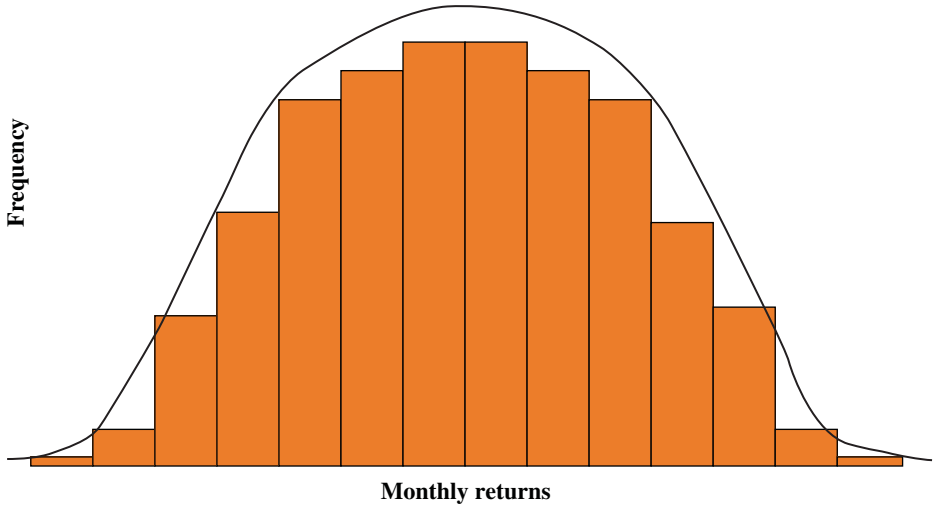

Figure 5.7 Kurtosis <3, broad peak with thin tails

Excess kurtosis (or Fisher's kurtosis)

Subtracting 3 from Equation 5.13, we obtain a measure of excess kurtosis (or Fisher's kurtosis). The terms *kurtosis* and *excess kurtosis* are often confused.

$$\text{Excess kurtosis } \kappa_E = \sum_{i=1}^{i=n} \left(\frac{r_i - \bar{r}}{\sigma} \right)^4 \times \frac{1}{n} - 3 \tag{5.14}$$

⚠ Caution

Care needs to be taken whenever calculating kurtosis to ensure that it is clear whether the calculation is excess kurtosis (subtracting 3) or not.

Sample kurtosis

Sample kurtosis is calculated using Bessel's correction as follows:

$$\text{Sample kurtosis } \widehat{\kappa} = \sum_{i=1}^{i=n} \left(\frac{r_i - \bar{r}}{\widehat{\sigma}} \right)^4 \times \frac{n \times (n+1)}{(n-1) \times (n-2) \times (n-3)} \tag{5.15}$$

Sample excess kurtosis is calculated also using Bessel's correction as follows:

$$\widehat{\kappa}_E = \sum_{i=1}^{i=n} \left(\frac{r_i - \bar{r}}{\widehat{\sigma}} \right)^4 \times \frac{n \times (n+1)}{(n-1) \times (n-2) \times (n-3)} - \frac{3 \times (n-1)^2}{(n-2) \times (n-3)} \tag{5.16}$$

Caution

The standard Excel® function labelled kurtosis is in fact sample excess kurtosis (or sample Fisher's kurtosis if I am more generous). The same arguments used to justify using the standard deviation of the population as opposed to a sample also apply to skewness and kurtosis.

A better understanding of the shape of the distribution of returns will aid in assessing the relative qualities of asset manager return distributions. Whether we prefer higher or lower kurtosis (or for that matter positive or negative skewness) will depend on the type of return series we want to see.

Equity markets tend to have fat tails; when markets fall portfolio managers tend to sell and when markets rise, portfolio managers tend to buy, and there is a higher probability of extreme events than the normal distribution would suggest. Therefore, any risk measure calculated using normal assumptions may well underestimate risk, especially outside the two-standard-deviation bands.

Note

Investors will naturally prefer positive skew and lower kurtosis with thinner tails.

Bera-Jarque statistic (or Jarque-Bera)

Since normal distributions should have skewness near 0 and kurtosis near 3, we can test for normality using the Bera-Jarque[22] test.

$$\text{Bera-Jarque statistic } BJ = \frac{n}{6} \times \left(\zeta^2 + \frac{\kappa_E^2}{4} \right) \tag{5.17}$$

With a confidence level of 95% we will reject the hypothesis that a distribution is normal if the Bera-Jarque statistic exceeds 5.99, and with a confidence level of 99% if it exceeds 9.21.

Note

Perfectly normal distributions will have a $BJ = 0$.

Portfolio skewness, kurtosis and the Bera-Jarque statistic of the portfolio data first shown in Table 5.1 are calculated in Exhibit 5.4 from summarised data contained in Table 5.3.

[22] Jarque and Bera, "A Test for Normality of Observations and Regression Residuals" (1987).

Exhibit 5.4 Skewness, kurtosis and the Bera-Jarque hypothesis test

Skewness $\zeta = \sum_{i=1}^{i=n} \left(\frac{r_i - \overline{r}}{\sigma} \right)^3 \times \frac{1}{n} = \frac{-312.79}{3.32^3} \times \frac{1}{36} = -0.24$

Kurtosis $\kappa = \sum_{i=1}^{i=n} \left(\frac{r_i - \overline{r}}{\sigma} \right)^4 \times \frac{1}{n} = \frac{12982.39}{3.32^4} \times \frac{1}{36} = 2.98$

Excess kurtosis $\kappa_E = \kappa - 3 = 2.98 - 3 = -0.02$

Bera-Jarque statistic $= \frac{n}{6} \times \left(\zeta^2 + \frac{\kappa_E^2}{4} \right) = \frac{36}{6} \times \left(-0.24^2 + \frac{-0.02^2}{4} \right) = 0.34$

It would appear that this return distribution is close to normal.
Sample skewness

$$\widehat{\zeta} = \sum_{i=1}^{i=n} \left(\frac{r_i - \overrightarrow{r}}{\widehat{\sigma}} \right)^3 \times \frac{n}{(n-1) \times (n-2)} = \frac{-312.79}{3.36^3} \times \frac{36}{(36-1) \times (36-2)} = -0.25$$

Sample kurtosis

$$\widehat{\kappa} = \sum_{i=1}^{i=n} \left(\frac{r_i - \overline{r}}{\widehat{\sigma}} \right)^4 \times \frac{n \times (n+1)}{(n-1) \times (n-2) \times (n-3)} = \frac{12982.39}{3.36^4} \times \frac{36 \times (36+1)}{(36-1) \times (36-2) \times (36-3)} = 3.44$$

Sample excess kurtosis

$$\widehat{\kappa}_E = \widehat{\kappa} - \frac{3 \times (n-1)^2}{(n-2) \times (n-3)} = 3.44 - \frac{3 \times (36-1)^2}{(36-2) \times (36-3)} = 0.16$$

Table 5.3 Portfolio skewness and kurtosis

Monthly return (%) r_i	Deviation cubed $(r_i - \overline{r})^3$	Fourth-power deviation $(r_i - \overline{r})^4$
0.3	−0.51	0.41
2.6	3.38	5.06
1.1	0.00	0.00
−0.9	−8.00	16.00
1.4	0.03	0.01
2.4	2.20	2.86
1.5	0.06	0.03
6.6	166.38	915.06
−1.4	−15.63	39.06
3.9	21.95	61.47
−0.5	−4.10	6.55
8.1	343.00	2401.00
4.0	24.39	70.73
−3.7	−110.59	530.84
−6.1	−373.25	2687.39
1.4	0.03	0.01
−4.9	−216.00	1,296.00
−2.1	−32.77	104.86
6.2	132.65	676.52
5.8	103.82	487.97

(Continued)

Table 5.3 (*Continued*)

Monthly return (%) r_i	Deviation cubed $(r_i - \overline{r})^3$	Fourth-power deviation $(r_i - \overline{r})^4$
−6.4	−421.88	3,164.06
1.7	0.22	0.13
−0.4	−3.38	5.06
−0.2	−2.20	2.86
−2.1	−32.77	104.86
1.1	0.00	0.00
4.7	46.66	167.96
2.4	2.20	2.86
3.3	10.65	23.43
−0.7	−5.83	10.50
4.7	46.66	167.96
0.6	−0.13	0.06
1.0	0.00	0.00
−0.2	−2.20	2.86
3.4	12.17	27.98
1.0	0.00	0.00
Total		**Total**
$\sum_{i=1}^{i=n}(r_i - \overline{r})^3 = -312.79$		$\sum_{i=1}^{i=n}(r_i - \overline{r})^4 = 12,982.39$

Skewness and kurtosis for the benchmark data in Table 5.2 are calculated in Exhibit 5.5 from the summarised data in Table 5.4.

Exhibit 5.5 Benchmark skewness and kurtosis

Skewness $$\zeta_b = \sum_{i=1}^{i=n}\left(\frac{b_i - \overline{b}}{\sigma_b}\right)^3 \times \frac{1}{n} = \frac{-495.94}{3.36^3} \times \frac{1}{36} = -0.36$$

Kurtosis $$\kappa_b = \sum_{i=1}^{i=n}\left(\frac{b_i - \overline{b}}{\sigma_b}\right)^4 \times \frac{1}{n} = \frac{14004.25}{3.36^4} \times \frac{1}{36} = 3.04$$

Excess kurtosis $\kappa_{Eb} = \kappa_b - 3 = 3.04 - 3 = 0.04$

Sample skewness

$$\widehat{\zeta} = i = \sum_{i=1}^{i=n}\left(\frac{b_i - \overline{b}}{\widehat{\sigma}_b}\right)^3 \times \frac{n}{(n-1)\times(n-2)} = \frac{-495.94}{3.41^3} \times \frac{36}{(36-1)\times(36-2)} = -0.38$$

Sample kurtosis

$$\widehat{\kappa} = \sum_{i=1}^{i=n}\left(\frac{b_i - \overline{b}}{\widehat{\sigma}_b}\right)^4 \times \frac{n\times(n+1)}{(n-1)\times(n-2)\times(n-3)} = \frac{14004.25}{3.41^4} \times \frac{36\times(36+1)}{(36-1)\times(36-2)\times(36-3)} = 3.51$$

Sample excess kurtosis

$$\widehat{\kappa}_E = \widehat{\kappa} - \frac{3\times(n-1)^2}{(n-2)\times(n-3)} = 3.51 - \frac{3\times(36-1)^2}{(36-2)\times(36-3)} = 0.24$$

Table 5.4 Benchmark skewness and kurtosis

Monthly return (%) b_i	Deviation cubed $(b_i - \overline{b})^3$	Fourth-power deviation $(b_i - \overline{b})^4$
0.2	–1.00	1.00
2.5	2.20	2.86
1.8	0.22	0.13
−1.1	−12.17	27.98
1.4	0.01	0.00
2.3	1.33	1.46
1.4	0.01	0.00
6.5	148.88	789.05
−1.5	−19.68	53.14
4.2	27.00	81.00
−0.3	–3.38	5.06
8.3	357.91	2,541.17
3.9	19.68	53.14
−3.8	−125.00	625.00
−6.2	−405.22	2,998.66
1.5	0.03	0.01
−4.8	−216.00	1296.00
−2.0	–32.77	104.86
6.0	110.59	530.84
5.6	85.18	374.81
−6.7	−493.04	3,895.01
1.9	0.34	0.24
−0.3	–3.38	5.06
−0.1	–2.20	2.86
−2.6	–54.87	208.51
0.7	–0.13	0.06
4.3	29.79	92.35
2.9	4.91	8.35
3.8	17.58	45.70
−0.2	−2.74	3.84
5.1	59.32	231.34
1.4	0.01	0.00
1.3	0.00	0.00
0.3	–0.73	0.66
3.4	10.65	23.43
2.1	0.73	0.66
	Total	**Total**
	$\sum_{i=1}^{i=n}(b_i - \overline{b})^3 = -495.94$	$\sum_{i=1}^{i=n}(b_i - \overline{b})^4 = 14,004.25$

Covariance

Covariance is a descriptive statistic that measures the tendency of two return streams to move together relative to their averages – for example, this could be the covariance between two portfolios, two indexes or, most commonly, between a portfolio and its benchmark.

$$\text{Covariance} = \frac{\sum_{i=1}^{i=n}(r_i - \overline{r}) \times (b_i - \overline{b})}{n} \tag{5.18}$$

where: b_i = benchmark return in period i
$\overline{b}$ = mean benchmark return

Equation 5.18 multiplies the period portfolio return difference from the mean portfolio return with the same period benchmark return difference from the mean benchmark return. If both are positive or negative this will make a positive contribution to covariance; if they are of different sign, it will make a negative contribution to covariance.

Therefore, a total positive covariance indicates that the returns are associated; they move together. A total negative covariance indicates that the returns move in opposite directions. A low or near zero covariance would indicate little relationship between portfolio and benchmark returns.

Sample covariance

Sample covariance is calculated using Bessel's correction as follows:

$$\text{Sample covariance} = \frac{\sum_{i=1}^{i=n}(r_i - \overline{r}) \times (b_i - \overline{b})}{n-1} \tag{5.19}$$

Correlation (ρ)

In isolation, covariance is a difficult statistic to interpret because it measures the "co-dispersion" relative to both the portfolio and its benchmark, which can be of unknown size. For example, is a covariance of 3 large or small? The answer depends on the standard deviations of both the portfolio and the benchmark.

We can standardise the covariance to a value between 1 and –1 by dividing by the product of the portfolio standard deviation and the benchmark standard deviation as follows:

$$\text{Correlation } \rho_{r,b} = \frac{\text{Covariance}}{\sigma \times \sigma_b} \tag{5.20}$$

where: $\rho_{r,b}$ is the coefficient of correlation between the portfolio return and benchmark return.

The closer the correlation is to 1, the stronger the linear relationship.

This standardisation has the immensely helpful feature that it allows us to compare the correlations between disparate portfolios and benchmarks on an apples-to-apples basis.

Caution

Interpreting correlation can sometimes be tricky. If a portfolio and its benchmark have a high correlation (for example, above 0.9) then it can be tempting to believe that the portfolio will move up (or down) as much as the benchmark. In such a case, if the benchmark moves up 1% it is **NOT** correct to expect the portfolio to move up by 1%.

The correlation measures the tendency of the portfolio to move up or down *relative to its own average and volatility* but not as an absolute measure. In this example, if the portfolio has a very small standard deviation, the expectation should be for it to move up just a little when the benchmark moves up by 1%.

Sample correlation

Bessel's correction applies equally to covariance and the product of the portfolio and benchmark standard deviations and therefore the correction in the numerator cancels with the correction in the denominator, eliminating the need to calculate sample correlation.

Covariance and correlation are calculated in Exhibit 5.6 from the standard portfolio data shown in Table 5.5.

Exhibit 5.6 Covariance and correlation

$$\text{Covariance} = \frac{\sum_{i=1}^{i=n}(r_i - \overline{r}) \times (b_i - \overline{b})}{n} = \frac{399.37}{36} = 11.09$$

$$\text{Correlation } \rho_{r,b} = \frac{\text{Covariance}}{\sigma \times \sigma_b} = \frac{11.09}{3.32 \times 3.36} = 0.995$$

Table 5.5 Covariance and correlation

Portfolio monthly return (%) r_i	Deviation from average $(r_i - \overline{r})$	Benchmark monthly return b_i	Deviation from average $(b_i - \overline{b})$	Portfolio deviation × Benchmark deviation $(r_i - \overline{r}) \times (b_i - \overline{b})$
0.3	−0.8	0.2	−1.0	0.80
2.6	1.5	2.5	1.3	1.95
1.1	0.0	1.8	0.6	0.00
−0.9	−2.0	−1.1	−2.3	4.60
1.4	0.3	1.4	0.2	0.06
2.4	1.3	2.3	1.1	1.43
1.5	0.4	1.4	0.2	0.08
6.6	5.5	6.5	5.3	29.15
−1.4	−2.5	−1.5	−2.7	6.75
3.9	2.8	4.2	3.0	8.40
−0.5	−1.6	−0.3	−1.5	2.40
8.1	7.0	8.3	7.1	49.7

(Continued)

Table 5.5 (*Continued*)

Portfolio monthly return (%) r_i	Deviation from average $(r_i - \bar{r})$	Benchmark monthly return b_i	Deviation from average $(b_i - \bar{b})$	Portfolio deviation × Benchmark deviation $(r_i - \bar{r}) \times (b_i - \bar{b})$
4.0	2.9	3.9	2.7	7.83
−3.7	−4.8	−3.8	−5.0	24.00
−6.1	−7.2	−6.2	−7.4	53.28
1.4	0.3	1.5	0.3	0.09
−4.9	−6.0	−4.8	−6.0	36.00
−2.1	−3.2	−2.0	−3.2	10.24
6.2	5.1	6.0	4.8	24.48
5.8	4.7	5.6	4.4	20.68
−6.4	−7.5	−6.7	−7.9	59.25
1.7	0.6	1.9	0.7	0.42
−0.4	−1.5	−0.3	−1.5	2.25
−0.2	−1.3	−0.1	−1.3	1.69
−2.1	−3.2	−2.6	−3.8	12.16
1.1	0.0	0.7	−0.5	0.00
4.7	3.6	4.3	3.1	11.16
2.4	1.3	2.9	1.7	2.21
3.3	2.2	3.8	2.6	5.72
−0.7	−1.8	−0.2	−1.4	2.52
4.7	3.6	5.1	3.9	14.04
0.6	−0.5	1.4	0.2	–0.10
1.0	−0.1	1.3	0.1	–0.01
−0.2	−1.3	0.3	−0.9	1.17
3.4	2.3	3.4	2.2	5.06
1.0	−0.1	2.1	0.9	–0.09
				Total
				$\sum_{i=1}^{i=n}(r_i - \bar{r}) \times (b_i - \bar{b}) = 399.37$

PERFORMANCE APPRAISAL

Asset owners are risk averse; given the same return, they would prefer the portfolio with less risk or less variability of return. Therefore, how do we evaluate portfolios with different returns and various levels of risk? We need composite risk measures that determine if the risk undertaken by the asset manager has been justified by the reward received. The dispersion, variability and tendency measures we have just defined allow us to appraise the performance of the asset manager by combining a measure of risk with a measure of reward ex-post.

Attempting to interpret a portfolio's performance without considering its risk is a misguided venture. Is an annualised performance of 8% a good return? One way to compare that number is relative to what other portfolios returned. Another, potentially more nuanced, way is to compare it to its own risk: a portfolio that delivers an 8% return while taking only 4% risk is clearly superior to a portfolio that delivers the same 8% return while taking 16% risk.

One way of thinking about the relationship between return and risk is that risk is one of the *costs* of investing. It is a *potential* cost that may (or may not) be realised and therefore should be interpreted probabilistically. Not taking it into account is a dangerous game.

Sharpe ratio (reward to variability, Sharpe index)

With two variables it is natural to resort to a graphical representation with return represented by the vertical axis and risk represented by the horizontal axis as shown in Figure 5.8 using data from Table 5.6.

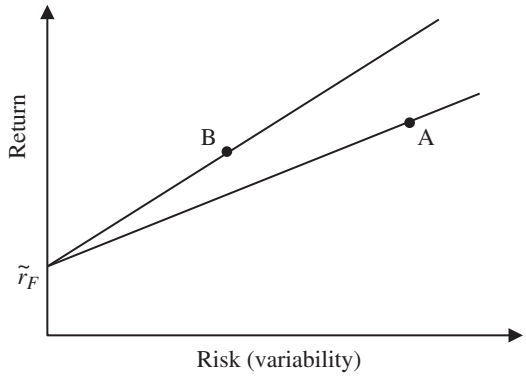

Figure 5.8 Sharpe ratio

Table 5.6 Sharpe ratio

	Portfolio A	Portfolio B	Benchmark
Annualised return	7.9%	6.9%	7.5%
Annualised risk	5.5%	3.2%	4.5%
Sharpe ratio			
(Risk-free rate = 2%) $SR = \frac{\tilde{r} - \tilde{r}_F}{\tilde{\sigma}}$	$\frac{7.9\% - 2.0\%}{5.5\%} = 1.07$	$\frac{6.9\% - 2.0\%}{3.2\%} = 1.53$	$\frac{7.5\% - 2.0\%}{4.5\%} = 1.22$

A straight line is drawn from a fixed point on the vertical axis to points A and B representing the annualised returns and annualised variability (risk) of portfolios A and B respectively.

This fixed point represents the natural starting point for all investors, the risk-free rate. The risk-free rate represents the return we should expect on a riskless asset, for example the interest return on cash or Treasury bills. An investor can achieve this return without any variability or risk.

⚠ Caution

It is important to ensure that the same risk-free rate is used for all portfolios for comparison purposes.

Clearly the investor will prefer to be in the top left-hand quadrant of this graph, representing high return and low risk. The gradient of the line determines how far toward the top left-hand quadrant each portfolio is positioned. The steeper the gradient, the further to the top left-hand side.

This gradient is called the Sharpe ratio, named after William Sharpe[23] and calculated as follows:

$$SR = \frac{\tilde{r} - \tilde{r}_F}{\tilde{\sigma}} \tag{5.21}$$

where: $\tilde{r}$ = annualised portfolio return
$\tilde{r}_F$ = annualised risk-free rate
$\tilde{\sigma}$ = annualised portfolio risk (variability, standard deviation of return)

The greater the Sharpe ratio, the steeper the gradient and hence a better combination of risk and return. The Sharpe ratio can be described as the return (or reward) per unit of variability (or risk).

Both graphically in Figure 5.8 and in the Sharpe ratios calculated in Table 5.6 we can see that portfolio B has a better risk-adjusted performance than either portfolio A or the benchmark even though portfolio B has a lower absolute return than both portfolio A and the benchmark.

Interpretation

Negative returns will generate negative Sharpe ratios, which despite the views of some commentators still retain meaning. Perversely for negative returns it is better to be more variable, not least because the chance of returning into positive territory is higher than when variability is low!

Although the concept of the Sharpe ratio is based on CAPM and the assumption that excess returns from risky assets cannot be negative (otherwise investors can simply invest in a risk-free asset), comparing two portfolios that both have a negative Sharpe ratio may still be meaningful in a way that a portfolio with a "less negative" Sharpe ratio has generated a smaller loss per unit of risk incurred. Akeda[24] concludes that the Sharpe ratio can be used as an indicator of performance evaluation, irrespective of its sign. For those who think higher variability is always less desirable, negative Sharpe ratios are difficult statistics to interpret. Some commentators[25] have suggested squaring the Sharpe ratio (or squaring the numerator of the Sharpe ratio) to eliminate negative ratios. Apart from not recognising the need to adjust for negative Sharpe ratios in the first place I can see no merit in squaring the Sharpe ratio and therefore it is not listed in this book.

Annualised portfolio and risk-free returns are used in the numerator rather than the arithmetic means, primarily because as performance measurers we are more concerned with annualised returns and in any event the mean of continuously compounded returns are equivalent to the annualised simple returns.

[23] Sharpe, "Mutual Fund Performance" (1966).
[24] Akeda, "Interpretation of Negative Sharpe Ratio" (2003).
[25] See Dowd, "Adjusting for Risk: An Improved Sharpe Ratio" (1999) and Kidd, "The Sharpe Ratio and the Information Ratio" (2011).

Interpretation

Sharpe ratios, like returns, are best used for comparison against the Sharpe ratios of benchmarks or other portfolios rather than analysed in isolation.

Roy ratio

The Sharpe ratio is regarded, quite correctly, as the grandfather of all risk-adjusted performance measures, despite the fact that the Treynor ratio predates the Sharpe by one year, and that Arthur Roy[26] suggested a similar measure as early as 1952:

$$\text{Roy ratio } RR = \frac{\tilde{r} - \tilde{r}_T}{\tilde{\sigma}} \tag{5.22}$$

where: $\tilde{r}_T$ = annualised minimum target return or "disaster level"

Risk-free rate

The risk-free rate of return is defined as the rate of return an investor can expect from a theoretically risk-free investment. Of course, no investments are genuinely risk free and even cash returns will display some variability. Investors' perceptions of the risk-free rate of return will differ, ranging from short-term rates of interest to long-term interest rates or even real risk-free rates adjusted for inflation. In the context of the reward-to-risk ratios covered in this chapter the actual choice of risk-free rate is not as crucial as might be imagined. Consider a magnet attached to the risk-free rate that allows movement up and down the vertical axis – although the calculated ratio may differ with different risk-free rates, the ranking of portfolios is not significantly impacted.

Caution

When comparing the Sharpe ratios of portfolios, it is important to ensure that risk-free rates are consistent.

Risk-free rates will vary depending on investor preferences, typically across currencies, and as we have observed in recent years may even be negative.

Caution

Negative interest rates still represent the natural starting point for investors – do not be tempted to replace negative rates with zero.

[26] Roy, "Safety First and the Holding of Assets" (1952).

Alternative Sharpe ratio

In the alternative Sharpe ratio the variability of the risk-free rate is explicitly considered.

$$\text{Alternative Sharpe ratio } r = \frac{\tilde{r} - \tilde{r}_T}{\tilde{\sigma} - \tilde{\sigma}_F} \tag{5.23}$$

where: $\tilde{\sigma}_F$ = the annualised variability of the risk-free rate

Even after allowing the risk-free rate anchor point to move away from the vertical axis to the right as shown in Figure 5.9, the natural starting point of investors will still inhabit a region close to the vertical access with low risk and relatively low return (if the risk-free rate of return is too high, investors will not be incentivised to take any risk). The gradients of the Sharpe ratio (or alternative Sharpe ratio) lines will vary without a significant impact on rankings.

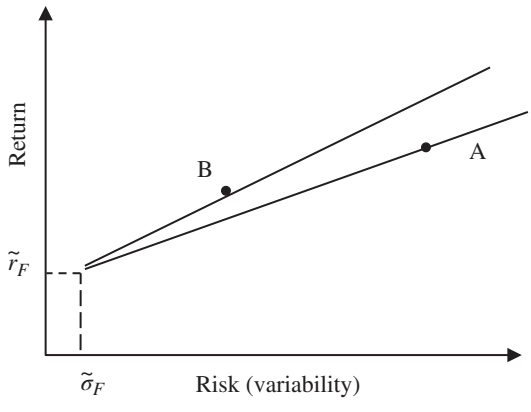

Figure 5.9 Alternative Sharpe ratio

Although acknowledging that there is some variability in the risk-free rate this version of the Sharpe ratio is rarely, if ever, used.

Revised Sharpe ratio

Sharpe revised his ratio in 1994,[27] acknowledging that the risk-free rate is not constant and varies over time as follows:

$$\text{Revised Sharpe ratio} = \frac{\tilde{r} - \tilde{r}_F}{\tilde{\sigma}(r_i - r_{Fi})} \tag{5.24}$$

The denominator of the revised Sharpe ratio is the variability of return above the risk-free rate, which of course will not be significantly different from the variability of the portfolio return if the variability of the risk-free rate is low.

Although technically more appropriate than the original version of the Sharpe ratio, this revised Sharpe ratio is not often used because the original is so ingrained and universal. The revised Sharpe ratio is shown in Figure 5.10; note that the revised Sharpe ratio lines are now

[27] Sharpe, "The Sharpe Ratio" (1994).

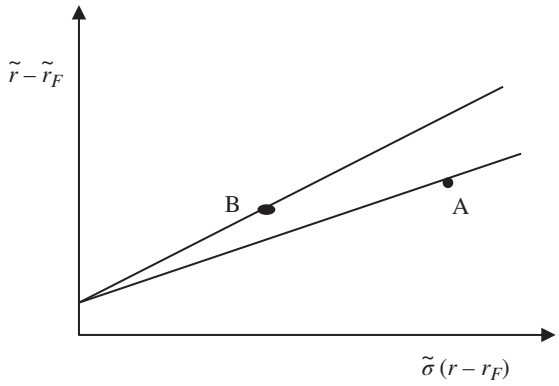

Figure 5.10 Revised Sharpe ratio

anchored through the origin because the x-axis has effectively been redefined to ensure zero offset.

Adjusted Sharpe ratio

Pezier[28] suggests using the adjusted Sharpe ratio (ASR), which explicitly adjusts for skewness and kurtosis by incorporating a penalty factor for negative skewness and excess kurtosis as follows:

$$\text{Adjusted Sharpe ratio } ASR = SR \times \left[1 + \left(\frac{\zeta}{6}\right) \times SR - \left(\frac{\kappa - 3}{24}\right) \times SR^2\right] \tag{5.25}$$

Interpretation

The adjusted Sharpe ratio rewards positive skew and kurtosis less than 3 and penalises negative skew and kurtosis greater than 3.

Although rarely used, the adjusted Sharpe ratio addresses the issue that portfolio returns are not normally distributed, and it is fairly easy to calculate. It should be used more often – it is demonstrably better than the Sharpe ratio.

Note

Any time variability, variance or standard deviation is used in a formula, there is an implicit assumption that returns are normally distributed, which we empirically know is not a particularly good assumption. For this reason, any analytic that adjusts for skewness and kurtosis will be an improvement on that analytic.

[28] Pezier and White, *The Relative Merits of Investable Hedge Fund Indices and of Funds of Hedge Funds in Optimal Passive Portfolios* (2006).

Skew-adjusted Sharpe ratio

A slightly easier calculation would adjust for skewness only in the skew-adjusted Sharpe ratio, ignoring kurtosis which, frankly, in most cases makes very little difference anyway:

$$\text{Skew-adjusted Sharpe ratio SASR} = \text{SR} \times \left[1 + \left(\frac{\zeta}{6}\right) \times SR\right] \tag{5.26}$$

The Sharpe ratio, alternative Sharpe ratio, adjusted Sharpe ratio and skew-adjusted Sharpe ratio for the standard portfolio data shown in Tables 5.1 and 5.3, together with the monthly risk-free rates in Table 5.7, are calculated in Exhibit 5.7. Revised Sharpe ratios are calculated in Exhibit 5.8 using data from Table 5.8.

Exhibit 5.7 Sharpe, alternative, skew-adjusted and adjusted Sharpe ratios

$$\text{Sharpe ratio} = \frac{\tilde{r} - \tilde{r}_F}{\tilde{\sigma}} = \frac{13.29\% - 2.43\%}{11.49\%} = 0.94$$

Note that the annualised risk-free rate is 2.43%.

$$\text{Alternative Sharpe ratio} = \frac{\tilde{r} - \tilde{r}_F}{\tilde{\sigma} - \tilde{\sigma}_F} = \frac{13.29\% - 2.43\%}{11.49\% - 0.23\%} = 0.97$$

Skew-adjusted Sharpe ratio:

$$\begin{aligned}\text{SASR} &= \text{SR} \times \left[1 + \left(\frac{\zeta}{6}\right) \times SR\right] \\ &= 0.94 \times \left[1 + \frac{-0.24}{6} \times 0.94\right] = 0.91\end{aligned}$$

Adjusted Sharpe ratio:

$$\begin{aligned}ASR &= SR \times \left[1 + \left(\frac{\zeta}{6}\right) \times SR - \left(\frac{\kappa - 3}{24}\right) \times SR^2\right] \\ &= 0.94 \times \left[1 + \frac{-0.24}{6} \times 0.94 - \left(\frac{2.98 - 3}{24}\right) \times 0.94^2\right] = 0.91\end{aligned}$$

Table 5.7 Risk-free rate variability

Portfolio monthly return (%) r_i	Monthly risk-free rate (%) r_{Fi}	Absolute deviation $\lvert r_{Fi} - \bar{r}_F \rvert$	Deviation squared $(r_{Fi} - \bar{r}_F)^2$
0.3	0.1	0.10	0.01
2.6	0.1	0.10	0.01
1.1	0.2	0.00	0.00
−0.9	0.2	0.00	0.00
1.4	0.2	0.00	0.00
2.4	0.2	0.00	0.00
1.5	0.2	0.00	0.00
6.6	0.3	0.10	0.01
−1.4	0.3	0.10	0.01
3.9	0.4	0.20	0.04
−0.5	0.4	0.20	0.04

Table 5.7 (*Continued*)

Portfolio monthly return (%) r_i	Monthly risk-free rate (%) r_{Fi}	Absolute deviation $\lvert r_{Fi} - \bar{r}_F \rvert$	Deviation squared $(r_{Fi} - \bar{r}_F)^2$
8.1	0.3	0.10	0.01
4.0	0.3	0.10	0.01
−3.7	0.3	0.10	0.01
−6.1	0.3	0.10	0.01
1.4	0.4	0.20	0.04
−4.9	0.2	0.00	0.00
−2.1	0.2	0.00	0.00
6.2	0.2	0.00	0.00
5.8	0.1	0.10	0.01
−6.4	0.1	0.10	0.01
1.7	0.1	0.10	0.01
−0.4	0.1	0.10	0.01
−0.2	0.1	0.10	0.01
−2.1	0.1	0.10	0.01
1.1	0.1	0.10	0.01
4.7	0.1	0.10	0.01
2.4	0.1	0.10	0.01
3.3	0.1	0.10	0.01
−0.7	0.2	0.00	0.00
4.7	0.2	0.00	0.00
0.6	0.2	0.00	0.00
1.0	0.2	0.00	0.00
−0.2	0.2	0.00	0.00
3.4	0.2	0.00	0.00
1.0	0.2	0.00	0.00
Total	**Total**	**Total**	**Total**
$\sum_{i=1}^{i=n} r_i = 39.6$	$\sum_{i=1}^{i=n} r_{Fi} = 7.2$	$\sum_{i=1}^{i=n} \lvert r_{Fi} - \bar{r}_F \rvert = 2.4$	$\sum_{i=1}^{i=n} (r_{Fi} - \bar{r}_F)^2 = 0.3$
	$\bar{r}_F = \frac{7.2\%}{36} = 0.2$		
	$\tilde{r}_F = 2.43$		

Exhibit 5.8 Revised Sharpe ratio

Standard deviation of excess return above risk-free rate

$$\sigma(r_i - r_{Fi}) = \sqrt{\frac{\sum_{i=1}^{i=n} ((r_i - r_{Fi}) - (\bar{r} - \bar{r}_F))^2}{n}} = \sqrt{\frac{395.62\%}{36}} = 3.315\%$$

$$\tilde{\sigma}(r_i - r_{Fi}) = 3.315\% \times \sqrt{12} = 11.48\%$$

$$\text{Revised Sharpe ratio} = \frac{\tilde{r} - \tilde{r}_F}{\tilde{\sigma}(r_i - r_{Fi})} = \frac{13.29\% - 2.43\%}{11.48\%} = 0.95$$

In this example the difference between the Sharpe ratio and the revised Sharpe ratio is so small because of the stability of the risk-free rate.

Table 5.8 Variability of portfolio excess return above risk-free rate

Portfolio monthly return (%) r_i	Monthly risk-free rate (%) r_{Fi}	Excess return (%) (risk-free rate) $(r_i - r_{Fi})$	Deviation $(r_i - r_{Fi}) - (\bar{r} - \bar{r}_F)$	Deviation squared $[(r_i - r_{Fi}) - (\bar{r} - \bar{r}_F)]^2$
0.3	0.1	0.2	−0.7	0.49
2.6	0.1	2.5	1.6	2.56
1.1	0.2	0.9	0.0	0.00
−0.9	0.2	−1.1	−2.0	4.00
1.4	0.2	1.2	0.3	0.09
2.4	0.2	2.2	1.3	1.69
1.5	0.2	1.3	0.4	0.16
6.6	0.3	6.3	5.4	29.16
−1.4	0.3	−1.7	−2.6	6.76
3.9	0.4	3.5	2.6	6.76
−0.5	0.4	−0.9	−1.8	3.24
8.1	0.3	7.8	6.9	47.61
4.0	0.3	3.7	2.8	7.84
−3.7	0.3	−4.0	−4.9	24.01
−6.1	0.3	−6.4	−7.3	53.29
1.4	0.4	1.0	0.1	0.01
−4.9	0.2	−5.1	−6.0	36.00
−2.1	0.2	−2.3	−3.2	10.24
6.2	0.2	6.0	5.1	26.01
5.8	0.1	5.7	4.8	23.04
−6.4	0.1	−6.5	−7.4	54.76
1.7	0.1	1.6	0.7	0.49
−0.4	0.1	−0.5	−1.4	1.96
−0.2	0.1	−0.3	−1.2	1.44
−2.1	0.1	−2.2	−3.1	9.61
1.1	0.1	1.0	0.1	0.01
4.7	0.1	4.6	3.7	13.69
2.4	0.1	2.3	1.4	1.96
3.3	0.1	3.2	2.3	5.29
−0.7	0.2	−0.9	−1.8	3.24
4.7	0.2	4.5	3.6	12.96
0.6	0.2	0.4	−0.5	0.25
1.0	0.2	0.8	−0.1	0.01
−0.2	0.2	−0.4	−1.3	1.69
3.4	0.2	3.2	2.3	5.29
1.0	0.2	0.8	−0.1	0.01
Total	**Total**			
$\sum_{i=1}^{i=n} r_i = 39.6$	$\sum_{i=1}^{i=n} r_{Fi} = 7.2$			**Total** = 395.62
$\bar{r} - \bar{r}_F = \frac{39.6\% - 7.2\%}{36} = 0.9$				

In this example the adjusted Sharpe ratio is less good, primarily because of the negative skewness of the return distribution. There is a tiny benefit from kurtosis just less than 3. The adjustment is small because the return distribution is close to normal with a slight negative skew.

RELATIVE RISK

The appraisal measures we have discussed so far are examples of absolute rather than relative risk measures, that is to say the returns and risks of the portfolio and benchmark are calculated separately and then used for comparison.

Relative risk measures, on the other hand, focus on the excess return of the portfolio against the benchmark. The variability of excess return calculated using standard deviation is called tracking error, tracking risk, relative risk or active risk.[29]

Tracking error is by far the most common of these terms used in the traditional asset management industry; however, it's probably not the best. The term is also used by the exchange-traded fund (ETF)[30] industry in particular and passive asset managers in general, but it means something slightly different. In the context of ETFs, the tracking error is the difference in the ETF's return from the index the fund is intending to track. Any difference is of course an error, so in this context the language "tracking error" makes sense. In the context of the volatility of excess return the language is less convincing.

Note

I prefer the term *relative risk*, rather than *tracking error*.

Caution

Of course, in isolation, low tracking error is not necessarily a good thing; it is a measure of consistency and might imply consistent underperformance.

Tracking error (or tracking risk, relative risk, active risk)

Tracking error is often forecast and since the calculation methods and meaning are quite different it is essential to clearly label whether you are using an ex-post or ex-ante tracking error.

$$\text{Tracking error } \sigma_A = \sqrt{\frac{\sum_{i=1}^{i=n} (a_i - \overline{a})^2}{n}} \tag{5.27}$$

[29] At this point I should perhaps introduce "Bacon's law": The more alternative names for a risk statistic or appraisal measure, the more useful it is.

[30] An ETF is an investment fund traded on stock exchanges. Most ETFs are designed to "track" a specific index.

where: σ_A = ex-post tracking error of arithmetic excess return
a_i = arithmetic excess return in month i, $(r_i - b_i)$
$\overline{a}$ = mean arithmetic excess return

Or, if you prefer geometric excess returns:

$$\text{Tracking error } \sigma_G = \sqrt{\frac{\sum_{i=1}^{i=n} (g_i - \overline{g})^2}{n}} \tag{5.28}$$

where: σ_G = ex-post tracking of geometric excess return
g_i = geometric excess return in month i, $\left(\frac{1 + r_i}{1 + b_i}\right) - 1$
$\overline{g}$ = mean of geometric excess returns

The arithmetic subtraction of continuously compounded or log returns is equivalent to the geometric excess of simple returns. Therefore, the geometric excess return is equivalent to the continuously compounded arithmetic difference and is thus more appropriate for use in a statistical context.

Information ratio

In exactly the same way we compared absolute return and absolute risk in the Sharpe ratio, you can compare excess return and tracking error (the standard deviation of excess return) graphically (see Figure 5.11).

The information ratio is similar to the Sharpe ratio except that instead of absolute return on the vertical axis we have excess return, and instead of absolute risk on the horizontal axis we have tracking error or relative risk, the standard deviation of excess return.

We have no need for a risk-free rate since we are dealing with excess returns; the natural starting point is the benchmark. The information ratio lines, therefore, must always originate

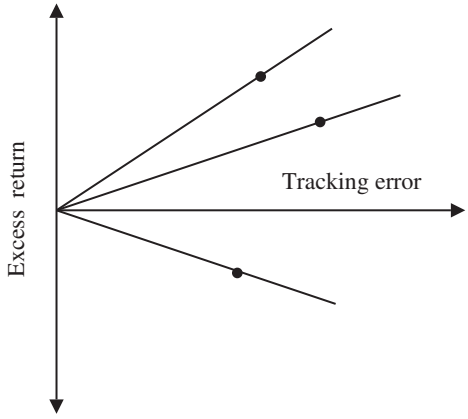

Figure 5.11 Information ratio

from the origin. The gradient of the line is simply the ratio of excess return and tracking error as follows:

$$\text{Information ratio } \mathrm{IR}_A = \frac{\widetilde{a}}{\widetilde{\sigma}_A} = \frac{\text{Annualised excess return}}{\text{Annualised tracking error}} \tag{5.29}$$

where: IR_A = information ratio using arithmetic excess return
$\widetilde{a}$ = annualised arithmetic excess return
$\widetilde{\sigma}_A$ = annualised ex-post tracking error of arithmetic excess return

The information ratio is directly related to the revised Sharpe ratio, simply replacing the risk-free rate with the benchmark.

Geometric information ratio

Using geometric excess returns we get the geometric information ratio:

$$\text{Information ratio } \mathrm{IR}_G = \frac{\widetilde{g}}{\widetilde{\sigma}_G} \tag{5.30}$$

where: $\widetilde{g}$ = annualised geometric excess return
$\widetilde{\sigma}_G$ = annualised ex-post tracking error of geometric excess return

Normally information ratios are calculated using annualised excess returns and annualised tracking errors. To aid comparison it is absolutely essential to disclose the method of calculation, i.e., frequency of data, overall time period, arithmetic or geometric excess returns, arithmetic or geometric means, population or sample (n or $n - 1$), ex-post or ex-ante, simple or continuously compounded returns.

Caution

When comparing information ratios, it is important to ensure that they are calculated using the same methodology and the same type of data.

Note

For the record, my preference is monthly data, over three years, using the geometric excess return, assuming a full population, calculated ex-post using simple returns – monthly returns because month-end valuations are typically of better quality and daily data can be a bit noisy, three years is long enough with monthly data to be statistically significant and short enough to reflect recent history, full population not a sample because we know the exact mean of the population we are analysing and using simple returns because they are commonly used and in truth we have bigger problems with the data than whether or not to use simple or continuously compounded returns.

If ex-ante tracking errors are used in the denominator, please remember that it is only a forecast based on a current snapshot of the portfolio. The portfolio manager will be able to "window dress"[31] the portfolio by reducing any "bets" at the point of measurement, thus reducing the forecast tracking error and therefore apparently improving the information ratio.

Note

Information ratio is a key statistic, used extensively by institutional asset managers; it's often described as the measure of a portfolio manager's skill. Called the information ratio because it holds so much information, like many financial statistics it is borrowed from other industries. Engineers might recognise it as the "signal-to-noise" ratio.

Views vary on what constitutes a good information ratio. In his research commentary, Thomas Goodwin[32] quotes Grinold and Kahn[33] stating that an information ratio of 0.5 is good, 0.75 is very good and 1.0 is exceptional; there is ample evidence to suggest that anything substantially above 1.0 is unsustainable or fraudulent. These numbers certainly accord with my personal experience if sustained over a substantial period (three to five years). Clearly a positive information ratio indicates outperformance and a negative information ratio indicates underperformance. Unlike the Sharpe ratio there is more consensus that if you are going to underperform, consistent underperformance (as indicated by a low tracking error) is worse than inconsistent underperformance (high tracking error).

It is quite easy to obtain a good information ratio for a single period; like all statistics the development of the information ratio should be viewed over time. Goodwin suggests that sustaining a high information ratio above 0.5 is more difficult than Grinold and Kahn suggest.

Note

You are likely to be statistically confident (i.e., reject the null hypothesis that the asset manager is not skilful) if the information ratio is sustained at 1.0 for 4 years, 0.75 for 7 years, and 0.5 for 16 years.

Of course, this statistic is most useful in measuring the performance of portfolios that are specifically required to outperform a benchmark.

Tracking error and information ratios are calculated in Exhibit 5.9 using standard example arithmetic data from Table 5.9 and geometric data from Table 5.10.

[31] "Window dressing" refers to actions taken by a portfolio manager prior to publication of portfolio information in order to (temporarily or artificially) improve the appearance of that information.

[32] Goodwin, "The Information Ratio: More Than You Ever Wanted to Know About One Performance Measure" (1998).

[33] Grinold and Khan, *Active Portfolio Management* (1999).

Exhibit 5.9 Arithmetic and geometric information ratios

Arithmetic tracking error $\sigma_A = \sqrt{\dfrac{\sum_{i=1}^{i=n}(a_i - \overline{a})^2}{n}} = \sqrt{\dfrac{4.36}{36}} = 0.348\%$

Annualised arithmetic tracking error $\widetilde{\sigma}_A = \sqrt{12} \times 0.348\% = 1.21\%$

Annualised portfolio return $\widetilde{r} = 13.29\%$

Annualised benchmark return $\widetilde{b} = 14.62\%$

Annualised arithmetic excess return $\widetilde{a} = 13.29\% - 14.62\% = -1.33\%$

Information ratio $\mathrm{IR}_A = \dfrac{\widetilde{a}}{\widetilde{\sigma}_A} = \dfrac{-1.33\%}{1.21\%} = -1.1$

Geometric tracking error $\sigma_G = \sqrt{\dfrac{\sum_{i=1}^{i=n}(g_i - \overline{g})^2}{n}} = \sqrt{\dfrac{4.24}{36}} = 0.343\%$

Annualised geometric tracking error $\widetilde{\sigma}_G = \sqrt{12} \times 0.343\% = 1.19\%$

Annualised geometric excess return $\widetilde{g} = \left(\dfrac{1+\widetilde{r}}{1+\widetilde{b}}\right) - 1 = \left(\dfrac{1.1329}{1.1462}\right) - 1 = -1.16\%$

Geometric information ratio $\mathrm{IR}_G = \dfrac{\widetilde{g}}{\widetilde{\sigma}_G} = \dfrac{-1.16\%}{1.19\%} = -0.98$

Table 5.9 Information ratio arithmetic excess return

Portfolio monthly return (%) r_i	Benchmark monthly return (%) b_i	Arithmetic excess return $a_i = r_i - b_i$	Deviation from average $(a_i - \overline{a})$	Deviation squared $(a_i - \overline{a})^2$
0.3	0.2	0.1	0.2	0.04
2.6	2.5	0.1	0.2	0.04
1.1	1.8	−0.7	−0.6	0.36
−0.9	−1.1	0.2	0.3	0.09
1.4	1.4	0.0	0.1	0.01
2.4	2.3	0.1	0.2	0.04
1.5	1.4	0.1	0.2	0.04
6.6	6.5	0.1	0.2	0.04
−1.4	−1.5	0.1	0.2	0.04
3.9	4.2	−0.3	−0.2	0.04
−0.5	−0.3	−0.2	−0.1	0.01
8.1	8.3	−0.2	−0.1	0.01
4.0	3.9	0.1	0.2	0.04
−3.7	−3.8	0.1	0.2	0.04
−6.1	−6.2	0.1	0.2	0.04
1.4	1.5	−0.1	0.0	0.00
−4.9	−4.8	−0.1	0.0	0.00
−2.1	−2.0	−0.1	0.0	0.00
6.2	6.0	0.2	0.3	0.09
5.8	5.6	0.2	0.3	0.09
−6.4	−6.7	0.3	0.4	0.16

(Continued)

Table 5.9 (*Continued*)

Portfolio monthly return (%) r_i	Benchmark monthly return (%) b_i	Arithmetic excess return $a_i = r_i - b_i$	Deviation from average $(a_i - \overline{a})$	Deviation squared $(a_i - \overline{a})^2$
1.7	1.9	−0.2	−0.1	0.01
−0.4	−0.3	−0.1	0.0	0.00
−0.2	−0.1	−0.1	0.0	0.00
−2.1	−2.6	0.5	0.6	0.36
1.1	0.7	0.4	0.5	0.25
4.7	4.3	0.4	0.5	0.25
2.4	2.9	−0.5	−0.4	0.16
3.3	3.8	−0.5	−0.4	0.16
−0.7	−0.2	−0.5	−0.4	0.16
4.7	5.1	−0.4	−0.3	0.09
0.6	1.4	−0.8	−0.7	0.49
1.0	1.3	−0.3	−0.2	0.04
−0.2	0.3	−0.5	−0.4	0.16
3.4	3.4	0.0	0.1	0.01
1.0	2.1	−1.1	−1.0	1.00
				Total
				$\sum_{i=1}^{i=n} (a_i - \overline{a})^2 = 4.36$

Table 5.10 Information ratio geometric excess return

Portfolio monthly return (%) r_i	Benchmark monthly return (%) b_i	Geometric excess return $g_i = \left(\frac{1 + r_i}{1 + b_i}\right) - 1$	Deviation from average $(g_i - \overline{g})$	Deviation squared $(g_i - \overline{g})^2$
0.3	0.2	0.10	0.2	0.04
2.6	2.5	0.10	0.19	0.04
1.1	1.8	−0.69	−0.59	0.35
−0.9	−1.1	0.20	0.30	0.09
1.4	1.4	0.00	0.10	0.01
2.4	2.3	0.10	0.19	0.04
1.5	1.4	0.10	0.20	0.04
6.6	6.5	0.09	0.19	0.04
−1.4	−1.5	0.10	0.20	0.04
3.9	4.2	−0.29	−0.19	0.04
−0.5	−0.3	−0.20	−0.10	0.01
8.1	8.3	−0.18	−0.09	0.01
4.0	3.9	0.10	0.19	0.04
−3.7	−3.8	0.10	0.20	0.04
−6.1	−6.2	0.11	0.20	0.04
1.4	1.5	−0.10	0.00	0.00
−4.9	−4.8	−0.11	−0.01	0.00
−2.1	−2.0	−0.10	−0.01	0.00
6.2	6.0	0.19	0.29	0.08

Table 5.10 (*Continued*)

Portfolio monthly return (%) r_i	Benchmark monthly return (%) b_i	Geometric excess return $g_i = \left(\frac{1+r_i}{1+b_i}\right) - 1$	Deviation from average $(g_i - \overline{g})$	Deviation squared $(g_i - \overline{g})^2$
5.8	5.6	0.19	0.29	0.08
−6.4	−6.7	0.32	0.42	0.17
1.7	1.9	−0.20	−0.10	0.01
−0.4	−0.3	−0.10	0.00	0.00
−0.2	−0.1	−0.10	0.00	0.00
−2.1	−2.6	0.51	0.61	0.37
1.1	0.7	0.40	0.49	0.24
4.7	4.3	0.38	0.48	0.23
2.4	2.9	−0.49	−0.39	0.15
3.3	3.8	−0.48	−0.38	0.15
−0.7	−0.2	−0.50	−0.40	0.16
4.7	5.1	−0.38	−0.28	0.08
0.6	1.4	−0.79	−0.69	0.48
1.0	1.3	−0.30	−0.20	0.04
−0.2	0.3	−0.50	−0.40	0.16
3.4	3.4	0.00	0.10	0.01
1.0	2.1	−1.08	−0.98	0.96
				Total
				$\sum_{i=1}^{i=n} (g_i - \overline{g})^2 = 4.24$

Modified information ratio

Some commentators would suggest that negative Sharpe and information ratios have no value, pointing out that if performance is negative the respective ratios actually reward higher variability and higher tracking error. While it's true that negative Sharpe and information ratios require a more nuanced interpretation, I disagree that they provide no value. It seems self-evident, to me at least, that if you are going to underperform it is better to underperform inconsistently rather than underperform consistently.

The modified information ratio suggested by Israelson[34] adjusts the information ratio to ensure that higher tracking errors are always penalised:

$$\text{Modified information ratio } \text{MIR}_A = \frac{\widetilde{a}}{\widetilde{\sigma}_A^{\frac{\widetilde{a}}{|\widetilde{a}|}}} \tag{5.31}$$

For negative excess return, Equation 5.31 means that the excess return is multiplied by the tracking error rather than divided as in the unmodified information ratio.

$$\text{Modified information ratio (negative returns) } \text{MIR}_A^- = \widetilde{a} \times \widetilde{\sigma}_A \tag{5.32}$$

[34] Israelsen, "A Refinement to the Sharpe Ratio and Information Ratio" (2005).

Obviously for geometric excess returns:

$$\text{Modified information ratio MIR}_G = \frac{\widetilde{g}}{\widetilde{\sigma}_G^{\frac{\widetilde{g}}{|\widetilde{g}|}}} \tag{5.33}$$

and for negative geometric excess return:

$$\text{Modified information ratio (negative returns) MIR}_G^- = \widetilde{g} \times \widetilde{\sigma}_G \tag{5.34}$$

REGRESSION ANALYSIS

Regression analysis is a statistical tool used for the investigation of relationships between variables. Regression analysis with a single explanatory variable is termed simple regression and multiple regression is a technique that allows additional factors to enter the analysis separately so that the effect of each can be estimated.

We can gain further information from a portfolio by plotting the portfolio returns against the corresponding benchmark returns in a scatter diagram; see Figure 5.12.

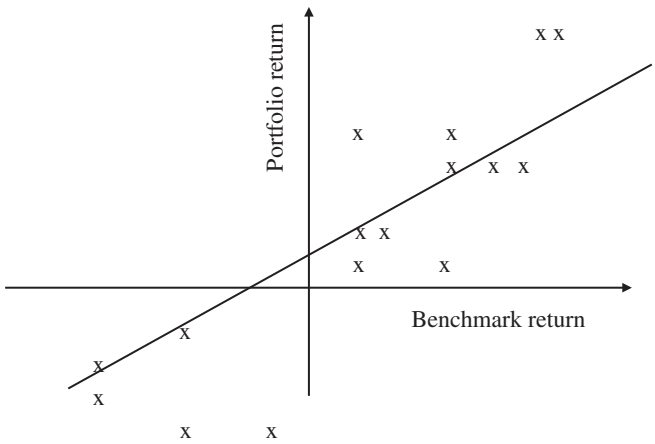

Figure 5.12 Regression analysis

We might expect portfolio returns to move in line with benchmark returns. If so, we can draw a line of best fit through these points, the aim of which is to minimise the (squares of) vertical distances of any one point from this line.

Regression equation

The formula of any straight line on a graph is given by the slope or gradient of the line plus the intercept with the vertical axis. Thus, the return of the portfolio might be described as:

$$r = \alpha_R + \beta_R \times b + \varepsilon_R \tag{5.35}$$

This equation is called the regression equation. The term *regression* was first coined by Francis Galton[35] in the nineteenth century to describe the biological phenomenon of the heights of tall ancestors to *regress* down toward a more normal average.

Regression alpha

The regression *alpha*, α_R, is the intercept of the regression equation with the vertical axis:

$$\text{Regression } \textit{alpha } \alpha_R = \bar{r} - \beta_R \times \bar{b} \tag{5.36}$$

Regression beta

The regression *beta*, β_R, is the slope or gradient of the regression equation.

The slope of the regression equation is given by:

$$\beta_R = \frac{\sum_{i=1}^{i=n}[(r_i - \bar{r}) \times (b_i - \bar{b})]}{\sum_{i=1}^{i=n}(b_i - \bar{b})^2} = \frac{\text{Covariance}}{\sigma_b^2} \tag{5.37}$$

Regression epsilon

The regression *epsilon*, ε_R, is an error term measuring the vertical distance between the return predicted by the equation and the real result.

$$\text{Regression epsilon } \varepsilon_{Ri} = r_i - \alpha_{Ri} - \beta_R \times b_i \tag{5.38}$$

The regression alpha and beta for the standard portfolio and benchmark data calculated in Exhibit 5.2 and Exhibit 5.6 are calculated in Exhibit 5.10.

Exhibit 5.10 Regression alpha and beta

$$\text{Covariance} = \frac{\sum_{i=1}^{i=n}(r_i - \bar{r}) \times (b_i - \bar{b})}{n} = \frac{399.37\%}{36} = 11.09\%$$

$$\text{Benchmark variance } \sigma_b^2 = \frac{\sum_{i=1}^{i=n}(b_i - \bar{b})^2}{n} = \frac{406.92\%}{36} = 11.30\%$$

Regression beta

$$\beta_R = \frac{\sum_{i=1}^{i=n}[(r_i - \bar{r}) \times (b_i - \bar{b})]}{\sum_{i=1}^{i=n}(b_i - \bar{b})^2} = \frac{\text{Covariance}}{\sigma_b^2} = \frac{11.09\%}{11.30\%} = 0.9814$$

$$\text{Regression alpha } \alpha_R = \bar{r} - \beta_R \times \bar{b} = 1.1\% - 0.9814 \times 1.2\% = -0.078\%$$

[35] Galton, "Kinship and Correlation" (reprinted 1989).

Capital asset pricing model (CAPM)

The capital asset pricing model (CAPM) links expected return with systematic risk. CAPM was published by Sharpe in 1964[36] and states that the expected return of the portfolio is:

$$r = r_F + \beta \times (b - r_F) \tag{5.39}$$

CAPM starts with the risk-free rate r_F, however described, to which is added the premium $(b - r_F)$ multiplied by volatility β or systematic risk.

The CAPM model in Equation 5.39 describes theoretical return. Any return above this theoretical return, the intercept with the vertical access, is alpha and is the result of specific risk. Incorporating the risk-free rate, we can use the following revised regression equation to calculate beta and a new alpha (Jensen's alpha[37]).

$$r - r_F = \alpha + \beta \times (b - r_F) + \varepsilon \tag{5.40}$$

⚠ Caution

You may have noticed the underlying assumption of linearity in the CAPM. While some portfolios exhibit a linear relationship with their benchmarks (especially index tracking funds), many portfolios do not. This is a significant source of criticism of the CAPM and its biggest limitation.

Beta (β) (systematic risk or volatility)

Beta was originally described as volatility and in some quarters may still be called volatility but these days volatility is more likely to refer to standard deviation, which is a real shame because I prefer the term variability in the context of standard deviation.

In the context of the CAPM model we should compare the excess return of the portfolio against the risk-free rate with the excess return of benchmark against the same risk-free rate as follows:

$$\beta = \frac{\sum_{i=1}^{i=n} \left[\left((r_i - r_{Fi}) - (\bar{r} - \bar{r}_F) \right) \times \left((b_i - r_{Fi}) - \left(\bar{b} - \bar{r}_F \right) \right) \right]}{\sum_{i=1}^{i=n} \left((b_i - r_{Fi}) - \left(\bar{b} - \bar{r}_F \right) \right)^2} \tag{5.41}$$

where: $\tilde{r}_F$ = mean risk-free rate
r_{Fi} = risk-free rate in month i
$\bar{b}$ = mean benchmark return

The result will be not too different from the regression beta (and indeed regression betas are often calculated because calculating the CAPM beta is perhaps a little tedious for little added value, particularly in a low-interest-rate environment), but technically we should consider the risk-free rate.

[36] Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk" (1964).
[37] M. Jensen, "The Performance of Mutual Funds in the Period 1945–1964" (1968).

Jensen's alpha (Jensen's measure or Jensen's differential return or ex-post alpha)

Jensen's alpha is the intercept of the regression equation in the capital asset pricing model and is in effect the excess return adjusted for systematic risk.

Ignoring the error term for ex-post calculations and using Equation 5.40:

$$\text{Jensen's alpha } \alpha = \overline{r} - \overline{r}_F - \beta \times (\overline{b} - \overline{r}_F) \tag{5.42}$$

Portfolio managers often talk in terms of alpha to describe their added value; rarely are they referring to either the regression or even Jensen's alpha. In all probability they are referring to their excess return above the benchmark. Confusingly, academics also frequently refer to excess return as the return above the risk-free rate.

In fact, both these terms are frequently abused; beta is often used by the financial media and portfolio managers alike to describe market returns when of course it is in fact the systematic risk relative to the market.

Due to the rise of passive investments, this changing terminology has become even more frequent. Today, beta is often used to indicate passive investment returns that take no skill to generate while alpha is used to indicate manager skill, justifying the fees charged by active managers.

Subtracting the benchmark return from both sides of Equation 5.39 we can see that the arithmetic excess return can be expressed by:

$$r - b = \alpha + (\beta - 1) \times (b - r_F) \tag{5.43}$$

Note that $b = r_F + (b - r_F)$

Annualised alpha

The alpha calculated in the regression equations is linked to the periodicity of the underlying data.

The annualised alpha can be calculated directly from annualised data as follows:

$$\text{Annualised regression alpha } \widetilde{\sigma}_R = \widetilde{r} - \beta_R \times \widetilde{b} \tag{5.44}$$

$$\text{Annualised Jensen's alpha } \widetilde{\alpha} = \widetilde{r} - \widetilde{r}_F - \beta(\widetilde{b} - \widetilde{r}_F) \tag{5.45}$$

Alternatively, and less accurately, alpha can be annualised either by multiplying by the periodicity of the data or by taking the power of the periodicity as follows:

$$\widetilde{\alpha} = t \times \alpha \tag{5.46}$$

or

$$\widetilde{\alpha} = (1 + \alpha)^t - 1 \tag{5.47}$$

Revised CAPM beta and Jensen's alpha are calculated in Exhibit 5.11 using data from Table 5.12 and Table 5.13. For completeness, the underlying data required to calculate the variability of portfolio returns above the risk-free rate is shown in Table 5.11.

Exhibit 5.11 CAPM beta and Jensen's alpha

$$\text{Beta } \beta = \frac{\sum_{i=1}^{i=n}[((r_i - r_{Fi}) - (\bar{r} - \bar{r}_F)) \times ((b_i - r_{Fi}) - (\bar{b} - \bar{r}_F))]}{\sum_{i=1}^{i=n}((b_i - r_{Fi}) - (\bar{b} - \bar{r}_F))^2} = \frac{398.66}{406.06} = 0.982$$

$$\text{Jensen's alpha} = \alpha = \bar{r} - \bar{r}_F - \beta \times (\bar{b} - \bar{r}_F) = 0.9 - 0.982 \times 1.0 = -0.082$$

Table 5.11 Portfolio excess return variability

Portfolio monthly return (%) r_i	Risk-free rate (%) r_{Fi}	Excess return (above risk-free rate) $(r_i - r_{Fi})$	Deviation $(r_i - r_{Fi}) - (\bar{r} - \bar{r}_F)$	Deviation squared $[(r_i - r_{Fi}) - (\bar{r} - \bar{r}_F)]^2$
0.3	0.1	0.8	−0.7	0.49
2.6	0.1	1.5	1.6	2.56
1.1	0.2	0.0	0.0	0.00
−0.9	0.2	2.0	−2.0	4.00
1.4	0.2	0.3	0.3	0.09
2.4	0.2	1.3	1.3	1.69
1.5	0.2	0.4	0.4	0.16
6.6	0.3	5.5	5.4	29.16
−1.4	0.3	2.5	−2.6	6.76
3.9	0.4	2.8	2.6	6.76
−0.5	0.4	1.6	−1.8	3.24
8.1	0.3	7.0	6.9	47.61
4.0	0.3	2.9	2.8	7.84
−3.7	0.3	4.8	−4.9	24.01
−6.1	0.3	7.2	−7.3	53.29
1.4	0.4	0.3	0.1	0.01
−4.9	0.2	6.0	−6.0	36.00
−2.1	0.2	3.2	−3.2	10.24
6.2	0.2	5.1	5.1	26.01
5.8	0.1	4.7	4.8	23.04
−6.4	0.1	7.5	−7.4	54.76
1.7	0.1	0.6	0.7	0.49
−0.4	0.1	1.5	−1.4	1.96
−0.2	0.1	1.3	−1.2	1.44
−2.1	0.1	3.2	−3.1	9.61
1.1	0.1	0.0	0.1	0.01
4.7	0.1	3.6	3.7	13.69
2.4	0.1	1.3	1.4	1.96
3.3	0.1	2.2	2.3	5.29
−0.7	0.2	1.8	−1.8	3.24
4.7	0.2	3.6	3.6	12.96
0.6	0.2	0.5	−0.5	0.25
1.0	0.2	0.1	−0.1	0.01
−0.2	0.2	1.3	−1.3	1.69
3.4	0.2	2.3	2.3	5.29
1.0	0.2	0.1	−0.1	0.01
		Total $\sum_{i=1}^{i=n}(r_i - r_{Fi}) = 32.4$		**Total 395.62**
		$(\bar{r} - \bar{r}_F) = \frac{32.4}{36} = 0.9$		

Table 5.12 Benchmark excess return variability

Benchmark monthly return (%) b_i	Risk-free rate (%) r_{Fi}	Excess return $(b - r_{Fi})$	Deviation $(b_i - r_{Fi}) - (\bar{b} - \bar{r}_F)$	Deviation squared $[(b_i - r_{Fi}) - (\bar{b} - \bar{r}_F)]^2$
0.2	0.1	0.1	−0.9	0.81
2.5	0.1	2.4	1.4	1.96
1.8	0.2	1.6	0.6	0.36
−1.1	0.2	−1.3	−2.3	5.29
1.4	0.2	1.2	0.2	0.04
2.3	0.2	2.1	1.1	1.21
1.4	0.2	1.2	0.2	0.04
6.5	0.3	6.2	5.2	27.04
−1.5	0.3	−1.8	−2.8	7.84
4.2	0.4	3.8	2.8	7.84
−0.3	0.4	−0.7	−1.7	2.89
8.3	0.3	8.0	7.0	49.00
3.9	0.3	3.6	2.6	6.76
−3.8	0.3	−4.1	−5.1	26.01
−6.2	0.3	−6.5	−7.5	56.25
1.5	0.4	1.1	0.1	0.01
−4.8	0.2	−5.0	−6.0	36.00
−2.0	0.2	−2.2	−3.2	10.24
6.0	0.2	5.8	4.8	23.04
5.6	0.1	5.5	4.5	20.25
−6.7	0.1	−6.8	−7.8	60.84
1.9	0.1	1.8	0.8	0.64
−0.3	0.1	−0.4	−1.4	1.96
−0.1	0.1	−0.2	−1.2	1.44
−2.6	0.1	−2.7	−3.7	13.69
0.7	0.1	0.6	−0.4	0.16
4.3	0.1	4.2	3.2	10.24
2.9	0.1	2.8	1.8	3.24
3.8	0.1	3.7	2.7	7.29
−0.2	0.2	−0.4	−1.4	1.96
5.1	0.2	4.9	3.9	15.21
1.4	0.2	1.2	0.2	0.04
1.3	0.2	1.1	0.1	0.01
0.3	0.2	0.1	−0.9	0.81
3.4	0.2	3.2	2.2	4.84
2.1	0.2	1.9	0.9	0.81
		Total $\sum_{i=1}^{i=n}(b_i - r_{Fi}) = 36.0$		**Total 406.06**
		$(\bar{b} - \bar{r}_F) = \frac{36.0}{36} = 1.0$		

Table 5.13 CAPM covariance

Portfolio monthly excess return (%) $(r_i - r_{Fi})$	Deviation from average $(r_i - r_{Fi}) - (\bar{r} - \bar{r}_F)$	Benchmark monthly excess return (%) $(b - r_{Fi})$	Deviation from average $(b_i - r_{Fi}) - (\bar{b} - \bar{r}_F)$	Portfolio deviation × Benchmark deviation $(r_i - r_{Fi}) - (\bar{r} - \bar{r}_F)$ × $(b_i - r_{Fi}) - (\bar{b} - \bar{r}_F)$
0.2	−0.7	0.10	−0.9	0.63
2.5	1.6	2.40	1.4	2.24
0.9	0.0	1.60	0.6	0.00
−1.1	−2.0	−1.30	−2.3	4.60
1.2	0.3	1.20	0.2	0.06
2.2	1.3	2.10	1.1	1.43
1.3	0.4	1.20	0.2	0.08
6.3	5.4	6.20	5.2	28.08
−1.7	−2.6	−1.80	−2.8	7.28
3.5	2.6	3.80	2.8	7.28
−0.9	−1.8	−0.70	−1.7	3.06
7.8	6.9	8.00	7.0	48.30
3.7	2.8	3.60	2.6	7.28
−4.0	−4.9	−4.10	−5.1	24.99
−6.4	−7.3	−6.50	−7.5	54.75
1.0	0.1	1.10	0.1	0.01
−5.1	−6.0	−5.00	−6.0	36.00
−2.3	−3.2	−2.20	−3.2	10.24
6.0	5.1	5.80	4.8	24.48
5.7	4.8	5.50	4.5	21.60
−6.5	−7.4	−6.80	−7.8	57.72
1.6	0.7	1.80	0.8	0.56
−0.5	−1.4	−0.40	−1.4	1.96
−0.3	−1.2	−0.20	−1.2	1.44
−2.2	−3.1	−2.70	−3.7	11.47
1.0	0.1	0.60	−0.4	–0.04
4.6	3.7	4.20	3.2	11.84
2.3	1.4	2.80	1.8	2.52
3.2	2.3	3.70	2.7	6.21
−0.9	−1.8	−0.40	−1.4	2.52
4.5	3.6	4.90	3.9	14.04
0.4	−0.5	1.20	0.2	–0.10
0.8	−0.1	1.10	0.1	–0.01
−0.4	−1.3	0.10	−0.9	1.17
3.2	2.3	3.20	2.2	5.06
0.8	−0.1	1.90	0.9	–0.09
				Total 398.66

Bull beta (β^+)

We need not restrict ourselves to fitting lines of best fit to all benchmark (or market) returns, both positive and negative. If we calculate a regression for only positive benchmark returns, we gain information on the behaviour of the portfolio in positive or "bull" markets.

Bear beta (β^-)

The beta for negative benchmark (or market) returns is described as the "bear" beta.

Note

Bull and bear betas are useful if you believe some stocks have persistently different betas depending on the direction of the market.

Beta timing ratio

Ideally, we would prefer a portfolio manager with a beta greater than 1 in rising markets and less than 1 in falling markets. In all likelihood such a manager would be a good timer of asset allocation decisions. Figure 5.13 demonstrates these two lines of best fit with a steeper gradient for positive benchmark returns than for negative benchmark returns. One measure of this desirable property of convexity would be the beta timing ratio.

$$\text{Beta timing ratio} = \frac{\beta^+}{\beta^-} \tag{5.48}$$

Bull and bear betas and the beta timing ratio are calculated from the data in Tables 5.14, 5.15 and 5.16 in Exhibit 5.12.

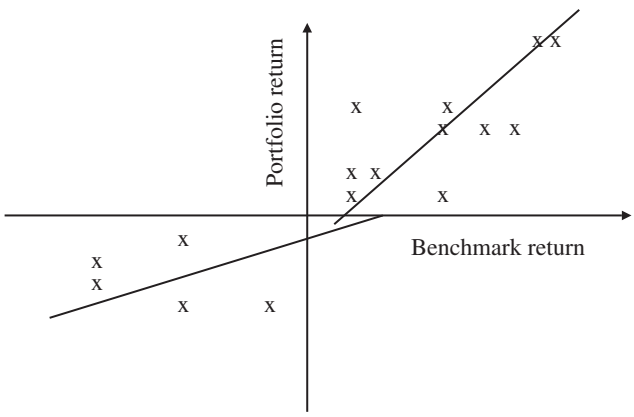

Figure 5.13 Beta timing ratio

Exhibit 5.12 Beta timing ratio

$$\text{Bull beta } \beta^{+} = \frac{104.17}{100.62} = 1.035$$

$$\text{Bear beta } \beta^{-} = \frac{59.11}{62.34} = 0.948$$

$$\text{Beta timing ratio} = \frac{\beta^{+}}{\beta^{-}} = \frac{1.035}{0.948} = 1.092$$

Table 5.14 Bull and bear deviations

Benchmark monthly excess return	Bull deviation	Bull deviation squared	Bear deviation	Bear deviation squared
0.10	−2.74	7.49		
2.40	−0.44	0.19		
1.60	−1.24	1.53		
−1.30			1.38	1.89
1.20	−1.64	2.68		
2.10	−0.74	0.54		
1.20	−1.64	2.68		
6.20	3.36	11.31		
−1.80			0.87	0.77
3.80	0.96	0.93		
−0.70			1.98	3.90
8.00	5.16	26.65		
3.60	0.76	0.58		
−4.10			−1.43	2.03
−6.50			−3.83	14.63
1.10	−1.74	3.02		
−5.00			−2.33	5.41
−2.20			0.47	0.23
5.80	2.96	8.78		
5.50	2.66	7.09		
−6.80			−4.13	17.02
1.80	−1.04	1.08		
−0.40			2.28	5.18
−0.20			2.48	6.13
−2.70			−0.03	0.00
0.60	−2.24	5.01		
4.20	1.36	1.86		
2.80	−0.04	0.00		
3.70	0.86	0.74		
−0.40			2.28	5.18
4.90	2.06	4.25		
1.20	−1.64	2.68		
1.10	−1.74	3.02		
0.10	−2.74	7.49		
3.20	0.36	0.13		
1.90	−0.94	0.88		
		Total 100.62		**Total 62.34**

Table 5.15 Bull covariance

Month	Portfolio monthly excess return	Deviation from average	Benchmark monthly excess return	Deviation from average	Portfolio deviation × Benchmark deviation
1	0.2	−2.48	0.10	−2.74	6.79
2	2.5	−0.18	2.40	−0.44	0.08
3	0.9	−1.78	1.60	−1.24	2.20
4					
5	1.2	−1.48	1.20	−1.64	2.42
6	2.2	−0.48	2.10	−0.74	0.35
7	1.3	−1.38	1.20	−1.64	2.26
8	6.3	3.62	6.20	3.36	12.18
9					
10	3.5	0.82	3.80	0.96	0.79
11					
12	7.8	5.12	8.00	5.16	26.44
13	3.7	1.02	3.60	0.76	0.78
14					
15					
16	1.0	−1.68	1.10	−1.74	2.92
17					
18					
19	6.0	3.32	5.80	2.96	9.84
20	5.7	3.02	5.50	2.66	8.04
21					
22	1.6	−1.08	1.80	−1.04	1.12
23					
24					
25					
26	1.0	−1.68	0.60	−2.24	3.76
27	4.6	1.92	4.20	1.36	2.62
28	2.3	−0.38	2.80	−0.04	0.01
29	3.2	0.52	3.70	0.86	0.45
30					
31	4.5	1.82	4.90	2.06	3.76
32	0.4	−2.28	1.20	−1.64	3.73
33	0.8	−1.88	1.10	−1.74	3.27
34	−0.4	−3.08	0.10	−2.74	8.43
35	3.2	0.52	3.20	0.36	0.19
36	0.8	−1.88	1.90	−0.94	1.76
					Total 104.17

Table 5.16 Bear covariance

Month	Portfolio monthly excess return	Deviation from average	Benchmark monthly excess return	Deviation from average	Portfolio deviation × Benchmark deviation
1					
2					
3					
4	−1.10	1.56	−1.30	1.38	2.14
5					
6					
7					
8					
9	−1.70	0.96	−1.80	0.87	0.84
10					
11	−0.90	1.76	−0.70	1.98	3.47
12					
13					
14	−4.00	−1.34	−4.10	−1.43	1.91
15	−6.40	−3.74	−6.50	−3.83	14.31
16					
17	−5.10	−2.44	−5.00	−2.33	5.68
18	−2.30	0.36	−2.20	0.47	0.17
19					
20					
21	−6.50	−3.84	−6.80	−4.13	15.85
22					
23	−0.50	2.16	−0.40	2.28	4.91
24	−0.30	2.36	−0.20	2.48	5.84
25	−2.20	0.46	−2.70	−0.03	–0.01
26					
27					
28					
29					
30	−0.90	1.76	−0.40	2.28	4.00
31					
32					
33					
34					
35					
36					
					Total 59.11

Note

The bull beta need not be greater than 1 to generate a desirable beta timing ratio greater 1, it is sufficient for the bull beta to be greater than the bear beta.

Market timing

There are additional methods to calculate market timing skill that utilise multiple regression. Merton and Henriksson[38] performed a multiple regression in which the portfolio excess return is regressed against the benchmark and up-market returns as follows:

$$r - r_F = \sigma_{MH} + \beta_{MH} \times (b - r_F) + \gamma_{MH} \times \max(0, b - r_F) + \varepsilon_{MH} \tag{5.49}$$

where: α_{MH} = intercept or alpha
β_{MH} = market sensitivity
γ_{MH} = market timing abilities
ε_{MH} = error term

If γ_{MH} is positive, then the portfolio manager is demonstrating good market timing ability.

Treynor and Mazuy[39] also utilised multiple regression to analyse market timing skill with a quadratic extension of the CAPM model. The first term is again the benchmark, and the second term is the value of excess return squared as follows:

$$r - r_F = \alpha_{TM} + \beta_{TM} \times (b - r_F) + \gamma_{TM} \times (b - r_F)^2 + \varepsilon_{TM} \tag{5.50}$$

where: α_{TM} = intercept or alpha
β_{TM} = market sensitivity
γ_{TM} = market timing abilities
ε_{TM} = error term

If γ_{TM} is positive then the estimated equation describes a convex upward-sloping regression curve demonstrating good market timing skill; see Figure 5.14.

Systematic risk

Michael Jensen[40] described beta as systematic risk. If we multiply beta by market risk, we obtain a measure of systematic risk calculated in the same units as variability. In my view this is a better definition of systematic risk.

$$\text{Systematic risk } \sigma_S = \beta_R \times \sigma_b \tag{5.51}$$

Either the regression or CAPM beta can be used.

[38] Henriksson and Merton, "On Market Timing and Investment Performance II. Statistical Procedures for Evaluating Forecast Skills" (1981).
[39] Treynor and Mazuy, "Can Mutual Funds Outguess the Market?" (1966).
[40] Jensen, "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios" (1960).

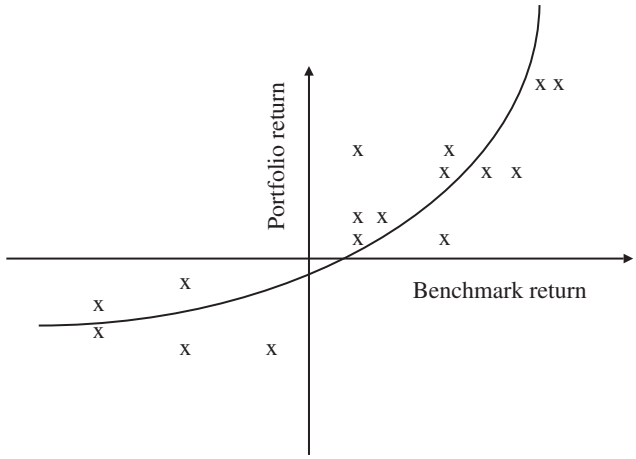

Figure 5.14 Market timing

Correlation

Correlation can also be derived by:

$$\rho_{r,b} = \frac{\text{Systematic risk}}{\text{Total risk}} \tag{5.52}$$

or

$$\rho_{r,b} = \frac{\beta \times \sigma_b}{\sigma}$$

Therefore, beta and correlation are linked by the formula:

$$\beta_R = \rho_{r,b} \times \frac{\sigma}{\sigma_b} \tag{5.53}$$

Correlation measures the variability in the portfolio that is systematic compared to the total variability.

If the correlation is high enough, we can approximate it with 1, leading to:

$$\beta_R \approx \frac{\sigma}{\sigma_b} \tag{5.54}$$

With sufficiently high correlation, a beta above 1 indicates that the portfolio is more volatile than the benchmark and a beta below 1 indicates it is less volatile. Note that this interpretation only holds if the correlation is very close to 1.

R^2 (or coefficient of determination)

R^2 is the proportion of variance in fund returns that is related to the variance of benchmark returns; it is a measure of portfolio diversification. Variance is the square of standard deviation or variability.

The closer R^2 is to 1 the more portfolio variance is explained by benchmark variance. A low R^2 would indicate that returns are more scattered and would indicate a less reliable line of best

fit leading to unstable alphas and betas. Therefore, if a portfolio has a low R^2, say much less than 0.7, then any alphas and betas and their derivative statistics should probably be ignored.

⚠ Caution

While alpha and beta are often calculated and quoted in reports, R^2 is often left out, possibly because it requires deeper understanding of regressions and statistics or possibly because analysts are concerned that the readers of the report will not know how to interpret R^2. In practice, this can lead (and has) to making portfolio decisions based on unsupported measures because of low R^2 values. I repeat the earlier warning: the only appropriate thing to do when R^2 is lower than about 0.7 is to ignore the associated alpha and beta measure and any appraisal measure derived from them.

$$R^2 = \frac{\sigma_S^2}{\sigma^2} = \frac{\beta_R^2 \times \sigma_b^2}{\sigma^2} = \rho_{r,b}^2 \tag{5.55}$$

R^2 in this context is correlation (ρ) squared.
Correlation and R^2 are calculated in Exhibit 5.13.

Exhibit 5.13 Correlation and R^2

$$\text{Correlation } \rho_{r,b} = \frac{\beta_R \times \sigma_b}{\sigma} = \frac{0.981 \times 3.36}{3.32} = 0.995$$

Or

$$\text{Correlation } \rho_{r,b} = \frac{\text{Covariance}}{\sigma \times \sigma_b} = \frac{11.09}{3.32 \times 3.36} = 0.995$$

$$R^2 = \frac{\sigma_S^2}{\sigma^2} = \frac{\beta_R^2 \times \sigma_b^2}{\sigma^2} = 0.989$$

Specific (or residual) risk

Residual or specific risk is not attributed to general market movements but is unique to the particular portfolio under consideration. It is represented by the standard deviation of the error term in the regression equation σ_ε.

Since specific risk and systematic risk are by definition independent, we can calculate total risk by using Pythagoras's equation:[41]

$$\text{Total risk}^2 = \text{Systematic risk}^2 + \text{Specific risk}^2$$

$$\sigma^2 = \sigma_S^2 + \sigma_\varepsilon^2 = \beta_R^2 \times \sigma_b^2 + \sigma_\varepsilon^2 \tag{5.56}$$

[41] Formally, in a right-angled triangle: the square of the hypotenuse is equal to the sum of the squares of the other sides. Since specific risk and systematic risk are independent, we can assume a right-angled triangle.

It follows that:

$$\widetilde{\sigma}^2 = \beta_R^2 \times \widetilde{\sigma}_b^2 + \widetilde{\sigma}_\varepsilon^2 \tag{5.57}$$

Rearranging Equation 5.56, specific risk and total risk can be calculated directly from:

$$\sigma_\varepsilon = \sqrt{\sigma^2 - \sigma_S^2} \tag{5.58}$$

$$\sigma = \sqrt{\sigma_s^2 + \sigma_\varepsilon^2} \tag{5.59}$$

Using the data in Table 5.17 we can demonstrate that Equation 5.59 holds for our standard portfolio and benchmark data from Table 5.1 in Exhibit 5.14.

Exhibit 5.14 Specific risk, systematic risk and total risk

Specific risk $\sigma_\varepsilon = \sqrt{\dfrac{\sum_{i=1}^{i=n} (\varepsilon_{Ri} - \overline{\varepsilon})^2}{n}} = \sqrt{\dfrac{\sum_{i=1}^{i=n} \varepsilon_{Ri}^2}{n}} = \sqrt{\dfrac{4.22}{36}} = 0.34$

since $\overline{\varepsilon} = 0$

Systematic risk $\sigma_S = \beta_R \times \sigma_b = 0.9814 \times 3.36 = 3.30$

Total risk $\sigma = \sqrt{\sigma_s^2 + \sigma_\varepsilon^2} = \sqrt{3.30^2 + 0.34^2} = 3.32$

And

Specific risk $\sigma_\varepsilon = \sqrt{\sigma^2 - \sigma_S^2} = \sqrt{3.32^2 - 3.30^2} = 0.34$

Table 5.17 Specific risk

Portfolio monthly return r_i	Regression residual or error term $\varepsilon_{Ri} = (r_i - \alpha_R - \beta_R \times b_i)$	Residual squared ε_{Ri}^2
0.3	0.18	0.03
2.6	0.22	0.05
1.1	−0.59	0.35
−0.9	0.26	0.07
1.4	0.10	0.01
2.4	0.22	0.05
1.5	0.20	0.04
6.6	0.30	0.09
−1.4	0.15	0.02
3.9	−0.14	0.02
−0.5	−0.13	0.02
8.1	0.03	0.00
4.0	0.25	0.06
−3.7	0.11	0.01
−6.1	0.06	0.00

Table 5.17 (*Continued*)

Portfolio monthly return r_i	Regression residual or error term $\varepsilon_{Ri} = (r_i - \alpha_R - \beta_R \times b_i)$	Residual squared ε_{Ri}^2
1.4	0.01	0.00
−4.9	−0.11	0.01
−2.1	−0.06	0.00
6.2	0.39	0.15
5.8	0.38	0.15
−6.4	0.25	0.06
1.7	−0.09	0.01
−0.4	−0.03	0.00
−0.2	−0.02	0.00
−2.1	0.53	0.28
1.1	0.49	0.24
4.7	0.56	0.31
2.4	−0.37	0.14
3.3	−0.35	0.12
−0.7	−0.43	0.18
4.7	−0.23	0.05
0.6	−0.70	0.48
1.0	−0.20	0.04
−0.2	−0.42	0.17
3.4	0.14	0.02
1.0	−0.98	0.97
Total	**0.00**	**4.22**

Treynor ratio (reward to volatility)

The Treynor[42] ratio (see Figure 5.15) is a performance appraisal measure similar to, but pre-dating by one year, the Sharpe ratio; the numerator (or vertical axis, graphically speaking) is identical but in the denominator (horizontal axis) instead of total risk we have systematic risk as calculated by beta.

$$\text{Treynor ratio} \quad TR = \frac{\tilde{r} - \tilde{r}_F}{\beta} \tag{5.60}$$

Presumably because it is included in most MBA studies the Treynor ratio is extremely well known but perhaps less frequently used because it ignores specific risk. If a portfolio is fully diversified with no specific risk the Treynor and Sharpe ratios will give the same ranking. Sharpe actually initially favoured the Treynor ratio because he felt any value gained from being not fully diversified was transitory. Unfortunately, the performance analyst does not have the luxury of ignoring specific risk when assessing historic returns, and of course active managers expect to generate *alpha* from specific risk.

[42] Treynor, "How to Rate Management of Invested Funds" (1965).

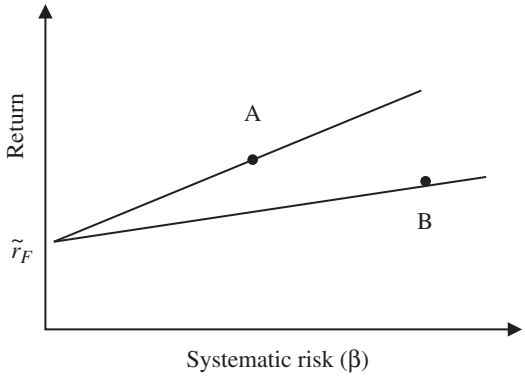

Figure 5.15 Treynor ratio

Appraisal ratio (or Treynor-Black ratio)

The appraisal ratio first suggested by Treynor and Black[43] is similar in concept to the information ratio, but using Jensen's alpha, excess return adjusted for systematic risk in the numerator is divided by specific risk (standard deviation of the error term), not tracking error.

$$\text{Appraisal ratio} = \frac{\tilde{\alpha}}{\tilde{\sigma}_{\varepsilon}} \tag{5.61}$$

The appraisal ratio measures the systematic risk-adjusted reward for each unit of specific risk taken. Figure 5.16 demonstrates the consistency of design of all of these composite statistics, a measure of reward in the vertical axis, in this case Jensen's *alpha*, and a measure of risk along the horizontal axis, in this case specific risk. The gradient of the line, from the origin in this case, determines which combination of risk and reward is positioned further into the top left of the quadrant.

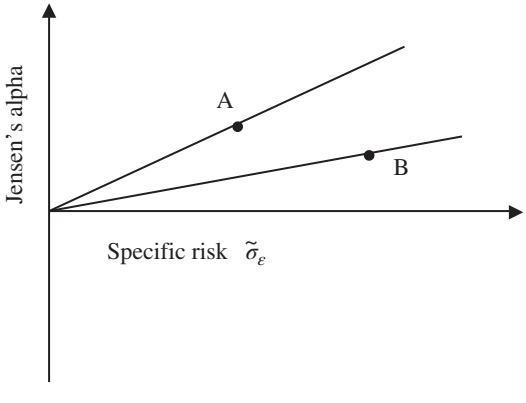

Figure 5.16 Appraisal ratio

[43] Treynor and Black, "How to Use Security Analysis to Improve Portfolio Selection" (1973).

Note

Although seldom used I must say the appraisal ratio appeals to me and perhaps should be given more consideration by more sophisticated investors (assuming of course a stable line of regression).

Caution

The appraisal ratio is often described as, and confused with, the information ratio. Assuming the beta and alphas are of good quality (high R^2) the appraisal measure is a good measure of skill, removing any return from systematic risk in the numerator and also removing systematic risk from the denominator.

FACTOR MODELS

Fama decomposition

Fama[44] extended the concept of Treynor's ratio in his paper "Components of Investment Performance" to further break down the return of a portfolio.

The excess return above the risk-free rate can be expressed as the selectivity (or Jensen's alpha) plus the return due to systematic risk as follows:

$$\underbrace{\bar{r} - \bar{r}_F}_{\text{Excess return}} = \underbrace{\bar{r} - \beta \times (\bar{b} - r_F) - \bar{r}_F}_{\text{Selectivity}} + \underbrace{\beta \times (\bar{b} - \bar{r}_F)}_{\text{Systematic risk}} \tag{5.62}$$

If a portfolio is completely diversified there is no specific risk and the total portfolio risk will equal the systematic risk. Portfolio managers will give up diversification seeking additional return. Selectivity can be broken down into net selectivity and the return required to justify the diversification given up.

Selectivity

Isolating selectivity in Equation 5.62 we notice that it is equivalent to Jensen's alpha from Equation 5.42:

$$\alpha = \bar{r} - \bar{r}_F - \beta \times (\bar{b} - \bar{r}_F) \tag{5.42}$$

Diversification

Diversification is the amount of return required to justify moving away from the benchmark and taking on specific risk. It calculates the return that would have been achieved simply by taking the same amount of systematic risk as the total risk of the portfolio. To calculate this return

[44] Fama, "Components of Investment Performance" (1972).

we first have to calculate the effective beta required so that the systematic risk is equivalent to the total portfolio risk. Call this the Fama beta, calculated as follows:

$$\beta_F = \frac{\sigma}{\sigma_b} \tag{5.63}$$

Therefore, the return required to justify not being fully diversified is calculated using the difference in the Fama beta from the portfolio beta as follows:

$$d = (\beta_F - \beta) \times (\bar{b} - \bar{r}_F) \tag{5.64}$$

Note the Fama beta will always be greater than or equal to the portfolio beta since total risk is greater than or equal to the systematic risk of the portfolio. If the benchmark return is greater than the risk-free rate the diversification required will be positive.

Net selectivity

Net selectivity is the remaining selectivity after deducting the amount of return required to justify not being fully diversified.

$$\text{Net selectivity } S_{\text{Net}} = \alpha - d \tag{5.65}$$

Obviously, if net selectivity is negative the portfolio manager has not justified the loss of diversification.

Fama decomposition is useful analysis if we only have access to total fund returns and are unable to perform more detailed analysis on the components of return – the mutual funds of competitors, for example.

Figure 5.17 illustrates Fama's decomposition for portfolio A. A' represents the return from systematic risk plus the risk-free rate and A'' represents the return from the Fama equivalent systematic risk plus the risk-free rate.

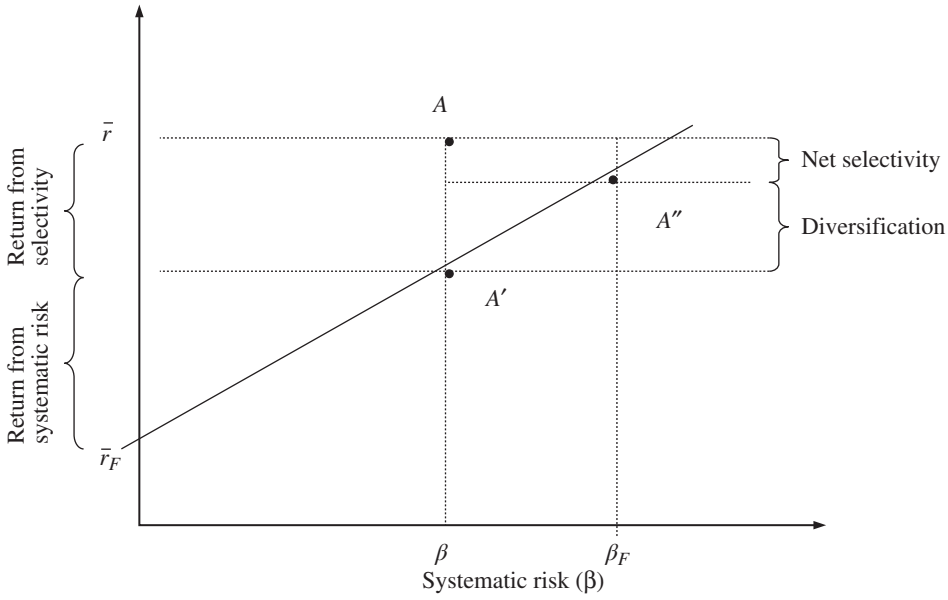

Figure 5.17 Fama decomposition

Fama-French three-factor model

The CAPM model uses just one factor, beta or systematic risk, to describe the relationship of portfolio returns to the benchmark. Eugene Fama and Kenneth French[45] observed that two classes of stocks tended to do better than the market as a whole: small-capitalisation stocks and value stocks. They added two factors to the CAPM model to reflect these exposures, resulting in the following Fama-French regression equation

$$r - r_F = \alpha_3 + \beta_3 \times (b - r_F) + \beta_S \times SMB + \beta_v \times HML + \varepsilon_3 \tag{5.66}$$

where:
α_3 = three-factor alpha
β_3 = three-factor systematic risk
ε_3 = three-factor error term
SMB = small minus big (market capitalisation)
HML = high minus low (book-to-market ratio)

β_S and β_V are coefficients determined by linear regression. β_S or size beta measures the portfolio's sensitivity to small-capitalisation stocks, a coefficient greater than 1 indicating greater sensitivity to small-capitalisation stocks compared to large-capitalisation stocks. β_V or value beta measures the portfolio's sensitivity to value stocks, a coefficient greater than 1 indicating greater sensitivity to value stocks rather than growth stocks.

Note

The historical series of returns for Fama and French's factors are available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Three-factor alpha (or Fama-French alpha)

The resulting alpha from the Fama-French regression equation is analogous to the CAPM alpha and can be used in exactly the same way. β_3 is similar to but not identical to CAPM β.

Carhart four-factor model

Carhart[46] extended the three-factor model to a four-factor model adding a momentum factor resulting in the following Carhart regression equation:

$$r - r_F = \alpha_4 + \beta_4 \times (b - r_F) + \beta_s \times SMB + \beta_v \times HML + \beta_m \times MOM + \varepsilon_4 \tag{5.67}$$

where:
α_4 = four-factor alpha
β_4 = four-factor systematic risk
ε_4 = four-factor error term
MOM = Momentum

β_m is a coefficient determined by linear regression. β_m or momentum beta measures a portfolio's sensitivity to momentum stocks.

[45] Fama and French, "Common Risk Factors in the Returns of Stocks and Bonds" (1993).
[46] Carhart, "On Persistence in Mutual Fund Performance" (1997).

Four-factor alpha (or Carhart's alpha)

Again, the resulting alpha from the Carhart regression equation is analogous to the CAPM and three-factor alpha and can be used in exactly the same way.

Multi-factor models

Further additional systematic risks or factors can be introduced with the objective of explaining the entire return and reducing any alpha to zero. Multi-factor models can be divided into three main categories:

(1) Fundamental
(2) Macroeconomic
(3) Statistical

Fundamental models compare portfolio returns to underlying microeconomic factors such as industry classification, market capitalisation, style, and so forth.

Macroeconomic models compare portfolio returns to economic factors such as employment, inflation and interest rates.

Statistical models start with no assumption about factors but attempt to determine which factors best explain portfolio returns by using statistical methods such as principal component analysis. Assigning economic interpretation to these factors can be difficult.

Caution

Ex-post factor analysis is an additional useful tool to understand better how asset managers are adding (or subtracting) value. Care is required: portfolios change through time, factors change through time, and individual securities' sensitivity to factors change through time. If you torture numbers long enough, eventually they will tell you what you want to hear.

DRAWDOWN

There are a surprisingly large number of appraisal measures based on drawdown and related concepts. Most in my view are fairly redundant, with the notable exception of maximum drawdown. Drawdown might be defined as either the peak-to-valley fall in performance or any continuous, uninterrupted losing return period. The choice of which definition to use will depend on the preferences of the asset owner, the time period and/or the periodicity of the data. Other definitions include active drawdown based on relative or active return rather than absolute return.

Note

High-net-worth investors typically using daily data would probably prefer a peak-to-valley definition, whereas institutional investors using monthly data might be more concerned by periods of continuous negative returns. Peak to valley is the most common.

Average drawdown

As its name suggests, the average drawdown is the average continuous negative return (or peak-to-trough return) over an investment period, three years being a typical period of measurement.

$$\text{Average drawdown } \overline{D} = \left| \sum_{j=1}^{j=d} \frac{D_j}{d} \right| \tag{5.68}$$

where: D_j = jth drawdown over entire period
d = total number of drawdowns in entire period

Some investors take the view that only the largest drawdowns in the return series are of any consequence and therefore restrict d to a predetermined maximum limit of, say, three or five, or even the single largest individual drawdown, thus enabling fair comparison between portfolios.

Maximum drawdown

The maximum drawdown (D_{Max}), not to be confused with the largest individual drawdown, is the maximum loss over a specific time period, typically three years. Maximum drawdown represents the maximum loss an investor could have suffered in the fund buying at the highest point (the high-water mark) and selling at the lowest (the low-water mark). Like any other statistic it is essential to compare performance over the same time period and of course this measure will be heavily influenced by outliers. Mini-max[47] is an alternative name for maximum drawdown. Maxi-min represents the maximum gain buying at the lowest point and selling at the highest.

Largest individual drawdown

As its name suggests the largest individual drawdown (D_{Lar}) is the largest individual uninterrupted or peak-to-valley loss in a return series.

Recovery time (or drawdown duration)

The recovery time or drawdown duration is the time taken to recover from an individual or maximum drawdown to the original level.

The recovery time may be measured from the peak or high-water mark or may also be measured from the subsequent valley or low-water mark.

⚠ Caution

I'm continually surprised how often drawdown and recovery statistics are requested without any guidance as to the time period in question; always disclose the basis of the analysis.

[47] Note that the terms mini-max and maxi-min are used differently in decision theory and game theory. Mini-max is a decision rule for minimising the worst-case loss, and maxi-min for maximising the lowest gain.

Individual and maximum drawdowns and recovery time from the high-water mark are illustrated in Figure 5.18 using the continuous interrupted loss definition. The largest individual loss is D_3. D_2 and D_3 would not be classified as separate drawdowns in the peak-to-trough definition since the high-water mark has not been recovered, just a small positive bump on the way down to a deeper trough.

Drawdown deviation

Drawdown deviation calculates a standard deviation-type statistic using individual drawdowns as follows:

$$\text{Drawdown deviation } DD = \sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{n}} \tag{5.69}$$

Ulcer index

The ulcer index, developed by Peter G. Martin in 1987,[48] (so called because of the worry suffered by both the portfolio manager and asset owner) is similar to drawdown deviation with the exception that the impact of time "under water" is combined with the depth of drawdown by selecting the negative return for each period below the previous peak or high-water mark. Deeper drawdowns will have a more significant impact since the calculation is squared. Naturally, in this index the definition of drawdown is peak to trough.

$$\text{Ulcer index } UI = \sqrt{\sum_{i=1}^{i=n} \frac{D'^2_i}{n}} \tag{5.70}$$

where: D'_i = drawdown since previous peak in period i

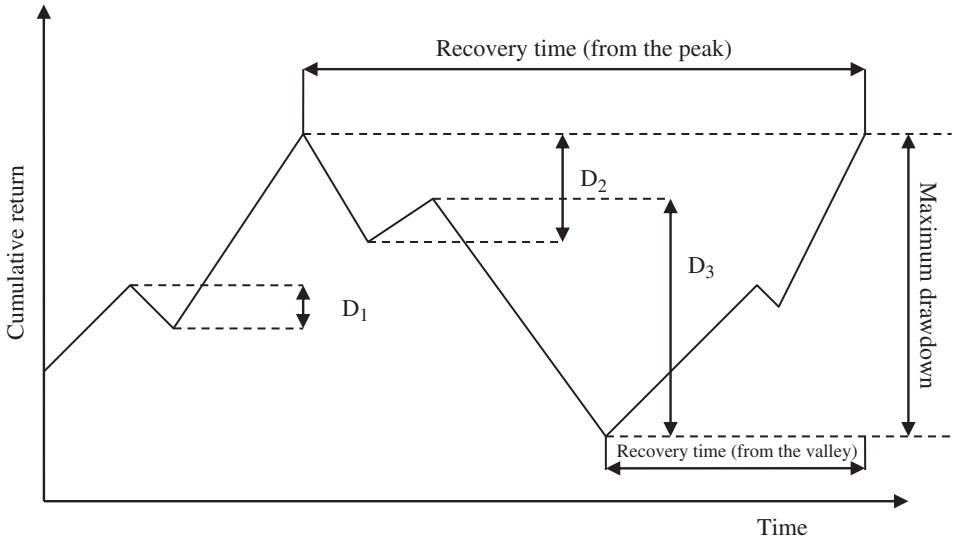

Figure 5.18 Drawdown statistics

[48] Martin and McCann, *The Investor's Guide to Fidelity Funds* (1989).

This approach is clearly sensitive to the frequency of time period and clearly penalises managers who take time to recover to previous highs considering both the depth and duration of drawdowns.

Pain index

If the drawdowns are not squared then the resulting pain index is very similar to the Zephyr pain index[49] as proposed by Thomas Becker:

$$\text{Pain index} \quad PI = \sum_{i=1}^{i=n} \frac{|D'_i|}{n} \tag{5.71}$$

This statistic combines the depth, duration and frequency of drawdowns in the portfolio manager's return series. Consider the manager's cumulative return series in Figure 5.19. The peaks or high-water marks in the return series act as dams holding back the lakes and increasing the volume of pain.

Calmar ratio (or drawdown ratio)

The Calmar ratio proposed by Young in 1991[50] is a Sharpe-type measure that uses maximum drawdown rather than standard deviation to reflect the investor's risk. In the context of hedge fund performance, it is easy to understand why investors might prefer the maximum possible loss from peak to valley as an appropriate measure of risk. Calmar is an acronym of **Cal**ifornia **M**anaged **A**nnual **R**eports and is always measured over 36 months.

$$\text{Calmar ratio} \quad CR = \frac{\widetilde{r} - \widetilde{r}_F}{D_{Max}} \tag{5.72}$$

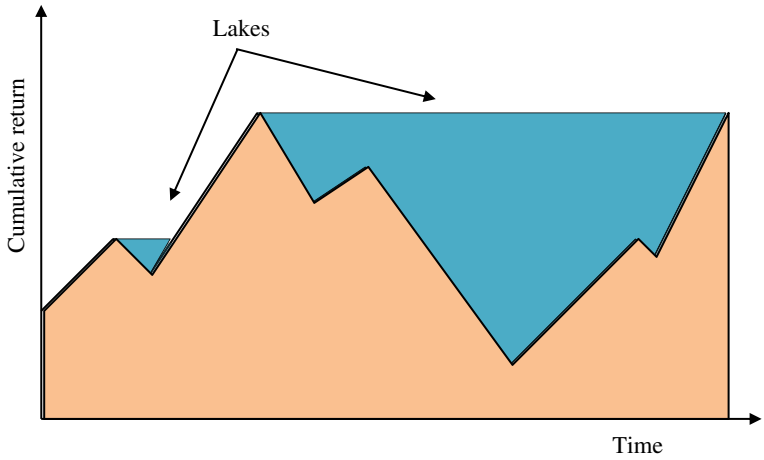

Figure 5.19 Pain index

[49] Becker, "The Zepher K-Ratio" (2010).
[50] Young, "Calmar Ratio: A Smoother Tool" (1991).

Note

Early versions of Calmar exclude the risk-free rate in the numerator. Unlike negative Sharpe and negative information ratios, I don't see a great deal of value in negative Calmar ratios.

MAR ratio

The MAR ratio promoted by a competitor newsletter to California Managed Annual Reports calculates the Calmar ratio since inception rather than over three years, which makes direct comparisons of portfolios with different inception dates difficult to evaluate.

Sterling ratio

The Sterling ratio replaces maximum drawdown in the Calmar ratio with the average drawdown over the period of analysis.

There are multiple variations of the Sterling ratio in common usage, perhaps reflecting its use across a range of differing asset categories and outside the field of finance.

The original definition, attributed to Deane Sterling Jones,[51] a company no longer in existence, appears to be:

$$\text{Original Sterling ratio} \quad OSR = \frac{\widetilde{r}}{\overline{D}_{Lar} + 10\%} \tag{5.73}$$

where: $\overline{D}_{Lar} =$ average largest drawdown

The denominator is defined as the average largest drawdown plus 10%. The addition of 10% is arbitrary, compensating for the fact that the average largest drawdown is inevitably smaller than the maximum drawdown. Typically, only a fixed number of the largest drawdowns are averaged, for example, the largest three individual drawdowns over three years. Adding additional small drawdowns clearly dilutes the impact of the largest drawdowns. With apologies to Deane Sterling Jones, I suggest that the definition be standardised to exclude the 10% (which is redundant in terms of comparison) but in Sharpe form as follows:

$$\text{Sterling ratio} \quad SR_d = \frac{\widetilde{r} - \widetilde{r}_F}{\overline{D}_{Lar}} = \frac{\widetilde{r} - \widetilde{r}_F}{\left| \sum_{j=1}^{j=d} \frac{D_j}{d} \right|} \tag{5.74}$$

The number of observations d is fixed to the investor's preference, which could be the largest single drawdown ($d = 1$) or, more typically, 3 or 5.

[51] McCafferty, *The Market Is Always Right* (2003).

Sterling-Calmar ratio

Perhaps the most common variation of the Sterling ratio uses the average annual maximum drawdown in the denominator over three years. To avoid confusion and to encourage consistent use across the industry I suggest the following standardised definition, a combination of Sterling and Calmar concepts:

$$\text{Sterling-Calmar ratio} \quad SCR = \frac{\widetilde{r} - \widetilde{r}_F}{\overline{D}_{\max}} \tag{5.75}$$

where: $\overline{D}_{Max}$ = average annual maximum drawdown

⚠ Caution

Given the variety of Sterling ratio definitions, great care should be taken to ensure that the same definition is used over the same time period using the same frequency of data when ranking portfolio performance.

Burke ratio

In his article "A Sharper Sharpe Ratio",[52] Burke suggested using the familiar concept of the square root of the sum of the squares of each drawdown in order to penalise major drawdowns as opposed to many mild ones.

$$\text{Burke ratio} \quad BR_d = \frac{\widetilde{r} - \widetilde{r}_F}{\sqrt{\sum_{j=1}^{j=d} D_j^2}} \tag{5.76}$$

Just like the Sterling ratio the number of drawdowns used can be restricted to a set number of the largest drawdowns.

Modified Burke ratio

For consistency with other Sharpe-type statistics, it might be more appropriate to define the modified Burke ratio using drawdown deviation in the denominator as follows:

$$\text{Modified Burke ratio } MBR_d = \frac{\widetilde{r} - \widetilde{r}_F}{\sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{n}}} \tag{5.77}$$

Clearly, both the modified and standard Burke ratios will generate identical portfolio rankings.

[52] Burke, "A Sharper Sharpe Ratio"(1994).

Martin ratio (or ulcer performance index)

If the duration of drawdowns is a concern for investors, the Martin ratio is similar to the modified Burke index but uses the ulcer index in the denominator:

$$\text{Martin ratio } MR = \frac{\widetilde{r} - \widetilde{r}_F}{\sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}}} \tag{5.78}$$

Pain ratio

The equivalent to the Martin ratio but using the pain index is the pain ratio:

$$\text{Pain ratio } PR = \frac{\widetilde{r} - \widetilde{r}_F}{\sum_{i=1}^{i=n} \frac{D_i'}{n}} \tag{5.79}$$

Both the pain and Martin ratios penalise managers with early high-water marks in their track record; therefore, it is far better to peak later rather than earlier when applying these ratios.

Peak-to-trough drawdown statistics are calculated for our standard portfolio data in Table 5.18 and used to calculate Calmar, Sterling, Burke, Sterling-Calmar, pain and ulcer ratios in Exhibit 5.15. Continuous, uninterrupted drawdown statistics are calculated in Exhibit 5.16. Figure 5.20 illustrates the drawdown for our standard portfolio data. This data is dominated by one large drawdown, month 14 to month 31, and a few smaller drawdowns. The maximum drawdown occurs through month 14 to month 18 in the manager's return series.

Exhibit 5.15 Drawdown (peak-to-trough definition)

$$\text{Calmar ratio } CR = \frac{\widetilde{r} - \widetilde{r}_F}{D_{Max}} = \frac{13.29\% - 2.43\%}{14.63\%} = 0.74$$

Largest individual drawdowns 14.6%, 1.4% and 0.9%

$$\text{Average drawdown } \overline{D}_{Lar} = \left| \sum_{j=1}^{j=d} \frac{D}{d} \right| = \frac{14.6\% + 1.4\% + 0.9\%}{3} = 5.64\%$$

Note that only the three largest drawdowns have been selected.

$$\text{Sterling ratio } SR_3 = \frac{\widetilde{r} - \widetilde{r}_F}{\left| \sum_{j=1}^{j=3} \frac{D_j}{3} \right|} = \frac{13.29\% - 2.43\%}{5.64\%} = 1.92$$

$$\text{Drawdown deviation } DD = \sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{n}} = \sqrt{\frac{214.04 + 1.96 + 0.81}{36}} = 2.45$$

$$\text{Modified Burke ratio } MBR_3 = \frac{\widetilde{r} - \widetilde{r}_F}{\sqrt{\sum_{j=1}^{j=3} \frac{D_j^2}{n}}} = \frac{13.29 - 2.43\%}{2.45} = 4.43$$

$$\text{Average maximum drawdown: } \overline{D}_{Max} = \frac{1.4\% + 14.63\% + 2.1\%}{3} = 6.04\%$$

(Continued)

Exhibit 5.15 (*Continued*)

Sterling-Calmar ratio $SCR = \dfrac{\tilde{r} - \tilde{r}_F}{\overline{D}_{\max}} = \dfrac{13.29\% - 2.43\%}{6.04\%} = 1.80$

Pain index $PI = \sum_{i=1}^{i=n} \dfrac{|D'_i|}{n} = \dfrac{135.33}{36} = 3.76$

Pain ratio $PR = \dfrac{\tilde{r} - \tilde{r}_F}{\sum_{i=1}^{i=n} \dfrac{D'_i}{n}} = \dfrac{13.29\% - 2.43\%}{3.76} = 2.89$

Ulcer index $UI = \sqrt{\sum_{i=1}^{i=n} \dfrac{D'^2_i}{n}} = \sqrt{\dfrac{1285.19}{36}} = 5.97$

Martin ratio $MR = \dfrac{\tilde{r} - \tilde{r}_F}{\sqrt{\sum_{i=1}^{i=n} \dfrac{D'^2_i}{n}}} = \dfrac{13.29\% - 2.43\%}{5.97} = 1.82$

Table 5.18 Drawdown statistics

Portfolio monthly return (%)	Continuous drawdown D_j	Continuous drawdown squared D_j^2	Drawdown from peak D'_i	Drawdown from peak squared D'^2_i
0.3				
2.6				
1.1				
−0.9	−0.9	0.81	−0.90	0.81
1.4				
2.4				
1.5				
6.6				
−1.4	−1.4	1.96	−1.40	1.96
3.9				
−0.5	−0.5	0.25	−0.50	0.25
8.1				
4.0				
−3.7			−3.70	13.69
−6.1	−9.6	91.67	−9.57	91.67
1.4			−8.31	69.03
−4.9			−12.80	163.87
−2.1	−6.9	47.57	−14.63	214.11
6.2			−9.34	87.23
5.8			−4.08	16.66

(Continued)

Table 5.18 (*Continued*)

Portfolio monthly return (%)	Continuous drawdown D_j	Continuous drawdown squared D_j^2	Drawdown from peak D'_i	Drawdown from peak squared D'^2_i
−6.4	−6.4	40.96	–10.22	104.45
1.7			−8.69	75.58
−0.4			−9.06	82.07
−0.2			−9.24	85.40
−2.1	−2.7	7.22	–11.15	124.25
1.1			–10.17	103.42
4.7			−5.95	35.37
2.4			−3.69	13.62
3.3			−0.51	0.26
−0.7	−0.7	0.49	−1.21	1.46
4.7				
0.6				
1.0				
−0.2	−0.2	0.04	−0.20	0.04
3.4				
1.0				
Maximum drawdown: 14.63% (months 14 to 18)			**Total**	**Total**
Maximum drawdown (year 1): 1.4% (month 9)			$\sum_{i=1}^{i=n} D'_i = -135.33$	$\sum_{i=1}^{i=n} D'^2_i = 1258.19$
Maximum drawdown (year 2): 14.63%				
Maximum drawdown (year 3): 2.1% (month 25)				

Exhibit 5.16 Drawdown (continuous uninterrupted definition)

Largest individual drawdowns 9.6%, 6.9% and 6.4%

Average drawdown $\overline{D}_{Lar} = \left|\sum_{j=1}^{j=d} \frac{D}{d}\right| = \frac{96\% + 6.9\% + 6.4\%}{3} = 7.62\%$

Note that all three of these "individual and uninterrupted" drawdowns are part of a much longer peak-to-trough drawdown. This is best observed in Figure 5.20.

Sterling ratio $SR_3 = \frac{\widetilde{r} - \widetilde{r}_F}{\left|\sum_{j=1}^{j=3} \frac{D_j}{3}\right|} = \frac{13.29\% - 2.43\%}{7.62\%} = 1.42$

Drawdown deviation $DD = \sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{n}} = \sqrt{\frac{91.67 + 47.57 + 40.96}{36}} = 2.24$

Modified Burke ratio $MBR_3 = \frac{\widetilde{r} - \widetilde{r}_F}{\sqrt{\sum_{j=1}^{j=3} \frac{D_j^2}{n}}} = \frac{13.29\% - 2.43\%}{2.24} = 4.85$

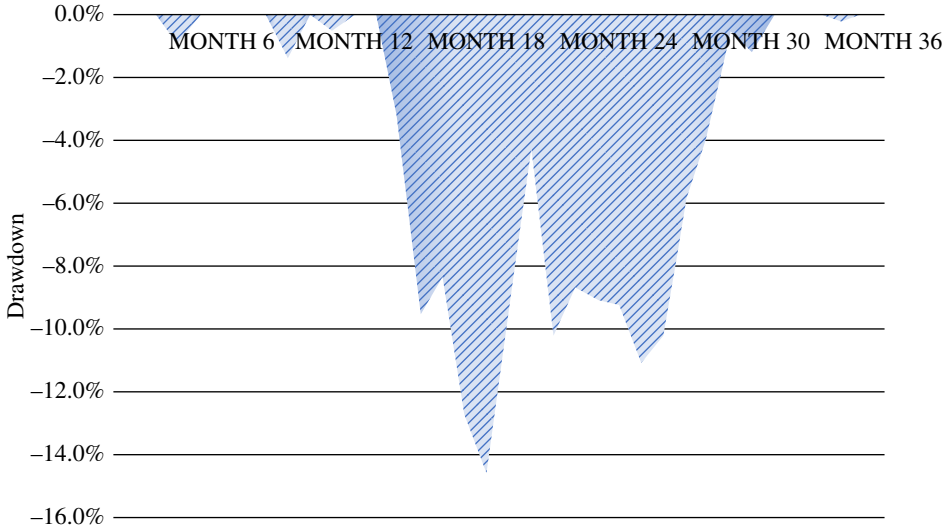

Figure 5.20 Drawdown

PARTIAL MOMENTS

Standard deviation and the symmetrical normal distribution are the foundations of modern portfolio theory. Post-modern portfolio theory recognises that asset owners prefer upside risk (or, if you prefer, upside uncertainty) rather than downside risk and utilises semi-standard deviation. Partial or one-sided moment measures allow asset owners to ignore desirable risk and focus on undesirable risk.

Downside risk (or semi-standard deviation)

Semi-standard deviation measures the variability of underperformance below a minimum target rate. The minimum target rate could be the risk-free rate, the benchmark, zero or any other fixed threshold required by the asset owner. All positive returns, above the minimum target rate, are included as zero in the calculation of semi-standard deviation or downside risk as follows:

$$\text{Downside risk } \sigma_D = \sqrt{\sum_{i=1}^{n} \frac{\min[(r_i - r_T), 0]^2}{n}} \tag{5.80}$$

where: $r_T =$ minimum target return

Clearly, since positive returns above the minimum target return are excluded there are potentially fewer or, in some cases, no observations less than the target return. Therefore, great care must be taken to ensure there are sufficient returns to ensure that the calculation is meaningful.

As should be expected downside variance is the square of downside risk:

$$\text{Downside variance } \sigma_D^2 = \sum_{i=1}^{n} \frac{\min[(r_i - r_T), 0]^2}{n} \tag{5.81}$$

Note

The downside risk measure is not directly comparable to standard deviation. For a normal distribution with a standard deviation of 1.0, the semi-standard deviation will be about 0.6.

Downside potential

Downside potential is the average sum of returns below target:

$$\text{Downside potential } \mu_D = \sum_{i=1}^{i=n} \frac{\min[(r_i - r_T), 0]}{n} \tag{5.82}$$

By convention the minus sign of downside potential is ignored. Therefore, perhaps a better way of expressing downside potential is as follows:

$$\text{Downside potential } \mu_D = \sum_{i=1}^{i=n} \frac{\max[(r_T - r_i), 0]}{n} \tag{5.83}$$

Note that the term $(r_i - r_T)$ has been reversed and replaced by $(r_T - r_i)$ and that:

$$\sum_{i=1}^{i=n} \max[(r_T - r_i), 0] = -1 \times \sum_{i=1}^{i=n} \min[(r_i - r_T), 0] \tag{5.84}$$

Downside variance is the second lower partial moment of return, and downside potential is the first lower partial moment.

Pure downside risk

Pure downside risk is the special case of $r_T = 0$; only returns less than zero are included in the calculation.

$$\text{Pure downside risk } \sigma_P = \sqrt{\sum_{i=1}^{i=n} \frac{\min[r_i, 0]^2}{n}} \tag{5.85}$$

Half variance (or semi-variance)

In half variance and half risk calculations, only returns lower than the mean are considered, i.e., $r_T = \overline{r}$.

$$\text{Half variance } \sigma_H^2 = \sum_{i=1}^{i=n} \frac{\min[(r_i - \overline{r}), 0]^2}{n} \tag{5.86}$$

$$\text{Half risk } \sigma_H = \sqrt{\sum_{i=1}^{i=n} \frac{\min[(r_i - \overline{r}), 0]^2}{n}} \tag{5.87}$$

Upside risk (or upside uncertainty)

The term "upside risk" may be disconcerting, but it simply refers to the variability of returns that exceed a given target. The equivalent upside statistics are as expected:

$$\text{Upside risk } \sigma_U = \sqrt{\sum_{i=1}^{i=n} \frac{\max[(r_i - r_T), 0]^2}{n}} \tag{5.88}$$

$$\text{Upside variance } \sigma_U^2 = \sum_{i=1}^{i=n} \frac{\max[(r_i - r_T), 0]^2}{n} \tag{5.89}$$

$$\text{Upside potential } \mu_U = \sum_{i=1}^{i=n} \frac{\max[(r_i - r_T), 0]}{0} \tag{5.90}$$

Note that in all the above equations the total number of observations, n, is always used irrespective of the number of observations above or below target.

Loss and gain standard deviation measure the variability of returns above or below target only and is typically not used in performance analysis.

$$\text{Loss standard deviation } \sigma_L = \sqrt{\sum_{i=1}^{i=n} \frac{\min[(r_i - r_T), 0]^2}{n_d}} \tag{5.91}$$

$$\text{Gain standard deviation } \sigma_G = \sqrt{\sum_{i=1}^{i=n} \frac{\max[(r_i - r_T), 0]^2}{n_U}} \tag{5.92}$$

where: $n_d =$ number of returns less than target
$n_U =$ number of returns greater than target

Mean absolute moment

Similarly, the mean absolute moment calculates the average of downside deviations or the average of upside deviations as follows:

$$\text{Mean downside deviation } MAM_D = \sum_{i=1}^{n} \frac{\min[(r_i - r_T), 0]}{n_d} \tag{5.93}$$

$$\text{Mean upside deviation } MAM_U = \sum_{i=1}^{n} \frac{\max[(r_i - r_T), 0]}{n_u} \tag{5.94}$$

Omega ratio (Ω)

In their article "A Universal Performance Measure", Shadwick and Keating[53] suggest a gain–loss ratio that also captures the information in the higher moments of a return distribution as follows:

$$\text{Omega ratio } \Omega = \frac{\text{Upside potential}}{\text{Downside potential}} = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0)}{\frac{1}{n} \times \sum \max(r_T - r_i, 0)} \tag{5.95}$$

[53] Shadwick and Keating, "A Universal Performance Measure" (2002).

When used as a ranking statistic, the higher the omega ratio, the better. The omega ratio equals 1 when r_T is the mean return.

Omega ratio implicitly adjusts for both skewness and kurtosis in the return distribution.

Bernardo and Ledoit (or gain–loss) ratio

The Bernardo and Ledoit[54] ratio is a special case of the omega ratio with $r_T = 0$:

$$\text{Bernardo–Ledoit ratio} = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(0 - r_i, 0)} \tag{5.96}$$

d ratio

The *d* ratio[55] is similar to the Bernardo and Ledoit ratio but inverted and taking into account the frequency of positive and negative returns:

$$d\,\text{ratio} = \frac{n'_d \times \sum_{i=1}^{i=n} \max(0 - r_i, 0)}{n'_u \times \sum \max(r_i, 0)} \tag{5.97}$$

where: $n'_d =$ number of returns less than zero
$n'_u =$ number of returns greater than zero

The *d* ratio will have values between zero and infinity and can be used to rank the performance of portfolios. The lower the *d* ratio the better the performance, a value of zero indicating that there are no returns less than zero and a value of infinity indicating that there are no returns greater than zero. Portfolio managers with positively skewed returns will have lower *d* ratios.

Omega–Sharpe ratio

The omega ratio can be converted to a ranking statistic in familiar form to the Sharpe ratio. Clearly, the average portfolio return less the target return is equal to the sum of the upside and downside potential:

$$\text{Omega–Sharpe ratio} = \frac{\tilde{r} - \tilde{r}_T}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} \tag{5.98}$$

$$\tilde{r} - \tilde{r}_T \approx \bar{r} - \bar{r}_T = \frac{1}{n} \times \sum_{i=1}^{i=n} r_i - r_T = \frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0) - \frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0) \tag{5.99}$$

[54] Bernardo and Ledoit, *Gain, Loss and Asset Pricing* (1996).
[55] Lavinio, *The Hedge Fund Handbook* (1999).

Substituting Equation 5.99 into Equation 5.98:

$$= \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0) - \frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} \tag{5.100}$$

$$= \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} - \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} = \Omega - 1 \tag{5.101}$$

$$\text{Omega–Sharpe ratio} \approx \Omega - 1 \tag{5.102}$$

Interpretation

The omega–Sharpe ratio will rank portfolios in the same order as the omega ratio. Although generating identical rankings, the omega–Sharpe ratio is at least in the familiar Sharpe-type format and therefore preferable.

Sortino ratio

A natural extension of the Sharpe and omega–Sharpe ratios is the Sortino ratio[56] suggested by Brian Rom in 1986, which uses downside risk in the denominator as follows:

$$\text{Sortino ratio} = \frac{(\tilde{r} - \tilde{r}_T)}{\tilde{\sigma}_D} \tag{5.103}$$

The Sortino ratio is shown graphically in Figure 5.21.

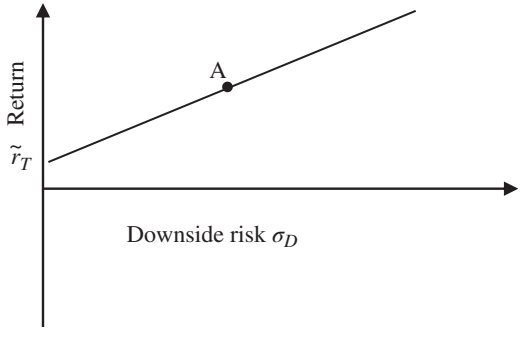

Figure 5.21 Sortino ratio

[56] Sortino and van der Meer, "Downside Risk" (1991).

Clearly, investors should be seeking returns greater than the risk-free rate (why take any risk otherwise?); therefore, the minimum accepted return in most cases should be greater than the risk-free rate.

Reward to half-variance

Reward to half-variance, suggested by Ang and Chua in 1979,[57] is similar to the Sortino ratio, in effect considering only returns less than the mean:

$$\text{Reward to half-variance} = \frac{(\widetilde{r} - \widetilde{r}_F)}{\widetilde{\sigma}_H^2} \tag{5.104}$$

where: $\widetilde{\sigma}_H^2$ = annualised half-variance

Downside-risk Sharpe ratio

Similar to the Sortino ratio but far less common is the downside-risk Sharpe ratio suggested by Ziemba in 2005,[58] which considers only losses less than zero or pure downside risk:

$$\text{Downside-risk Sharpe ratio} = \frac{(\widetilde{r} - \widetilde{r}_F)}{\widetilde{\sigma}_P} \tag{5.105}$$

where: $\widetilde{\sigma}_P$ = annualised pure downside risk

Sortino–Satchell ratio

Rachev, Stoyanov and Fabozzi,[59] Eling, Farinelli, Rossello and Tibiletti,[60] and Biglova, Orotobelli, Rachev and Stoyanov[61] all describe the generalised downside risk-adjusted Sortino–Satchell ratio. Kaplan and Knowles,[62] in their paper "Kappa: A Generalised Downside Risk-Adjusted Performance Measure," describe the same generalised ratio but name it kappa. They demonstrate that both the Sortino ratio and the omega–Sharpe ratio are special cases of kappa:

$$\text{Sortino–Satchell} = K_l = \frac{\widetilde{r} - \widetilde{r}_T}{\sqrt[l]{\sum_{i=1}^{i=n} \frac{[(r_T - r_i),0]^l}{n}}} \tag{5.106}$$

Interpretation

For $l = 1$, K_1 is the omega–Sharpe ratio and for $l = 2$, K_2 is the Sortino ratio. l need not be an integer and is set according to investor preference; a greater l suggests a higher penalty for more extreme results, suitable for more risk-averse investors than the kappa ratio.

[57] Ang and Chua, "Composite Measures for the Evaluation of Investment Performance" (1979).
[58] Ziemba, "The Symmetric Downside-Risk Sharpe Ratio" (2005).
[59] Rachev, Stoyanov and Fabozzi, *Advanced Stochastic Models, Risk Assessment and Portfolio Optimization* (2008).
[60] Eling, Farinelli, Rossello and Tibiletti, "One-Size or Tailor-Made Performance Ratios for Ranking Hedge Funds" (2009).
[61] Biglova, Ortobelli, Rachev and Stoyanov, "Different Approaches to Risk Estimation in Portfolio Theory" (2004).
[62] Kaplan and Knowles, "Kappa: A Generalized Downside Risk-adjusted Performance Measure" (2004).

In particular, K_3, the third lower partial moment, is known specifically as the kappa ratio. It perhaps makes most sense to use Sortino–Satchell to name the generalised ratio and kappa to describe K_3.

$$\text{Kappa ratio} = K_3 = \frac{\tilde{r} - \tilde{r}_T}{\sqrt[3]{\sum_{i=1}^{i=n} \frac{[(r_T - r_i),0]^3}{n}}} \tag{5.107}$$

Upside potential ratio

The upside potential ratio suggested by Sortino, Van de Meer and Plantinga[63] can also be used to rank portfolio performance and combines upside potential with downside risk as follows:

$$UPR = \frac{\text{Upside potential}}{\text{Downside risk}} = \frac{\mu_U}{\tilde{\sigma}_D} = \frac{\sum_{i=1}^{i=n} \frac{\max[(r_i - r_T),0]}{n}}{\tilde{\sigma}_D} \tag{5.108}$$

Interpretation

By using the first partial moment on the upside and the second partial moment on the downside, investors are expressing a preference: they dislike the extreme losses on the downside more than they like the extreme gains on the upside.

Note

The Sortino ratio is conceptually more consistent with the Sharpe ratio in design and certainly in more common usage than the upside potential ratio and therefore is my preference of the two ratios.

Volatility skewness

A similar measure to omega but using the second partial moment is volatility skewness,[64] the ratio of the upside variance compared to the downside variance. Values greater than 1 would indicate positive skewness and values less than 1 would indicate negative skewness:

$$\text{Volatility skewness} = \frac{\tilde{\sigma}_U^2}{\tilde{\sigma}_D^2} \tag{5.109}$$

[63] Sortino, van de Meer and Plantinga, "The Dutch Triangle: A Framework to Measure Upside Potential Relative to Downside Rsk" (1999).

[64] Rom and Ferguson, "A Software Developer's View: Using Post-Modern Portfolio Theory to Improve Investment Performance Measurement" (2001).

Interpretation

This measure, in particular, rewards extreme positive events and penalises extreme negative events. For those like me who are unsure about the merits of over-rewarding extreme (potentially one-off) positive events, but are still concerned by negative extreme events, then perhaps the upside potential ratio is more appropriate.

Variability skewness

Although the rankings will be identical for consistency with other measures typically used with asset management, I prefer the square root of volatility skewness. To differentiate the term, I use the name variability skewness.

$$\text{Variability skewness} = \frac{\text{Upside risk}}{\text{Downside risk}} = \frac{\sigma_U}{\sigma_D} \tag{5.110}$$

Portfolio downside risk, Sortino ratio, upside and downside potential, omega, omega–Sharpe ratio and upside potential ratio and variability skewness are calculated in Exhibit 5.17 based on data in Table 5.19.

Exhibit 5.17 Downside and upside partial moments

$$\text{Downside potential } \mu_D = \sum_{i=1}^{n} \frac{\max[(r_T - r_i), 0]}{n} = \frac{36.3\%}{36} = 1.01\%$$

$$\text{Downside risk } \sigma_D = \sqrt{\sum_{i=1}^{n} \frac{\max[(r_T - r_i), 0]^2}{n}} = \sqrt{\frac{161.33}{36}} = 2.12\%$$

$$\text{Annualised downside risk } \widetilde{\sigma}_D = \sqrt{t} \times \sigma_D = \sqrt{12} \times 2.12\% = 7.33\%$$

$$\text{Upside potential } \mu_U = \sum_{i=1}^{n} \frac{\max[(r_i - r_T), 0]}{n} = \frac{57.9\%}{36} = 1.61\%$$

$$\text{Upside risk } \sigma_U = \sqrt{\sum_{i=1}^{n} \frac{\max[(r_i - r_T), 0]^2}{n}} = \sqrt{\frac{247.81}{36}} = 2.62\%$$

$$\text{Annualised upside risk } \widetilde{\sigma}_U = \sqrt{t} \times \sigma_U = \sqrt{12} \times 2.62\% = 9.09\%$$

Omega ratio

$$\Omega = \frac{\text{Upside potential}}{\text{Downside potential}} = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0)}{\frac{1}{n} \times \sum \max(r_T - r_i, 0)} = \frac{1.61\%}{1.01\%} = 1.6$$

$$\text{Sortino ratio} = \frac{(\widetilde{r} - \widetilde{r}_T)}{\widetilde{\sigma}_D} = \frac{13.29\% - 6.17\%}{7.33\%} = 0.97$$

Upside potential ratio

$$UPR = \frac{\text{Upside potential}}{\text{Downside risk}} = \frac{\sum_{i=1}^{i=n} \max(r_i - r_T, 0)/n}{\widetilde{\sigma}_D} = \frac{1.61\%}{7.33\%} = 0.22$$

$$\text{Variability skewness} = \frac{\text{Upside risk}}{\text{Downside risk}} = \frac{\widetilde{\sigma}_U}{\widetilde{\sigma}_D} = \frac{9.09\%}{7.33\%} = 1.24$$

Table 5.19 Portfolio downside risk

Monthly minimum target return = 0.5%, annual minimum target return = 6.17%				
Portfolio monthly return (%) r_i	Deviation against target (downside only) $\min[(r_i - r_T), 0]$	Squared deviation (downside only) $\min[(r_i - r_T), 0]^2$	Upside only $\max[(r_i - r_T), 0]$	Squared deviation (upside only) $\max[(r_i - r_T), 0]^2$
0.3	−0.2	0.04		
2.6			2.1	4.41
1.1			0.6	0.36
−0.9	−1.4	1.96		
1.4			0.9	0.81
2.4			1.9	3.61
1.5			1	1
6.6			6.1	37.21
−1.4	−1.9	3.61		
3.9			3.4	11.56
−0.5	−1.0	1		
8.1			7.6	57.76
4			3.5	12.25
−3.7	−4.2	17.64		
−6.1	−6.6	43.56		
1.4			0.9	0.81
−4.9	−5.4	29.16		
−2.1	−2.6	6.76		
6.2			5.7	32.49
5.8			5.3	28.09
−6.4	−6.9	47.61		
1.7			1.2	1.44
−0.4	−0.9	0.81		
−0.2	−0.7	0.49		
−2.1	−2.6	6.76		
1.1			0.6	0.36
4.7			4.2	17.64
2.4			1.9	3.61
3.3			2.8	7.84
−0.7	−1.2	1.44		
4.7			4.2	17.64
0.6			0.1	0.01
1			0.5	0.25
−0.2	−0.7	0.49		
3.4			2.9	8.41
Total	$\sum_{i=1}^{i=n} \min[(r_i - r_T), 0] =$ **–36.3**	$\sum_{i=1}^{i=n} \min[(r_i - r_T), 0]^2 =$ **161.33**	$\sum_{i=1}^{i=n} \max[(r_i - r_T), 0] =$ **57.9**	$\sum_{i=1}^{i=n} \max[(r_i - r_T), 0]^2$ **247.81**

Farinelli–Tibiletti ratio

Farinelli and Tibiletti[65] suggest a generalised measure similar to Sortino-Satchell but referencing both the upside and downside:

$$F - T_l^u = \frac{\sqrt[u]{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0)^u}}{\sqrt[l]{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)^l}} \tag{5.111}$$

Interpretation

When u and l are both equal to 1, the Farinelli–Tibiletti ratio is equivalent to the omega ratio and when u and l are both equal to 2, equivalent to variability skewness. $l = 2$ and $u = 1$ is the upside potential ratio. u and l need not be integers and can be set according to investor preference; $u < 1$ and $l > 1$ is risk averse and $l < 1$ and $u > 1$ is risk seeking.

Both the Farinelli–Tibiletti ratio and Sortino–Satchell offer significant flexibility to investors to design ratios that meet their particular requirements, although it requires considerable investment in time, experience and copious quantities of data to add value.

Caution

For asset managers these ratios offer significant opportunities to "data mine"[66] in order to find favourable results. Like all appraisal ratios they are calculated ex-post but of course should be implemented ex-ante.

Prospect ratio

Watanabe notes that people tend to feel loss more acutely than the equivalent gain,[67] a well-known phenomenon described by prospect theory.[68] Watanabe suggests utilising a Sharpe-type ratio, really an adapted Sortino ratio, which penalises losses more than it rewards gains, as shown below.

$$\text{Prospect ratio} = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} (\max[r_i, 0] + \lambda \times \min[r_i, 0]) - r_T}{\sigma_D} \tag{5.112}$$

[65] Farinello and Tibiletti, "Sharpe Thinking in Asset Ranking with One-Sided Measures" (2008).
[66] Data mine (or *data dredge*): the bad practice of analysing data for the exclusive purpose of identifying favourable outcomes.
[67] Watanabe, "New Prospect Ratio: Application to Hedge Funds with Higher Order Moments." (2014).
[68] Kahneman and Tversky, "Prospect Theory: An Analysis of Decision under Risk" (1979).

λ reflects investor preference and measures the extent of loss aversion and the greater weight given to losses than gains. If the investor does not exhibit loss aversion, $\lambda = 0$. $\lambda < 0$ implies gain-seeking behaviour. Based on empirical research, Watanabe suggests investors dislike losses two and a quarter times as much as they enjoy gains and sets λ to 2.25. Users might wish to select their own preferences.

Note

In many ways the prospect ratio is similar to kappa but with investor preferences expressed in the numerator of the ratio rather than in the denominator in the form of higher partial moments.

FIXED INCOME RISK

Pricing fixed income instruments

In many ways it is easier to measure the risk of fixed income instruments or bonds; they consist of (for the most part) predictable future cash flows in the form of coupon payments and a final redemption value.

The price of a bond is the sum of present value of the future cash flows:

$$P = \sum_{i=1}^{n} F_i \times d^{t_i} \tag{5.113}$$

where: n = number of future coupon and capital repayments
F_i = ith future coupon and capital repayment(s)
t_i = time in years to the ith coupon or capital repayment
d = discount factor

Redemption yield (yield to maturity)

The redemption yield of a bond is the internal rate of return (IRR), or the constant rate of return at which the current value of the bond is equal to the sum of discounted values (or present values) of each cash flow.

Weighted average cash flow

Clearly, the price of a bond is influenced by discount rates, the amount of each cash flow and, crucially, the time over which these cash flows are discounted. Naturally, we are interested in the sensitivity to changes in the future discount rates or yields. Time is crucial because the change in yield will have a greater effect over longer time periods. One such measure is the weighted average cash flow of the bond:

$$\text{Weighted average cash flow } \frac{\sum_{i=1}^{i=n} t_i \times F_i}{\sum_{i=1}^{i=n} F_i} \tag{5.114}$$

Duration (effective mean term, discounted mean term or volatility)

A more accurate measure of sensitivity replaces the future cash flow with the present value of the cash flow and is called duration.

Duration is defined as the average life of the present values of all future cash flows, but it is interpreted as an estimate of the bond's price sensitivity to changes in interest rates. In other words, it expresses the bond's exposure to interest rate risk. For example, a bond with a duration of 13 years would be expected to gain approximately 13% in market value if interest rates fall by 1% (100 basis points) and to lose the same amount if interest rates rise by 1%. Duration is thus a systematic risk or volatility measure for bonds. In calculating the present value of the future cash flows, a discount rate equal to the redemption yield is used.

Macaulay duration

Frederick Macaulay[69] was among the first to suggest the use of duration for studying the returns on bonds. Using present values in Equation 5.114 we get:

$$\text{Macaulay duration } D = \frac{\sum_{i=1}^{i=n} t_i \times F_i \times d^{t_i}}{\sum_{i=1}^{i=n} F_i \times d^{t_i}} \tag{5.115}$$

where:

$$d = \frac{1}{(1+y)} \tag{5.116}$$

where: y = yield to maturity or redemption yield

The denominator in Equation 5.115 is equal to the present value of future coupon and capital repayments of the bond or, in other words, the price P.

Substituting Equation 5.113 into Equation 5.115 we have the formula for Macaulay duration:

$$D = \frac{\sum_{i=1}^{n} F_i \times t_i \times d^{t_i}}{P} \tag{5.117}$$

Macaulay–Weil duration

There is no reason the discount rate should be constant for all coupon payments; in fact, interest rates are likely to be different for different time periods. Macaulay–Weil duration uses the appropriate spot rate for each cash flow and is therefore slightly more accurate.

[69] Macaulay, "Some Theoretical Problems Suggested by the Movement of Interest Rates, Bond Yields and Stock Prices in the US since 1856" (1938).

Modified duration

Both Macaulay and Macaulay–Weil durations assume that coupon payments are reinvested continuously, which is not the case. Duration can be modified for the actual payment frequency as follows:

$$MD = \frac{D}{1 + \frac{y}{k}} \tag{5.118}$$

where: k = number of cash flows or coupons per year.

Note that for $k = \infty$, i.e., continuous compounding, modified duration is equal to the Macaulay duration. For a normal bond the duration will always be between zero and the maturity of the bond. The duration will be equal to the time to maturity for a zero-coupon bond, a bond with a single cash flow. Clearly all normal bonds are simply a series of zero-coupon bonds.

Weighted average cash flow, Macaulay, modified and Macaulay–Weil durations for the simple 20-year bond with a 6% annual coupon payment shown in Table 5.20 are calculated in Exhibit 5.18. Macaulay duration is graphically illustrated in Figure 5.22. The fulcrum marks the balancing point.

Exhibit 5.18 Duration

Weighted average cash flow $\frac{3260}{220} = 14.82$ years

Macaulay duration $\frac{1692.7}{128.91} = 13.13$ years

Modified duration $= \frac{D}{1 + \frac{y}{k}} = \frac{13.13}{1 + \frac{1.0389}{1}} = 12.64$ years

Macaulay–Weil duration $= \frac{1679.67}{128.91} = 13.03$ years

Modified Macaulay–Weil duration $= \frac{D}{1 + \frac{y}{k}} = \frac{13.03}{1 + \frac{1.0389}{1}} = 12.54$ years

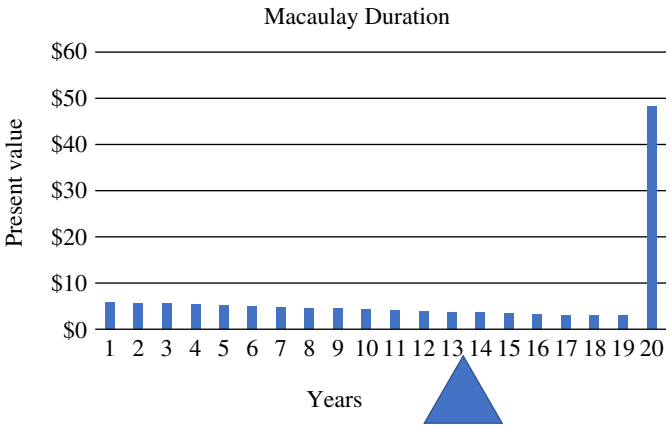

Figure 5.22 Macaulay duration

Table 5.20 Price, Macaulay, modified and Macaulay–Weil durations

20-year bond 6% coupon			Macaulay		Macaulay–Weil		
Period	Cash flow	Time × Cash flow	Present value	Time × Present value	Spot rate	Present value	Time × Present value
1	6	6	5.78	5.78	3.5%	5.80	5.8
2	6	12	5.56	11.12	3.5%	5.60	11.2
3	6	18	5.35	16.05	3.5%	5.41	16.23
4	6	24	5.15	20.60	3.5%	5.23	20.91
5	6	30	4.96	24.79	3.5%	5.05	25.26
6	6	36	4.77	28.63	3.5%	4.88	29.29
7	6	42	4.59	32.15	3.5%	4.72	33.01
8	6	48	4.42	35.36	3.5%	4.56	36.45
9	6	54	4.25	38.29	3.75%	4.31	38.77
10	6	60	4.1	40.95	3.75%	4.15	41.52
11	6	66	3.94	43.36	3.75%	4.00	44.02
12	6	72	3.79	45.53	3.75%	3.86	46.29
13	6	78	3.65	47.48	3.75%	3.72	48.33
14	6	84	3.52	49.21	3.75%	3.58	50.17
15	6	90	3.38	50.75	3.75%	3.45	51.81
16	6	96	3.26	52.11	3.75%	3.33	53.27
17	6	102	3.13	53.29	4.0%	3.08	52.36
18	6	108	3.02	54.31	4.0%	2.96	53.31
19	6	114	2.9	55.18	4.0%	2.85	54.11
20	106	2120	49.39	987.75	4.0%	48.38	967.54
Total	**220**	**3260**	**128.91**	**1692.70**	**Price**	**128.91**	**1679.67**
IRR (y)		3.89%		IRR (y)	3.89%		

Portfolio duration

Durations in a fixed income portfolio or benchmark are additive. The entire duration of the portfolio can be computed directly from the weighted sum of each bond in the portfolio:

$$\text{Portfolio duration } D_r = \sum_{i=1}^{i=n} w_i \times MD_i \tag{5.119}$$

Effective duration (or option-adjusted duration)

Modified duration does not calculate the effective duration of the bond if there is any optionality in future payments.[70] To calculate the effective duration, the estimated price must be calculated for both a positive and negative change in interest rates:

$$ED = \frac{P_- - P_+}{2 \times P \times \Delta y} \tag{5.120}$$

[70] Callable bonds, for example.

where: Δy = change in interest rates
P_- = estimated price if interest rate is decreased by Δy
P_+ = estimated price if interest rate is increased by Δy

If there is no optionality in future payments, effective duration will be identical to modified duration. Technically, although numerically similar to modified duration, effective duration is a direct measure of sensitivity and not measured in years.

The effective duration of the bond used in Table 5.20 is calculated in Exhibit 5.19 based on a parallel change of 25 basis points using the revised data in Table 5.21. Notice that the answer is close to modified Macaulay–Weil duration, as expected.

Exhibit 5.19 Effective duration

Effective duration $ED = \dfrac{P_- + P_+}{2 \times P \times \Delta y}$

$$ED = \frac{133.04 - 124.96}{2 \times 128.91 \times 0.25\%} = 12.55$$

Table 5.21 Effective duration

20-year bond 6% coupon	+0.25% parallel shift		−0.25% parallel shift	
Cash flow	Spot rate	Present value	Spot rate	Present value
6	3.75%	5.78	3.25%	5.81
6	3.75%	5.57	3.25%	5.63
6	3.75%	5.37	3.25%	5.45
6	3.75%	5.18	3.25%	5.28
6	3.75%	4.99	3.25%	5.11
6	3.75%	4.81	3.25%	4.95
6	3.75%	4.64	3.25%	4.80
6	3.75%	4.47	3.25%	4.65
6	4.0%	4.22	3.5%	4.40
6	4.0%	4.05	3.5%	4.25
6	4.0%	3.90	3.5%	4.11
6	4.0%	3.75	3.5%	3.97
6	4.0%	3.60	3.5%	3.84
6	4.0%	3.46	3.5%	3.71
6	4.0%	3.33	3.5%	3.58
6	4.0%	3.20	3.5%	3.46
6	4.25%	2.96	3.75%	3.21
6	4.25%	2.84	3.75%	3.09
6	4.25%	2.72	3.75%	2.98
106	4.25%	46.11	3.75%	50.76
Price		**124.96**	**Price**	**133.04**

Duration to worst

Duration to worst is identical to modified duration except that if there is any optionality (for example, if the bond is callable) the cash flow used in the calculation will be that which results in the worst yield for the investor.

Convexity

Duration is only the first-order approximation in the change of the bond's price. The approximation is due to the fact that a curved line represents the relationship between bond prices and interest rates (see Figure 5.23). Duration assumes there is a linear relationship. This approximation can be improved by using a second approximation, convexity:

$$C = \frac{\sum_{i=1}^{n} F_i \times t_i \times (t_i + 1) \times d^{t_i}}{P} \tag{5.121}$$

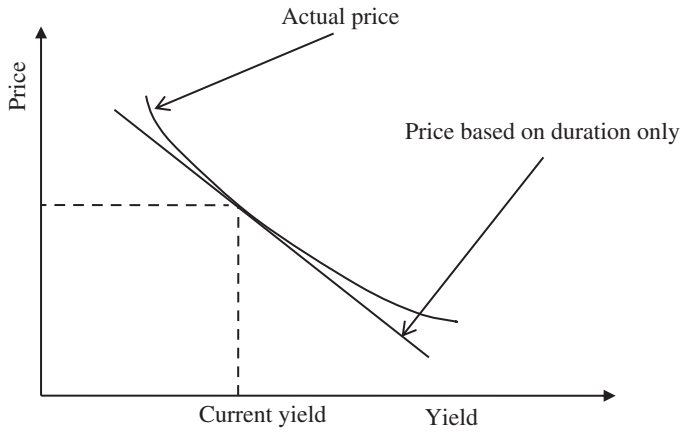

Figure 5.23 Convexity

Modified convexity

Convexity can be modified just like duration for the actual payment frequency of coupons as follows:

$$MC = \left(\frac{1}{1 + \frac{y}{k}}\right)^2 \times \frac{\sum_{i=1}^{n} F_i \times t_i \times (t_i + 1) \times d^{t_i}}{P} \tag{5.122}$$

Modified convexity is often erroneously just called convexity.

Effective convexity

Again, modified convexity does not calculate the effective convexity of the bond if there is any optionality in future payments. Using estimated prices to calculate effective convexity:

$$EC = \frac{P_- + P_+ - 2 \times P}{P \times (\Delta y)^2} \tag{5.123}$$

Convexity, modified convexity, and effective convexity based on the same 25-basis-point change in yield are shown in Table 5.22 and calculated in Exhibit 5.20.

Table 5.22 Convexity

Period	Cash flow	Present value	$t \times (t+1)$	$t \times (t+1) \times$ Present value
1	6	5.80	2	11.59
2	6	5.60	6	33.61
3	6	5.41	12	64.94
4	6	5.23	20	104.57
5	6	5.05	30	151.56
6	6	4.88	42	205.00
7	6	4.72	56	264.09
8	6	4.56	72	328.07
9	6	4.31	90	387.70
10	6	4.15	110	456.73
11	6	4.00	132	528.27
12	6	3.86	156	601.75
13	6	3.72	182	676.67
14	6	3.58	210	752.55
15	6	3.45	240	828.97
16	6	3.33	272	905.55
17	6	3.08	306	942.55
18	6	2.96	342	1,012.92
19	6	2.85	380	1,082.18
20	106	48.38	420	20,318.35
		Price 128.91		**Total 29,657.64**

Exhibit 5.20 Convexity

$$\text{Convexity } C = \frac{\sum_{i=1}^{n} F_i \times t_i \times (t_i + 1) \times d^{t_i}}{P}$$

$$C = \frac{29657.64}{128.91} = 230.06 \text{ years}$$

$$\text{Modified convexity } MC = \left(\frac{1}{1 + \frac{y}{k}}\right)^2 \times \frac{\sum_{i=1}^{n} F_i \times t_i \times (t_i + 1) \times d^{t_i}}{P}$$

$$MC = \left(\frac{1}{1 + \frac{3.89\%}{1}}\right)^2 \times 230.06 = 213.14 \text{ years}$$

$$\text{Effective convexity } EC = \frac{P_- + P_+ - 2 \times P}{P \times (\Delta y)^2} = \frac{133.04 + 124.96 - 2 \times 128.91}{128.91 \times (0.0025)^2} = 213.00$$

Portfolio convexity

Just like duration, convexity in a fixed income portfolio or benchmark is additive. The convexity of the portfolio can be computed directly from the weighted sum of each bond in the portfolio:

$$\text{Portfolio convexity } C_r = \sum_{i=1}^{i=n} w_i \times C_i \tag{5.124}$$

Bond returns

Duration measures the sensitivity of a bond's price to changes in yield. Price changes can be estimated by:

$$\Delta P = -MD \times \Delta y \tag{5.125}$$

A better estimate of the return of a bond would include return from income through time, duration and convexity as follows:

$$r_B = y \times \Delta t - MD \times \Delta y + \frac{C}{2} \times (\Delta y)^2 \tag{5.126}$$

The price return for the bond in Table 5.20 is estimated in Exhibit 5.21 using both modified and effective duration and convexity for both a 25-basis-point increase and decrease in yields.

Exhibit 5.21 Estimated bond returns

Actual price change

$\frac{133.04}{128.91} - 1 = 3.2\%$ –0.25% change in yield

$\frac{124.96}{128.91} - 1 = -3.07\%$ +0.25% change in yield

Using modified duration and modified convexity

For a –0.25% change in yield:

$$r_B = -MD \times \Delta y + \frac{C}{2} \times (\Delta y)^2 = -12.54 \times (-0.25\%) + \frac{213.14}{2} \times (-0.25\%)^2 = 3.2\%$$

For a 0.25% change in yield:

$$r_B = -MD \times \Delta y + \frac{C}{2} \times (\Delta y)^2 = -12.54 \times (0.25\%) + \frac{213.14}{2} \times (0.25\%) = -3.07\%$$

Using effective duration and effective convexity

For a –0.25% change in yield:

$$r_B = -MD \times \Delta y + \frac{C}{2} \times (\Delta y)^2 = -12.55 \times (-0.25\%) + \frac{213.00}{2} \times (-0.25\%)^2 = 3.2\%$$

For a 0.25% change in yield:

$$r_B = -MD \times \Delta y + \frac{C}{2} \times (\Delta y)^2 = -12.55 \times (0.25\%) + \frac{213.00}{2} \times (0.25\%)^2 = -3.07\%$$

Both the modified and effective calculations are good estimates of the actual return of bond.

Duration beta

The ratio of the portfolio's duration with that of the benchmark duration provides a measure equivalent to a regression beta:

$$D_\beta = \frac{D_r}{D_b} \tag{5.127}$$

where: D_r = portfolio duration
D_b = benchmark duration

Reward to duration

Reward to duration is the Treynor ratio for fixed income portfolios with modified duration replacing beta as the measure of systematic risk.

$$\text{Reward to duration} = \frac{\tilde{r} - \tilde{r}_F}{MD} \tag{5.128}$$

MISCELLANEOUS RISK MEASURES

Hurst index (or Hurst exponent)

The Hurst index[71] is a useful statistic for detecting whether a portfolio manager's returns are mean reverting (anti-persistent), totally random or persistent. It is estimated as follows:

$$\text{Hurst index } H = \frac{\log(R/S)}{\log(n)} \tag{5.129}$$

where:

$$\frac{R}{S} = \frac{[\max({}_k r) - \min({}_k r)]}{\sigma} \tag{5.130}$$

n = number of observations

$${}_k r = \sum_{i=1}^{i=k} (r_i - \bar{r}) \tag{5.131}$$

(R/S) is the range of the maximum cumulative deviation from the mean to the minimum cumulative deviation from the mean divided by the standard deviation of portfolio returns, otherwise known as the rescaled range or simply (R/S).

H. E. Hurst originally developed the Hurst index to help in the difficult task of establishing optimal water storage along the Nile.[72] He noted that (R/S) scales by a power law as time increases:

$$\frac{R}{S} = C \times n^H \tag{5.132}$$

[71] R. Clarkson, "FARM: A Financial Actuarial Risk Model" (2001).

[72] B. Qian and K. Rasheed, "Hurst Exponent and Financial Market Predictability," *Proceedings of the Second IASTED International Conference on Financial Engineering and Applications* (1 January 2004).

Taking logarithms, we can obtain an estimate of the Hurst index as follows:

$$\log\left(\frac{R}{s} = \log \mathrm{C} + H \times \log(n)\right) \tag{5.133}$$

The Hurst index is the gradient of the plot of $\log(R/S)$ against $\log(n)$; the intercept with the vertical axis is the constant log C, which for most time periods shorter than four or five years we can assume is zero, as shown in Figure 5.24.

A Hurst index between 0 and 0.5 would suggest that a portfolio manager's series of returns is mean-reverting (anti-persistent).

A Hurst index of 0.5 would suggest that the series of returns was totally random.

A Hurst index between 0.5 and 1 would suggest that the series of returns are persistent, i.e., there is memory in the return series.

Interpretation

Clearly, persistent positive returns or excess returns would be a desirable property for any asset manager; a Hurst index close to 1 would suggest either exceptional performance or fraud. Nile floods are extremely persistent as evidenced by a Hurst index of 0.9; equity markets have a Hurst index of around 0.7.

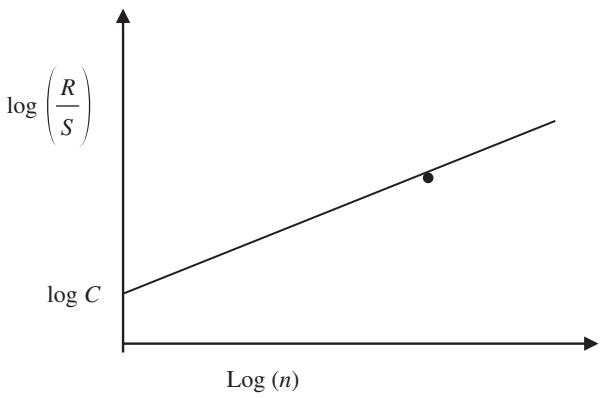

Figure 5.24 Hurst index

Bias ratio

The bias ratio is a simple but ingenious ratio devised by Adil Abdulali[73] of Protégé Partners, published in 2006 to provide some insight into the distribution of returns near zero. It is defined as the number of returns equal to and closely exceeding zero, divided by the number of returns close to but less than zero. The definition of close is set to the investor's preference but is typically within one standard deviation.

$$\text{Bias ratio } BR = \frac{\text{Count}\langle r_i | r_i \in [0, \sigma]\rangle}{1 + \text{Count}\langle r_i + | r_i \in [-\sigma, 0]\rangle} \tag{5.134}$$

[73] A. Abdulali, "The Bias Ratio™, Measuring the Shape of Fraud" (2006).

The bias ratio is a useful indicator of stale prices associated with illiquid assets and perhaps a subjective pricing policy. Hedge fund, real estate and private equity managers are very keen to deliver constant positive returns and may be reluctant to crystallise negative returns, potentially leading to an element of smoothing, avoiding if possible small negative revaluations and only "biting the bullet" if a negative write-down is unavoidable. Apart from the obvious impact of hiding volatility, such activity may be an indicator of outright fraud.

A bias ratio of less than 1 would certainly not be an indicator of fraud but would be undesirable nevertheless. Equity indexes tend to generate bias ratios between 1 and 1.5; Ponzi schemes would in all likelihood exhibit very high bias ratios.

Note

Interestingly, I calculate the bias ratio for the Fairfield Sentry Hedge Fund Ltd, a feeder fund for the well-known Bernie Madoff Ponzi scheme,[74] over the period 31/12/1990 to 31/10/2008 to be 5.2!

Caution

Bias ratios should not be analysed in isolation or out of context but used as an additional measure by comparison to an appropriate benchmark that reflects the underlying investment strategy. Cash and near cash strategies will naturally lead to a large number of small positive returns and few or no negative returns. If genuine, a high bias ratio is desirable.

Active share

Active share is a descriptive risk measure suggested by Cremers and Petajisto[75,76] in 2006 that is perhaps easy to dismiss as being too simple to be of any value or perhaps should not be described as a risk measure at all. Active share is a measure of the percentage difference of security holdings in the portfolio as opposed to the benchmark defined as:

$$\text{Active share} = \frac{1}{2} \times \sum_{j=1}^{j=n} |PW_j - IW_j| \tag{5.135}$$

where: PW_j = weight in portfolio of security j
IW_j = weight in the index (or benchmark) of security j

[74] "The Madoff Case: A Timeline," *Wall Street Journal*, 6 March 2009.
[75] Cremers and Petajisto, "How Active Is Your Fund Manager? A New Measure That Predicts Performance" (2006).
[76] Cremers and Petajisto, "How Active Is Your Fund Manager? A New Measure That Predicts Performance" (2009).

An active share of 0% represents full replication between the portfolio and index and 100% represents no overlap.

Cremers and Petajisto analysed some 2,647 US mutual funds from 1980 to 2003 and concluded:

(1) Manager active share is persistent.
(2) Funds with high active share tend to outperform.
(3) Funds with low active share tend to underperform.

Using active share and tracking error, they classified managers into four groups as shown in Table 5.23 and illustrated in Figure 5.25.

Not unsurprisingly, portfolios with high active share and low tracking error (stock pickers), for example portfolio managers prepared to take big bets yet nevertheless controlling relative risk, were the best-performing group. Portfolios with high active share and high tracking error (concentrated portfolios) also performed well, but portfolios with low active share and low tracking error (pejoratively described as closet indexers) tend to underperform and portfolios with low active share and high tracking error (portfolios betting on systematic factors generating high tracking error without taking big individual stock bets against the index, described as factor bet portfolios) also underperform.

Table 5.23 Active share and tracking error

Active share	Tracking error	Classification
High	High	Concentrated
High	Low	Stock pickers
Low	High	Factor bets
Low	Low	Closet indexers

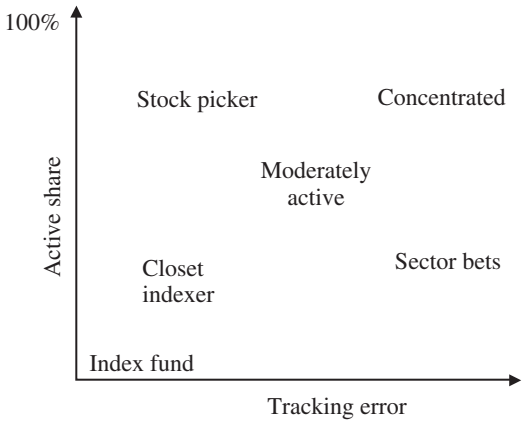

Figure 5.25 Active share versus tracking error

Interpretation

Suggesting that active share is a good predictor of outperformance is perhaps a little optimistic, and as such, has been criticised; see Morrison[77] and Frazzini, Friedman and Pomorski.[78] However, using active share as a filter rather than a predictor of performance seems reasonable. Regulators are increasingly using active share as a measure to determine whether investment strategies are active (and hence justify charging active fees). The Danish Investment Funds Association requires the publication of both active share and tracking error in UCITS (Undertakings for Collective Investment in Transferable Securities) annual reports and recommends funds with active share less than 50% and tracking error less than 3% to justify their investment strategies.[79]

In subsequent analysis Cremers, Fulkerson and Riley[80] (2022) suggest that for separate accounts with high active share there is evidence of positive performance persistence. In other words, a combination of good past performance and active share can be used to identify asset managers who might be expected to outperform.

Value at risk (VaR)

Value at risk, usually abbreviated as VaR, is an extreme risk measure originally developed in the 1990s by investment banks to manage their daily trading risk.[81] VaR measures the worst expected loss over a given time interval under normal market conditions at a given confidence level. For example, an annual value of risk of £5 million at a 95% confidence level for a portfolio would suggest that only once in 20 years would the annual loss exceed £5 million.

VaR, like tracking error and variability, can be calculated both ex-post and ex-ante. Typically, VaR is forecast ex-ante although again, like tracking error, it is useful to also calculate ex-post to provide a comparison and monitor risk efficiency.[82] Recall that ex-post measures rely on the realised historical total returns of the portfolio, whereas ex-ante measures rely on a snapshot of the current holdings of securities and instruments within a portfolio and, based on their historic relationship with each other, forecast the prospective risk.

In the context of portfolio returns, return at risk or performance at risk is perhaps a more appropriate term, although the term VaR is used universally to represent both values and returns at risk.

[77] Morrison, "Key Insight or Flawed Measure?" (2016).
[78] Frazzini, Friedman and Pomorski, "Deactivating Active Share" (2016).
[79] Finanstilsynet "Active/Passive Management in Danish UCITS," The Danish FSA (2016).
[80] Cremers, Fulkerson and Riley, "Active Share and the Predictability of the Performance of Separate Accounts" (2022).
[81] Sir Dennis Weatherstone, former CEO of J. P. Morgan, requested a 4:15 report that aggregated the firm's risk on one page within 15 minutes of market close at 4 p.m. VaR was developed to satisfy his request.
[82] See risk efficiency ratio Equation 8.5.

Note

Although we are concerned with extreme losses and negative returns, VaR by convention is presented as a positive number.

Caution

Both the time horizon and confidence levels will impact the interpretation of VaR significantly and consistency is essential for comparison.

The appealing simplicity of VaR has led to its adoption as the standard risk measure for many trading institutions, retail banks, insurance companies, pension funds and regulators.

RISK-ADJUSTED RETURN

The Sharpe ratio is sometimes erroneously described as a risk-adjusted return, but the clue is in its name – it is a ratio, not a return. We can rank portfolios in order of preference with the Sharpe ratio, but it is difficult to judge the size of relative performance. We need a risk-adjusted return measure to gain a better feel of risk-adjusted outperformance. Ratios all suffer the same problem: they allow us to rank portfolios in order of preference but do not inform in terms of the quantity of relative performance. Risk-adjusted returns, on the other hand, convert ratios to a return metric we fully understand and can freely use to make a judgement about relative performance.

M^2

M^2 is perhaps the most commonly used and easiest to calculate risk-adjusted return. M^2 is graphically represented in Figure 5.26. A straight line is drawn vertically through the risk of the benchmark $\widetilde{\sigma}_b$; the intercept with the Sharpe ratio line of portfolio B would give the return of the portfolio with the same Sharpe ratio of portfolio B but at the risk of the benchmark. This return is called M^2, a genuinely risk-adjusted return, extremely useful for comparing portfolios with various levels of risk.

In Figure 5.26, M^2 for portfolio B is clearly higher than for portfolio A, even though portfolio A has a higher absolute return. This is a crucial point captured by the M^2 measure: it adjusts each portfolio's returns to what it *would have been* had the portfolio managers taken the same amount of risk. In Figure 5.26, portfolio B's return is adjusted up (because it did not take as much risk as the benchmark) while portfolio A's return is adjusted down (because it has taken more risk than the benchmark).

$$M^2 = \widetilde{r} + SR \times (\widetilde{\sigma}_b - \widetilde{\sigma}) \tag{5.136}$$

where: σ_b = market risk (variability, standard deviation of benchmark return)

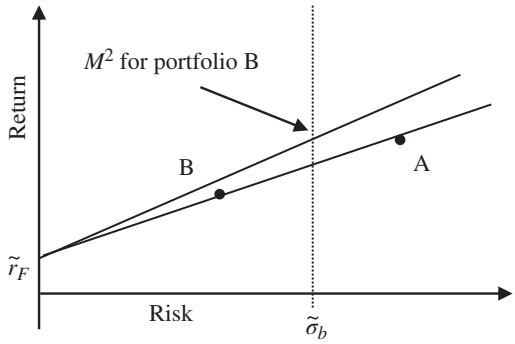

Figure 5.26 M^2

M^2 is so called not because any element of the calculation is squared, but because it was first proposed by the partnership of Leah Modigliani and her grandfather, Professor Franco Modigliani.[83]

Alternatively, you might see the equation for M^2 expressed as:

$$M^2 = (\tilde{r} - \tilde{r}_F) \times \frac{\tilde{\sigma}_b}{\tilde{\sigma}} + \tilde{r}_F \tag{5.137}$$

Although Equations 5.136 and 5.137 will give the same answer, I prefer the presentation of Equation 5.136, which is directly linked to the geometry of Figure 5.26 and very clearly captures the penalty if portfolio risk is greater than the benchmark risk and the reward if portfolio risk is less than benchmark risk (assuming of course the Sharpe ratio is positive).

Note

Of course, M^2 can be calculated using a variety of different measures of risk, simply relabelling the horizontal access. Return adjusted for both systematic and extreme risks are obvious examples, although variability is certainly the most common.

Using the data from Table 5.6, M^2 is calculated in Table 5.24.

M^2 excess return

Exactly the same arguments apply to geometric or arithmetic M^2 excess returns as they do to normal excess returns. Simple geometry from Figure 5.26 might suggest that arithmetic excess return would be more appropriate; however, it is easy to argue that continuously compounded returns should be used. For consistency I prefer the geometric definition.

$$M^2 \text{ excess return} = \frac{(1 + M^2)}{(1 + \tilde{b})} - 1 \tag{5.138}$$

or, arithmetically:

$$M^2 \text{ excess return} = M^2 - \tilde{b} \tag{5.139}$$

[83] Modigliani, "Risk-Adjusted Performance, Part 1: The Time for Risk Measurement Is Now" (1997).

Table 5.24 M^2

	Portfolio A	Portfolio B
Annualised return	7.9%	6.9%
Annualised risk	5.5%	3.2%
Sharpe ratio	1.07	1.53
Risk-free rate	2.0%	2.0%
$M^2 = \tilde{r} + SR \times (\tilde{\sigma}_b - \tilde{\sigma})$	$7.9\% + 1.07 \times (4.5\% - 5.5\%)$ $= 6.83\%$	$6.9\% + 1.53\% \times (4.5\% - 3.2\%)$ $= 8.89\%$
$M^2 = (\tilde{r} - \tilde{r}_F) \times \frac{\tilde{\sigma}_b}{\tilde{\sigma}} + r_F$	$(7.9\% - 2.0\%) \times \frac{4.5\%}{5.5\%} + 2.0\%$ $= 6.83\%$	$(6.9\% - 2.0\%) \times \frac{4.5\%}{3.2\%} + 2.0\%$ $= 8.89\%$

Differential return

The differential return is similar in concept to M^2 excess return except that the benchmark return is adjusted to the risk of the portfolio. The differential return is the difference between the portfolio return and the adjusted benchmark return. For the same portfolio, the M^2 excess return and the differential return will differ because the Sharpe ratio lines of the portfolio and benchmark will diverge over time.

The adjusted benchmark return $\tilde{b}'$ is calculated as follows:

$$\tilde{b}' = \tilde{r}_F + \left(\frac{\tilde{b} - \tilde{r}_F}{\tilde{\sigma}_b} \right) \times \tilde{\sigma} \tag{5.140}$$

Therefore, subtracting the adjusted benchmark return from the portfolio return we derive the differential return (see Figure 5.27):

$$\text{Differential return} \quad DR = \tilde{r} - \tilde{r}_F - \left(\frac{\tilde{b} - \tilde{r}_F}{\tilde{\sigma}_b} \right) \times \tilde{\sigma} \tag{5.141}$$

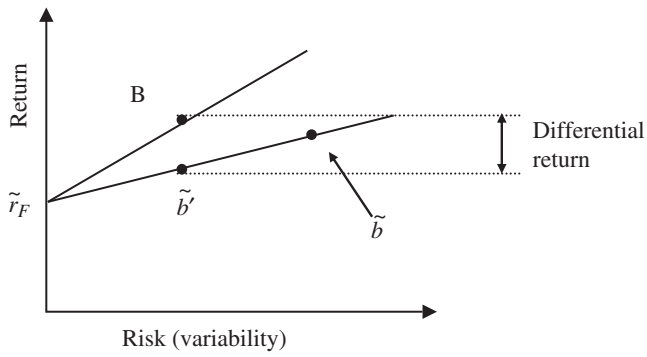

Figure 5.27 Differential return

Differential returns are calculated in Table 5.25.

Table 5.25 Differential return

	Portfolio A	Portfolio B
Annualised return	7.9%	6.9%
Annualised risk	5.5%	3.2%
M^2	6.83%	8.89%
M^2 excess return	$6.83\% - 7.5\% = -0.67\%$	$8.89\% - 7.5\% = 1.39\%$
(Arithmetic) $M^2 - \widetilde{b}$		
Differential return	$7.9\% - 2.0\% - \frac{7.5\% - 2.0\%}{4.5\%} \times 5.5\%$ $= -0.82\%$	$6.9\% - 2.0\% - \frac{7.5\% - 2.0\%}{4.5\%} \times 3.2\%$ $= +0.99\%$
$\widetilde{r} - \widetilde{r}_F - \left(\frac{\widetilde{b} - \widetilde{r}_F}{\widetilde{\sigma}_b}\right) \times \widetilde{\sigma}$		

Differential return is less useful for comparing multiple portfolios because multiple risk-adjusted benchmark returns are required to be calculated; for M^2 the benchmark returns are consistent for all portfolios. M^2 is a demonstrably better measure than either the Sharpe ratio from which it is derived or differential return. It is possible to calculate differential return geometrically.

Adjusted M^2

Assuming benchmark returns are normally distributed, the Sharpe ratio can be replaced with a line of different gradient using the adjusted Sharpe ratio and a risk-adjusted return calculated that also incorporates the third and fourth moments of the return distribution,[84] as shown in Figure 5.28. In this example, the asset manager is penalised with a shallower gradient,

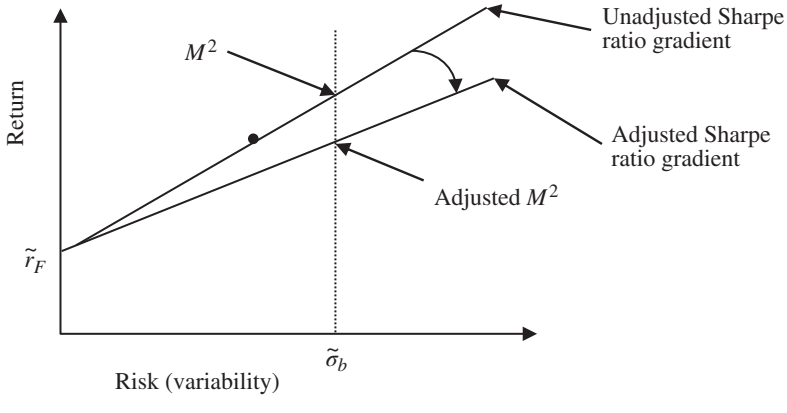

Figure 5.28 Adjusted M^2

[84] Bacon, "Introducing Adjusted M^2" (2013).

represented by an adjusted Sharpe ratio reflecting less desirable skewness and kurtosis. Therefore, the intercept with the variability of the benchmark is that bit lower, producing a lower risk-adjusted return.

$$\text{Adjusted } M^2 = \tilde{r} + ASR \times (\tilde{\sigma}_b - \sigma) \tag{5.142}$$

Interpretation

Adjusted M^2 is a very appealing statistic that incorporates all four moments of the return distribution and converts to a readily understood single return metric.

Skew-adjusted M^2

Skew-adjusted M^2, as its name suggests, incorporates just the third moment (skewness) and not the fourth moment (kurtosis).

$$\text{Skew-adjusted } M^2 = \tilde{r} + SASR \times (\tilde{\sigma}_b - \sigma) \tag{5.143}$$

M^2, adjusted M^2 and skew-adjusted M^2 for the standard portfolio are calculated in Exhibit 5.22 using data from Exhibit 5.7.

Exhibit 5.22 M^2, skew-adjusted M^2 and adjusted M^2

$M^2 = \tilde{r} + SR \times (\tilde{\sigma}_b - \tilde{\sigma}) = 13.29\% + 0.94 \times (11.65\% - 11.49\%) = 13.44\%$

Skew-adjusted

$M^2 = \tilde{r} + SASR \times (\tilde{\sigma}_b - \tilde{\sigma}) = 13.29\% + 0.91 \times (11.65\% - 11.49\%) = 13.43\%$

Adjusted $M^2 = \tilde{r} + ASR \times (\tilde{\sigma}_b - \tilde{\sigma}) = 13.29\% + 0.91 \times (11.65\% - 11.49\%) = 13.43\%$

There is only a tiny difference, lost in rounding, in this example because the distribution is close to normal.

TYPES OF EXCESS RETURN (OR ALPHA)

Confusingly we now have a number of different definitions of excess return and alpha; which should we use? Ultimately, it depends on the preferences of the asset owner and what exactly they are looking for. Alpha, the excess return adjusted for systematic risk, is often described as the holy grail of investment management.

Various definitions of excess return are listed in Table 5.26.

Table 5.26 Definitions of excess return

Type	Definition
Excess return	Excess return above the risk-free rate. The numerator of the Sharpe ratio.
Arithmetic excess	$(r - b)$ is the most common. Often confused with alpha. See Equation 4.24.
Geometric excess	$\left(\frac{1+r}{1+b}\right) - 1$. The technically correct definition of excess return. See Equation 4.25.
Regression alpha	See Equation 5.36.
Jensen's alpha	See Equation 5.42. Excess return adjusted for systematic risk. My preferred definition of alpha as opposed to excess return.
Three-factor alpha	See Equation 5.66.
Four-factor alpha	See Equation 5.67.
M^2 excess	See Equations 5.138 and 5.139 (arithmetic and geometric versions).
Skew-adjusted M^2 excess	Excess return adjusted for risk and skew preference. See Equation 5.143.
Adjusted M^2 excess	Excess return adjusted for risk and preferences of skew and kurtosis. See Equation 5.142, my strong preference for calculating performance fees.
Differential return	See Equation 5.141.
Net selectivity	See Equation 5.65.

A PERIODIC TABLE OF RISK MEASURES

Dimitri Mendeleev first published his periodic table of chemical elements in 1869.[85] This extremely useful table, familiar to all students of science, orders chemical elements according to atomic number, electron configuration and chemical properties. Mendeleev wasn't the first to publish such a table, but his table received recognition because it left gaps in the table and predicted the properties of then-undiscovered elements that would eventually fill those gaps.

A periodic table for risk measures is such a good idea I wish I'd thought of it first, but sadly I can't claim credit as the initial inspiration resulted from a rare invasion of my son's bedroom in which I found and studied a wall chart detailing a periodic table of cocktail drinks. This basic idea, applying the concept of the periodic table to anything other than chemical elements, was further reinforced in a presentation by Bruce Feibel at a CFA conference in 2012.[86] There are so many different risk measures available to performance analysts that to fully appreciate the relative merits of each measure and their relationship with each other, it makes sense to attempt to categorise them into some sort of logical structure. To be fair, Cogneau and Hubner attempt with some success to do this,[87,88] classifying over 100 performance appraisal measures based on their objectives, properties and degree of generalisation. The periodic table shown in Table 5.27 is my attempt to build on Bruce's idea. This is the third version of a periodic table of risk measures that I have attempted; the second was published in the *Journal of Performance*

[85] Mendeleev, "Uber die Beziehungen der Eigenschaften zu den Atomgewichten der Elemente" (1869).
[86] CFA GIPS Conference, Boston September 2012.
[87] Cogneau and Hubner, "The (more than) 100 ways to Measure Portfolio Performance Part 1" (2009).
[88] Cogneau and Hubner, "The (more than) 100 ways to Measure Portfolio Performance Part 2" (2009).

Table 5.27 Periodic table of risk measures

Descriptive Statistics		Absolute	Relative	Partial Moments: Downside	Partial Moments: Gain-Loss	Prospect	Drawdown	Risk-adjusted returns
First moment	Average	MAD ratio	Relative batting average	Omega–Sharpe ratio	Omega	Omega–prospect ratio	Sterling ratio	Excess return
Second moment	Variability	Sharpe ratio	Information ratio	Sortino ratio	Variability skewness	Prospect ratio	Burke ratio	M^2
Third moment	Skewness	Skew-adjusted Sharpe ratio	Skew-adjusted information ratio	Kappa	Gain–loss skewness	Skew-adjusted prospect ratio	Calmar ratio	Skew-adjusted M^2
Fourth moment	Kurtosis	Adjusted Sharpe ratio	Adjusted information ratio	Sortino–Satchell ratio	Farnelli–Tibiletti ratio	New prospect ratio	Pain ratio	Adjusted M^2
Systematic risk	Beta	Treynor ratio	Appraisal ratio	Return to duration	Timing ratio	Generalised Z ratio	Ulcer ratio	Alpha
Extreme risk	VaR	Reward to VaR	Reward to relative VaR	Conditional Sharpe ratio	Rachev ratio	Generalised Rachev ratio	Reward to conditional drawdown	M^2 for Var
Miscellaneous Risk measures		K ratio	Upside capture	Downside capture	Capture ratio	R^2	Bias ratio	Factor alpha
		Absolute batting average	Risk efficiency ratio	Tail ratio	Convexity	Bera–Jarque statistic	Active share	Omega excess

Measurement.[89] I used the first attempt to clarify my own ideas, and, just like the original Mendeleev periodic table, to identify new measures and gaps in my own knowledge worthy of further research.

Periodic table design

The main body of the table contains most, but not all, of the performance appraisal measures or composite ratios used to assess the performance of asset managers. They are either in typical Sharpe form, the acknowledged grandfather of all appraisal measures, "reward to risk" or in a "gain to loss" format.

Vertically the main body of the table is split into six columns based on different types of appraisal measures:

(1) Absolute: Measures based on the absolute return of the portfolio.
(2) Relative: Measures based on the relative return against a benchmark.
(3) Downside: Using partial moments, measures based on downside risk.
(4) Gain–loss: Measures reflecting upside in the numerator and downside in the denominator.
(5) Prospect: Measures based on prospect theory (or other asset owner preference measures).
(6) Drawdown: Measures based on drawdown.

Horizontally, the key descriptive statistics are shown in six rows, the four moments of the return distribution in descending order of importance:

The first moment or average return
The second moment, variability or standard deviation
The third moment, skewness
The fourth moment, kurtosis
Systematic risk
And finally risk measures focusing on the tail of the return distribution (extreme risk)

In earlier versions I included extreme risk measures as an additional column, which to be honest always felt a little uncomfortable. I've attempted in this third iteration to construct a table that extreme risk measures can be catered for in an additional row of the table, not a column.

Why measure ex-post risk?

Many commentators would argue that there is little value and obviously no uncertainty when measuring ex-post or historic risk, so why bother? I maintain there are good reasons to measure risk ex-post:

(1) Ex-post portfolio performance is two-dimensional. It is simply not acceptable to focus only on return; risk must also be assessed. If an asset manager achieved a reasonable but unexciting return, but took very high risk to achieve this return, then the asset owner should be very disappointed. The return achieved should justify the risk taken. Virtually all performance fee arrangements are based on return only; theoretically they should be calculated using risk-adjusted return.

[89] Bacon, "A Periodic Table of Risk Measures – Version 2" (2015).

(2) Asset owners must articulate their risk preferences in advance: they should determine which risk measure they want their asset manager to maximise and then they should monitor performance against that chosen risk measure ex-post.
(3) Both asset owners and asset managers should compare realised or ex-post risk against the forecast or ex-ante risk. It is essential that forecasts of risk are consistently compared against actual realised risk. Risk controllers need to be grounded in reality; it is essential that they compare realised risk with their previous forecasts of risk.

Which risk measures to use?

Risk, like beauty, is very much in the eye of the beholder. The objectives and preferences of the asset owner should determine which risk measure to use. Although most ex-post risk measures are easy to calculate, they are not all easy to interpret and are often contradictory.

There is little value in monitoring multiple risk measures. The ideal measure is determined in advance, consistent with investment objectives and easily understood by all parties. Additional descriptive statistics can be useful, but asset managers should be allowed to focus and maximise a single reward-to-risk appraisal measure.

Most appraisal measures are fundamentally the same structure: a ratio, the numerator of which is a measure of reward desired by the asset owner corresponding to the vertical axis of a two-dimensional graph and the denominator of which is a measure of risk that most concerns the asset owner corresponding to the horizontal axis. The reward-to-risk ratio corresponds to the slope of the line from a starting point on the vertical axis to the combined reward and risk of the portfolio.

Reward-to-risk ratios are most suitable for ranking portfolio performance. The most appropriate ratios take account of asset owners' risk tolerances. For downside risk measures, the minimum acceptable or target return should be consistent with the asset manager's investment objectives and strategy. There is relatively little value in using a range of minimum target returns to analyse portfolio performance if the asset manager is targeting relative return.

Ratios all suffer the same problem: they allow us to rank portfolios in order of preference but do not inform in terms of the quantity of relative performance. Risk-adjusted returns, on the other hand, convert ratios to a return metric we do understand.

Hedge funds

The more complex risk measures tend to be reserved for alternative investment strategies such as hedge funds that expect non-normal return distributions.

There are numerous definitions of hedge funds. My favourite is from Ineichen:

> A hedge fund constitutes an investment program whereby the managers or partners seek absolute returns by exploiting investment opportunities while protecting principal from potential financial loss.[90]

Predominately hedge fund management styles are designed to be asymmetric in their return patterns. If successful, this leads to variability of returns on the upside but not on the downside. Investors are less concerned with variability on the upside but of course are extremely

[90] Ineichen, *Absolute Returns: The Risk and Opportunities of Hedge Fund Investing* (2003).

concerned about variability on the downside. This leads to an extended family of risk-adjusted measures reflecting the downside risk tolerances of investors seeking absolute, not relative, returns. Investors should prefer high average returns, lower variance or standard deviation, positive skewness and lower kurtosis.

Hedge funds can be further differentiated by:

(1) More flexible investment strategies and the ability to employ leverage.
(2) Substantial personal holding by the portfolio manager and performance fees.
(3) Lighter regulation.
(4) Restricted liquidity – investors are typically subject to a lock-up period.

Smoothing

All these return measures assume accurate valuations of portfolios. Of course, if valuations are inaccurate then the risk calculations will be incorrect; if there is any smoothing employed in the valuation process then volatility will be hidden and lower than it should be.[91] This is an obvious issue for real estate and private equity investment strategies but also for some less liquid hedge fund strategies. Investors should be cognisant of this issue and should take care not to compare liquid and illiquid investment strategies. Care should also be taken when using measures that attempt to adjust for smoothing. Smoothing is one issue when comparing illiquid and liquid investment strategies; there are others. Adjusting for smoothing offers yet another opportunity for asset managers to self-select favourable measures.

⚠ Caution

Avoid comparing performance appraisal measures of liquid and illiquid strategies. Always compare like with like.

Outliers

In engineering or agriculture, it may be appropriate to ignore extreme results assuming they are measurement errors and not real returns. This is almost never appropriate for finance: extreme results represent real extreme losses. It may represent an unlikely event (such as Porsche building an undisclosed stake in VW using options and literally catching many short sellers desperately short of stock[92]), but this extreme event resulted in real losses.

⚠ Caution

Outliers should never be excluded from portfolio risk calculations.

[91] Some risk measures can be adjusted for smoothing. See Lo, "The Statistics of Sharpe Ratios" (2002) and Bacon, *Practical Risk-adjusted Performance Measurement* (2022).

[92] W. Boston, "Hedge Funds Shorting VW Stung by Porsche," *Time* magazine, 29 October 2008.

Data mining

Clearly with such a large number of risk measures available it is extremely tempting to data mine after the fact and find the appraisal measure that favours the asset manager in the best possible way. This of course is not useful; appraisal measures will generate different rankings according to asset owner preferences. These preferences must be established in advance and articulately expressed and documented. The asset manager's role is to maximise the chosen appraisal measure. Normally one measure, expressed over time, will suffice; multiple measures will lead to potential conflicts of objectives.

Caution

Avoid the temptation to present multiple appraisal measures.

Time period

Performance appraisal is a compromise: we must find a balance between long measurement periods that are more informative but slow to reflect structural changes and short recent measurement periods that reflect performance in recent market conditions but are less accurate. Three years is probably the minimum period to judge the performance of an asset manager (retaining a zero tolerance for breaching any investment mandate), but five years, seven years or longer is better.

Most performance appraisal measures should be compared over the same time period, assessing asset managers in the same market environment. Some "skill" measures, such as information ratio, appraisal ratio and alpha, that adjust for the relevant benchmark allow, with caution, assessment over different time periods. We can use statistical measures, such as the t-statistic,[93] to determine the probability that an asset manager is skilled. If we begin with the null hypothesis that the asset manager is not skilled, the asset manager will have a low t-statistic. With a t-statistic greater than 1.96 (say 2 as a rule of thumb), there is only a 2.5% chance that the asset manager is not skilled.

The t-statistic for alpha is:

$$t = \frac{\alpha}{\sigma \times \sqrt{n}} \tag{5.144}$$

The t-statistic for the information ratio is[94]:

$$t = \sqrt{n} \times \frac{\widetilde{g}}{\widetilde{\sigma}_G} \tag{5.145}$$

The t-statistics for three asset managers using the data in Table 5.28 are calculated in Exhibit 5.23.

[93] Shorthand for "hypothesis test statistic."
[94] Blatt, "An In-Depth Look at the Information Ratio" (2004).

Table 5.28 *t*-statistic

	Information ratio	Length of track record (years)
Manager A	1.0	3
Manager B	0.75	5
Manager C	0.5	16

Exhibit 5.23 *t*-statistic

$$t = \sqrt{n} \times \frac{\tilde{g}}{\tilde{\sigma}_G}$$

Manager A $t = \sqrt{3} \times 1.0 = 1.73$

Manager B $t = \sqrt{5} \times 0.75 = 1.67$

Manager C $t = \sqrt{16} \times 0.5 = 2.0$

We have the greatest statistical confidence in manager C; in fact, with an information ratio of 0.5 we need a 16-year track record before rejecting the null hypothesis the asset manager is not skilled.

⚠ Caution

Statistical confidence in an asset manager's track record is one thing, but comparison of asset manager performance across the same market conditions, a full understanding of the drivers of risk and return and evidence of a sustainable investment decision process in the future are more important.

⚠ Caution

Typically, fundamental changes in market conditions, major changes in investment decision processes, changes in regulation and structural changes in asset management firms occur in shorter time periods than the time required to obtain statistical confidence. Unfortunately, statistical confidence is a luxury that rarely exists in performance appraisal.

⚠ Caution

If you are comparing asset managers with different length track records, compare directly over the same time period. If an asset manager's track record is too short, eliminate them from the selection process.

6

Return Attribution

Never try to walk across a river just because it has an average depth of four feet.
Milton Friedman (1912–2006)

WHAT IS ATTRIBUTION?

Definition

The objective of return attribution, as stated by Menchero,[1] is to explain portfolio return relative to a benchmark, identify the sources of excess return, and relate these to active decisions by the asset manager. Hensel, Ezra and Ilkiw[2] define attribution as the mathematical process of explaining an investment return by relating it to the different risk-taking decisions implicit in the portfolio, and the extent to which each of those risks was rewarded or penalised in the capital markets. Colin[3] says that the purpose of attribution is to disentangle the fund's overall return into its component returns generated by each risk.

In other words, return attribution measures which of your investment decisions about the portfolio's underlying risks worked, and which did not. This is critical business intelligence for anyone involved in selecting, managing, controlling or marketing investments. In general, it is not the absolute but the relative return that will be analysed.

Note

The terms *performance attribution*, *return attribution* or simply *attribution* are interchangeable. Performance attribution is perhaps in more common usage but as already discussed performance is not one-dimensional; it is, at the very least, two-dimensional – the combination of risk and return. Therefore, I much prefer the term return attribution.

Specifically, I define return attribution as:

A technique used to quantify the excess return of a portfolio against its benchmark into the active decisions of the investment decision process.

Return attribution itself is, therefore, perhaps a misleading term: as the definition suggests, it is the excess, active or relative return that is examined, not the whole of the return

[1] Menchero, "A Fully Geometric Approach to Performance Attribution" (2000/2001).
[2] Hensel, Ezra and Ilkiw, "The Importance of the Asset Allocation Decision" (1991).
[3] Colin, *Attribution in Finance* (2014).

(or absolute return). Relative return attribution is perhaps better terminology. Absolute attribution, or simply contribution analysis, would explain the sources of return of a portfolio in isolation rather than in comparison to a benchmark.

⚠ Caution

It is possible to undertake a comparison of contribution to return from an investment category in the portfolio compared to the contribution to return from the same investment category in the benchmark. This type of analysis tends to conflate allocation and selection decisions and should be avoided.

Attribution as an asset management tool

Attribution analysis is a key management tool for several key stakeholders in the asset management process. Above all it is the key tool for performance analysts: crucially, it allows the analyst to ensure that the return is correct; it allows them to participate in the investment decision process, enabling them to demonstrably add value and thus justifying their salary and effort. Return attribution, together with risk analysis, is the tool that allows the analyst to understand the sources of return in the portfolio and to communicate that understanding to portfolio managers, senior management and asset owners.

Asset managers, and the portfolio managers they employ, are major users of attribution analysis. Attribution analysis provides a good starting point for a dialogue with asset owners engaging in a discussion of the positive and negative aspects of recent performance. It is possible to use attribution analysis aggressively, identifying underperformance early and providing asset owners with a thorough explanation of underperformance and hopefully a rationale for why performance might improve. It is crucial in gaining the confidence of the asset owner by demonstrating a good understanding of the drivers of performance.

Portfolio managers inhabit the front office of asset management firms; typically, performance analysts inhabit the middle office. However, the back office is also able to utilise daily, security-level attribution as a diagnostic tool for identifying pricing and transaction errors.

Senior management take an active interest in attribution analysis to provide them with a tool to monitor their portfolio managers. They will be keen to identify performance outliers – good or bad – and to ensure that value is added consistently across the firm.

Hence, attribution analysis has a broad use across a wide range of asset management stakeholders.

Lord[4] suggested that the characteristics of a good attribution system should include:

- A consistent approach for analysing returns
- A solid theoretical framework for valuation
- The ability to analyse portfolios and indexes, consistently
- Attribution factors that are consistent with decision variables of the portfolio strategy process
- Summary attribution analysis that is constructed from security detail, which itself should be available for inspection
- Transactions
- Robust analytic systems and high-quality data

[4] Lord, "The Attribution of Portfolio and Index Returns in Fixed Income" (1997).

Murira and Sierra[5] suggest that there is general agreement that an attribution framework must meet two key requirements:

(1) It should be consistent with the investment decision-making process of the portfolio. Performance attribution is most useful if it mirrors the investment management process, as it enables the identification and explanation of the sources of excess return.
(2) It should be compatible with the organisation's performance measurement and risk measurement systems. It would be difficult to make effective use of the information provided by the performance attribution process if the data being explained by this process are inconsistent with the data originating from the internal risk and performance systems.

Early development

Fama was the first to fully delve into the subject of attribution analysis in his "Components of Investment Performance", published in 1972.[6] In this seminal paper, Fama suggests breaking down observed return into the part resulting from ability to pick the best securities at a given level of risk (which he at the time called "selectivity", now often referred to as selection return), and the part that is due to predictions of general market price movements, the return from systematic risk (see Equation 5.62).

At the same time, the Society of Investment Analysts in the UK instituted a working group to make recommendations on the subject of portfolio performance measurement for pension funds. Their approach, eventually published in 1972,[7] had two aims: the determination of a standard method of calculating a portfolio's return, so that funds could be fairly compared, and the decomposition of the performance of the fund into two components – selection of individual stocks and selection of stock sectors. The first aim is achieved by calculating the time-weighted rate of return using a simple approximation (which later became known as the analyst's test method – see Equation 3.29). The second aim, the analysis of performance, is produced by constructing "notional funds" based on market indices and comparing these with the actual fund.

While this early work dealt with equity-market sectors as the macro level of selection and securities within the sectors as the micro level, it is now common practice to regard asset classes (equities, fixed income, private assets, etc.) as the macro level and securities within the asset class as the micro level.

The working group established two notional funds as follows:

(1) Fully restrained fund (index returns, benchmark weights)
(2) Partly restrained fund (index returns, actual weights)

Note that the actual fund consists of actual returns and actual weights.

The fully restrained fund (sometimes described as the fully notional fund) represents the return on the policy portfolio – that is, the strategic benchmark or policy weights set by the asset owner multiplied by the market index returns for each asset class.

$$\text{Fully restrained fund} \qquad b = \sum_{i=1}^{i=n} W_i \times b_i \qquad (6.1) \text{ or } (4.8)$$

[5] Murira and Sierra, "Fixed Income Attribution: A Unified Framework – Part 1" (2006).
[6] Fama, "Components of Investment Performance" (1972).
[7] Society of Investment Analysts (1972).

where: W_i = weight of the benchmark in the ith asset class (note $\sum_{i=1}^{i=n} W_i = 1$)

b_i = return of the index in the ith asset class

Note

For attribution to be complete and accurate it is essential that the total benchmark return is the sum of its parts. Thankfully, this is exactly how most commercial indexes are calculated. Equation 6.1 is a key foundation of attribution analysis.

The partly restrained fund (sometimes described as the semi-notional fund) represents the intermediate return resulting from the actual weights determined by the manager multiplied by market index returns.

$$\text{Partly restrained fund} \qquad b_S = \sum_{i=1}^{i=n} w_i \times b_i \tag{6.2}$$

where: w_i = weight of the portfolio in the ith asset class (note $\sum_{i=1}^{i=n} w_i = 1$)

The actual portfolio return is:

$$\text{Portfolio return} \qquad r = \sum_{i=1}^{i=n} w_i \times r_i \qquad (6.3) \text{ or } (3.41)$$

where: r_i = return of the portfolio assets in the ith asset class

Note

For attribution to be complete and accurate it is essential that the total portfolio return is the sum of its parts. This is technically possible for most return methodologies but not for the internal rate of return (IRR). Equation 6.3 is a key foundation of attribution analysis.

The performance of a fund thus depends on two factors: the selection of asset classes (typically now described as asset allocation) and the selection of securities within an asset class. The contribution of each of these components can be found by breaking down the transition from the fully restrained fund to the actual fund as shown in Table 6.1.

The success of the manager's asset allocation is measured by comparing the performance of the partly restrained fund to the fully restrained fund.

The success of the manager's security selection is measured by comparing the performance of the actual fund to the partly restrained fund.

Table 6.1 Transition from fully restrained fund to actual fund

Excess return	Actual fund minus the fully restrained fund
Selection of asset classes (asset allocation)	Partly restrained fund minus the fully restrained fund
Selection of securities	Actual fund minus the partly restrained fund

The Society of Investment Analysts paper is thus crucial because it introduces the concepts of macro and micro levels of manager decision-making, and intermediate notional portfolios to calculate these effects.

Holbrook[8] in effect pulled together the BAI report, Fama's decomposition, and the report of the Society of Investment Analysts. This virtually unknown paper provides the foundation for much of the development in performance analysis that follows. Directing his advice to pension fund trustees, Holbrook defines the decision-making process as follows:

> *The practical management of a pension fund investment portfolio involves decisions at three levels.*
>
> *(i) Policy, i.e., the proportions of the assets which, as a long-term aim, the trustees wish to hold in the different investment markets. The fundamental element of policy is the split between equity-type and fixed-interest vestments. Definition of policy may extend to a split of equity-type investments between UK equities, overseas equities and properties, specific proportions being set for each. If the trustees are not experts in long-term investment, they will tend to formulate policy in discussion with specialist advisers; it is gratifying to record that actuaries who advise on the financing of pension schemes are increasingly becoming involved in such discussions.*
>
> *(ii) Strategy, i.e., decisions to depart from the policy proportions in the light of current market conditions, including decisions to hold part of the fund on short-term deposit. The trustees will usually seek advice from experts in the various markets regarding the short-term outlook, up to the time of the next investment meeting and over the next year or so. Except where trustees delegate strategic decisions to managers and advisers, investment meetings are usually held as short regular intervals, and arrangements are made for special meetings whenever it is thought that there has been a material change in market prospects. Often trustees authorize the managers concerned to determine the timing of purchases; thus, a manager is able to retain part of the assets entrusted to him in the form of short-term deposits.*
>
> *(iii) Selection, i.e., the choice of the particular investments to hold, buy or sell within the various markets. Again, practice varies; many trustees give their managers carte blanche in this area – some keep close control over day-to-day transactions.*

Holbrook goes on to describe the trustees' needs. First, they need to know if the fund performed well, moderately or badly. This requires the calculation of return, comparisons with market returns and consideration of risk. The performance of the fund as a whole, and within the different markets and sectors, must be examined separately. Second, the trustees need to be advised on the reasons for their fund's performance; this may be investigated by analysing the results to show contributions to total performance made by policy, strategy and selection decisions. Finally, the trustees need help in interpreting the results as a guide to

[8] Holbrook, "Investments Performance of Pension Funds" (1977).

future action. Identification of points of strength and weakness in past performance may lead to reconsideration of policy and practice, of the decision-making process, and possibly to a change of manager.

To facilitate the breakdown of returns into policy, strategic and selection decisions Holbrook suggests the return calculation of three notional funds:

r_m = based on the model
r_p = based on the trustees' policy proportions
r_A = based on actual allocations

Then the geometric excess return, g, for the whole fund is given by:

$$(1+g) = \frac{(1+r)}{(1+r_m)} = \frac{(1+r_p)}{(1+r_m)} \times \frac{(1+r_A)}{(1+r_p)} \times \frac{(1+r)}{(1+r_A)} \tag{6.4}$$

The first factor represents the contribution to total performance resulting from the decision to adopt a policy that differs from the equity and fixed-interest proportions in the model (the industry-standard allocation between equity and fixed income at the time); the second, the contribution from strategic decision to depart from policy proportions; the third, the contribution from selection decisions for the fixed-interest and equity sectors taken together. This was the first articulation of successive notional portfolios, each isolating one step in the investment decision process.

Note

The fact that this was written in 1977 shows how far ahead of his time Holbrook was.

Holbrook links this successive notional portfolio approach to Fama's decomposition by equating the first factor to the investor's return from systematic risk, the second factor to the manager's return from systematic risk, and the third factor to selectivity.

TYPES OF RETURN ATTRIBUTION

Fama decomposition is a type of attribution derived from the pattern of returns of the portfolio – called returns-based or factor attribution. Fama decomposition is a simple, single-market-factor model; Fama and French is a three-factor model that includes a general market factor, together with a size and value factor; and Carhart is a four-factor model introducing a momentum factor. Sharpe[9] uses a 12-factor model; other models use even more factors.

Interpretation

A factor is any characteristic that helps explain the risk and return of an asset or portfolio.

Traditional attribution analysis, called asset-grouping (or variance type) attribution, is based on the holdings of the portfolio – *holdings-based attribution*, and based both on holdings and

[9] Sharpe, "Asset Allocation: Management Style and Performance Measurement" (1992).

transactions – *transaction-based attribution*. Thus, there are three main types of attribution, returns-based, holdings-based and transaction-based attribution, each with their own advantages and disadvantages.

Returns-based (regression or factor) attribution

Returns-based attribution uses regression analysis to identify the sources of return of a portfolio. These sources of return or factors can either be statistically derived from principal component analysis, macroeconomic factors or fundamental factors. Stock-specific factors are normally the residual of the chosen approach. Although useful information can be derived from factor models, accuracy is never guaranteed. Amnec, Sfeir and Martellini[10] describe four types of factor models:

(1) *Implicit Factor Model*. In this approach, some factor analysis (e.g., a principal component analysis) is performed to statistically extract the factors from the return's time series. These factors may not be easily interpretable.
(2) *Explicit Macro Factor Model*. In this approach, macroeconomic variables are used as factors.
(3) *Explicit Micro Factor Model*. In this approach, microeconomic attributes are used as factors.
(4) *Explicit Index Factor Model*. In this approach, stock market indices (including style sub-indices of the market) are used as factors.

Returns-based or factor attribution:

(1) Does not necessarily require the composition of the portfolio;
(2) Complements traditional holdings- or transaction-based attribution;
(3) Is relatively easy and cost effective to implement; and
(4) Is potentially more consistent with the investment decision process of the asset manager (e.g., with a factor-based decision process).

But, on the other hand:

(1) Is less accurate, particularly for concentrated strategies;
(2) Relies on the appropriate definition of factors; and
(3) Assumes a constant exposure over the analysis period.

Holdings-based (or buy/hold) attribution

To simplify the attribution process, some practitioners suggest an approximate approach to attribution which ignores the impact of transactions during the period of measurement called holdings-based attribution.

In this approach the beginning weights of securities and sectors together with their returns are used to calculate attribution effects. The returns are usually derived from another source and not from the actual portfolio return.

The advantage of this method is its ease of implementation; only the holdings need to be input into the attribution system. Proponents argue that returns are only estimates anyway; valuations are uncertain and cannot be achieved if the portfolio manager wished to sell anyway. Why worry too much about the accuracy of the methodology if the valuation is incorrect?

[10] Amnec, Sfeir and Matellini, "An Integrated Framework for Style Analysis and Performance Measurement" (2003).

The big disadvantage of this approach is that the holdings-based return will not reconcile with the actual return (good estimate or not) of the portfolio. This will lead to a residual between the actual portfolio return and the return explained by attribution. A small residual will be of no concern but often the residual will grow over time to become the single largest factor thus invalidating the entire analysis.

Residuals will tend to be larger for:

(1) More active managers
(2) Large cash flows
(3) Illiquid assets
(4) Longer measurement periods

The smaller the measurement period the better, but even daily analysis can result in significant residuals.

If transaction information is not captured, there is no opportunity to investigate the value added by the trading department. Since performance analysts using holdings-based attribution are not expecting the attribution to reconcile to the portfolio return, operational errors will not be spotted.

Transaction-based attribution

Transaction-based attribution is the most complete form of attribution identifying all sources of return and reconciling exactly to the published return. Transaction-based attribution is defined as:

Return attribution calculated directly from transaction and holdings-based data.

Because transaction data is sourced directly, including the transaction price, all the sources of return can be measured, including timing effects within the measurement period, bid/offer spread and transaction costs.

The decision on whether to use returns-based, holdings-based or transaction-based attribution will ultimately depend on the purpose for which the attribution will be used, investment decision process, investment objective, quality and availability of data, cost and time considerations, asset type, complexity and reporting objective. Holdings-based attribution may be acceptable for the exclusive internal use of the asset manager but asset owners are not always tolerant of attribution residuals. Returns-based attribution might complement more traditional asset-grouping models of attribution, in that it might reveal unexpected biases and make communication easier for asset managers with a more factor-based investment decision process.

Note

Pragmatism is a key tool in the performance analyst's armoury; however, in this instance I would not compromise. Performance analysts rely on good-quality data; reconciled transaction-based attribution improves the quality of back-office processes by providing a tool that quickly identifies operational errors and improves the general risk control environment of the firm. I have no hesitation in recommending transaction-based attribution despite the additional operational overhead.

The advantages and disadvantages of the different types of attribution are summarised in Table 6.2.

Table 6.2 Advantages and disadvantages of types of attribution analysis

Type of attribution	Advantages	Disadvantages
Returns-based	Easy to implement and cost effective. Consistent with factor-based investment decision process. Complements traditional asset-grouping attribution. Does not require detailed composition of portfolio.	Less accurate, not complete, residuals. Relies on appropriate definition of factors. Assumes constant exposure over period of analysis. Can't be used as a diagnostic tool to identify errors.
Holdings-based	Easy to implement and cost effective. Can use alternative pricing sources.	Not complete, does not reconcile to client return. Residuals might overwhelm other factors. Errors won't be spotted.
Transaction-based	Reconciles to return reported to asset owner. Identifies all sources of excess return. Can be used as a diagnostic tool to identify errors.	Expensive and difficult to implement. Data must be complete and accurate at granular level. Highlights back-office inefficiencies.

ARITHMETIC ATTRIBUTION

For asset-grouping models, both holdings- and transaction-based, the foundations of return attribution are established in two articles published in 1986 and 1985 by Brinson, Hood and Beebower[11] and Brinson and Fachler,[12] now collectively known as the Brinson model. These articles build on the work of the UK Society of Investment Analysts and Holbrook and the assumption that the total portfolio returns and benchmark returns are the sum of their parts; in other words, both portfolio and benchmark returns can be disaggregated.

Recall that the total portfolio return from Equation 6.3 is the sum product of portfolio weights (w_i) and portfolio returns (r_i) for category or sector i (asset category, geographic region, country, industrial sector, factor or individual security).

$$r = \sum_{i=1}^{i=n} w_i \times r_i \tag{6.3}$$

Recall also that the total benchmark return from Equation 6.1 (the fully restrained or fully notional fund) is the sum product of benchmark weights (W_i) and benchmark returns (b_i), the same formula used by most commercial indexes from Equation 4.8.

$$b = \sum_{i=1}^{i=n} W_i \times b_i \qquad (6.1) \text{ or } (4.8)$$

[11] Brinson, Hood and Beebower, "Determinants of Portfolio Performance" (1986).
[12] Brinson and Fachler, "Measuring Non-US Equity Portfolio Performance" (1985).

The challenge for single-period attribution is to quantify each of the portfolio manager's active decisions that contribute to the difference between the portfolio return r and the benchmark return b.

Brinson, Hood and Beebower

Brinson, Hood and Beebower suggested a model to break down the arithmetic excess return ($r - b$), assuming a standard investment decision process in which the asset manager seeks to add value through both *asset allocation* and *security selection*.

In asset allocation (or simply just *allocation*), the asset manager (or asset allocator) will seek to add value by taking different asset category (or sector) weights in the portfolio in comparison to category (or sector) benchmark weights. A category weight in the portfolio greater than the equivalent benchmark category weight would be described as *overweight* and a lesser weight would be described as *underweight*.

Clearly the asset allocator will aim to overweight good-performing categories and underweight poor-performing categories. In their original article, Brinson, Hood and Beebower called this impact "timing"; asset or market allocation is now a more common and appropriate description.

In *security selection* (or simply just *selection*) the asset manager (or stock picker) will seek to add value by selecting individual securities within the asset category.

Again, the asset manager will aim to be overweight in good-performing securities and underweight in poor-performing securities.

Asset allocation

To identify the added value from asset allocation we must calculate the return of an intermediate fund called the "allocation notional fund or semi-notional fund or partially restrained fund", which is one step away from the benchmark portfolio, one step toward the actual portfolio.

Note

I prefer the term *semi-notional fund*. It represents the intermediate (semi) step between the fully notional fund and actual fund.

In the semi-notional fund, the asset allocation weights of the actual fund are applied to index returns within each category. By definition the return on this notional fund reflects the asset manager's asset allocation "bets" but since index returns are used within the asset category it includes no stock selection.

$$\text{Allocation or semi-notional fund} \qquad b_S = \sum_{i=1}^{i=n} w_i \times b_i \tag{6.2}$$

The contribution from asset allocation is therefore the difference between the semi-notional fund and the benchmark fund or:

$$b_S - b = \sum_{i=1}^{i=n} w_i \times b_i - \sum_{i=1}^{i=n} W_i \times b_i = \sum_{i=1}^{i=n} (w_i - W_i) \times b_i \tag{6.5}$$

The contribution to asset allocation in the *i*th category is:

$$A_i = (w_i - W_i) \times b_i \tag{6.6}$$

Note that:

$$\sum_{i=1}^{i=n} A_i = b_S - b \tag{6.7}$$

Security (or stock) selection

Similarly, to identify the added value from security selection we must calculate the return of a different intermediate fund called the "selection notional fund", which is also by definition one step away from the benchmark return. In the selection notional fund, the weights of the benchmark are kept static and applied to the category returns within the actual portfolio. By definition the return on this notional fund reflects the portfolio manager's stock selection since real returns are applied to index weights, but excludes any contribution from asset allocation.

$$\text{Selection notional fund} \qquad r_S = \sum_{i=1}^{i=n} W_i \times r_i \tag{6.8}$$

The contribution from stock selection is therefore the difference between the selection notional fund and the benchmark fund or:

$$r_S - b = \sum_{i=1}^{i=n} W_i \times r_i - \sum_{i=1}^{i=n} W_i \times b_i = \sum_{i=1}^{i=n} W_i \times (r_i - b_i) \tag{6.9}$$

The contribution to stock selection in category *i* is:

$$S_i = W_i \times (r_i - b_i) \tag{6.10}$$

Note that:

$$\sum_{i=1}^{i=n} S_i = r_S - b \tag{6.11}$$

Interaction

In this, the "classical", definition of attribution, stock selection and asset allocation do not explain the arithmetic difference completely, a third term is required.

$$\text{Stock selection} + \text{Asset allocation} = r_S - b + b_S - b$$

$$\text{or} = r_S + b_S - 2 \times b \tag{6.12}$$

To achieve $r - b$ we must add a third term, called interaction:

$$\underbrace{r_s - b}_{\text{Stock selection}} + \underbrace{b_s - b}_{\text{Asset allocation}} + \underbrace{r - r_s - b + b}_{\text{Interaction}} = r - b \tag{6.13}$$

In their article, Brinson, Hood and Beebower described this term as "other"; *interaction* is perhaps a better description and is in common usage today.

$$r - r_s - b_s + b = \sum_{i=1}^{i=n} w_i \times r_i - \sum_{i=1}^{i=n} W_i \times r_i - \sum_{i=1}^{i=n} w_i \times b_i + \sum_{i=1}^{i=n} W_i \times b_i \tag{6.14}$$

This simplifies to:

$$\sum_{i=1}^{i=n}(w_i - W_i) \times (r_i - b_i) \tag{6.15}$$

It can be seen from Equation 6.15 that interaction is the combination of asset allocation and stock selection effects.

The contribution to interaction in category i is:

$$I_i = (w_i - W_i) \times (r_i - b_i) \tag{6.16}$$

Note that:

$$\sum_{i=1}^{i=n} I_i = r - r_S - b_S + b \tag{6.17}$$

Interpretation

Interaction is not a residual or a balancing item. It is a mathematically defined term – the combination of allocation and selection decisions.

Figure 6.1 illustrates the Brinson framework for return attribution.

Allocation \ Selection	Actual	Passive
Actual	**Quadrant IV** Portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$	**Quadrant II** Semi-notional $b_s = \sum_{i=1}^{i=n} w_i \times b_i$
Passive	**Quadrant III** Selection notional $r_s = \sum_{i=1}^{i=n} W_i \times r_i$	**Quadrant I** Benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$

Figure 6.1 Brinson framework

Excess returns due to:	
Asset allocation	II – I
Security selection	III – I
Interaction	IV – III – II + I
Total	IV – I

Graphically, Figure 6.2 illustrates the attribution factors for each category i. The contribution to total portfolio return from category i is the area $r_i \times w_i$; the contribution from the benchmark in this example is the smaller area $b_i \times W_i$. In this particular example the return in the portfolio (r_i) for category i is both positive and greater than the return in the benchmark (b_i) and the weight in the portfolio (w_i) is greater than the weight in the benchmark (W_i).

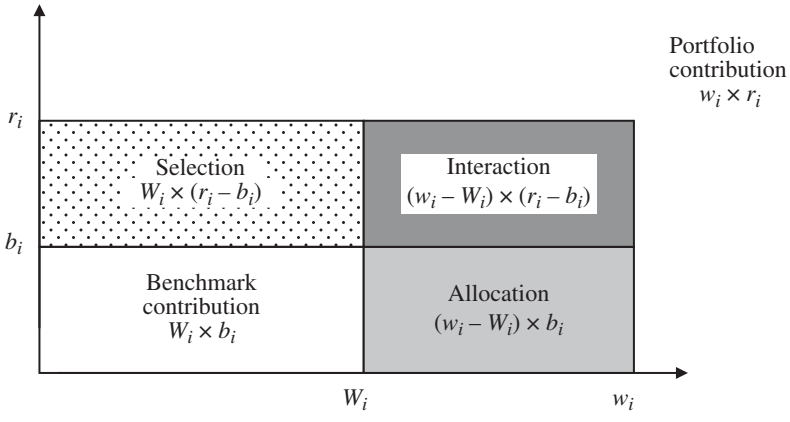

Figure 6.2 Attribution factors $(r_i - b_i) > 0$, $(w_i - W_i) > 0$

The contribution to excess return in category i is the sum of the areas representing selection $W_i \times (r_i - b_i)$, allocation $(w_i - W_i) \times b_i$, and interaction $(w_i - W_i) \times (r_i - b_i)$. In this particular example all three areas are positive contributions because $(r_i - b_i) > 0$ and $(w_i - W_i) > 0$.

Should $(r_i - b_i) < 0$ then it is less obvious whether category i in the portfolio or the benchmark is making the greater contribution (see Figure 6.3). Because $(r_i - b_i) < 0$ then the areas representing selection is negative. Because $(r_i - b_i) < 0$ and $(w_i - W_i) > 0$ then the area representing interaction is also negative but because $(w_i - W_i) > 0$ and $b_i > 0$ then allocation is a positive contribution. The contribution to excess return in category i is still the sum of the areas representing selection $W_i \times (r_i - b_i)$, allocation $(w_i - W_i) \times b_i$ and interaction $(w_i - W_i) \times (r_i - b_i)$.

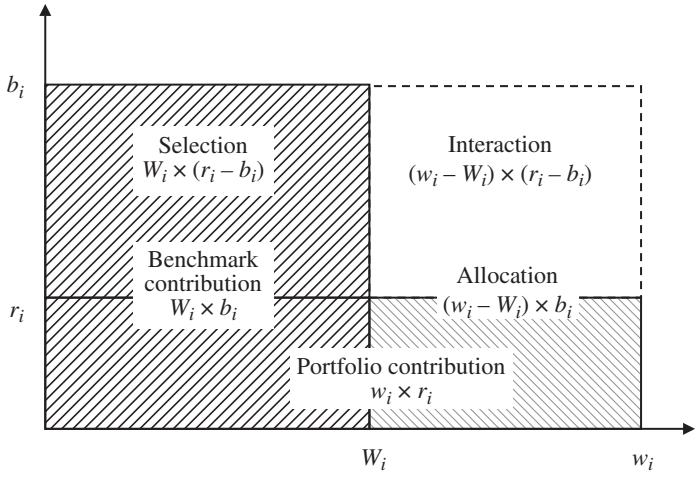

Figure 6.3 Attribution factors $(r_i - b_i) < 0$, $(w_i - W_i) > 0$

Similar problems arise if $(r_i - b_i) > 0$ *and* $(w_i - W_i) < 0$ (see Figure 6.4). Because $(r_i - b_i) > 0$ then the area representing selection is positive. Because $(r_i - b_i) > 0$ and $(w_i - W_i) < 0$ then the area representing interaction is negative, reducing some of the positive contribution from selection. Because $(w_i - W_i) < 0$ and $b_i > 0$ then allocation is a negative contribution in this example. The contribution to excess return in category i is still the sum of the areas representing selection $W_i \times (r_i - b_i)$, allocation $(w_i - W_i) \times b_i$ and interaction $(w_i - W_i) \times (r_i - b_i)$.

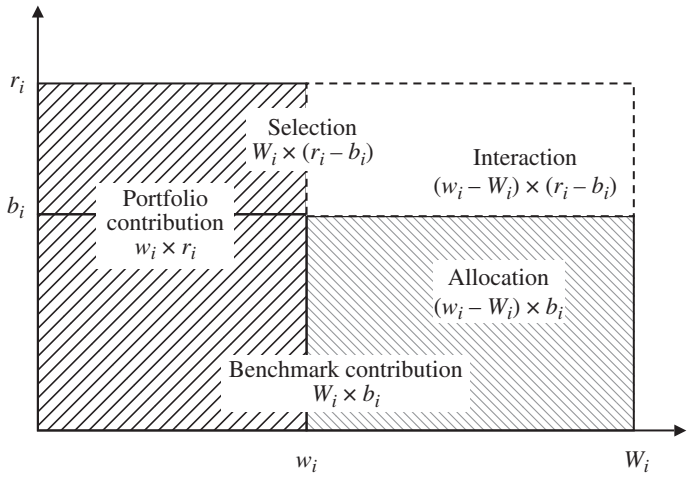

Figure 6.4 Attribution factors $(r_i - b_i) > 0$, $(w_i - W_i) < 0$

Finally, if $(r_i - b_i) < 0$ and $(w_i - W_i) < 0$ and the benchmark return is positive, then clearly the benchmark contribution in category i is greater than the portfolio contribution from category i (see Figure 6.5). In this example, the area representing selection $W_i \times (r_i - b_i)$ is

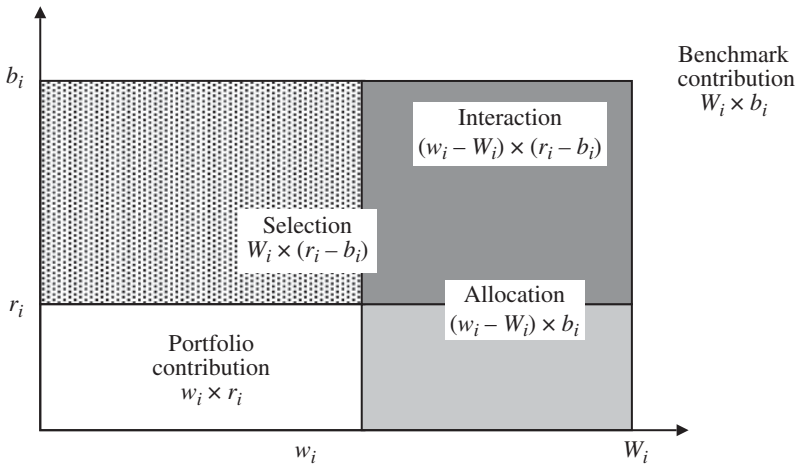

Figure 6.5 Attribution factors $(r_i - b_i) < 0$, $(w_i - W_i) < 0$

negative and the area representing allocation $(w_i - W_i) \times b_i$ is negative. In this example, the area representing interaction $(w_i - W_i) \times (r_i - b_i)$ represents the overlap between selection and allocation. Since this contribution cannot be lost twice over, it is a positive contribution to excess return.

Table 6.3 provides the data for a simple numerical example of a three-category portfolio consisting of UK, Japanese and US equities.

Table 6.3 Sample three-category portfolio

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)
UK equities	40	40	20	10
Japanese equities	30	20	−5	−4
US equities	30	40	6	8
Total	**100**	**100**	**8.3**	**6.4**

Total portfolio, benchmark and notional funds are calculated in Exhibit 6.1.

Exhibit 6.1 Return calculations

Using the data from Table 6.3 the portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ is:

$$r = 40\% \times 20\% + 30\% \times -5\% + 30\% \times 6\% = 8.3\%$$

The benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ is:

$$b = 40\% \times 10\% + 20\% \times -4\% + 40\% \times 8\% = 6.4\%$$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 40\% \times 10\% + 30\% \times -4\% + 30\% \times 8\% = 5.2\%$$

The selection notional return $r_S = \sum_{i=1}^{i=n} W_i \times r_i$ is:

$$r_S = 40\% \times 20\% + 20\% \times -5\% + 40\% \times 6\% = 9.4\%$$

The challenge of attribution analysis is to break down and quantify the decisions made by the portfolio manager contributing to the arithmetic excess return of 1.9%.

Using Equation 6.6 we can derive the asset allocation effects shown in Exhibit 6.2 for the data in Table 6.3.

Exhibit 6.2 Asset allocation

Total asset (or country) allocation:

$$b_S - b = 5.2\% - 6.4\% = -1.2\%$$

Individual country asset allocation effects are:

$$A_i = (w_i - W_i) \times b_i$$

UK equities	$(40\% - 40\%) \times 10\% = 0.0\%$
Japanese equities	$(30\% - 20\%) \times -4.0\% = -0.4\%$
US equities	$(30\% - 40\%) \times 8\% = -0.8\%$
Total	$0.0\% - 0.4\% - 0.8\% = -1.2\%$

In Exhibit 6.2 the portfolio weight in UK equities is exactly in line with the benchmark weight, therefore there is no contribution to asset allocation in this category.

There is, however, an overweight position of 10% in Japanese equities which, when applied to the negative market return in Japanese equities of –4.0%, results in a negative contribution of −0.4%.

If there is an overweight position in a portfolio it follows that there must be a least one underweight position. In Table 6.3 there is a 10% underweight position in US equities, resulting in a negative contribution of −0.8% when applied to the positive market return in the US market of 8.0%.

The total contribution to arithmetic excess return from asset allocation is –1.2%.

Using Equation 6.10 stock selection effects are calculated in Exhibit 6.3.

Exhibit 6.3 Stock selection

Total stock selection:

$$r_S - b = 9.4\% - 6.4\% = 3.0\%$$

Individual country stock selection effects are:

$$S_i = W_i \times (r_i - b_i)$$

UK equities	$40\% \times (20\% - 10\%) = 4.0\%$
Japanese equities	$20\% \times (-5.0\% + 4.0\%) = -0.2\%$
US equities	$40\% \times (6.0\% - 8.0\%) = -0.8\%$
Total	$4.0\% - 0.2\% - 0.8\% = 3.0\%$

UK equity performance is very strong, outperforming the benchmark by 10%; the benchmark suggests that 40% of the portfolio should be invested in this category resulting in a 4.0% contribution to arithmetic excess return.

Japanese equities underperformed by 1%; the benchmark suggested a 20% weighting, therefore resulting in a negative contribution of −0.2% from Japanese stock selection.

US equity performance is also poor, underperforming by 2%; the benchmark suggests a 40% weighting, therefore generating a negative contribution of −0.8% from US stock selection.

Total contribution to arithmetic excess return from stock selection is +3.0%. Combining asset allocation of –1.2% and stock selection of +3.0%, 1.8% of added value is explained. The remaining term is interaction calculated by Equation 6.16 as demonstrated in Exhibit 6.4.

Exhibit 6.4 Interaction

Total interaction in Table 6.3 is:

$$r - r_S - b_S + b = 8.3\% - 9.4\% - 5.2\% + 6.4\% = 0.1\%$$

Individual interaction effects are:

$$I_i = (w_i - W_i) \times (r_i - b_i)$$

UK equities	$(40\% - 40\%) \times (20\% - 10\%) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times (-5.0\% + 4.0\%) = -0.1\%$
US equities	$(30\% - 40\%) \times (6.0\% - 8.0\%) = 0.2\%$
Total	$0.0\% - 0.1\% + 0.2\% = 0.1\%$

The overall contribution from interaction is small. For UK equities the portfolio weight is in line with the benchmark weight and therefore there is no contribution to interaction.

For Japanese equities there is an asset allocation bet of 10%; we have 10% more of this underperforming asset category than suggested by the benchmark, therefore causing a further negative impact of −0.1%.

In US equities there is an underweight bet of 10% in this underperforming category. There is less of this underperforming category than the benchmark suggests, therefore the combined effect of an underweight position in an underperforming category is an added value of +0.2%. Total contribution from interaction is +0.1%.

The arithmetic excess is now –1.2% asset allocation, plus 3.0% stock selection, plus 0.1% interaction, totalling 1.9%.

The attribution results are summarised in Table 6.4.

Table 6.4 Brinson, Hood and Beebower attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times b$ (%)	Stock selection $W_i \times (r_i - b_i)$ (%)	Interaction $(w_i - W_i) \times (r_i - b_i)$ (%)
UK equities	40	40	20	10	0.0	4.0	0.0
Japanese equities	30	20	−5	−4	−0.4	−0.2	−0.1
US equities	30	40	6	8	−0.8	−0.8	0.2
Total	**100**	**100**	**8.3**	**6.4**	**−1.2**	**3.0**	**0.1**
Arithmetic excess return			**1.9**				

Clearly, this attribution model successfully breaks down the sources of arithmetic excess return – but does it reflect the investment decision process of the portfolio manager?

For the most part, asset allocation decisions are taken in the context of an overall benchmark return; the asset allocator is not seeking to be overweight in positive markets but rather to be overweight in markets that outperform the overall benchmark. The asset allocator will have lost value by being overweight in a market with a positive return that nevertheless returns less than the overall benchmark. We therefore need an attribution model that follows this decision process.

BRINSON AND FACHLER

In the Brinson, Hood and Beebower model,[13] all overweight positions in positive markets will generate positive attribution factors irrespective of the overall benchmark return while all overweight positions in negative markets will generate negative attribution factors.

[13] In fact, the original Brinson, Hood and Beebower article does not attempt to attribute returns to individual categories. In all likelihood I do not believe the authors intended their top-level formulae to be applied to individual categories as shown; however, over the years many practitioners have done just that.

Clearly, if the asset allocator is overweight in a negative market that has outperformed the overall benchmark, there should be a positive effect.

The Brinson and Fachler model solves this problem by modifying the asset allocation factor to compare returns against the overall benchmark as follows:

$$b_s - b = \sum_{i=1}^{i=n}(w_i - W_i) \times b_i = \sum_{i=1}^{i=n}(w_i - W_i) \times (b_i - b) \tag{6.18}$$

Since $\sum_{i=1}^{i=n} w_i = \sum_{i=1}^{i=n} W_i = 1$ the contribution to asset allocation in the ith category is now:

$$A_i = (w_i - W_i) \times (b_i - b) \tag{6.19}$$

Graphically extending Figure 6.2 to include the benchmark return in Figure 6.6 we observe no change to the areas representing selection and interaction, but allocation is now described by the area $(w_i - W_i) \times (b_i - b)$.

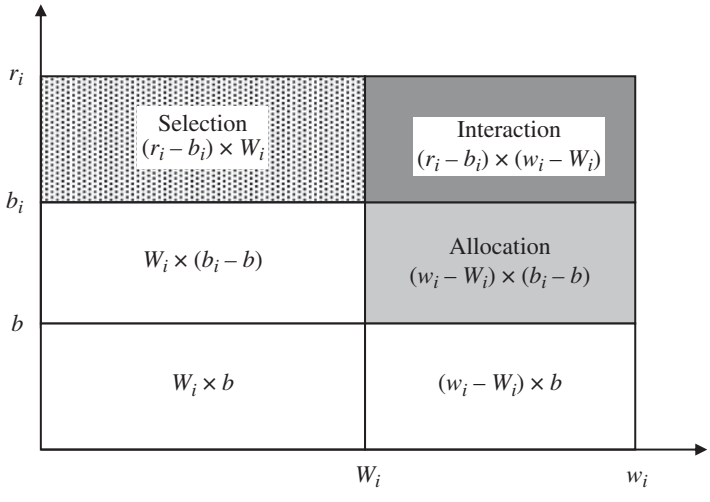

Figure 6.6 Brinson and Fachler

Equation 6.18 demonstrates that the sum of allocation areas in Figure 6.6 for all categories is equal to the sum of allocation areas in Figure 6.2.

Since $\sum_{i=1}^{n=1} W_i = 1$ the benchmark return is derived by the sum of areas $W_i \times b$ for all categories:

$$\sum_{i=1}^{i=n} W_i \times b = b \tag{6.20}$$

and since both $\sum_{i=1}^{n=1} W_i = 1$ and $\sum_{i=1}^{n=1} w_i = 1$ the sum of areas $(w_i - W_i) \times b$ reduces to zero:

$$\sum_{i=1}^{n=1}(w_i - W_i) \times b = 0 \tag{6.21}$$

Revised Brinson and Fachler asset allocation effects are calculated in Exhibit 6.5.

Exhibit 6.5 Brinson and Fachler asset allocation

$b_S - b = 5.2\% - 6.4\% = -1.2\%$	
Individual country asset allocation effects are:	
$A_i = (w_i - W_i) \times (b_i - b)$	
UK equities	$(40\% - 40\%) \times (10.0\% - 6.4\%) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times (-4.0\% - 6.4\%) = -1.04\%$
US equities	$(30\% - 40\%) \times (8.0\% - 6.4\%) = -0.16\%$
Total	$0.0\% - 1.04\% - 0.16\% = -1.2\%$

The impact in Japanese equities is much greater. In addition to being overweight in a negative market, which costs −0.4%, we are also rightly penalised the opportunity cost of not being invested in the overall market return of 6.4%, generating a further cost of $10\% \times -6.4\% = -0.64\%$, resulting in a total impact of –1.04%.

The impact in US equities is much smaller. Although being underweight in a positive market costs −0.8%, we must add back the opportunity cost of being invested in the overall market return of 6.4%, generating a contribution of $-10\% \times -6.4\% = 0.64\%$ resulting in a total impact of −0.16%. The total asset allocation remains –1.2%; only the allocation between individual categories is altered.

The revised attribution effects are summarised in Table 6.5. The total arithmetic excess return, of course, remains 1.9%.

Table 6.5 Brinson and Fachler attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times (b_i - b)$ (%)	Stock selection $W_i \times (r_i - b_i)$ (%)	Interaction $(w_i - W_i) \times (r_i - b_i)$ (%)
UK equities	40	40	20	10	0.0	4.0	0.0
Japanese equities	30	20	−5	−4	−1.04	−0.2	−0.1
US equities	30	40	6	8	−0.16	−0.8	0.2
Total	**100**	**100**	**8.3**	**6.4**	**−1.2**	**3.0**	**0.1**
Arithmetic excess return			**1.9**				

Note

I have a strong preference for Brinson and Fachler over Brinson, Hood and Beebower – it is consistent with most investment decision processes.

INTERACTION

A flaw of both Brinson models is the inclusion of the interaction or other term. Interaction is not part of the investment decision process; you are unlikely to identify any individual portfolio manager responsible for adding value through interaction.

While it is true that interaction reflects the combined effect of asset allocation bets with stock selection decisions, asset managers simply do not seek to add value through interaction. For most investment decision processes, the asset allocation decision comes first and stock selection decisions are taken after the cash has been allocated.

For genuine bottom-up stock pickers, asset allocation decisions are not made; therefore, the attribution model should reflect this process and measure the contribution of each stock decision to the overall performance ignoring asset allocation.

Because interaction is not well understood, presumably because it is not intuitively part of the investment decision process, it is often abused. It may be ignored and not shown, randomly allocated to other factors, split proportionally or simply split 50:50 between stock selection and asset allocation and therefore potentially misleading the user.

Assuming asset allocation decisions are taken first, then the contribution from stock selection must be:

$$r - b_s = \sum_{i=1}^{i=n} w_i \times r_i - \sum_{i=1}^{i=n} w_i \times b_i = \sum_{i=1}^{i=n} w_i \times (r_i - b_i) \tag{6.22}$$

Equivalent to quadrant IV – quadrant II from Figure 6.1.

The contribution to stock selection in the *i*th category is now:

$$S_i = w_i \times (r_i - b_i) \tag{6.23}$$

In effect, interaction is now included in the stock selection effect. This is not an arbitrary decision: for most investment decision processes this results directly from the natural order of investment decisions.

Note

I do not recommend the independent calculation of interaction. No asset manager seeks to add value specifically from interaction; it is not part of the investment decision process. Stock selection can be calculated directly without calculating interaction – because of the order of most investment decision processes, interaction naturally belongs in stock selection.

Figure 6.7 demonstrates graphically the revised impact on individual categories.

Revised stock selection effects, including interaction, are calculated in Exhibit 6.6.

Actual portfolio weights are now used to calculate stock selection effects. Stock pickers rightly point out that their contribution to stock selection is impacted by the weight of assets chosen by the asset allocator. This is true for either portfolio or benchmark weight; there is no particular advantage in calculating the stock selection impact had the portfolio been at the benchmark weight. The individual stock picker should be judged by the performance within the asset category, not their contribution to overall performance. The revised results are summarised in Table 6.6.

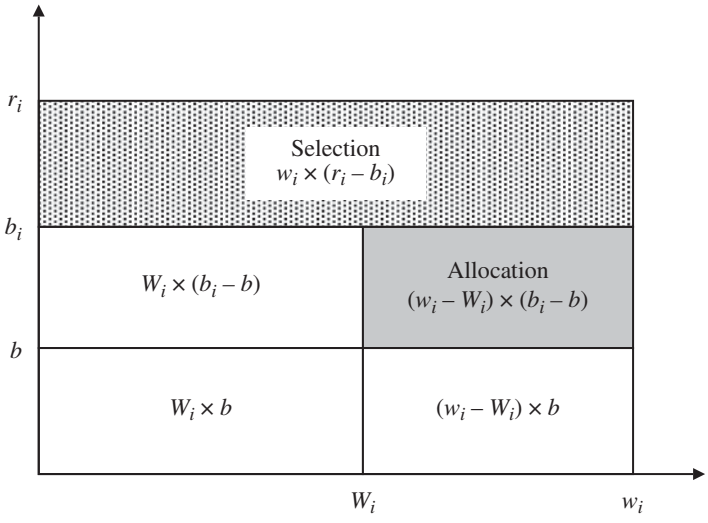

Figure 6.7 Brinson and Fachler (selection and interaction combined)

Exhibit 6.6 Stock selection, including interaction

Total stock selection, including interaction, using the data from Table 6.3 is:

$$r - b_S = 8.3\% - 5.2\% = 3.1\%$$

Individual country stock selection effects are:

UK equities	$40\% \times (20\% - 10\%) = 4.0\%$
Japanese equities	$30\% \times (-5.0\% + 4.0\%) = -0.3\%$
US equities	$30\% \times (6.0\% - 8.0\%) = -0.6\%$
Total	$4.0\% - 0.3\% - 0.6\% = 3.1\%$

Table 6.6 Brinson and Fachler (stock selection and interaction combined)

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times (b_i - b)$ (%)	Stock selection $w_i \times (r_i - b_i)$ (%)
UK equities	40	40	20	10	0.0	4.0
Japanese equities	30	20	−5	−4	−1.04	−0.3
US equities	30	40	6	8	−0.16	−0.6
Total	**100**	**100**	**8.3**	**6.4**	**−1.2**	**3.1**
Arithmetic excess return			**1.9**			

Rather than the Brinson quadrants of Figure 6.1, I prefer to think in terms of the steps of the investment decision process illustrated in Figure 6.8.

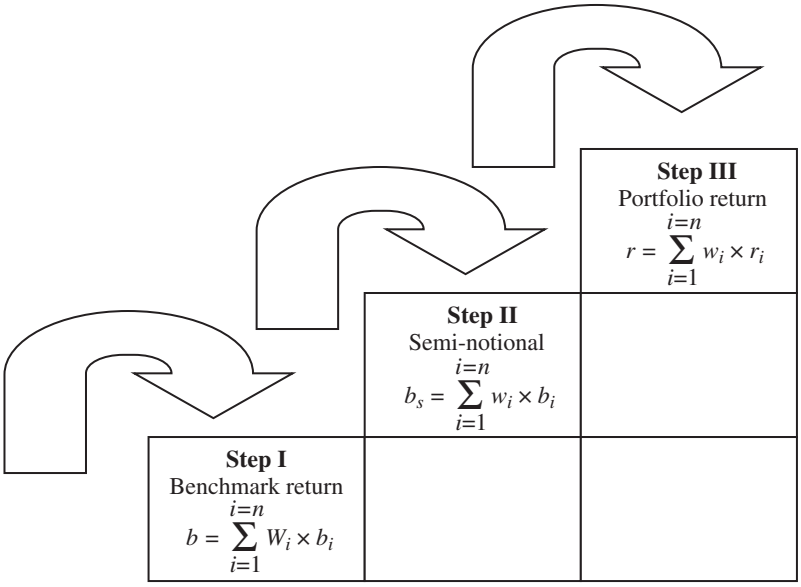

Figure 6.8 Steps of the investment decision process

GEOMETRIC EXCESS RETURN ATTRIBUTION

The Brinson models described so far quantify arithmetic excess return. In Chapter 4 an alternative geometric definition of excess return was proposed to measure the added value of asset managers.

A number of geometric excess return attribution models (geometric methods) have been developed over the years (Allen[14]; Bain[15]; Burnie, Knowles and Teder[16]; and Bacon[17]). These methods are similar and in most cases were in use long before being published externally.

The Brinson model can be easily extended to break down the geometric excess return:

$$\frac{(1+r)}{(1+b)} - 1 \tag{4.25}$$

Asset allocation

To identify the contribution from asset allocation we can use the same intermediate or semi-notional fund we used in the Brinson method, but this time using the geometric rather than the arithmetic difference.

$$\frac{(1+b_S)}{(1+b)} - 1 \tag{6.24}$$

[14] Allen, "Performance Attribution of Global Equity Portfolios" (1991).
[15] Bain, *Investment Performance Measurement* (1996).
[16] Burnie, Knowles and Teder, "Arithmetic and Geometric Attribution" (1998).
[17] Bacon, "Excess Returns – Arithmetic or Geometric" (2002).

The contribution to geometric asset allocation in the ith category is now:

$$A^G{}_i = (w_i - W_i) \times \left(\frac{1 + b_i}{1 + b} - 1 \right) \tag{6.25}$$

Note the total geometric asset allocation A^G:

$$A^G = \sum_{i=1}^{i=n} A_i{}^G = \frac{(1 + b_S)}{1 + b} - 1 \tag{6.26}$$

Formally, this type of method might be described as a mixed geometric arithmetic method since the individual category allocations sum to the total geometric asset allocation. In a pure geometric model, the individual category allocations would compound to the total geometric allocation.

Equation 6.24 is analogous to Equation 6.19; the geometric difference of the category return against the overall benchmark is used rather than the arithmetic difference. A more detailed proof can be found in Appendix A.

Stock selection

Similarly, to identify the total contribution to stock selection we can use the ratio of the portfolio return compared to the semi-notional return:

$$\frac{(1 + r)}{(1 + b_S)} - 1 \tag{6.27}$$

The contribution to geometric stock selection in the ith category is now:

$$S^G{}_i = w_i \times \left(\frac{1 + r_i}{1 + b_i} - 1 \right) \times \frac{(1 + b_i)}{(1 + b_S)} \tag{6.28}$$

Equation 6.28 is not quite as expected extending from Equation 6.23; there is an unexpected term, $\frac{1 + b_i}{1 + b_S}$. This term is required because outperformance in a category whose benchmark return is already performing well will add more value geometrically than the equivalent outperformance in a category whose benchmark return is not performing well. Again, a more detailed proof can be found in Appendix A.

Equation 6.28 simplifies to Equation 6.29 the arithmetic difference divided by the semi-notional fund preferred by Burnie, Knowles and Teder:

$$S^G{}_i = w_i \times \frac{(r_i - b_i)}{(1 + b_S)} \tag{6.29}$$

Note the total geometric stock selection S^G:

$$S^G = \sum_{i=1}^{i=n} S_i{}^G = \frac{(1 + r)}{(1 + b_S)} - 1 \tag{6.30}$$

The total stock selection and asset allocation effects compound together to produce the geometric excess return:

$$\frac{(1 + r)}{(1 + b_S)} \times \frac{(1 + b_S)}{(1 + b)} - 1 = \frac{(1 + r)}{(1 + b)} - 1 \tag{6.31}$$

or

$$(1 + S^G) \times (1 + A^G) - 1 = \frac{(1 + r)}{(1 + b)} - 1 = g \tag{6.32}$$

The contribution to geometric excess return from asset allocation is calculated in Exhibit 6.7.

Exhibit 6.7 Geometric asset allocation

Geometric asset allocation for data in Table 6.3 is:

$$\frac{(1 + b_S)}{(1 + b)} - 1 = \frac{1.052}{1.064} - 1 = -1.13\%$$

Individual country asset allocation effects are:

$$A^G{}_i = (w_i - W_i) \times \left(\frac{1 + b_i}{1 + b} - 1\right)$$

UK equities	$(40\% - 40\%) \times \left(\frac{1.10}{1.064} - 1\right) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times \left(\frac{0.96}{1.064} - 1\right) = -0.98\%$
US equities	$(30\% - 40\%) \times \left(\frac{1.08}{1.064} - 1\right) = -0.15\%$
Total	$0.0\% - 0.98\% - 0.15\% = -1.13\%$

Given that the benchmark return is positive the geometric excess return is less than the arithmetic excess return. The contributions to asset allocation are of the same order but slightly less than the arithmetic asset allocation. The sign will always be the same. The contribution to geometric excess return from stock selection is calculated in Exhibit 6.8.

Exhibit 6.8 Geometric stock selection

Geometric stock selection including interaction for data in Table 6.3 is:

$$\frac{(1 + r)}{(1 + b_S)} - 1 = \frac{1.083}{1.052} - 1 = 2.95\%$$

Individual country stock selection effects are:

$$S^G{}_i = w_i \times \left(\frac{1 + r_i}{1 + b_i} - 1\right) \times \frac{(1 + b_i)}{(1 + b_S)}$$

UK equities	$40\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.052} = 3.80\%$
Japanese equities	$30\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.96}{1.052} = -0.28\%$
US equities	$30\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.052} = -0.57\%$
Total	$3.80\% - 0.28\% - 0.57\% = 2.95\%$

Again, as expected, the geometric stock selection effects are similar to the arithmetic stock selection effects but slightly smaller in magnitude. As shown in Exhibit 6.9 the allocation and stock selection effects compound to provide the total geometric excess return.

Exhibit 6.9 Total geometric excess return

Total geometric excess return:

$$\frac{(1+r)}{(1+b)} - 1 = \frac{1.083}{1.064} - 1 = 1.79\%$$

$$\frac{(1+r)}{(1+b)} - 1 = \frac{(1+r)}{(1+b_S)} \times \frac{(1+b_S)}{(1+b)} - 1 = \frac{1.083}{1.052} \times \frac{1.052}{1.064} - 1 = 1.79\%$$

The geometric attribution effects are summarised in Table 6.7.

Note

Because of my strong preference for geometric excess return, I naturally prefer geometric attribution methodologies. The majority of the remaining attribution examples are therefore geometric.

SECTOR WEIGHTS

In the Brinson-type models (arithmetic or geometric) we have assumed from Equation 6.3 that the sum of the weights and returns for each asset category will equal the total portfolio return and that the weights will sum to 100%.

To ensure that Equation 6.3 holds we must use return calculation methodologies from Chapter 3 that can be disaggregated, that is, broken down into the contributions to return from each active decision of the investment process.

The return methodology for each category, segment or sector must be identical to that used for the overall return. Because internal rates of return assume a single constant force of return throughout the period of measurement this methodology is not suitable for attribution.

The Dietz methodologies readily fit into the Brinson model since the returns can be disaggregated. Although the total return will be completely explained, transactions during the period of measurement may result in the contributions to asset allocation and stock selection being calculated incorrectly due to the Dietz method weighting assumptions. The more detailed analysis of our standard attribution in Table 6.8 illustrates this point.

The portfolio returns used to calculate the attribution effects summarised in Table 6.8 have actually been calculated using a simple Dietz formula. Using time-weighted returns, summarised for each sub-period in Table 6.8, it would appear that US equities outperform rather than underperform the benchmark.

To calculate the true attribution, we must calculate attribution effects for the period immediately before the cash flow and then after the cash flow, summarised in Table 6.9.

Over the entire period asset allocation effects are minimal; all markets performed equally well in the first period and the bet sizes were small for the second period.

Our first attribution did not capture the portfolio's move to a neutral asset allocation position at the time of the cash flow. Although the original attribution reconciled, in this example the Dietz-type returns do not provide the full picture, as there has been a transfer of effects between asset allocation and stock selection caused by the choice of methodology.

Table 6.7 Geometric attribution (stock selection and interaction combined)

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i)\times\left(\frac{1+b_i}{1+b}-1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_i}{1+b_i}-1\right)\times\frac{(1+b_i)}{(1+b_S)}$ (%)
UK equities	40	40	20	10	0.0	3.8
Japanese equities	30	20	−5	−4	−0.98	−0.28
US equities	30	40	6	8	−0.15	−0.57
Total	**100**	**100**	**8.3**	**6.4**	**−1.13**	**2.95**
Geometric excess return			**1.79**			

Table 6.8 More detailed analysis of standard attribution

	UK equities	Japanese equities	US equities
Start value	£400	£400	£200
End value	£480	£185	£418
Cash flow	£0	−£200	£200
Market value at time of cash flow	n/a	£420	£220
Simple Dietz	$\frac{480-400}{400} = 20.0\%$	$\frac{185-400+200}{400-\frac{200}{2}} = -5\%$	$\frac{418-200-200}{200+\frac{200}{2}} = 6.0\%$
Time-weighted	$\frac{480}{400} - 1 = 20.0\%$	$\frac{420}{400} \times \frac{185}{220} - 1 = -11.7\%$	$\frac{220}{200} \times \frac{418}{420} - 1 = 9.48\%$

Table 6.9 Attribution effects for the period before and after the cash flow

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
First period						
UK equities	40	40	10	10	0.0	0.0
Japanese equities	40	20	5	10	0.0	−1.8
US equities	20	40	10	10	0.0	0.0
Total	**100**	**100**	**8.0**	**10.0**	**0.0**	**−1.8**
Second period						
UK equities	40.74	40.0	9.1	0.0	0.0	3.8
Japanese equities	20.37	20	−15.9	−8.6	−0.0	−0.7
US equities	38.89	40	−0.5	−1.8	−0.0	0.5
Total	**100**	**100**	**0.3**	**−2.4**	**−0.00**	**3.7**
Combined period						
UK equities	**n/a**	**n/a**	20.0	10	0.0	3.8
Japanese equities	**n/a**	**n/a**	−11.7	−4.0	0.0	−2.5
US equities	**n/a**	**n/a**	9.5	8.0	0.0	0.5
Total			**8.3**	**6.4**	**−0.0**	**1.8**

In reality, US stock selection is much better than initially presented and Japanese stock selection much worse. The more frequent the attribution, ideally daily, the more accurate the result.

FREQUENCY OF ANALYSIS

The superiority of more frequent attribution analysis is not unchallenged. DiBartolomeo[18] challenges the assumption that aggregated daily attribution analysis is more appropriate than monthly aggregated analysis and suggests that applying typical attribution methods to daily

[18] DiBartolomeo, "Just Because We Can Doesn't Mean We Should" (2003).

data draws on assumptions that are highly biased and unreliable. Beyond just observing the attributed returns, we must be able to judge their statistical significance. DiBartolomeo suggests there are four fallacies of daily observation:

(1) Daily performance is economically meaningful
Not necessarily – for example, information is lost by adjusting for the timing of cash flow in time-weighted returns.
(2) Daily returns are themselves accurate
Not necessarily – many securities are traded on multiple exchanges or electronic markets at different times, so exactly which closing price is used to compute the daily returns can have a sizeable impact. It is also difficult to coordinate with index providers the timing of corporate actions such as dividends, spin-offs and mergers.
(3) The assumption of independent observations
Investment portfolios do not start as a fresh set of management decisions every trading day. The reasonableness of an independent "start fresh" assumption is weak. Substantial path dependencies are present in portfolio management in practice, reducing any justification for the assumption of independent periodic observations of manager skill.
(4) Parametric distributional assumptions
Most of the academic literature illustrates that the imperfections of our assumptions with respect to quarterly or monthly return data are small, while for daily data we must consistently reject the correctness of the assumptions. In general, daily security returns have fat-tailed distributions with large return events occurring far more often than would be consistent with a normal distribution. This is less of a problem with quarterly or monthly returns.

DiBartolomeo concludes that daily performance attribution compounds to a more accurate measure of the time-weighted return than attribution using less frequent data, but at a tremendous cost in our ability to judge the statistical significance of the results.

Darling and MacDougall[19] certainly question the benefits of more frequent and more detailed performance measurement, suggesting that the cost of daily, stock-level attribution is difficult to justify.

⚠ Caution

Given recent developments in technology and workflow aids, I suspect Darling and MacDougall's views may have changed. Just a short time ago technology was behind the curve, but now I fear performance analysts are behind the curve with more data and workflow issues to solve than they have capacity.

Darling and MacDougall suggest that monthly modified Dietz calculations may actually be near the optimum balance.

On the other hand, Frongello[20] makes the case for daily attribution analysis. He explains that, in the real world, risk factor mismatches may occur unintentionally due to neglect, accidents or simply to changes in the risk profile of securities. Performance attribution should seek to

[19] Darling and MacDougall, "Using Performance Statistics: Have Measurers Lost the Plot?" (2002/2003).
[20] Frongello, "Linking of Attribution Results" (2006).

explain performance due to all risk factor mismatches. Daily attribution is the best method for monitoring the status and consequence of risk factor mismatches.

Zangari and Bayraktar[21] suggest that, if done methodically, daily attribution should produce results that are superior to monthly attribution. They point out that, regardless of the methodology employed, ultimately it is the process governing attribution that is critical. The five core components of what they consider to be a good return attribution process are as follows:

(1) Data management and quality control. This is the most important. Positions, prices, corporate actions, etc. must be accurately captured, processed and incorporated.
(2) Infrastructure. Without a proper infrastructure (efficient application of hardware and databases), the chance of errors and delays increases.
(3) Analytics. Algorithms that perform the attribution calculations.
(4) Methods. For example, daily buy and hold, monthly buy and hold, and transaction-based attribution.
(5) Reports. The way in which information is presented is important, requiring timely, accurate, and easy-to-understand attribution reports.

They conclude that process plays a critical part in determining the accuracy of attribution results. Return attribution should be based on a process that monitors key input data and evaluates attribution results on a daily basis.

Note

I have no doubt that daily calculation of attribution effects is more accurate and, with appropriate technology, easily achievable; therefore, I strongly recommend its use. Daily analysis, on the other hand, is less appropriate: we are in the main dealing with asset owners with long-term time horizons; daily calculation is merely the foundation for accurate long-term analysis. Daily attribution is an essential diagnostic tool for the middle and back office, and any-day to any-day reporting, facilitated by daily calculation, is useful for both the front and middle office for analysis and communication related to periods of extreme results.

Security-level attribution

If the investment decision process is genuinely "bottom up" then it is not appropriate to calculate asset or sector allocation effects. For security-level attribution the investment decision process is effectively over- or underweighting an individual security; therefore to measure the contribution at security level we can use the asset allocation formula in Equations 6.19 and 6.25, respectively, for arithmetic and geometric attribution. The traditional stock selection formula, Equations 6.23 and 6.28, effectively measures the ability to add value within that security by timing trades. This timing attribution effect includes transaction costs, the impact of bid/offer spread and gain or loss resulting from transacting the trade at prices other than end of period, typically within the day.

Security-level attribution is demonstrated using data from Table 6.10, which contains five securities measured against a simple five security benchmark.

[21] Zangari and Bayraktar, "Which Is Better: Daily or Monthly Attribution?" (2005/2006).

Table 6.10 Security-level attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Stock allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Timing $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
Security 1	10.2	9.3	15.0	15.0	0.08	0.0
Security 2	5.2	0.1	−25.0	−25.0	−1.48	0.0
Security 3	15.0	20.8	3.4	3.4	0.11	0.0
Security 4	22.0	14.7	−5.2	−5.2	−0.74	0.0
Security 5	47.6	55.1	6.9	7.5	−0.15	−0.28
Total	**100**	**100**	**2.88**	**5.46**	**−2.17**	**−0.28**
Geometric excess return			**−2.44**			

Detailed calculations are shown in Exhibits 6.10 and 6.11.

Exhibit 6.10 Geometric "bottom up" security-level allocation

Geometric stock allocation for the data in Table 6.10 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.0317}{1.0546} - 1 = -2.17\%$$

Individual security (or stock) allocation effects are:

Security 1	$(10.2\% - 9.3\%) \times \left(\frac{1.15}{1.0546} - 1\right) = 0.08\%$
Security 2	$(5.2\% - 0.1\%) \times \left(\frac{0.75}{1.0546} - 1\right) = -1.48\%$
Security 3	$(15.0\% - 20.8\%) \times \left(\frac{1.034}{1.0546} - 1\right) = 0.11\%$
Security 4	$(22.0\% - 14.7\%) \times \left(\frac{0.948}{1.0546} - 1\right) = -0.74\%$
Security 5	$(47.6\% - 55.1\%) \times \left(\frac{1.075}{1.0546} - 1\right) = -0.15\%$
Total	$0.08\% - 1.48\% + 0.11\% - 0.74\% - 0.15\% = -2.17\%$

Exhibit 6.11 Geometric "bottom up" security-level allocation

Geometric timing for the data in Table 6.10 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.0288}{1.0317} - 1 = -0.28\%$$

Individual security timing effects are:

Security 1	$10.2\% \times \left(\frac{1.15}{1.15} - 1\right) \times \left(\frac{1.15}{1.0317}\right) = 0.0\%$
Security 2	$5.2\% \times \left(\frac{0.75}{0.75} - 1\right) \times \left(\frac{0.75}{1.0317}\right) = 0.0\%$
Security 3	$15.0\% \times \left(\frac{1.034}{1.034} - 1\right) \times \left(\frac{1.034}{1.0317}\right) = 0.0\%$
Security 4	$22.0\% \times \left(\frac{0.948}{0.948} - 1\right) \times \left(\frac{0.948}{1.0317}\right) = 0.0\%$
Security 5	$47.6\% \times \left(\frac{1.069}{1.075} - 1\right) \times \left(\frac{1.075}{1.0317}\right) = -0.28\%$
Total	$0.0\% + 0.0\% + 0.0\% + 0.0\% - 0.28\% = -0.28\%$

Interpretation of this result is fairly straightforward; geometrically, the portfolio has underperformed the benchmark by 2.44%, the largest contributor to which is an overweight position in underperforming Security 2, contributing –1.48% of geometric excess return.

The contribution from timing is negative and relatively small. Without any transactions it should be expected that the return of each security within the portfolio will be the same as the return in the benchmark, particularly if the same pricing source has been used. In this example for Security 5 there is timing contribution of −0.28%. This indicates some trading activity within the period; while it is possible to generate a positive trading or timing effect,

this would require trading at a sufficiently advantageous price to cover both transaction costs and the bid/offer spread. Therefore, typically timing effects will be negative.

It is inappropriate to roll up individual security allocation and security effects into sectors for genuine bottom-up decision processes. Sector attribution is calculated by reference to sector indexes; bottom-up stock selection is essentially a different investment process referencing individual security prices. The sum of individual stock allocations will not equal total stock selection within the sector.

Stock- or security-level attribution is clearly required for bottom-up portfolio managers who are taking investment decisions at an individual security level. Performance analysts learn early in their careers that the attribution analysis must replicate the investment decision process. It seems awkward to use the traditional Brinson and Fachler allocation formulae to measure the impact of individual stock decisions, but this does replicate the investment decisions taken by the bottom-up style portfolio manager. The manager is making investment decisions at the stock level, choosing to be underweight or overweight individual securities, which the traditional allocation formulae measures precisely.

In the context of stock-level analysis, the traditional Brinson and Fachler selection formulae actually measures the impact of the security return within the portfolio differing from the return of that same security within the benchmark. This of course could be the result of an alternative pricing source or corporate action factor used in the portfolio compared to the benchmark, but is most likely caused by a transaction during the period.

But stock-level attribution is not only for bottom-up stock pickers; it's also valuable to "top-down" managers, and other stakeholders in the investment decision process. Attribution analysis of all forms is useful to portfolio managers, particularly when performance is poor. Client meetings can be difficult enough at the best of times but if the portfolio manager can't articulate why they've underperformed and, crucially, why the prospects for future outperformance are better, then there is a good chance the client will be lost. Stock-level analysis provides the possibility of fully explaining performance and, in desperation, the prospect of clutching a few straws and pointing out isolated good decisions, or perhaps highlighting a single bad decision that is unlikely to be repeated.

In the middle office, inhabited by performance analysts and risk controllers, stock-level attribution helps ensure that the actual performance return is accurate. In the context of the back office, daily stock-level attribution analysis can be used as a diagnostic tool to identify pricing and other operational errors.

Note

Admittedly, this is an old-fashioned view, but from my perspective the first responsibility of the performance analyst is to ensure that the calculated return is correct. With the foundation of a correct return, accurate risk and attribution analysis can then be undertaken.

Stock-level attribution is the vital cog in the asset manager's toolkit, providing quality control of the investment decision process, allowing effective communication with asset owners, understanding the behaviour of the portfolio, ensuring the return is accurately calculated and identifying errors.

Stock-level attribution provides large volumes of extremely valuable data, so much that it is easy to be overwhelmed. The key is to focus on the big impacts, perhaps the largest five positive contributions from overweight positions and the largest five negative contributions from overweight positions together with the largest five positive contributions from underweight positions and the largest five negative contributions from underweight positions. Special cases include "dodged bullets" and "missed opportunities"; these are zero-weight holdings in the portfolio for securities that are included in the benchmark. In concentrated portfolios these positions will be common, but will also exist in mildly active portfolios. Dodged bullets are positive contributions to excess return from securities held in the benchmark, but not in the portfolio, and have performed badly in the period. Missed opportunities are negative contributions to excess return from securities held in the benchmark, but not in the portfolio, that performed well in the period. From time to time asset managers will invest in securities not included in the benchmark, an off-benchmark decision. Cash is typically always present in portfolios, rarely included in benchmarks and therefore in effect is an off-benchmark decision. In rising markets, the impact of cash will be negative and in falling markets, positive. It is crucially important to attribute all impacts within a portfolio, particularly cash which over time is typically a large contributor to relative return. When measuring the impact of off-benchmark decisions it is important to choose the correct comparative benchmark for analysis (see the section on off-benchmark attribution).

Transaction costs

Almost all transaction-based attribution methodologies include transaction costs in the stock selection effect by default. Asset allocation effects are measured by reference to the category index and the overall benchmark only, with no allowance for transaction costs. Asset allocation decisions when implemented clearly generate transaction costs. These costs can be significant particularly for illiquid assets such as emerging markets, and should be allocated to the asset allocator, not the stock selector.

Off-benchmark (or zero-weight sector) attribution

From time to time, if allowed by their investment mandate, asset managers may invest in securities, or investment categories, not included in their assigned benchmark (off-benchmark decisions). The impact of the off-benchmark decision can be challenging to assess, since category benchmarks may not be pre-assigned. Depending on the investment decision process there are three appropriate approaches to attributing the impact of off-benchmark decisions:

(1) Stock selection only
(2) Asset allocation only
(3) Stock and asset allocation

Table 6.11 provides the data for an off-benchmark decision; the asset manager, presumably permitted by the investment mandate, has invested in emerging market securities that are not in the assigned benchmark.

If the rationale behind the asset manager's decision to buy an individual security in a country outside the benchmark is based solely on the merits of the security, then this is a pure security selection decision and therefore should be reflected as stock selection only. To represent a stock selection only decision correctly, the category benchmark for emerging markets should

Table 6.11 Off-benchmark decision

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)
UK equities	40	40	20	10
Japanese equities	30	20	−5	−4
US equities	20	40	6	8
Emerging markets	10	0	8	n/a
Total	**100**	**100**	**8.5**	**6.4**

use the overall benchmark return, as shown in Table 6.12. Since the strategic benchmark weight is 0%, the choice of category benchmark will not impact the total benchmark return. Return calculations are shown in Exhibit 6.12. Geometric asset allocation effects are calculated in Exhibit 6.13. Geometric stock selection effects are calculated in Exhibit 6.14.

Interpreting the results presented in Table 6.12, the asset manager has outperformed, contributing 3.29% from selection (including 0.15% from emerging markets) but losing –1.28% from allocation; a total geometric excess return of 1.97%. In line with the stock selection only investment decision process, any allocation to emerging markets will only generate stock selection effects and never any asset allocation effects because of using the total benchmark for the emerging markets benchmark as well.

Table 6.12 Off-benchmark decision – stock selection only

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
UK equities	40	40	20	10	0.00	3.81
Japanese equities	30	20	−5	−4	−0.98	−0.29
US equities	20	40	6	8	−0.30	−0.38
Emerging earkets	10	0	8	6.4	0.00	0.15
Total	**100**	**100**	**8.5**	**6.4**	**−1.28**	**3.29**
Geometric excess return			**1.97**			

Exhibit 6.12 Geometric off-benchmark – stock selection only return calculations

Using the data from Table 6.12 the portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ is:

$$r = 40\% \times 20\% + 30\% \times -5\% + 20\% \times 6\% + 10\% \times 8\% = 8.5\%$$

The benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ is:

$$b = 40\% \times 10\% + 20\% \times -4\% + 20\% \times 8\% + 0\% \times 6.4\% = 6.4\%$$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 40\% \times 10\% + 30\% \times -4\% + 20\% \times 8\% + 10\% \times 6.4\% = 5.0\%$$

Exhibit 6.13 Geometric off-benchmark (stock selection only) asset allocation

Geometric asset allocation for the data in Table 6.12 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.050}{1.064} - 1 = -1.28\%$$

Individual country asset allocation effects are:

UK equities	$(40\% - 40\%) \times \left(\frac{1.10}{1.064} - 1\right) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times \left(\frac{0.96}{1.064} - 1\right) = -0.98\%$
US equities	$(20\% - 40\%) \times \left(\frac{1.08}{1.064} - 1\right) = -0.30\%$
Emerging markets	$(10\% - 0\%) \times \left(\frac{1.064}{1.064} - 1\right) = 0.0\%$
Total	$0.0\% - 0.98\% - 0.30\% + 0.0\% = -1.28\%$

Exhibit 6.14 Geometric off-benchmark (stock selection only) stock selection

Geometric stock selection including interaction for the data in Table 6.12 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.085}{1.050} - 1 = 3.29\%$$

Individual country stock selection effects are:

UK equities	$40\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.05} = 3.81\%$
Japanese equities	$30\% \times \left(\frac{0.94}{0.95} - 1\right) \times \frac{0.95}{1.05} = -0.29\%$
US equities	$20\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.05} = -0.38\%$
Emerging markets	$10\% \times \left(\frac{1.08}{1.064} - 1\right) \times \frac{1.064}{1.05} = 0.15\%$
Total	$3.81\% - 0.29\% - 0.38\% + 0.15\% = 3.29\%$

If, however, the asset manager wishes to invest in a sector or country outside the benchmark, because of the merits of that sector, country or region rather than the merits of an individual security, then this is an asset allocation decision and should be measured accordingly. In order to reflect the entire contribution of this decision as asset allocation, the actual portfolio return in this category should be chosen as the benchmark for this category (recall that this decision will not impact the total benchmark return because the category weight is 0%). In practice this might be a decision to invest in an index-tracking ETF. The resulting attribution is shown in Table 6.13. Note that the attribution effects for UK, Japanese and US equities are unchanged. Geometric allocation and stock selection effects are calculated in Exhibit 6.15.

The 0.15% added value from stock selection only previously included in the emerging markets stock selection has in effect been transferred to asset allocation, reflecting the added value of choosing an outperforming off-benchmark category.

Table 6.13 Off-benchmark decision – asset allocation only

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
UK equities	40	40	20	10	0.00	3.80
Japanese equities	30	20	−5	−4	−0.98	−0.29
US equities	20	40	6	8	−0.30	−0.38
Emerging markets	10	0	8	8	0.15	0.00
Total	**100**	**100**	**8.5**	**6.4**	**−1.13**	**3.14**
Geometric excess return			**1.97**			

Exhibit 6.15 Off-benchmark attribution – asset allocation only

Asset allocation:

Emerging markets $(10\% - 0\%) \times \left(\frac{1.08}{1.064} - 1\right) = 0.15\%$

Stock selection:

Emerging markets $10\% \times \left(\frac{1.08}{1.08} - 1\right) \times \frac{1.08}{1.052} = 0.00\%$

It's worth noting that the semi-notional fund for asset allocation only (5.2%) is different from the semi-notional fund for stock selection only (5.0%).

$$b_S = 40\% \times 10\% + 30\% \times -4\% + 20\% \times 8\% + 10\% \times 8\% = 5.2\%$$

If, however, the asset manager is in effect taking an asset allocation decision first and then choosing securities with the emerging markets category second, then the attribution must reflect this investment decision process. A representative index must be chosen for emerging markets. Because of the zero-benchmark category weight, this choice does not impact the overall benchmark return, but it does reallocate between allocation and selection effects; see Table 6.14 and Exhibit 6.16.

The 0.15% added value exclusively included in stock selection for selection-only decisions, and exclusively included in asset allocation for allocation-only decisions, is now distributed −0.13% to stock selection and 0.28% to asset allocation. The asset allocation decision, in fact

Table 6.14 Off-benchmark decision – stock selection and asset allocation

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
UK equities	40	40	20	10	0.00	3.80
Japanese equities	30	20	−5	−4	−0.98	−0.29
US equities	20	40	6	8	−0.30	−0.38
Emerging markets	10	0	8	9.4	0.28	−0.13
Total	**100**	**100**	**8.5**	**6.4**	**−1.00**	**3.00**
Geometric Excess Return			**1.97**			

Exhibit 6.16 Off-benchmark decision – stock selection and asset allocation

Asset allocation:

Emerging markets $(10\% - 0\%) \times \left(\frac{1.094}{1.064} - 1\right) = 0.28\%$

Stock selection:

Emerging markets $10\% \times \left(\frac{1.08}{1.094} - 1\right) \times \frac{1.094}{1.0534} = -0.13\%$

It's worth noting that the semi-notional fund for asset allocation and stock-level decisions (5.34%) is different from the semi-notional fund for stock selection only (5.0%) or asset allocation only (5.2%).

$b_S = 40\% \times 10\% + 30\% \times -4\% + 20\% \times 8\% + 10\% \times 9.4\% = 5.34\%$

created more value when an appropriate representative index is chosen and the stocks within the category underperformed within the period.

Off-benchmark decisions require performance analysts to make decisions with regard to category benchmarks which are not necessarily documented. These decisions will be based on the actual investment decision process; is the asset manager making a selection decision, an allocation decision or both? The default decision of using a benchmark return of 0% because no benchmark has been formally assigned may generate completely misleading results; see Table 6.15 and Exhibit 6.17.

Table 6.15 Off-benchmark decision – default 0% benchmark return

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
UK equities	40	40	20	10	0.00	3.83
Japanese equities	30	20	−5	−4	−0.98	−0.29
US equities	20	40	6	8	−0.30	−0.38
Emerging markets	10	0	8	0	−0.60	0.77
Total	**100**	**100**	**8.5**	**6.4**	**−1.88**	**3.93**
Geometric excess return			**1.97**			

Exhibit 6.17 Off-benchmark decision – default 0% benchmark return

Asset allocation:

Emerging markets $(10\% - 0\%) \times \left(\frac{1.0}{1.064} - 1\right) = -0.60\%$

Stock selection:

Emerging markets $10\% \times \left(\frac{1.08}{1.0} - 1\right) \times \frac{1.00}{1.044} = 0.77\%$

The semi-notional return for the default decision is:

$$b_S = 40\% \times 10\% + 30\% \times -4\% + 20\% \times 8\% + 10\% \times 0\% = 4.4\%$$

Using 0% for the off-benchmark decision results in the completely erroneous suggestion of good stock selection contributing 0.77% and a poor asset allocation contributing −0.60%. In the context of this example the error might seem obvious, but smaller results in a larger volume of data are easily ignored, leading to potentially misleading conclusions.

Attribution consistent with the investment decision process

It is essential that attribution analysis is consistent with the investment decision process. Asset managers should be able to articulate their decision process, performance analysts should be able to replicate that decision process in their analysis, and asset owners have the expectation that their assets are being managed according to their investment mandate and that the results of the investment decision process are communicated accurately. For example, take the European security data in Table 6.16 – this could be analysed in several different ways, depending on the actual investment decision process. The investment decision process might be country-based, industry-based, market capitalisation-based, factor based or bottom-up security level. For simplicity in this example the benchmark is an equal-weighted index of the 10 securities held in the portfolio shown in Table 6.17.

Table 6.16 European equity portfolio

Security	Country	Sector	Size	Factor	Weight (%)	Return (%)
Security 1	France	Energy	Large	Value	13	7.0
Security 2	France	Healthcare	Mid	Growth	4	3.5
Security 3	France	Financial	Small	Growth	11	4.0
Security 4	Germany	Energy	Small	Growth	3	−3.0
Security 5	Germany	Healthcare	Large	Value	20	2.5
Security 6	Germany	Financial	Mid	Growth	9	6.0
Security 7	Italy	Energy	Mid	Value	12	10.0
Security 8	Italy	Healthcare	Small	Growth	8	5.6
Security 9	Italy	Energy	Large	Value	7	8.1
Security 10	Italy	Financial	Mid	Value	13	−5.7
				Total	**100**	**3.91**

Table 6.17 European equity portfolio – equal-weighted benchmark

Security	Weight (%)	Return (%)
Security 1	10	7.0
Security 2	10	3.5
Security 3	10	4.2
Security 4	10	−3.0
Security 5	10	2.5
Security 6	10	6.4
Security 7	10	10.0
Security 8	10	5.2
Security 9	10	8.1
Security 10	10	−5.7
	100	**3.82**

Using the data in Table 6.16, if the asset manager is employing a country decision process, then the appropriate attribution analysis is presented in Table 6.18. Attribution effects are calculated in Exhibits 6.18 and 6.19.

Table 6.18 European equity portfolio – country decision process

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
France	28	30	5.32	4.90	−0.02	0.11
Germany	32	30	2.91	1.97	−0.04	0.31
Italy	40	40	3.69	4.40	0.00	−0.28
Total	**100**	**100**	**3.91**	**3.82**	**−0.06**	**0.15**
Geometric excess return			**0.09**			

Exhibit 6.18 Geometric asset allocation – country decision process

Geometric asset allocation (country allocation) for the data in Table 6.18 is:

France $(28\% - 30\%) \times \left(\frac{1.049}{1.0382} - 1\right) = -0.02\%$

Germany $(32\% - 30\%) \times \left(\frac{1.0197}{1.0382} - 1\right) = -0.04\%$

Italy $(40\% - 40\%) \times \left(\frac{1.044}{1.0382} - 1\right) = 0.00\%$

Total $-0.02\% - 0.04\% + 0.00\% = -0.06\%$

Exhibit 6.19 Geometric stock selection – country decision process

Geometric stock selection for the data in Table 6.18 is:

France $28\% \times \left(\frac{1.0532}{1.049} - 1\right) \times \frac{1.049}{1.0376} = 0.11\%$

Germany $32\% \times \left(\frac{1.0291}{1.0197} - 1\right) \times \frac{1.0197}{1.0376} = 0.31\%$

Italy $40\% \times \left(\frac{1.0369}{1.044} - 1\right) \times \frac{1.044}{1.0376} = -0.28\%$

Total $0.11\% + 0.31\% - 0.28\% = 0.15\%$

Note the semi-notional return for the country decision process $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 28\% \times 4.9\% + 32\% \times 1.97\% + 40\% \times 4.4\% = 3.76\%$$

The asset manager has lost value via country allocation in the period: −0.02% in France and −0.04% in Germany. Stock selection, however, is positive overall: 0.11% in France, 0.31% in Germany, partially offset by −0.28% in Italy.

Table 6.19 represents an industry decision process. Attribution effects are calculated in Exhibits 6.20 and 6.21.

Table 6.19 European equity portfolio – industry decision process

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
Energy	35	40	7.39	5.53	−0.08	0.63
Health	32	30	3.40	3.73	0.00	−0.10
Financials	33	30	0.72	1.63	−0.06	−0.29
Total	**100**	**100**	**3.91**	**3.82**	**−0.15**	**0.24**
Geometric excess return			**0.09**			

Exhibit 6.20 Geometric asset allocation – industry decision process

Geometric asset allocation (industry sector allocation) for the data in Table 6.19 is:

Energy $(35\% - 40\%) \times \left(\frac{1.0553}{1.0382} - 1\right) = -0.08\%$

Health $(32\% - 30\%) \times \left(\frac{1.0373}{1.0382} - 1\right) = 0.00\%$

Financials $(33\% - 30\%) \times \left(\frac{1.0163}{1.0382} - 1\right) = -0.06\%$

Total $-0.08\% + 0.00\% - 0.06\% = -0.15\%$

Exhibit 6.21 Geometric stock selection – industry decision process

Geometric stock selection for the data in Table 6.19 is:

Energy $35\% \times \left(\frac{1.0739}{1.0553} - 1\right) \times \frac{1.0553}{1.0367} = 0.63\%$

Health $32\% \times \left(\frac{1.034}{1.0373} - 1\right) \times \frac{1.0373}{1.0367} = -0.10\%$

Financials $33\% \times \left(\frac{1.0072}{1.0163} - 1\right) \times \frac{1.0163}{1.0367} = -0.29\%$

Total $0.63\% - 0.10\% - 0.29\% = 0.24\%$

Note the semi-notional return for the industry decision process $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$b_S = 35\% \times 5.53\% + 32\% \times 3.73\% + 33\% \times 1.63\% = 3.67\%$

The asset manager has lost value via industry sector allocation in the period: −0.08% in energy and −0.06% in financials. Stock selection, however, is positive overall: 0.63% in energy, partially offset by −0.1% in health, and −0.29% in financials.

Table 6.20 represents a size (market capitalisation) decision process. Attribution effects are calculated in Exhibits 6.22 and 6.23.

Table 6.20 European equity portfolio – size decision process

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
Large	40	30	4.94	5.87	0.20	−0.35
Mid	38	40	3.00	3.55	0.01	−0.20
Small	22	30	3.63	2.13	0.13	0.32
Total	**100**	**100**	**3.91**	**3.82**	**0.33**	**−0.24**
Geometric excess return			**0.09**			

Exhibit 6.22 Geometric asset allocation – size decision process

Geometric asset allocation (size allocation) for the data in Table 6.20 is:

Large $(40\% - 30\%) \times \left(\frac{1.0587}{1.0382} - 1\right) = 0.20\%$

Mid $(38\% - 40\%) \times \left(\frac{1.0355}{1.0382} - 1\right) = 0.01\%$

Small $(22\% - 30\%) \times \left(\frac{1.0213}{1.0382} - 1\right) = 0.13\%$

Total $+0.20\% + 0.01\% + 0.13\% = 0.33\%$

Exhibit 6.23 Geometric stock selection – size decision process

Geometric stock selection for the data in Table 6.20 is:

Large $40\% \times \left(\frac{1.0494}{1.0587} - 1\right) \times \frac{1.0587}{1.0417} = -0.35\%$

Mid $38\% \times \left(\frac{1.03}{1.0355} - 1\right) \times \frac{1.0355}{1.0417} = -0.20\%$

Small $22\% \times \left(\frac{1.0363}{1.0213} - 1\right) \times \frac{1.0213}{1.0417} = 0.32\%$

Total $-0.35\% - 0.20\% + 0.32\% = -0.24\%$

Note that the semi-notional return for the size decision process $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 40\% \times 5.87\% + 38\% \times 3.55\% + 22\% \times 2.13\% = 4.17\%$$

The asset manager has added value via size (market capitalisation) allocation in the period: 0.2% in large-cap, 0.01% in mid-cap and 0.13% in small-cap. Stock selection for this investment decision process, though, is negative overall: −0.35% in large-cap, −0.2% in mid-cap, partially offset by 0.32% in small-cap.

Table 6.21 represents a factor decision process. Attribution effects are calculated in Exhibits 6.24 and 6.25.

Table 6.21 European equity portfolio – factor decision process

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
Growth	53	50	5.01	5.50	0.05	−0.25
Value	47	50	2.67	2.14	0.05	0.24
Total	**100**	**100**	**3.91**	**3.82**	**0.10**	**−0.01**
Geometric excess return			**0.09**			

Exhibit 6.24 Geometric asset allocation – factor decision process

Geometric asset allocation (factor allocation) for the data in Table 6.21 is:

Growth $(53\% - 50\%) \times \left(\frac{1.055}{1.0382} - 1\right) = 0.05\%$

Value $(47\% - 50\%) \times \left(\frac{1.0214}{1.0382} - 1\right) = 0.05\%$

Total $+0.05\% + 0.05 = 0.10\%$

Exhibit 6.25 Geometric stock selection – factor decision process

Geometric stock selection for the data in Table 6.21 is:

Growth $53\% \times \left(\frac{1.0501}{1.055} - 1\right) \times \frac{1.055}{1.0392} = -0.25\%$

Value $47\% \times \left(\frac{1.0267}{1.0214} - 1\right) \times \frac{1.0214}{1.0392} = 0.24\%$

Total $-0.25\% + 0.24\% = -0.01\%$

Note that the semi-notional return for the factor decision process $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 53\% \times 5.50\% + 47\% \times 2.14\% = 3.92\%$$

The asset manager has added value via factor allocation in the period: 0.05% in growth and 0.05% in value. Stock selection for this investment decision process, though, is just negative: −0.25% in growth, almost offset by 0.24% in value.

Note

In any two-category portfolio the asset allocation decision effects will either be both positive or both negative.

Table 6.22 represents a bottom-up security-level decision process. Attribution effects are calculated in Exhibits 6.26 and 6.27.

Table 6.22 European equity portfolio – bottom-up security selection

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Stock selection (%)	Timing (%)
Security 1	13	10	7.0	7.0	0.09	0.00
Security 2	4	10	3.5	3.5	0.02	0.00
Security 3	11	10	4.0	4.2	0.00	−0.02
Security 4	3	10	−3.0	−3.0	0.46	0.00
Security 5	20	10	2.5	2.5	−0.13	0.00
Security 6	9	10	6.0	6.4	−0.03	−0.03
Security 7	12	10	10.0	10.0	0.12	0.00
Security 8	8	10	5.6	5.2	−0.03	0.03
Security 9	7	10	8.1	8.1	−0.12	0.00
Security 10	13	10	−5.7	−5.7	−0.28	0.00
Total	**100**	**100**	**3.91**	**3.82**	**0.12**	**−0.03**
Geometric excess return			**0.09**			

Exhibit 6.26 European equity "bottom-up" security-level allocation

Geometric stock allocation for the data in Table 6.22 is:

Security 1 $(13\% - 10\%) \times \left(\frac{1.07}{1.0382} - 1\right) = 0.09\%$

Security 2 $(4\% - 10\%) \times \left(\frac{1.035}{1.0382} - 1\right) = 0.02\%$

Security 3 $(11\% - 10\%) \times \left(\frac{1.042}{1.0382} - 1\right) = 0.00\%$

Security 4 $(3\% - 10\%) \times \left(\frac{0.97}{1.0382} - 1\right) = 0.46\%$

Security 5 $(20\% - 10\%) \times \left(\frac{1.015}{1.0382} - 1\right) = -0.13\%$

Security 6 $(9\% - 10\%) \times \left(\frac{1.064}{1.0382} - 1\right) = -0.03\%$

Security 7 $(12\% - 10\%) \times \left(\frac{1.1}{1.0382} - 1\right) = 0.12\%$

Security 8 $(8\% - 10\%) \times \left(\frac{1.052}{1.0382} - 1\right) = -0.03\%$

Security 9 $(7\% - 10\%) \times \left(\frac{1.081}{1.0382} - 1\right) = -0.12\%$

Security 10 $(13\% - 10\%) \times \left(\frac{0.943}{1.0382} - 1\right) = -0.28\%$

Total $0.10\% - 0.02\% + 0.00\% + 0.48\% - 0.13\%$
$-0.03\% + 0.12\% - 0.03\% - 0.13\% - 0.29\% = 0.12\%$

Exhibit 6.27 European equity portfolio "bottom-up" security-level allocation

Geometric timing for the data in Table 6.22 is:

Only three securities suffered timing effects:

Security 3	$11\% \times \left(\frac{1.04}{1.042} - 1\right) \times \left(\frac{1.042}{1.0394}\right) = -0.02\%$
Security 6	$9\% \times \left(\frac{1.06}{1.064} - 1\right) \times \left(\frac{1.064}{1.0394}\right) = -0.03\%$
Security 8	$15\% \times \left(\frac{1.056}{1.052} - 1\right) \times \left(\frac{1.052}{1.0394}\right) = 0.03\%$
Total	$-0.02\% - 0.03\% + 0.03\% = -0.03\%$

The most significant positive contributor in this security-level decision process is the large underweight position in Security 4 contributing 0.46%. The most significant negative contributor is the overweight position in Security 10 contributing −0.28%. Timing effects are the result of transactions in individual securities; the resulting return in the portfolio will be different (negative in most cases) from the security return in the benchmark. Transaction costs and timing have cost −0.02% in Security 3 and −0.03% in Security 6. The timing effect in Security 8 is a rare positive 0.03% caused by obtaining such a good price at the time of the trade that transaction costs and bid/offer spreads have been more than offset.

Market-neutral attribution

Market-neutral funds, as their name suggests, are designed to be neutral against market movements. In theory they consist of a 100% long allocation offset by a 100% short position in the same market. The resultant exposure should be 100% cash plus the excess return from stock selection on the long side and the excess return from the stock selection on the short side. Thus, good stock pickers should be able to generate double the excess return without any market exposure; a very attractive and easily understood absolute return strategy.

Attribution is relatively straightforward if the long and short parts of the portfolio are treated as separate asset categories. It is essential that the performance of underlying assets retains the appropriate sign. The profits generating by assets that are falling in value are the result of negative weights compounded with negative returns. Asset allocation and stock selection effects are calculated for a market-neutral portfolio using data from Table 6.23 in Exhibits 6.29 and 6.30, respectively. Returns for the portfolio, benchmark and semi-notional fund are calculated in Exhibit 6.28.

Table 6.23 Market-neutral attribution

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
Long UK equities	40	40	20	10	−0.68	2.93
Long Japanese equities	70	60	−5.0	−4.0	−0.68	−0.68
Short UK equities	−30	−40	6.0	10.0	0.68	1.17
Short Japanese equities	−60	−60	−7.0	−4.0	0.0	1.76
Cash	90	100	3.0	3.0	0.0	0.0
Total	**100**	**100**	**7.6**	**3.0**	**−0.68**	**5.18**

Exhibit 6.28 Market-neutral return calculations

Using the data from Table 6.23 the portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ is:

$$r = 30\% \times 20\% + 70\% \times -5\% - 30\% \times 6\% - 60\% \times -7\% + 90\% \times 3\% = 7.6\%$$

The benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ is:

$$b = 40\% \times 10\% + 60\% \times -4\% - 40\% \times 10\% - 60\% \times -4\% + 100\% \times 3\% = 3.0\%$$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 30\% \times 10\% + 70\% \times -4\% - 30\% \times 10\% - 60\% \times -4\% + 90\% \times 3\% = 2.3\%$$

Exhibit 6.29 Market-neutral asset allocation

Geometric asset allocation for the data in Table 6.23 is:

$$\frac{(1 + b_S)}{(1 + b)} - 1 = \frac{1.023}{1.03} - 1 = -0.68\%$$

Individual country asset allocation effects are:

Long UK equities	$(30\% - 40\%) \times \left(\frac{1.10}{1.03} - 1\right) = -0.68\%$
Long Japanese equities	$(70\% - 60\%) \times \left(\frac{0.96}{1.03} - 1\right) = -0.68\%$
Short UK equities	$(-30\% + 40\%) \times \left(\frac{1.10}{1.03} - 1\right) = 0.68\%$
Short Japanese equities	$(-60\% + 60\%) \times \left(\frac{0.96}{1.03} - 1\right) = 0.0\%$
Cash	$(90\% - 100\%) \times \left(\frac{1.03}{1.03} - 1\right) = 0.0\%$
Total	$-0.68\% - 0.68\% + 0.68\% + 0.0\% + 0.0\% = -0.68\%$

Exhibit 6.30 Market-neutral stock selection

Geometric stock selection including interaction for the data in Table 6.23 is:

$$\frac{(1 + r)}{(1 + b_S)} - 1 = \frac{1.076}{1.023} - 1 = 5.18\%$$

Individual country stock selection effects are:

Long UK equities	$30\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.023} = 2.93\%$
Long Japanese equities	$70\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.95}{1.023} = -0.68\%$
Short UK equities	$-30\% \times \left(\frac{1.06}{1.10} - 1\right) \times \frac{1.10}{1.023} = 1.17\%$
Short Japanese equities	$-60\% \times \left(\frac{0.93}{0.96} - 1\right) \times \frac{0.96}{1.023} = 1.76\%$
Cash	$90\% \times \left(\frac{1.03}{1.03} - 1\right) \times \frac{1.03}{1.023} = 0.0\%$
Total	$2.93\% - 0.68\% + 1.17\% + 1.76\% + 0.0\% = 5.18\%$

Interpreting the attribution results, the long and short UK positions offset each other, leaving a small negative allocation of −0.68% from a net 10% long position in Japanese equities. Asset allocation from cash will always be zero regardless of the weight since the overall benchmark return is also a cash benchmark. The excess return from this portfolio is derived from stock selection, positive on long UK equities and negative on long Japanese equities. In the short UK equity portion of the portfolio, although generating a positive return, the portfolio manager has selected stocks that have underperformed the benchmark, resulting in a positive contribution. In the short Japanese portfolio, the manager again selected stocks that underperformed the benchmark which when combined with the negative weight generated a positive contribution. The stock selection from cash is unsurprisingly neutral; stock lending fees which are suffered in order to generate the short positions can be charged either to the cash sector or to the appropriate short equity sector as negative income.

Attribution for 130/30 funds (or extended short funds)

130/30 funds (including other weighting variations such as 120/20, etc.) are a more recent variation of market-neutral funds with a 30% short position offset by a 130% long position, thus retaining market exposure but allowing the asset manager to short a limited percentage of assets.

Attribution for a 130/30 fund is shown in Exhibits 6.31 and 6.32 based on data from Table 6.24.

Table 6.24 130/30 fund attribution

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
UK equities	120	130	14	10	0.00	4.39
Short UK equities	−30	−30	9.0	10	0.00	0.27
Cash	10	0	3.0	3.0	−0.64	0.00
Total	**100**	**100**	**14.4**	**10.0**	**−0.64**	**4.67**

Exhibit 6.31 130/30 Asset allocation

Geometric asset allocation for data in Table 6.24 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.093}{1.1} - 1 = -0.64\%$$

Individual country asset allocation effects are:

UK equities	$(120\% - 130\%) \times \left(\frac{1.10}{1.10} - 1\right) = 0.00\%$
Short UK equities	$(-30\% + 30\%) \times \left(\frac{1.10}{1.10} - 1\right) = 0.00\%$
Cash	$(10\% - 0\%) \times \left(\frac{1.03}{1.10} - 1\right) = -0.64\%$
Total	$0.0\% + 0.0\% - 0.64\% = -0.64\%$

Exhibit 6.32 130/30 Stock selection

Geometric stock selection including interaction for the data in Table 6.24 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.144}{1.93} - 1 = 4.67\%$$

Individual country stock selection effects are:

UK equities	$120\% \times \left(\frac{1.14}{1.10} - 1\right) \times \frac{1.10}{1.093} = +4.39\%$
Short UK equities	$-30\% \times \left(\frac{1.09}{1.10} - 1\right) \times \frac{1.10}{1.093} = +0.27\%$
Cash	$10\% \times \left(\frac{1.03}{1.03} - 1\right) \times \frac{1.03}{1.093} = 0.0\%$
Total	$4.39\% + 0.27\% + 0.0\% = 4.67\%$

Note that the semi-notional return is:

$$b_S = 120\% \times 10\% - 30\% \times 10\% + 10\% \times 3 = 9.3\%$$

It's worth noting in this attribution that since the UK equity benchmark is equal to the overall benchmark there are no asset allocation effects in the long or short positions, only from any allocation to cash; in this case an overweight position leading to a negative allocation of −0.64%. However, outperformance in the long sector is leveraged by 20% with a 120% allocation and there is an additional contribution because the shorted stocks underperformed the benchmark. Notice the benchmark retained 100% allocation to UK equities.

Leverage (or gearing)

Leverage is created if the total value of physical and economic assets (excluding notional cash) is greater than the total value of the portfolio. The amount of leverage is calculated as follows:

$$\text{Leverage} = \frac{\text{Total value of economic assets}}{\text{Total portfolio value}} - 1 \tag{6.33}$$

One is subtracted from Equation 6.33 because the natural exposure of the portfolio is 100%. In effect, leverage measures the degree to which an investor is utilising borrowed money.

The opposite of leverage is damping, representing the total amount of cash, notional cash and cash equivalent instruments relative to the total portfolio value.

$$\text{Damping} = \frac{\text{Total value of cash assets}}{\text{Total portfolio value}} \tag{6.34}$$

The representation of leverage in attribution analysis very much depends on the investment decision process. From the perspective of a UK investment trust it is often the trust board that takes decisions with regard to leverage (normally implemented using loans to buy further assets) while the management company takes the remaining investment decisions. This decision process can be replicated as a series of notional funds as shown in Exhibit 6.33.

Exhibit 6.33 Investment trust attribution (including leverage)

Benchmark		**+10.0%**
Stock selection	*+1.7%*	
Asset allocation	*−0.5%*	
Currency	*−0.3%*	
Gross assets		**+11.0%**
Leverage	*+1.3%*	
Total assets (before fees)		**+12.4%**
Fees, expenses	*−0.4%*	
Total assets (net of fees)		**+12.0%**
Discount, other corporate	*+1.2%*	
Client performance		**+13.4%**

The difference between each step in the decision process by definition defines the contribution for that factor. For example, the contribution from gearing is simply the geometric difference between the total asset performance including gearing and gross assets excluding gearing:

$$\frac{1.124}{1.11} - 1 = 1.3\%$$

Alternatively, leverage decisions may be part of the natural investment decision process at the category or sector level. An alternative demonstration of leverage is shown in Exhibits 6.34, 6.35 and 6.36 using revised data from Table 6.25.

Table 6.25 Attribution including leverage

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
UK equities	150	40	20	10	3.72	13.04
Japanese equities	50	20	−5.0	−4.0	−2.93	−0.43
US equities	100	40	6.0	8.0	0.90	−1.74
Cash	−200	0	3.0	3.0	6.39	0.00
Total	**100**	**100**	**27.5**	**6.4**	**8.08**	**10.87**

Exhibit 6.34 Return calculations including leverage

Using the data from Table 6.25 the portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ is:

$$r = 150\% \times 20\% + 50\% \times -5\% + 100\% \times 6\% - 200\% \times 3\% = 27.50\%$$

The benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ is:

$$b = 40\% \times 10\% + 20\% \times -4\% + 40\% \times 8\% = 6.4\%$$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 150\% \times 10\% + 50\% \times -4\% + 100\% \times 8\% - 200\% \times 3.0\% = 15.0\%$$

Exhibit 6.35 Asset allocation with leverage

Geometric asset allocation for the data in Table 6.25 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.15}{1.064} - 1 = 8.08\%$$

Individual country asset allocation effects are:

UK equities	$(150\% - 40\%) \times \left(\frac{1.10}{1.064} - 1\right) = 3.72\%$
Japanese equities	$(50\% - 20\%) \times \left(\frac{0.96}{1.064} - 1\right) = -2.93\%$
US equities	$(100\% - 40\%) \times \left(\frac{1.08}{1.064} - 1\right) = 0.90\%$
Cash	$(-200\% - 0\%) \times \left(\frac{1.03}{1.064} - 1\right) = 6.39\%$
Total	$3.72\% - 2.93\% + 0.9\% + 6.39\% = 8.08\%$

Note that total leverage is $\frac{300\%}{100\%} - 1 = 200\%$

Exhibit 6.36 Stock selection with leverage

Geometric stock selection including interaction for the data in Table 6.25 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.275}{1.15} - 1 = 10.87\%$$

Individual country stock selection effects are:

UK equities	$150\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.15} = 13.04\%$
Japanese equities	$50\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.96}{1.15} = -0.43\%$
US equities	$100\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.15} = -1.74\%$
Cash	$-200\% \times \left(\frac{1.03}{1.03} - 1\right) \times \frac{1.03}{1.15} = 0.00\%$
Total	$13.04\% - 0.43\% - 1.74\% + 0.0\% = 10.87\%$

Exhibit 6.35 demonstrates that apart from using leverage to increase the weight in UK and US equities and generate positive asset allocation, additional value is added from the short position in cash or leverage of 200%. Stock selection in Exhibit 6.36 is also magnified by virtue of increased weights in outperforming categories (UK equities) and underperforming categories (US equities). Therefore, value is added both from the leverage itself and by increased exposure to the assets bought with the cash made available.

ATTRIBUTION INCLUDING DERIVATIVES

Attribution including equity index futures

Like any other asset, the attribution of equity index futures must reflect the investment decision process.

The most effective way of representing this decision process is to use notional assets. Gains and losses in an equity index future are realised daily in the variation margin account, which remains an asset of the portfolio. Both the economic exposures of the investment decision process and the return can be disaggregated as two notional assets of equal size: one long and one short, one representing the appropriate equity category and one cash. The effect of interest rates is included by applying benchmark interest rates[22] to the notional cash asset; this synthetic income, positive or negative, represents part of the realised return in the variation margin account.

The impacts of these notional assets are demonstrated in Exhibits 6.37, 6.38, and 6.39 using data from Table 6.26.

Table 6.26 Attribution with futures

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
UK equities	40	50	20	10	−0.15	3.71
US equities	55	40	6.0	8.0	−0.05	−1.02
Cash	5	10	1.4	1.5	0.32	0.00
UK equities	5	0.0	9.5	9.9	0.07	−0.02
US futures	−15	0.0	8.6	8.5	−0.02	−0.01
UK economic cash	−5	0.0	1.5	1.5	0.32	0.0
US economic cash	15	0.0	1.2	1.2	−0.99	0.0
Total	**100**	**100**	**10.66**	**8.35**	**−0.51**	**2.65**

Exhibit 6.37 Attribution with futures – return calculations

Using the data from Table 6.26 the portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ is:

$$r = 40\% \times 20\% + 55\% \times 6\% + 5\% \times 1.4\% + 5\% \times 9.5\%$$
$$-15\% \times 8.6\% - 5\% \times 1.5\% + 15\% \times 1.2 = 10.66\%$$

The benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ is:

$$b = 50\% \times 10\% + 40\% \times 8\% + 10\% \times 1.5\% = 8.35\%$$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 40\% \times 10\% + 55\% \times 8\% + 5\% \times 1.5\% + 5\% \times 9.9\%$$
$$-5\% \times 1.5\% + 15\% \times 1.2\% = 7.8\%$$

[22] The interest rate benchmark will be written into the futures contract itself. Until recently most derivative instruments used Libor. However, following manipulation by the banking organisations responsible for setting rates, Libor is gradually being replaced by a range of heavily regulated alternatives.

Exhibit 6.38 Asset allocation with futures

Geometric asset allocation for data in Table 6.26 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.078}{1.0835} - 1 = -0.51\%$$

Individual country asset allocation effects are:

UK equities	$(40\% - 50\%) \times \left(\frac{1.10}{1.0835} - 1\right) = -0.15\%$
US equities	$(55\% - 40\%) \times \left(\frac{1.08}{1.0835} - 1\right) = -0.05\%$
Cash	$(5\% - 10\%) \times \left(\frac{1.015}{1.0835} - 1\right) = 0.32\%$
UK futures	$(5\% - 0\%) \times \left(\frac{1.099}{1.0835} - 1\right) = 0.07\%$
US futures	$(-15\% - 0\%) \times \left(\frac{1.085}{1.0835} - 1\right) = -0.02\%$
UK economic cash	$(-5\% - 0\%) \times \left(\frac{1.015}{1.0835} - 1\right) = 0.32\%$
US economic cash	$(15\% - 0\%) \times \left(\frac{1.012}{1.0835} - 1\right) = -0.99\%$
Total	

$-0.15\% - 0.05\% + 0.32\% + 0.07\% - 0.02\% + 0.32\% - 0.99\% = -0.51\%$

Exhibit 6.39 Stock selection with futures

Geometric stock selection including interaction for data in Table 6.26 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.1066}{1.078} - 1 = 2.65\%$$

Individual country stock selection effects are:

UK equities	$40\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.078} = 3.71\%$
US equities	$55\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.078} = -1.02\%$
Cash	$5\% \times \left(\frac{1.014}{1.015} - 1\right) \times \frac{1.015}{1.078} = 0.00\%$
UK futures	$5\% \times \left(\frac{1.095}{1.099} - 1\right) \times \frac{1.099}{1.078} = -0.02\%$
US futures	$-15\% \times \left(\frac{1.086}{1.085} - 1\right) \times \frac{1.085}{1.078} = -0.01\%$
UK economic cash	$-5\% \times \left(\frac{1.015}{1.015} - 1\right) \times \frac{1.015}{1.078} = 0.00\%$
US economic cash	$15\% \times \left(\frac{1.012}{1.012} - 1\right) \times \frac{1.012}{1.078} = 0.00\%$
Total	$3.71\% - 1.02\% + 0.0\% - 0.02\% - 0.01\% + 0.0\% + 0.0\% = 2.65\%$

The data shown in Table 6.26 is in long form for illustration. The investment decision process is unlikely to treat the futures contracts in isolation, and we should find some way of consolidating the data consistent with the investment decision process. Analysing the attribution effects, we notice that the return of UK futures has underperformed the UK futures

benchmark; this is most likely to be a matter of timing but could also be due to the fact that the futures contract does not necessarily trade in line with the cash index. It's also worth noting that both the US and UK futures benchmarks are not the same as the underlying asset category benchmarks. This is fairly common; there may not be a futures index contract for the actual sector benchmark of the account. The impact from this mismatch is unavoidable but can be measured. Finally, it's worth noting that the return on US economic cash is different from that on UK economic cash; this is because in this example the US interest rate is lower than the UK interest rate.

The presentation we use largely depends on the investment decision process. If the portfolio managers responsible for UK equities and US equities were responsible for the futures contract it would be reasonable to combine the effects, as shown in Table 6.27 and Exhibits 6.40, 6.41 and 6.42.

Table 6.27 Attribution with futures (alternative presentation)

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
UK equities	45	50	18.83	10	−0.08	3.69
US equities	40	40	5.03	8.0	0.00	−1.10
Cash	15	10	1.17	1.5	−0.32	−0.05
Total	**100**	**100**	**10.66**	**8.35**	**−0.39**	**2.54**

Exhibit 6.40 Adjusted return calculations including futures

Combining sector returns using data from Table 6.27:

UK equities $$\frac{40\% \times 20.0\% + 5\% \times 9.5}{40\% + 5\%} = 18.83\%$$

US equities $$\frac{55\% \times 6.0\% - 15\% \times 8.6}{55\% - 15\%} = 5.03\%$$

Cash $$\frac{5\% \times 1.4\% - 5\% \times 1.5 + 15\% \times 1.2}{5\% - 5\% + 15\%} = 1.17\%$$

Using these combined returns, the portfolio return still reconciles as follows:

$$r = 45\% \times 18.83\% + 40\% \times 5.03\% + 15\% \times 1.17\% = 10.66\%$$

The benchmark return is clearly unchanged:

$$b = 50\% \times 10\% + 40\% \times 8\% + 10\% \times 1.5\% = 8.35\%$$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ however must be revised to reflect this slightly different investment decision process:

$$b_S = 45\% \times 10\% + 40\% \times 8\% + 15\% \times 1.5\% = 7.93\%$$

Exhibit 6.41 Revised asset allocation with futures

Geometric asset allocation for data in Table 6.27 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.0793}{1.0835} - 1 = -0.39\%$$

Individual country asset allocation effects are:

UK equities	$(45\% - 50\%) \times \left(\frac{1.10}{1.0835} - 1\right) = -0.08\%$
US equities	$(40\% - 40\%) \times \left(\frac{1.08}{1.0835} - 1\right) = 0.00\%$
Cash	$(15\% - 10\%) \times \left(\frac{1.015}{1.0835} - 1\right) = -0.32\%$
Total	$-0.08\% + 0.0\% - 0.32\% = -0.39\%$

Exhibit 6.42 Revised stock selection with futures

Geometric stock selection including interaction for data in Table 6.27 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.1066}{1.0793} - 1 = 2.54\%$$

Individual country stock selection effects are:

UK equities	$45\% \times \left(\frac{1.188}{1.10} - 1\right) \times \frac{1.10}{1.0793} = 3.69\%$
US equities	$40\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.0793} = -1.10\%$
Cash	$15\% \times \left(\frac{1.012}{1.015} - 1\right) \times \frac{1.015}{1.0793} = -0.05\%$
Total	$3.69\% - 1.10\% - 0.05\% = 2.54\%$

If the asset allocator and not the individual sector manager is responsible for the futures contracts then we must reflect this in our attribution analysis, including any futures index mismatch between the contract index used and the underlying benchmark (see Table 6.28). Asset allocation is identical to before as in Exhibit 6.41, but reflecting the investment decision process the impact of futures is separated from stock selection. Stock selection for the physical assets is the same as before in Exhibit 6.39, but there are two new factors:

(1) Futures timing – This represents the timing effect, trading or difference between fair value of the futures contract and is the selection effect calculated in Exhibit 6.39.
(2) Futures mismatch – This represents the difference between the futures index contract and the underlying benchmark and is calculated in Exhibit 6.43.

The asset allocation effects, of course, include both physical and futures notional assets. The futures timing loss of three basis points is simply slight underperformance of the UK futures contract against its own index and a further loss of a basis point because the short US futures contract slightly outperformed. The −0.07% loss due to index mismatch in the short US futures position was because the futures contract index outperformed the underlying benchmark index. The −0.04% loss in cash was due to the fact that the net cash position was exposed to US dollar interest rates with a lower return than UK interest rates.

Table 6.28 Attribution with futures (preferred alternative presentation)

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Futures mismatch (%)	Stock selection (%)	Futures timing (%)
UK equities	45	50	20	10	−0.08	0.00	3.71	−0.02
US equities	40	40	6.0	8.0	0.00	−0.07	−1.02	−0.01
Cash	15	10	1.4	1.5	−0.32	−0.04	0.00	0.00
Total	**100**	**100**	**10.66**	**8.35**	**−0.39**	**−0.12**	**2.69**	**−0.03**

Exhibit 6.43 Futures mismatch

UK futures	$5\% \times \left(\frac{1.099}{1.10} - 1\right) \times \frac{1.10}{1.078} = 0.00\%$
US equities	$-15\% \times \left(\frac{1.085}{1.08} - 1\right) \times \frac{1.08}{1.078} = -0.07\%$
UK economic cash	$-5\% \times \left(\frac{1.015}{1.015} - 1\right) \times \frac{1.015}{1.078} = 0.00\%$
US economic cash	$15\% \times \left(\frac{1.012}{1.015} - 1\right) \times \frac{1.015}{1.078} = -0.04\%$
Total	$0.0\% - 0.07\% + 0.0\% - 0.04\% = -0.12\%$

Attribution analysis using options

The key to attribution analysis with options is to include the appropriate economic exposure, for which we use the option's delta. The option itself will provide part of the valuation; the remainder of the option's economic value must be achieved by using notional assets. The fundamentals of generating notional assets for options are laid out by Stannard[23]:

$$\text{Option economic exposure} = \delta \times \text{number of options} \times \text{stock price} \tag{6.35}$$

$$\text{Option economic exposure} = \text{option valuation} + \text{required notional asset} \tag{6.36}$$

The delta and hence the exposure is continuously changing. However, if the notional asset exposures are recalculated on a daily basis the impact of gamma can be significantly reduced.

Return calculations and attribution analysis for a portfolio with a single long call option plus cash are shown in Exhibits 6.44 and 6.45 and summarised in Table 6.29.

Adjusting for appropriate economic exposure, Table 6.29 demonstrates that the asset manager is overweight the reference asset and underweight cash, generating an added value of 19 basis points. Stock selection also made a small contribution of 0.04%; this might come from any of the Greeks associated with the option other than delta – no attempt is made to identify which of the Greeks added the most value. The synthetic asset return of 2.08% outperformed the reference asset return of 2.0%. For standard portfolios including options, this type of analysis is sufficient to quantify the manager's decision process; for option-only portfolios, more detailed analysis might be required.

[23] Stannard, "Measuring Investment Returns of Portfolios Containing Futures and Options" (1996).

Exhibit 6.44 Performance measurement for options

Type of option	Long call		
Underlying	Massive Firm Inc. (the reference asset)		
Current share price	\$100		
Exercise price	\$100		
Current option price	\$10		
Option delta	50%		
Share end price	\$102		
Option end price	\$11		
Interest on cash for period of measurement = 0.1%			
	SMV	EMV	Return
Options	1,000	1,100	10.0%
Cash	9,000	9,009	0.1%
Total	10,000	10,109	1.09%
Option economic value = 0.5 × 100 × 100 = 5,000(from Equation 6.35)			
Notional cash asset required 5,000 – 1,000 = 4,000 (from Equation 6.36)			
	SMV	EMV	Return
Options	1,000	1,100	10.0%
Notional exposure	4,000	4,004	0.1%
Combined	5,000	5,104	2.08%
Cash	9,000	9,009	0.1%
Notional cash	(4,000)	(4,004)	0.1%
Combined	5,000	5,005	0.1%
Total	10,000	10,109	1.09%
The options and required notional exposure assets have combined to produce a "synthetic" asset return of 2.08%.			

Exhibit 6.45 Attribution including options

Using data from Exhibit 6.44 and assuming a benchmark consisting of 40% exposure to the underlying asset of the option and 60% cash attribution is calculated as follows:

Portfolio return

$$50\% \times 2.08\% + 50\% \times 0.1\% = 1.09\%$$

Benchmark return

$$40\% \times 2.0\% + 60\% \times 0.1\% = 0.86\%$$

Semi-notional return

$$50\% \times 2.0\% + 50\% \times 0.1\% = 1.05\%$$

Geometric asset allocation

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.0105}{1.0086} - 1 = 0.19\%$$

Options $(50\% - 40\%) \times \left(\frac{1.02}{1.0086} - 1\right) = 0.11\%$

Cash $(50\% - 60\%) \times \left(\frac{1.001}{1.0086} - 1\right) = 0.08\%$

(Continued)

Exhibit 6.45 (*Continued*)

Geometric **stock selection**

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.0109}{1.0105} - 1 = 0.04\%$$

$$\text{Options } 50\% \times \left(\frac{1.0208}{1.02} - 1\right) \times \frac{1.02}{1.0105} = 0.04\%$$

$$\text{Cash } 50\% \times \left(\frac{1.001}{1.001} - 1\right) \times \frac{1.001}{1.0105} = 0.00\%$$

Table 6.29 Attribution including options

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
Options	50	40	2.08	2.0	0.11	0.04
Cash	50	60	0.10	0.10	0.08	0.00
Total	**100**	**100**	**1.09**	**0.86**	**0.19**	**0.04**
Geometric excess return			**0.23**			

MULTI-CURRENCY ATTRIBUTION

Multi-currency attribution is generally accepted to add several layers of additional complexity to an already complex and controversial area. Many of us have the luxury of ignoring multi-currency issues in single-currency portfolios or perhaps the currency decision is simply an implicit decision within the asset allocation process. However, for genuinely multi-currency investment decision processes we must face these issues head-on. These issues include but are not limited to:

(1) The compounding effects of market and currency returns
(2) Interest rate differentials
(3) Unrealised gain/losses in forward currency contracts
(4) Securities denominated in currencies other than the currency of economic exposure

Early papers addressing multi-currency attribution include Allen,[24] which offers a geometric approach, and Ankrim and Hensel,[25] which is arithmetic.

Ankrim and Hensel

Ankrim and Hensel recognised that the currency return is comprised of two components: the unpredictable "currency surprise" and the predictable interest rate differential or "forward premium" between the appropriate currencies.

[24] Allen, "Performance Attribution of Global Equity Portfolios" (1991).
[25] Ankrim and Hensel, "Multi-currency Performance Attribution" (1992).

Let:

S_i^t = the spot rate of currency i at time t
F_i^{t+1} = the forward exchange rate of currency i at time t for conversion through a forward contract at time $t+1$

Then the return of currency i is:

$$c_i = \frac{S_i^{t+1} - S_i^t}{S_i^t} = \frac{S_i^{t+1}}{S_i^t} - 1 \tag{6.37}$$

Expanding Equation 6.37:

$$c_i = \frac{S_i^{t+1} - F_i^{t+1} + F_i^{t+1} - S_i^t}{S_i^t} \tag{6.38}$$

We can break down the currency return into:

$$\text{Currency surprise in currency } i \qquad e_i = \frac{S_i^{t+1} - F_i^{t+1}}{S_i^t} \tag{6.39}$$

and

$$\text{Forward premium in currency } i \qquad d_i = \frac{F_i^{t+1} - S_i^t}{S_i^t} = \frac{F_i^{t+1}}{S_i^t} - 1 \tag{6.40}$$

The currency return is the sum of the currency surprise plus the forward premium or interest rate differential:

$$c_i = e_i + d_i \tag{6.41}$$

We can now expand Equation 6.3 as follows:

$$r = \sum_{i=1}^{i=n} w_i \times (r_i - e_i - d_i) + \sum_{i=1}^{i=n} w_i \times e_i + \sum_{i=1}^{i=n} w_i \times d_i \tag{6.42}$$

If the portfolio includes forward currency contracts these can be isolated separately as shown in Equation 6.43:

$$r = \sum_{i=1}^{i=n} w_i \times (r_i - e_i - d_i) + \sum_{i=1}^{i=n} w_i \times e_i + \sum_{i=1}^{i=n} w_i \times d_i + \sum_{i=1}^{i=n} \widetilde{w}_i \times f_i \tag{6.43}$$

where:

The return to forward currency contracts is:

$$f_i = \frac{S_i^{t+1} - F_i^{t+1}}{F_i^{t+1}} = \frac{S_i^{t+1}}{F_i^{t+1}} - 1 \tag{6.44}$$

$\widetilde{w}_i$ = weight of currency forward contract in currency i

Note that the forward return is linked to the currency surprise by the formula:

$$f_i = \frac{e_i}{(1 + d_i)} \tag{6.45}$$

In effect, forward currency contracts economically consist of two currencies, one with a positive weight and one negative.

Assuming that the currency returns, currency forward returns, currency surprise and forward premiums are the same in the portfolio and the benchmark, then it follows that the benchmark return can be constructed as:

$$b = \sum_{i=1}^{i=n} W_i \times (b_i - e_i - d_i) + \sum_{i=1}^{i=n} W_i \times e_i + \sum_{i=1}^{i=n} W_i \times d_i + \sum_{i=1}^{i=n} \widetilde{W}_i \times f_i \tag{6.46}$$

where: $\widetilde{W}_i$ = benchmark weight of currency forward contract in currency i

Applying the standard Brinson approach to Equation 6.43 and Equation 6.46 we derive the following attribution effects:

Asset allocation

$$A_i = (w_i - W_i) \times (l_i - l) \tag{6.47}$$

where:

$$l_i = b_i - e_i - d_i = b_i - c_i \tag{6.48}$$

Equation 6.48 is the arithmetic difference between the benchmark return in base currency and the currency return; not quite the local return, which should only be derived by using the geometric difference. The revised weighted average benchmark return adjusting for currency is now:

$$l = \sum_{i=1}^{i=n} w_i \times l_i \tag{6.49}$$

Security selection excluding interaction:

$$S_i = W_i \times (k_i - l_i) \tag{6.50}$$

where:

$$k_i = r_i - e_i - d_i = r_i - c_i \tag{6.51}$$

Interaction:

$$I_i = (w_i - W_i) \times (k_i - l_i) \tag{6.52}$$

Or, if you prefer, security selection including stock selection:

$$S_i = w_i \times (k_i - l_i) \tag{6.53}$$

The contribution from currency is analogous to asset allocation:

$$C_i = \underbrace{(w_i - W_i) \times (e_i - e)}_{\text{Underlying assets}} + \underbrace{(\widetilde{w}_i - \widetilde{W}_i) \times (f_i - e)}_{\text{Currency forwards}} \tag{6.54}$$

where: C_i = contribution to currency from currency i

$$e = \sum_{i=1}^{i=n} W_i \times e_i \text{ weighted average currency surprise} \tag{6.55}$$

The final term, forward premium, is also analogous to asset allocation:

$$D_i = (w_i - W_i) \times (d_i - d) \tag{6.56}$$

where: D_i = contribution to forward premium in currency i

The weighted average benchmark forward premium:

$$d = \sum_{i=1}^{i=n} W_i \times d_i \tag{6.57}$$

Extending the basic data in Table 6.3 to Table 6.30 creates a multi-currency account including currency forward contracts, with a base currency of sterling. Exhibit 6.46 calculates the portfolio and benchmark returns for the data in Table 6.30.

Table 6.30 Ankrim and Hensel attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio base return r_i (%)	Benchmark base return b_i (%)	Currency return c_i (%)	Currency surprise e_i (%)	Forward premium d_i (%)
UK equities	40	40	20.0	10.0	0	0	0
Japanese equities	30	20	4.5	5.6	10	9	1
US equities	30	40	27.2	29.6	20	18	2
	$\widetilde{w}_i$	$\widetilde{W}_i$		f_i			
Sterling forward contracts	+20	+30		0			
Yen forward contracts	−15	−10		8.91			
US dollar forward contracts	−5	−20		17.65			
Total	100	100	15.29	12.54			

Ankrim and Hensel asset allocation effects are calculated in Exhibit 6.47. The asset allocation bets are exactly as before but the individual country returns and the revised benchmark return now include the compounding effect of market and currency returns thus impacting the

Exhibit 6.46 Return calculations for Ankrim and Hensel

Using Table 6.30 the portfolio return is:

$$r = 40\% \times 20\% + 30\% \times 5.6\% + 30\% \times 29.6\% - 15\% \times 8.91\% - 10\% \times 17.65 = 15.29\%$$

The benchmark return is:

$$b = 40\% \times 10\% + 20\% \times 5.6\% + 40\% \times 29.6\% - 10\% \times 8.91 - 20\% \times 17.65 = 12.54\%$$

The weighted average benchmark currency surprise is:

$$e = 40\% \times 0.0\% + 20\% \times 9.0\% + 40\% \times 18\% = 9.0\%$$

The weighted average benchmark forward premium is:

$$d = 40\% \times 0.0\% + 20\% \times 1.0\% + 40\% \times 2.0\% = 1.0\%$$

The revised weighted average benchmark return premium is:

$$l = 40\% \times (10.0\% - 0.0\%) + 20\% \times (5.6\% - 10.0\%) + 40\% \times (29.6\% - 20.0\%) = 6.96\%$$

Exhibit 6.47 Asset allocation for Ankrim and Hensel

	$(w_i - W_i) \times (l_i - l)$
UK equities	$(40\% - 40\%) \times [(10.0\% - 0.0\%) - 6.96\%] = 0.0\%$
Japanese equities	$(30\% - 20\%) \times [(5.6\% - 10.0\%) - 6.96\%] = -1.14\%$
US equities	$(30\% - 40\%) \times [(29.6\% - 20\%) - 6.96\%] = -0.26\%$
Total	$0.0\% - 1.14\% - 0.26\% = -1.4\%$

calculation of asset allocation effects. This compounding effect has changed the total asset allocation effect in Exhibit 6.5 from −1.2% to −1.4%.

Security selection effects are calculated in Exhibit 6.48. There is no compounding effect for UK equities, therefore the result is the same as in Exhibit 6.6. However, the Japanese and US equities stock selection has been impacted by currency compounding, dramatically in the case of US equities, changing the effect from −0.6% to −0.96. For completeness the revised interaction effects are shown in Exhibit 6.49.

Exhibit 6.48 Stock selection for Ankrim and Hensel

	$W_i \times (k_i - l_i)$
UK equities	$40\% \times [(20.0\% - 0.0\%) - (10.0\% - 0.0\%)] = 4.0\%$
Japanese equities	$20\% \times [(-4.5\% - 10.0\%) - (-4.4\% - 10.0\%)] = -0.22\%$
US equities	$40\% \times [(27.2\% - 20.0\%) - (29.6\% - 20.0\%)] = -0.96\%$
Total	$4.0\% - 0.22\% - 0.96\% = 2.82\%$

Exhibit 6.49 Interaction for Ankrim and Hensel

	$(w_i - W_i) \times (k_i - l_i)$
UK equities	$(40\% - 40\%) \times [(20.0\% - 0.0\%) - (10.0\% - 0.0\%)] = 0.0\%$
Japanese equities	$(30\% - 20\%) \times [(4.5\% - 10\%) - (5.6\% - 10.0\%)] = -0.11\%$
US equities	$(30\% - 40\%) \times [(27.2\% - 20\%) - (29.6\% - 20\%)] = 0.24\%$
Total	$0.0\% - 0.11\% + 0.24\% = 0.13\%$

Currency allocation is calculated in Exhibit 6.50. From the underlying assets there is no contribution from sterling since there is no bet and no contribution from yen since, although there is a 10% overweight position, the currency surprise in yen is equal to the weighted average benchmark surprise. There is, however, a −0.9% contribution from a 10% underweight position in the strongly performing US dollar.

The currency forward contracts also contribute to the added value from currency management. Even though this is a sterling-based account it is possible to add value by being underweight sterling, in this example caused by being 10% underweight the benchmark hedged position, sterling underperforming both the yen and the US dollar. In the benchmark the currency positions caused by the index weights in the underlying markets has been 50% hedged. Very little impact is caused by the underweight yen bet, but the overweight US dollar forward contract more than offsets the underweight underlying assets position and contributes +1.3%.

Exhibit 6.50 Currency for Ankrim and Hensel

Underlying assets	$(w_i - W_i) \times (e_i - e)$
Sterling	$(40\% - 40\%) \times (0.0\% - 9.0\%) = 0.0\%$
Yen	$(30\% - 20\%) \times (9.0\% - 9.0\%) = 0.0\%$
US dollar	$(30\% - 40\%) \times (18.0\% - 9.0\%) = -0.9\%$
Currency forwards	$(\widetilde{w}_i - \widetilde{W}_i) \times (f_i - e)$
Sterling	$(20\% - 30\%) \times (0.0\% - 9.0\%) = 0.9\%$
Yen	$(-15\% - 10\%) \times (8.91\% - 9.0\%) = 0.0\%$
US dollar	$(-5\% - 20\%) \times (17.65\% - 9.0\%) = 1.3\%$
Total	$0.0\% + 0.0\% - 0.9\% + 0.9\% + 0.0\% + 1.3\% = 1.3\%$

Forward premium effects tend to be small, particularly over short periods.

In Exhibit 6.51 the impact of being underweight the US dollar caused by the underlying asset allocations contributes −0.1%.

Exhibit 6.51 Forward premium for Ankrim and Hensel

	$(w_i - W_i) \times (d_i - d)$
Sterling	$(40\% - 40\%) \times (0.0\% - 1.0\%) = 0.0\%$
Yen	$(30\% - 20\%) \times (1.0\% - 1.0\%) = 0.0\%$
US dollar	$(30\% - 40\%) \times (2.0\% - 1.0\%) = -0.1\%$
Total	$0.0\% + 0.0\% - 0.1\% = -0.1\%$

Total attribution effects are summarised in Table 6.31.

Table 6.31 Ankrim and Hensel attribution

	Asset allocation $(w_i - W_i) \times (l_i - l)$ (%)	Stock selection $W_i \times (k_i - l_i)$ (%)	Forward premium $(w_i - W_i) \times (d_i - d)$ (%)	Currency management $(w_i - W_i) \times (e_i - e)$ (%)	Interaction $(w_i - W_i) \times (k_i - l_i)$ (%)
UK equities	0.0	4.0	0.0	0.0	0.0
Japanese equities	−1.14	−0.22	0.0	0.0	−0.11
US equities	−0.26	−0.96	−0.10	−0.9	0.24
				$(\widetilde{w}_i - \widetilde{W}_i) \times (f_i - e)$	
Sterling forwards				0.9	
Yen forwards				0.0	
US dollar forwards				1.3	
Total	**−1.40**	**2.82**	**−0.10**	**1.30**	**0.13**
				Total excess return	**2.75**

There are a number of problems with the Ankrim and Hensel approach:

(1) The main problem is the use of arithmetic return premium k. This ignores the compounding effect between market and currency returns and distributes this effect across asset allocation, stock selection and interaction.
(2) The forward premium returns are isolated as a separate factor. In reality this effect is always a consequence of asset allocation decisions. The asset allocator should be cognisant of forward premiums' effects to avoid abuse. This effect should always be included with asset allocation.
(3) The reference benchmark currency effect e is unaffected by hedging changes to the benchmark.

Karnosky and Singer

Perhaps the single most important paper published in attribution is Karnosky and Singer in 1994.[26] Not only does this paper explain why managing multi-currency portfolios is suboptimal if currency is not managed independently, it also provides a framework for calculating attribution effects, taking into account interest rate differentials. They resolve the issue of compounding by using continuously compounded returns in their model and solve the forward premium concern by thinking in terms of "return premium" above local interest rates.

[26] Karnosky and Singer, *Global Asset Management and Performance Attribution* (1994).

They defined the total return on the portfolio as:

$$r = \sum_{i=1}^{i=n} w_i \times r_{Li} + \sum_{i=1}^{i=n} w_i \times c_i \tag{6.58}$$

where: r_{Li} = return in local currency for currency i

Note

Karnosky and Singer have neatly sidestepped compounding issues and greatly simplified the mathematics by using continuously compounded or log returns. Unfortunately, asset owners won't accept reporting in continuously compounded returns.

Expanding Equation 6.58:

$$r = \sum_{i=1}^{i=n} w_i \times (r_{Li} - x_i) + \sum_{i=1}^{i=n} w_i \times (c_i + x_i) \tag{6.59}$$

where: x_i = interest rate in currency i

If the portfolio includes forward currency contracts these can be isolated separately as shown in Equation 6.60.

$$r = \sum_{i=1}^{i=n} w_i \times (r_{Li} - x_i) + \sum_{i=1}^{i=n} w_i \times (c_i + x_i) + \sum_{i=1}^{i=n} \widetilde{w}_i \times f_i \tag{6.60}$$

Note that using continuously compounded returns:

$$f_i = c_i + x_i - x_B$$

where: x_B = interest rate in base currency

If $\sum_{i=1}^{i=n} \widetilde{w}_i = 0$ then $\sum_{i=1}^{i=n} \widetilde{w}_i \times f_i = \sum_{i=1}^{i=n} c_i + x_i$ and Equation 6.60 simplifies to:

$$r = \sum_{i=1}^{i=n} w_i \times (r_{Li} - x_i) + \sum_{i=1}^{i=n} (w_i + \widetilde{w}_i) \times (c_i + x_i) \tag{6.61}$$

It follows that the benchmark can be produced in similar form:

$$b = \sum_{i=1}^{i=n} W_i \times (b_{Li} - x_i) + \sum_{i=1}^{i=n} (W_i + \widetilde{W}_i) \times (c_i + x_i) \tag{6.62}$$

where: b_{Li} = benchmark return in local currency for currency i

Subtracting Equation 6.62 from Equation 6.61:

$$r - b = \underbrace{\sum_{i=1}^{i=n} w_i \times \left(r_{Li} - x_i\right) - \sum_{i=1}^{i=n} W_i \times (b_{Li} - x_i)}_{\text{Local premium attribution}}$$
$$\underbrace{+\sum_{i=1}^{i=n} (\mathrm{w_i} + \widetilde{w}_i) \times (c_i + x_i) - \sum_{i=1}^{i=n} (W_i + \widetilde{W}_i) \times (c_i + x_i)}_{\text{Currency attribution}} \tag{6.63}$$

Applying the standard Brinson and Fachler approach separately to the local premium part and currency part of Equation 6.27 illustrated in Figure 6.9 we derive the following attribution effects:

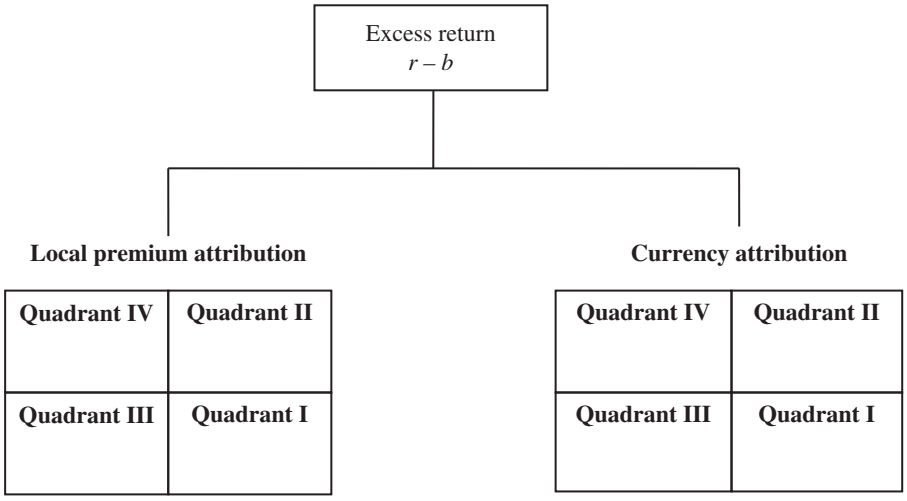

Figure 6.9 Karnosky and Singer multi-currency attribution

Asset allocation:

$$A_i = (w_i - W_i) \times (l'_i - l') \tag{6.64}$$

where:

$$l'_i = b_{Li} - x_i \text{ (the benchmark return premium)}$$

$$l' = \sum_{i=1}^{i=n} W \times l'_i \text{ (the average benchmark return premium)} \tag{6.65}$$

This definition of asset allocation includes the forward premium effect in the benchmark return premium.

Security selection excluding interaction:

$$S_i = W_i \times (k'_i - l'_i) \tag{6.66}$$

where:

$$k'_i = r_{Li} - x_i \text{ (the portfolio return premium)} \tag{6.67}$$

Interaction:

$$I_i = (w_i - W_i) \times (k'_i - l'_i) \tag{6.68}$$

Or, if you prefer, security selection including interaction:

$$S_i = w_i \times (k'_i - l'_i) \tag{6.69}$$

The contribution from currency is analogous to asset allocation:

$$C_i = \underbrace{(w_i - W_i) \times (c_i + x_i - c')}_{\text{Underlying assets}} + \underbrace{(\widetilde{w}_i - \widetilde{W}_i) \times (c_i + x_i - c')}_{\text{Currency forwards}} \tag{6.70}$$

where:

$$c' = \sum_{i=1}^{i=n} W_i \times (c_i + x_i) \tag{6.71}$$

From the data in Table 6.32 we can calculate total portfolio and benchmark returns and the local benchmark return premium and currency benchmark in Exhibit 6.52.

Table 6.32 Karnosky and Singer

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio local return r_{Li} (%)	Benchmark local return b_{Li} (%)	Local interest rates x_i (%)	Currency return c_i (%)
UK equities	40	40	15.0	10.0	1.5	0
Japanese equities	30	20	5.0	6.0	0.5	5
US equities	30	40	−8.0	−6.0	1.0	10
	$\widetilde{w}_i$	$\widetilde{W}_i$				
Sterling forward contracts	+40	+30			1.5	0
Yen forward contracts	−30	−10			0.5	5
US dollar forward contracts	−10	−20			1.0	10
Total	**100**	**100**	**5.1**	**2.8**		
Total base currency return	**7.45**	**5.5**				

Exhibit 6.52 Total portfolio and benchmark returns for Karnosky and Singer

Using Table 6.32 the portfolio return is:

$$r = 40\% \times (15.0\% + 0.0\%) + 30\% \times (6.0\% + 5.0\%) + 30\% \times (-6.0\% + 10.0\%) + 40\% \times (1.5\% + 0.0\%) - 30\% \times (0.5\% + 5.0\%) - 10\% \times (1.0\% + 10.0\%) = 7.45\%$$

The benchmark return is:

$$b = 40\% \times (10.0\% + 0.0\%) + 20\% \times (6.0\% + 5.0\%) + 40\% \times (-6.0\% + 10.0\%) + 30.0\% \times (1.5\% + 0.0\%) - 10\% \times (0.5\% + 5.0\%) - 20\% \times (1.0\% + 10.\%) = 5.5\%$$

The average local benchmark return premium is:

$$l' = 40\% \times (10.0\% - 1.5\%) + 20\% \times (6.0\% - 0.5\%) + 40\% \times (-6.0\% - 1.0\%) = 1.7\%$$

The currency plus interest benchmark return is:

$$c' = 40\% \times (1.5\% + 0.0\%) + 20\% \times (0.5\% + 5.0\%) + 40\% \times (1\% + 10\%) + 30\% \times (1.5\% + 0.0\%) - 10\% \times (0.5\% + 5\%) - 20\% \times (1\% + 10\%) = 3.8\%$$

Asset allocation effects using the return premium are calculated in Exhibit 6.53.

Exhibit 6.53 Asset allocation for Karnosky and Singer

	$(w_i - W_i) \times (l'_i - l')$
UK equities	$(40\% - 40\%) \times [(10.0\% - 1.5\%) - 1.7\%] = 0.0\%$
Japanese equities	$(30\% - 20\%) \times [(6.0\% - 0.5\%) - 1.7\%] = 0.38\%$
US equities	$(30\% - 40\%) \times [(-6.0\% - 1.0\%) - 1.7\%] = 0.87\%$
Total	$0.0\% + 0.38\% + 0.87\% = 1.25\%$

Stock selection effects effectively based on local portfolio returns are calculated in Exhibit 6.54.

Exhibit 6.54 Stock selection (including Interaction) for Karnosky and Singer

	$w_i \times (k'_i - l'_i)$
UK equities	$40\% \times [(15.0\% - 1.5\%) - (10.0\% - 1.5\%)] = 2.0\%$
Japanese equities	$30\% \times [(5.0\% - 0.5\%) - (6.0\% - 0.5\%)] = -0.3\%$
US equities	$30\% \times [(-8.0\% - 1.0\%) - (-6.0\% - 1.0\%)] = -0.6\%$
Total	$2.0\% - 0.3\% - 0.6\% = 1.1\%$

The currency management effect, including forward currency contracts, is calculated in Exhibit 6.55. Note that the currency benchmark is essentially currency plus cash.

Exhibit 6.55 Currency for Karnosky and Singer

Underlying assets	$(w_i - W_i) \times (c_i + x_i - c')$
Sterling	$(40\% - 40\%) \times [(0.0\% + 1.5\%) - 3.8\%] = 0.0\%$
Yen	$(30\% - 20\%) \times [(5.0\% + 0.5\%) - 3.8\%] = 0.17\%$
US dollar	$(30\% - 40\%) \times [(10.0\% + 1.0\%) - 3.8\%] = -0.72\%$
Currency forwards	$(\widetilde{w}_i - \widetilde{W}_i) \times (c_i + x_i - c')$
Sterling	$(40\% - 30\%) \times [(0.0\% + 1.5\%) - 3.8\%] = -0.23\%$
Yen	$(-30\% + 10\%) \times [(5.0\% + 0.5\%) - 3.8\%] = -0.34\%$
US dollar	$(-10\% + 20\%) \times [(10.0\% + 1.0\%.) - 3.8\%] = 0.72\%$
Total	$0.0\% + 0.17\% - 0.72\% - 0.23\% - 0.34\% + 0.72\% = -0.40\%$

The Karnosky and Singer attribution effects are summarised in Table 6.33. This portfolio added value through stock selection and asset allocation but lost value via currency management. Although the physical assets were in line with the benchmark, the currency manager chose to use currency forwards to overweight sterling, resulting in a loss of −0.23%. Even though the base currency of the account is sterling it is still possible to overweight sterling against a benchmark containing other currency exposures. The other currencies both outperformed sterling even after taking into account higher sterling interest rates. The US dollar currency forward overweight position of 10% exactly matched the underweight US equity physical asset position, therefore the attribution effects cancelled. Unfortunately, the overweight Japanese equity position was more than matched by the underweight yen currency forward position, leading to a net loss of −0.17%. The total loss due to currency management is therefore −0.4%.

Table 6.33 Karnosky and Singer attribution

	Asset allocation $(w_i - W_i) \times (l'_i - l')$ (%)	Stock selection $w_i \times (k'_i - l'_i)$ (%)	Currency management $(w_i - W_i) \times (c_i + x_i - c')$ (%)
UK equities	0.0	2.0	0.0
Japanese equities	0.38	−0.30	0.17
US equities	0.87	−0.60	−0.72
			$(\widetilde{w}_i - \widetilde{W}_i) \times (c_i + x_i - c')$
Sterling forward			−0.23
Yen forwards			−0.34
US dollar forwards			0.72
Total	**1.25**	**1.1**	**−0.40**
Total excess return			**1.95**

GEOMETRIC MULTI-CURRENCY ATTRIBUTION

Karnosky and Singer overcame the arithmetic deficiencies evident in Ankrim and Hensel by using continuously compounded returns. If we wish to continue using simple returns, because

of the geometric relationship between market and currency returns it is essential to use the geometric definition of excess return for multi-currency attribution.

Naïve currency attribution

Deriving the contribution from currency attribution at the top level is a relatively straightforward calculation.

We have already established the total portfolio returns in the base currency as:

$$r = \sum_{i=1}^{i=n} w_i \times r_i \qquad \text{(3.41) or (6.3)}$$

and in local currency:

$$r_L = \sum_{i=1}^{i=n} w_i \times r_{Li} \qquad (3.45)$$

Similarly for the benchmark:

$$b = \sum_{i=1}^{i=n} W_i \times b_i \qquad \text{(6.1) or (4.8)}$$

and in local currency the weighted average local benchmark return:

$$b_L = \sum_{i=1}^{i=n} W_i \times b_{Li} \qquad (6.72)$$

and defining in local currency the semi-notional local return:

$$b_{SL} = \sum_{i=1}^{i=n} w_i \times b_{Li} \qquad (6.73)$$

By definition the currency performance of the portfolio must be the relative difference between the portfolio performance in base currency and the weighted total local return of the portfolio.

$$r'_C = \frac{1 + r}{1 + r_L} - 1 \qquad (6.74)$$

Likewise, the currency performance of the benchmark is defined as the relative difference between the performance of the benchmark in base currency and the performance of the benchmark in local currency.

$$b'_C = \frac{1 + b}{1 + b_L} - 1 \qquad (6.75)$$

The ratio of the currency return of the portfolio relative to the currency return of the benchmark, called naïve attribution, is:

$$\left(\frac{1 + r}{1 + r_L} \Big/ \frac{1 + b}{1 + b_L} \right) - 1 \qquad (6.76)$$

This can be rewritten as:

$$\text{Naive currency attribution } \left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1 \tag{6.77}$$

This simple version of currency attribution is described as naïve because it does not take into account interest rate differentials in the manner so excellently described by Karnosky and Singer to which we will return later.

In this naïve version of currency attribution, we can calculate our normal attribution effects of stock selection and allocation in local currency as follows:

$$\text{Stock selection } \frac{1+r_L}{1+b_{SL}} - 1 \tag{6.78}$$

$$\text{Asset allocation } \frac{1+b_{SL}}{1+b_L} - 1 \tag{6.79}$$

$$\text{Total currency effects } \left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1 \tag{6.80}$$

We can then see that these factors compound to give:

$$\left(\frac{1+r_L}{1+b_{SL}}\right) \times \left(\frac{1+b_{SL}}{1+b_L}\right) \times \left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1 = \frac{1+r}{1+b} - 1 \tag{6.81}$$

Note

Naïve currency attribution is so straightforward and complete currency attribution is so complex that I genuinely wonder if the extra complexity is worth the effort. My natural instincts as a performance analyst is to explain everything, accurately. Attribution analysis must reflect the investment decision process. Karnosky and Singer tell us, correctly, that we must take interest rate differentials into account. However, I'm not sure many asset managers take interest rate differentials into account when making allocation decisions. I'm trapped between the technically correct approach and actual practice. The effects also are typically quite small.

extending our existing example in Table 6.34, including currency but with no forward currency contracts and expressed in the base currency of sterling. Base currency returns are calculated in Exhibit 6.56.

Table 6.34 Multi-currency geometric

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio local return r_{Li} (%)	Benchmark local return b_{Li} (%)	Portfolio base return r_i (%)	Benchmark base return b_i (%)	Currency return c_i (%)
UK equities	40	40	20	10	20.0	10.0	0
Japanese equities	30	20	−5	−4	4.5	5.6	10
US equities	30	40	6	8	27.2	29.6	20
Total	100	100	8.3	6.4	17.51	16.96	

Exhibit 6.56 Multi-currency portfolio returns

Using Table 6.5 the local portfolio return is:

$$r_L = 40\% \times 20\% + 30\% \times -5\% + 30\% \times 6\% = 8.3\%$$

The weighted average local benchmark return is:

$$b_l = 40\% \times 10\% + 20\% \times -4\% + 40\% \times 8\% = 6.4\%$$

The local semi-notional return is:

$$b_{SL} = 40\% \times 10\% + 30\% \times -4\% + 30\% \times 8\% = 5.2\%$$

The base currency (£) return of the portfolio:

$$r = 40\% \times 20\% + 30\% \times 4.5\% + 30\% \times 27.2\% = 17.51\%$$

The base currency (£) return of the benchmark:

$$b = 40\% \times 10\% + 20\% \times 4.6\% + 40\% \times 29.6\% = 16.96\%$$

Naïve currency attribution is a fairly straightforward calculation geometrically, as shown in Exhibit 6.57. Using the data in Table 6.35 the return from currency in the portfolio is 8.5% and the return from currency in the benchmark is 9.92%, therefore the added value from currency management must be the ratio of the portfolio currency return with the benchmark currency return, –1.29%.

Exhibit 6.57 Naïve currency attribution

The currency return in the portfolio is:

$$r'_C = \frac{1+r}{1+r_L} - 1 = \frac{1.1751}{1.083} - 1 = 8.50\%$$

The currency return in the benchmark is:

$$b'_C = \frac{1+b}{1+b_L} - 1 = \frac{1.1696}{1.064} - 1 = 9.92\%$$

The naïve currency allocation is therefore:

$$\frac{1+r'_C}{1+b'_C} - 1 = \frac{1+r}{1+r_L} \Big/ \frac{1+b}{1+b_L} - 1 = \frac{1.0850}{1.0992} - 1 = -1.29\%$$

The asset allocation and stock selection shown in Exhibits 6.58 and 6.59 are unchanged from Exhibits 6.7 and 6.8.

Exhibit 6.58 Multi-currency asset allocation (naïve)

From Exhibit 6.7 the geometric asset allocation is:

$$\frac{(1+b_{SL})}{(1+b)} - 1 = \frac{1.052}{1.064} - 1 = -1.13\%$$

Individual country asset allocation effects are:

UK equities	$(w_i - W_i) \times \left(\frac{1+b_{Li}}{1+b_L} - 1\right) = (40\% - 40\%) \times \left(\frac{1.10}{1.064} - 1\right) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times \left(\frac{0.96}{1.064} - 1\right) = -0.98\%$
US equities	$(30\% - 40\%) \times \left(\frac{1.08}{1.064} - 1\right) = -0.15\%$
Total	$0.0\% - 0.98\% - 0.15\% = -1.13\%$

Exhibit 6.59 Multi-currency stock selection

From before in Exhibit 6.8 total stock selection in local currency is:

$$\frac{1+r_L}{1+b_{SL}} - 1 = \frac{1.083}{1.052} - 1 = 2.95\%$$

Individual country stock selection effects are:

UK equities	$w_i \times \left(\frac{1+r_{Li}}{1+b_{Li}} - 1\right) \times \frac{1+b_{Li}}{1+b_{SL}} = 30\% \times \left(\frac{1.2}{1.1} - 1\right) \times \frac{1.1}{1.052} = 3.8\%$
Japanese equities	$30\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.96}{1.052} = -0.29\%$
US equities	$40\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.052} = -0.57\%$
Total	$3.8\% - 0.29\% - 0.57\% = 2.95\%$

Note that the currency, asset allocation and stock selection compound to provide the total geometric excess return as shown in Exhibit 6.60.

Exhibit 6.60 Naïve currency attribution

$$\left(\frac{1+r_L}{1+b_{SL}}\right) \times \left(\frac{1+b_{SL}}{1+b_L}\right) \times \left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1$$

$$= \frac{1.083}{1.052} \times \frac{1.052}{1.064} \times \frac{1.1751}{1.083} \times \frac{1.064}{1.1696} - 1$$

$$= \frac{1+r}{1+b} - 1 = \frac{1.1751}{1.1696} - 1 = 0.47\%$$

or

$$1.0295 \times 0.9887 \times 0.9871 - 1 = 0.47\%$$

The naïve currency attribution results are summarised in Table 6.35.

Table 6.35 Naïve currency attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_{Li}}{1+b_L} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_{Li}}{1+b_{Li}} - 1\right) \times \frac{1+b_{Li}}{1+b_{SL}}$ (%)	Naïve currency allocation $\left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1$ (%)
UK equities	40	40	20	10	0.0	3.8	
Japanese equities	30	20	4.7	5.6	−0.98	−0.28	
US equities	30	40	28.0	29.6	−0.15	−0.57	
Total	**100**	**100**	**17.51**	**16.96**	**−1.13**	**2.95**	**−1.29**

Compounding effects

An unavoidable but calculable complication in multi-currency portfolios is the impact of changing currency exposure due to the changing market values of the underlying assets.

Genuine "currency overlay" managers are unaware of changing market values (either in the portfolio or benchmark) and are only obliged to respond when they are informed of market value changes. Therefore, if the currency management is independent, we must isolate these compounding effects.

The currency return of the benchmark in the *i*th currency can be calculated as follows:

$$c_i = \frac{1 + b_i}{1 + b_{Li}} - 1 \tag{6.82}$$

Because most commercial international indexes use the same spot rates[27] benchmark currency returns for each currency will be consistent and can be derived from base and local index returns using Equation 6.82.

Currency returns from Table 6.35 are calculated in Exhibit 6.61.

Exhibit 6.61 Currency returns

Yen	$\frac{1.056}{0.96} - 1 = 10.0\%$
\$	$\frac{1.296}{1.08} - 1 = 20.0\%$
At this stage for simplicity, I've assumed the currency return within the portfolio is the same as within the benchmark:	
Yen	$\frac{1.045}{0.95} - 1 = 10.0\%$
\$	$\frac{1.272}{1.06} - 1 = 20.0\%$

We might expect the implied currency return in the benchmark to be equal to the sumproduct of the benchmark weights and currency returns:

$$\text{Implied benchmark currency return } b_C = \sum_{i=1}^{i=n} W_i \times c_i \tag{6.83}$$

Similarly, we might expect the implied currency return in the portfolio to be equal to the sumproduct of the portfolio weights and currency returns:

$$\text{Implied portfolio currency return } b_{SC} = \sum_{i=1}^{i=n} w_i \times c_i \tag{6.84}$$

[27] WM Reuters 4 o'clock London close.

Implied currency returns are calculated in Exhibit 6.62.

Exhibit 6.62 Implied currency returns

Implied benchmark currency return:

$$b_C = 40\% \times 0\% + 20\% \times 10.0\% + 40\% \times 20.0\% = 10.0\%$$

Implied portfolio currency return:

$$b_{SC} = 40\% \times 0\% + 30\% \times 10.0\% + 30\% \times 20.0\% = 9.0\%$$

Both the implied benchmark and portfolio currency returns in Exhibit 6.62 appear to differ from the actual currency returns calculated in Exhibit 6.57, which at first seems somewhat unexpected. This is caused by the changing market values of the underlying assets during the period of measurement. In this example the falling Japanese market reduces the exposure to the raising yen, which is only partially offset by the increased exposure to the US dollar caused by the rising US market. The derivation of the real benchmark and portfolio currency returns adjusted for changing market values are shown in Exhibit 6.63.

Exhibit 6.63 Currency return

Real benchmark currency return	$b'_C = \dfrac{1+b}{1+b_L} - 1 = 9.92\%$
Sterling	$40\% \times 0.0\% \times \left(\dfrac{1.1}{1.064}\right) = 0\%$
Japanese equities	$20\% \times 10.0\% \times \left(\dfrac{0.96}{1.064}\right) = 1.80\%$
US equities	$40\% \times 20.0\% \times \left(\dfrac{1.08}{1.064}\right) = 8.12\%$
	$0.0\% + 1.8\% + 8.12\% = 9.92\%$
Real portfolio currency return	$r'_C = \dfrac{1+r}{1+r_L} - 1 = 8.5\%$
Sterling	$40\% \times 0.0\% \times \left(\dfrac{1.20}{1.083}\right) = 0\%$
Yen	$30\% \times 10.0\% \times \left(\dfrac{0.95}{1.083}\right) = 2.63\%$
	$0.0\% + 2.63\% + 5.87\% = 8.50\%$
US dollar	$30\% \times 20.0\% \times \left(\dfrac{1.06}{1.083}\right) = 5.87\%$

The difference between implied and real currency returns is called compounding. The total impact of compounding within the benchmark is:

$$\frac{1+b_C}{1+b'_C} - 1 \tag{6.85}$$

The total impact of compounding within the portfolio is:

$$\frac{1 + r'_C}{1 + b_{SC}} - 1 \tag{6.86}$$

The total compounding effects in our sample portfolio using data from Table 6.35 are calculated in Exhibit 6.64.

Exhibit 6.64 Compounding effects

Benchmark compounding:

$$\frac{1 + b_C}{1 + b'_C} - 1 = \frac{1.1}{1.0992} - 1 = 0.07\%$$

Portfolio compounding:

$$\frac{1 + r'_C}{1 + b_{SC}} - 1 = \frac{1.085}{1.09} - 1 = -0.45\%$$

Clearly, it is inappropriate to allocate this effect to currency overlay managers unaware of market movements between points of measurement. The effects in this sample portfolio are unusually large but normally only amount to a few basis points, particularly if measurement periods are maintained as short as possible, preferably daily.

Other geometric attribution multi-currency methodologies (Bain[28] and McLaren[29]) recognise this impact but suggest addressing this issue by adjusting market weights to calculate revised stock and allocation effects.[30]

Geometric currency allocation

Using the implied currency returns implicit in Equations 6.84 and Equation 6.83, we can calculate currency allocation effects from the perspective of the currency overlay manager, excluding compounding effects.

Currency allocation is analogous to asset allocation and we can use a similar formula:

$$(w_i - W_i) \times \left(\frac{1 + c_i}{1 + b_C} - 1 \right) \tag{6.87}$$

So, the total currency allocation performance is:

$$\frac{1 + b_{SC}}{1 + b_C} - 1 \tag{6.88}$$

[28] Bain, *Investment Performance Measurement* (1996).
[29] McLaren, "A Geometric Methodology for Performance Attribution" (2001).
[30] McLaren provides a choice of which factor to impact determined by the order of investment decision process.

Currency allocation effects are calculated in Exhibit 6.65.

Exhibit 6.65 Geometric currency allocation

Total currency allocation:

$$\frac{1+b_{SC}}{1+b_C} - 1 = \frac{1.09}{1.10} = -0.91\%$$

Calculating for individual currencies:

Sterling (£)	$= (40\% - 40\%) \times \left(\frac{1.0}{1.10} - 1\right) = 0.00\%$
Yen	$= (30\% - 20\%) \times \left(\frac{1.1}{1.1} - 1\right) = 0.00\%$
US dollar ($)	$= (30\% - 40\%) \times \left(\frac{1.20}{1.10} - 1\right) = -0.91\%$
Total	$0.00\% + 0.00\% - 0.91\% = -0.91\%$

The currency allocation effects and both compounding effects combine to produce the total naïve currency attribution as follows:

$$\underbrace{\left(\frac{1+r'_C}{1+b_{SC}}\right)}_{\text{Portfolio compounding}} \times \underbrace{\left(\frac{1+b_C}{1+b'_C}\right)}_{\text{Benchmark compounding}} \times \underbrace{\left(\frac{1+b_{SC}}{1+b_C}\right)}_{\text{Currency allocation}} - 1 = \frac{1+r'_C}{1+b'_c} - 1 \tag{6.89}$$

Currency timing

Up to now we have assumed that the return for each currency in the portfolio is the same as the benchmark; in real portfolios, this is rarely the case. Transactions will take place at exchange rates other than the exchange rates used to calculate indexes. This effect is analogous to stock selection and can be called currency selection or currency timing.

Most performance measurement systems actually derive local returns directly from the base return using benchmark currency returns, therefore a currency timing effect is never observed (one could say lost). More accurately, the local and base currency returns should be calculated separately using market values and cash flows in local and base currency, respectively. From the underlying assets point of view this will most likely reflect issues in calculation methodology and data rather than genuine currency timing effects but for forward currency contracts genuine timing effects are likely to be observed and are likely to be important.

Note that b_{SC} is in effect a semi-notional return for currency in that it does not reflect any currency timing effects.

Including currency timing effects, the implied currency return of the portfolio should be:

$$r_C = \sum_{i=1}^{i=n} w_i \times c'_i \tag{6.90}$$

where the portfolio currency return in each market is defined as:

$$c'_i = \frac{1+r_i}{1+r_{Li}} - 1 \tag{6.91}$$

The formula for currency timing is analogous to stock selection in the ith currency as follows:

$$w_i \times \left(\frac{1 + c'_i}{1 + c_i} - 1 \right) \times \left(\frac{1 + c_i}{1 + b_{SC}} \right) \tag{6.92}$$

So the total currency timing effect is:

$$\frac{1 + r_C}{1 + b_{SC}} - 1 \tag{6.93}$$

Equation 6.89 must be expanding to include currency timing:

$$\underbrace{\left(\frac{1 + r'_C}{1 + r_C} \right)}_{\substack{\text{Portfolio} \\ \text{compounding}}} \times \underbrace{\left(\frac{1 + b_C}{1 + b'_C} \right)}_{\substack{\text{Benchmark} \\ \text{compounding}}} \times \underbrace{\left(\frac{1 + b_{SC}}{1 + b_C} \right)}_{\substack{\text{Currency} \\ \text{allocation}}} \times \underbrace{\left(\frac{1 + r_C}{1 + b_{SC}} \right)}_{\substack{\text{Currency} \\ \text{timing}}} - 1 = \frac{1 + r'_C}{1 + b'_c} - 1$$

$$= \frac{1 + r}{1 + r_L} \times \frac{1 + b_L}{1 + b} - 1 \tag{6.94}$$

The combination of currency allocation and currency timing can be genuinely allocated to the currency overlay manager.

Equation 6.81 can now be expanded to:

$$\underbrace{\frac{1 + r_L}{1 + b_{SL}}}_{\substack{\text{Stock} \\ \text{selection}}} \times \underbrace{\frac{1 + b_{SL}}{1 + b_L}}_{\substack{\text{Asset} \\ \text{allocation}}} \times \underbrace{\frac{1 + r_C}{1 + b_{SC}} \times \frac{1 + b_{SC}}{1 + b_C}}_{\substack{\text{Currency} \\ \text{overlay}}} \times \underbrace{\frac{1 + r'_C}{1 + r_C} \times \frac{1 + b_C}{1 + b'_C}}_{\text{Compounding}} = 1 - \frac{1 + r}{1 + b} - 1 \tag{6.95}$$

The data in Table 6.36 is adapted from Table 6.34 to illustrate the impact of portfolio currency returns differing from benchmark currency returns. Revised portfolio and portfolio implied currency returns are calculated in Exhibit 6.66; benchmark returns are unchanged.

Exhibit 6.66 Revised portfolio returns

Portfolio base currency return

$$r = 40\% \times 20\% + 30\% \times 4.7\% + 30\% \times 28.0\% = 17.81\%$$

Note that in Table 6.37 the portfolio currency return in Japanese equities is:

$\frac{1.047}{0.95} - 1 = 10.2\%$ This differs from the benchmark currency of 10.0%.

Likewise, the portfolio currency in US dollars is:

$\frac{1.28}{1.06} - 1 = 20.8\%$ This differs from the benchmark currency of 20.0%.

The implied portfolio currency return is now:

$$r_C = 40\% \times 0\% + 30\% \times 10.2\% + 30\% \times 20.8\% = 9.29\%$$

Table 6.36 Multi-currency geometric

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio local return r_{Li} (%)	Benchmark local return b_{Li} (%)	Portfolio base return r_i (%)	Benchmark base return b_i (%)	Portfolio currency return c_i' (%)	Benchmark currency return c_i (%)
UK equities	40	40	20	10	20.0	10.0	0	0
Japanese equities	30	20	−5	−4	4.7	5.6	10.2	10
US equities	30	40	6	8	28.0	29.6	20.8	20
Total	**100**	**100**	**8.3**	**6.4**	**17.81**	**16.96**		

The currency timing effect can now be calculated in Exhibit 6.67.

Exhibit 6.67 Currency timing effect

$$\frac{1+r_C}{1+b_{SC}} - 1 = \frac{1.0929}{1.09} - 1 = 0.27\%$$

Calculating the impact per currency:

Sterling	$40\% \times \left(\frac{1.0}{1.0} - 1\right) \times \frac{1.0}{1.09} = 0\%$
Yen	$30\% \times \left(\frac{1.102}{1.10} - 1\right) \times \frac{1.10}{1.09} = 0.06\%$
US dollar	$30\% \times \left(\frac{1.208}{1.20} - 1\right) \times \frac{1.20}{1.09} = 0.21\%$
Total	$0\% + 0.06\% + 0.21\% = 0.27\%$

The impact of currency timing will change both the portfolio currency return and the compounding effects as demonstrated in Exhibit 6.68.

Exhibit 6.68 Impact of currency timing

Portfolio currency return:

$$\frac{1+r}{1+r_L} - 1 = \frac{1.178}{1.083} - 1 = 8.78\%$$

The total impact of compounding within the portfolio is:

$$\frac{1+r'_C}{1+r_C} - 1 = \frac{1.0878}{1.0929} - 1 = -0.47\%$$

From before the compounding impact in the benchmark is:

$$\frac{1+b_C}{1+b'_C} = \frac{1.1}{1.0992} - 1 = 0.07\%$$

Revised attribution effects are summarised in Exhibit 6.69.

Exhibit 6.69 Attribution summary

Stock selection:	$\frac{1+r_L}{1+b_{SL}} - 1 = \frac{1.083}{1.052} - 1 = 2.95\%$
Asset allocation:	$\frac{1+b_{SL}}{1+b_L} - 1 = \frac{1.052}{1.064} - 1 = -1.13\%$
Naïve currency attribution:	$\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1 = \frac{1.178}{1.083} \times \frac{1.064}{1.1696} = -1.04\%$

(Continued)

Exhibit 6.69 (*Continued*)

We can then see that these factors compound again to give:

$$\underbrace{\frac{1+r_L}{1+b_{SL}}}_{\text{Stock}} \times \underbrace{\frac{1+b_{SL}}{1+b_L}}_{\text{Asset}} \times \underbrace{\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b}}_{\text{Native currency attribution}} - 1 = \frac{1+r}{1+b} - 1$$

$$\underbrace{\frac{1.083}{1.052}}_{\text{Stock}} \times \underbrace{\frac{1.052}{1064}}_{\text{Asset}} \times \underbrace{\frac{1.1781}{1083} \times \frac{1.064}{1.1696}}_{\text{Native currency attribution}} - 1 = \frac{1.1781}{1.1696} - 1 = 0.73\%$$

Or

$$1.0295 \times 0.9887 \times 0.9896 - 1 = 0.73\%$$

The naïve currency effect can be broken down further to:

$$\underbrace{\frac{1+r_C}{1+b_{SC}} \times \frac{1+b_{SC}}{1+b_C}}_{\text{Currency overlay}} \times \underbrace{\frac{1+r'_C}{1+r_C} \times \frac{1+b_C}{1+b'_C}}_{\text{Compounding}} - 1$$

$$\underbrace{\frac{1.0929}{1.09} \times \frac{1.09}{1.1}}_{\text{Currency overlay}} \times \underbrace{\frac{1.0878}{1.0929} \times \frac{1.1}{1.0992}}_{\text{Compounding}} - 1 = -1.04\%$$

INTEREST RATE DIFFERENTIALS

A further complicating factor in multi-currency portfolios is the exposure of currency managers to interest rate differentials between currencies as they take currency "bets".

Ankrim and Hensel recognised the impact of interest rate differentials by identifying a separate forward premium effect; Karnosky and Singer recognised this effect by using the return premium of the local market return above local interest rates.

One way to understand why interest rate differentials are important is to focus on asset allocation within the investment decision process. Assuming there is a separate country and currency allocation process, if the country manager decides to overweight Japanese equities, that will inevitably create a long position in yen; the currency manager wishing to keep a neutral yen position may decide to hedge the exposed yen position using currency forwards. If the base currency of the portfolio is sterling, the currency manager will sell yen and buy sterling. In effect, the currency manager is borrowing yen to buy sterling – there is a cost or benefit attached to this, depending on the interest rate differential between the two currencies at the time; currently sterling interest rates are higher than yen, hence it's a benefit. This is why the "carry trade" is so beneficial for Japanese investors at present; they have been selling yen and buying high-yielding currencies such as the New Zealand dollar to great advantage,

clearly an effect that cannot be ignored in attribution analysis. The process of maintaining a neutral currency position within the portfolio is described as "hedged to neutral".

Apart from hedging exposed positions caused by country managers or achieving hedged positions implied by hedged benchmarks, the currency manager may well be seeking to generate active currency positions – in effect, this can only be achieved by forward currency contracts or other derivative instruments that implicitly include interest rate differentials.

Forward currency contracts are priced by reference to the interest rate differential between the relevant currencies. Therefore, any currency manager wishing to take a currency allocation "bet" must be exposed to the costs (or benefits) of these interest rate differentials. In other words, the forward currency return rather than the spot currency return must be used to measure currency allocation effects. Crucially, this cost or benefit should be borne by the country allocator, not the currency manager.

The currency return between spot rates can be broken down to the forward currency return and the forward premium (or interest rate differential) as follows:

$$c_i = \frac{S_i^{t+1}}{S_i^t} - 1 \tag{6.96}$$

Defining the return on a benchmark forward currency contract as:

$$f_i = \frac{S_i^{t+1}}{F_i^{t+1}} - 1 \tag{6.97}$$

Note that the interest rate differential or forward premium in between currency i and the base currency of the portfolio is:

$$d_i = \frac{F_i^{t+1}}{S_i^t} - 1 \tag{6.98}$$

The currency return is therefore:

$$\frac{S_i^{t+1}}{S_i^t} = \frac{S_i^{t+1}}{F_i^{t+1}} \times \frac{F_i^{t+1}}{S_i^t} = (1 + f_i) \times (1 + d_i) = (1 + c_i) \tag{6.99}$$

The hedged benchmark return b_{Hi} in currency i is defined as:

$$b_{Hi} = \frac{(1 + b_i)}{(1 + f_i)} - 1 \tag{6.100}$$

Note that base, local, hedged, currency, forward returns and interest rate differentials are linked as follows:

$$b_{Hi} = (1 + b_{Li}) \times (1 + d_i) - 1 \tag{6.101}$$

$$b_{Hi} = \frac{(1 + b_i)}{(1 + f_i)} - 1 = (1 + b_{Li}) \times (1 + d_i) - 1 \tag{6.102}$$

Thus, the hedged return is either the local return compounded with the interest rate differential or equivalently the base return divided by the forward currency return:

$$\frac{(1+b_i)}{(1+b_{Li})} - 1 = (1+f_i) \times (1+d_i) - 1 = c_i \tag{6.103}$$

Hedged benchmark returns are normally calculated slightly differently in commercial indexes (and therefore incorrectly). Typically, monthly hedged returns are calculated based on a notional forward contract at the start of the month. Markets will rise or fall but the forward contract is unaltered, thus causing residual currency exposure in the calculation.

Revised currency allocation

We must recognise that any "bet" caused by the currency manager must be generated by a forward currency contract (notional or actual) or other currency derivative; then the forward currency rate, not the spot rate, must be used to measure the contribution of that decision.

Therefore, the implied currency return or semi-notional currency return must be adjusted for any variation from the benchmark as follows:

$$b_{SC} = \sum_{i=1}^{i=n} W_i \times c_i + \sum_{i=1}^{i=n} (w_i - W_i) \times f_i \tag{6.104}$$

Equation 6.87 must be adapted to use forward rates rather than spot rates:

$$(w_i - W_i) \times \left(\frac{1+f_i}{1+b_C} - 1 \right) \tag{6.105}$$

The total currency allocation effect remains:

$$\frac{1+b_{SC}}{1+b_C} - 1 \tag{6.88}$$

To continue our analysis, we must supplement the data in Table 6.36 with the additional forward currency data[31] included in Table 6.37. Revised currency allocation is calculated in Exhibit 6.70.

Table 6.37 Forward currency data

	Benchmark hedged return b_{Hi} (%)	Forward currency (%)	Benchmark forward return f_i (%)
UK equities	10.0	Sterling	0.0
Japanese equities	−3.0	Yen	8.87
US equities	10.0	US $	17.82

[31] Note that the interest rate differentials of sterling against the yen and US dollar are: Yen $\frac{1.10}{1.0887} - 1$ or $\frac{0.97}{0.96} - 1 = 1.04\%$ (UK interest rates, 1.04% higher than Japan); US dollar $\frac{1.20}{1.1782} - 1$ or $\frac{1.1}{1.08} - 1 = 1.85\%$ (UK interest rates, 1.85% higher than US).

Exhibit 6.70 Revised currency allocation using forward rates

Revised implied currency return (or semi-notional currency return):

$$b_{SC} = 40\% \times 0\% + 20\% \times 10.0\% + 40\% \times 20.0\% + (40\% - 40\%) \times 0\% + (30\% - 20\%) \times 8.87\% + (30\% - 40\%) \times 17.82\% = 9.10\%$$

Revised total currency allocation:

$$\frac{1 + b_{SC}}{1 + b_C} - 1 = \frac{1.091}{1.10} = 0.81\%$$

Calculating for individual currencies

Sterling (£)	$= (40\% - 40\%) \times \left(\frac{1.0}{1.10} - 1\right) = 0.00\%$
Yen	$= (30\% - 20\%) \times \left(\frac{1.089}{1.10} - 1\right) = -0.10\%$
US dollar ($)	$= (30\% - 40\%) \times \left(\frac{1.178}{1.0555} - 1\right) = -0.71\%$
Total	$0.00\% - 0.10\% - 0.71\% = -0.81\%$

Revised country allocation

The cost or benefit caused by physical currency positions must be borne by the country allocator. To do this we can use hedged indexes rather than local indexes to measure the true impact of the country allocator. The cost or benefit is in effect transferred from the currency overlay manager to the country allocator.

The revised semi-notional return, including the cost of hedge to neutral, is:

$$b_{SH} = \sum_{i=1}^{i=n} W_i \times b_{Li} + (w_i - W_i) \times b_{Hi} \tag{6.106}$$

This represents a minor departure from the Karnosky and Singer approach in which any deviation from the base currency is hedged. In this approach only deviations from the benchmark currency position are hedged.

Equation 6.25 must be adapted to use hedged indexes as follows:

$$(w_i - W_i) \times \left(\frac{1 + b_{Hi}}{1 + b_L} - 1\right) \tag{6.107}$$

Total revised asset allocation, including the cost of hedging:

$$\frac{1 + b_{SH}}{1 + b_L} - 1 \tag{6.108}$$

The revised semi-notional fund and revised asset allocation is calculated in Exhibit 6.71. The revised asset allocation effect is –1.22%. This differs very slightly from the original asset allocation effect of –1.13% earlier. The difference between the two is simply the cost

of hedging the exposed asset allocation bets of the country allocator. Generally, there is no requirement to break down this cost per currency but for completeness this calculation is shown in Exhibit 6.72. There is a benefit in Japan because of the "carry trade"; in effect, low-yielding yen are sold for high-yielding sterling. In this example, low-yielding dollars must be borrowed from high-yielding sterling. The interest rate differential between dollars and sterling is greater than the interest rate differential between yen and sterling; therefore, the overall cost is negative.

Exhibit 6.71 Revised asset allocation

Revised semi-notional return:

$$b_{SH} = 40\% \times 10.0\% + 20\% \times -4.0\% + 40\% \times 8.0\% + (40\% - 40\%) \times 10\%$$
$$+ (30\% - 20\%) \times -3.0\% + (30\% - 40\%) \times 10.0\% = 5.1\%$$

Revised country allocation:

$$\frac{1 + b_{SH}}{1 + b_L} - 1 = \frac{1.051}{1.064} - 1 = -1.22\%$$

UK allocation	$= (40\% - 40\%) \times \left(\frac{1.1}{1.064} - 1\right) = 0.0\%$
Japanese allocation	$= (30\% - 40\%) \times \left(\frac{0.97}{1.064} - 1\right) = -0.88\%$
US allocation	$= (30\% - 40\%) \times \left(\frac{1.1}{1.064} - 1\right) = -0.34\%$
Total	$0.0\% - 0.88\% - 0.34\% = -1.22\%$

Exhibit 6.72 Cost of hedging to neutral

$$\frac{1 + b_{SH}}{1 + b_{SL}} - 1 = \frac{1.051}{1.052} - 1 = -0.09\%$$

The cost in each market can be calculated as follows, although this level of detail is rarely required:

UK	$= (40\% - 40\%) \times \left(\frac{1.10}{1.10} - 1\right) \times \left(\frac{1.10}{1.052}\right) = 0.00\%$
Japan	$= (30\% - 20\%) \times \left(\frac{0.97}{0.96} - 1\right) \times \left(\frac{0.96}{1.052}\right) = 0.10\%$
US	$= (30\% - 40\%) \times \left(\frac{1.08}{1.10} - 1\right) \times \left(\frac{1.10}{1.052}\right) = -0.19\%$
Total	$0.00\% + 0.10\% - 0.19\% = -0.09\%$

Stock selection effects are calculated in local currency and remain exactly as before.

We now have attributed relative performance to stock selection, asset allocation and currency using forward not spot rates as summarised in Exhibit 6.73.

Exhibit 6.73 Revised attribution effects including the cost of hedging

Stock selection $\dfrac{1+r_L}{1+b_{SL}} - 1 = \dfrac{1.083}{1.052} - 1 = 2.95\%$

Revised asset allocation $\dfrac{1+b_{SH}}{1+b_L} - 1 = \dfrac{1.051}{1.064} - 1 = -1.22\%$

Total currency effects $\dfrac{1+b_{SL}}{1+b_{SH}} \times \dfrac{1+r'_C}{1+b'_C} - 1$ or

$$\frac{1+b_{SL}}{1+b_{SH}} \times \frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1$$

We can then see that these factors compound to give:

$$\underbrace{\frac{1+r_L}{1+b_{SL}}}_{\text{Stock}} \times \underbrace{\frac{1+b_{SH}}{1+b_L}}_{\text{Asset}} \times \underbrace{\frac{1+b_{SL}}{1+b_{SH}}}_{\substack{\text{Hedging}\\ \text{cost}\\ \text{transferred}}} \times \underbrace{\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b}}_{\substack{\text{Native}\\ \text{currency}\\ \text{attribution}}} - 1 = \frac{1+r}{1+b} - 1$$

$$\underbrace{\frac{1.083}{1.052}}_{\text{Stock}} \times \underbrace{\frac{1.051}{1.064}}_{\text{Asset}} \times \underbrace{\frac{1.052}{1.051}}_{\substack{\text{Hedging}\\ \text{cost}\\ \text{transferred}}} \times \underbrace{\frac{1.1781}{1.083} \times \frac{1.064}{1.190}}_{\substack{\text{Native}\\ \text{currency}\\ \text{attribution}}} - 1 = \frac{1.178}{1.696} - 1 = 0.73\%$$

Incorporating forward currency contracts

Actual portfolio and notional benchmark forward contracts can be introduced in our standard return formula as follows:

$$r = \sum_{i=1}^{i=n} w_i \times r_i + \sum_{i=1}^{i=n} \widetilde{w}_i \times f'_i \tag{6.109}$$

and

$$b = \sum_{i=1}^{i=n} W_i \times b_i + \sum_{i=1}^{i=n} \widetilde{W}_i \times f_i \tag{6.110}$$

where: f'_i = return on portfolio forward currency contracts in currency i
f_i = return on benchmark forward currency contracts in currency i
$\widetilde{w}_i$ = weight of portfolio forward currency contracts in currency i
$\widetilde{W}_i$ = weight of benchmark forward currency contracts in currency i (or, put another way, the benchmark-hedged weight)

Forward currency contracts are included for our standard example in Table 6.38. Since we have only introduced physical forward contracts into our example, the stock selection, asset allocation and cost of hedging results remain as before. Clearly there are new currency allocations and currency timing effects but also revised compounding effects.

Extending Equations 6.83 and 6.84 to include currency forwards, the revised implied currency benchmark return and implied portfolio currency return are:

$$b_C = \sum_{i=1}^{i=n} W_i \times c_i + \sum_{i=1}^{i=n} \widetilde{W}_i \times f_i \tag{6.111}$$

$$b_{SC} = \sum_{i=1}^{i=n} W_i \times c_i + \sum_{i=1}^{i=n} (w_i - W_i + \widetilde{w}_i) \times f_i \tag{6.112}$$

Revised portfolio, benchmark and currency benchmark and currency semi-notional returns are calculated in Exhibit 6.74.

Exhibit 6.74 Revised returns incorporating forward currency contracts

The base currency (£) return of the portfolio is now:

$$r = 40\% \times 20\% + 30\% \times 4.5\% + 30\% \times 27.2\% + 20\% \times 0\% - 15\% \times 9.5\% - 5\% \times 17.0\% = 15.54\%$$

The base currency (£) return of the benchmark is now:

$$b = 40\% \times 10\% + 20\% \times 4.6\% + 40\% \times 29.6\% + 30\% \times 0\% - 10\% \times 8.87\% + -20\% \times 17.82\% = 12.51\%$$

The revised benchmark currency return is now:

$$b_C = 40\% \times 0\% + 20\% \times 10.0\% + 40\% \times 20.0\% + 30\% \times 0\% - 10\% \times 8.87\% - 20 \times 17.82 = 5.55\%$$

The revised currency semi-notional return is now:

$$b_{SC} = 40\% \times 0\% + 20\% \times 10.0\% + 40\% \times 20.0\% + (40\% - 40\% + 20\%) \times 0\% + (30\% - 20\% - 15\%) \times 8.87\% + (30\% - 40\% - 5\%) \times 17.82\% = 6.88\%$$

Note that spot rate returns are used for the equity market exposures; physical assets generate the exposure, not forward currency contracts. Forward currency rates are only used if there is an element of hedging in the benchmark description. This differs slightly from the Karnosky and Singer method, which assumes any investment in overseas assets generates a return premium, i.e., must be hedged back to the base currency; in most investment strategies only the variance from benchmark should be measured this way. The currency exposure in the benchmark is obtained without the use of forward currency contracts and is not exposed to interest rate differentials, unless, of course, the benchmark includes an element of hedging.

Equation 6.105 can be extended to specifically include forward contracts:

$$((w_i + \widetilde{w}_i) - (W_i + \widetilde{W}_i)) \times \left(\frac{1 + f_i}{1 + b_C} - 1 \right) \tag{6.113}$$

Table 6.38 Forward currency contracts

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio local return r_{Li} (%)	Benchmark local return b_{Li} (%)	Portfolio base return r_i (%)	Benchmark base return b_i (%)	Benchmark currency return c_i (%)
UK equities	40	40	20	10	20.0	10.0	0
Japanese equities	30	20	−5	−4	4.7	5.6	10
US equities	30	40	6	8	28.0	29.6	20
	$\widetilde{w}_i$	$\widetilde{W}_i$			Portfolio forward return f_i'	Benchmark forward return f_i	
Sterling forward contracts	+20	+30	n/a	n/a	0	0	
Yen forward contracts	−15	−10	n/a	n/a	9.5	8.9	
US dollar forward contracts	−5	−20	n/a	n/a	17.0	17.8	
Total	100	100	8.3	6.4	15.54	12.51	

Revised currency allocation is calculated in Exhibit 6.75. In this example the portfolio has benefited from being underweight sterling and overweight both the US dollar and Japanese yen.

Exhibit 6.75 Currency allocation including forwards

Total currency allocation:

$$\frac{1+b_{SC}}{1+b_C} - 1 = \frac{1.0688}{1.0555} = 1.26\%$$

Calculating for individual currencies:

$$((w_i + \widetilde{w}_i) - (W_i + \widetilde{W}_i)) \times \left(\frac{1+f_i}{f+b_C} - 1\right)$$

Sterling (£)	$= (60\% - 70\%) \times \left(\frac{1.0}{1.0555} - 1\right) = 0.53\%$
Yen	$= (15\% - 10\%) \times \left(\frac{1.089}{1.0555} - 1\right) = 0.15\%$
US dollar ($)	$= (25\% - 20\%) \times \left(\frac{1.178}{1.0555} - 1\right) = 0.58\%$
Total	$0.53\% + 0.15\% + 0.58\% = 1.26\%$

To calculate currency timing effects, we need to adapt Equation 6.109 to measure pure timing effects excluding the impact of hedging to neutral:

$$r_{SC} = \sum_{i=1}^{i=n} w_i \times c_i + \sum_{i=1}^{i=n} \widetilde{w}_i \times f_i \tag{6.114}$$

Portfolio currency returns and pure currency semi-notional returns are calculated in Exhibit 6.76.

Exhibit 6.76 Revised portfolio currency returns including forwards

The portfolio currency return is now:

$$r_C = 40\% \times 0\% + 30\% \times 10.2\% + 30\% \times 20.8\% + 20\% \times 0\% - 15\% \times 9.5\% + -5\% \times 17.0\% = 7.01\%$$

From the perspective of the currency overlay manager the semi-notional currency return is:

$$r_{SC} = 40\% \times 0\% + 30\% \times 10.0\% + 30\% \times 20.0\% + 20\% \times 0\% - 15\% \times 8.87\% - 5\% \times 17.82\% = 6.78\%$$

The timing effect of forward currency contracts and physical assets can be included by adapting Equation 6.92.

For physical assets:

$$w_i \times \left(\frac{1+c_i'}{1+c_i} - 1\right) \times \left(\frac{1+c_i}{1+r_{SC}}\right) \tag{6.115}$$

For forward contracts:

$$\widetilde{w}_i \times \left(\frac{1+f_i'}{1+f_i} - 1 \right) \times \left(\frac{1+f_i}{1+r_{SC}} \right) \tag{6.116}$$

Timing attribution is calculated in Exhibit 6.77. The timing effects in forward currency are genuine currency timing effects caused by the timing decisions of the currency overlay manager. To the extent that local returns of underlying assets are derived in performance systems, not calculated separately, typically timing decision are only observed in forward currency contracts and therefore appropriately attributed to the overlay manager.

Exhibit 6.77 Currency timing effects including forwards

	$\frac{1+r_C}{1+r_{SC}} = \frac{1.0701}{1.0678} - 1 = 0.22\%$
	$w_i \times \left(\frac{1+c_i'}{1+c_i} - 1 \right) \times \left(\frac{1+c_i}{1+r_{SC}} \right)$
Sterling	$40\% \times \left(\frac{1.0}{1.0} - 1 \right) \times \frac{1.0}{1.0678} = 0\%$
Yen	$30\% \times \left(\frac{1.102}{1.10} - 1 \right) \times \frac{1.10}{1.0678} = 0.06\%$
US dollar	$30\% \times \left(\frac{1.208}{1.20} - 1 \right) \times \frac{1.20}{1.0678} = 0.21\%$
The timing effects in forward contracts are:	
	$\widetilde{w}_i \times \left(\frac{1+f_i'}{1+f_i} - 1 \right) \times \left(\frac{1+f_i}{1+b_{SC}} \right)$
Sterling forwards	$20\% \times \left(\frac{1.0}{1.0} - 1 \right) \times \frac{1.0}{1.0678} = 0\%$
Yen forwards	$-15\% \times \left(\frac{1.095}{1.089} - 1 \right) \times \frac{1.089}{1.0678} = -0.09\%$
US dollar forwards	$-5\% \times \left(\frac{1.17}{1.178} - 1 \right) \times \frac{1.17}{1.0678} = 0.04\%$
Total	$0\% + 0.06\% + 0.21\% + 0\% - 0.09\% + 0.04\% = 0.22\%$

Summarising

We now have attributed relative performance to the following factors:

$$\text{Stock selection } \frac{1+r_L}{1+b_{SL}} - 1 \tag{6.42}$$

$$\text{Asset allocation } \frac{1+b_{SH}}{1+b_L} - 1 \tag{6.70}$$

$$\text{Total currency effects } \frac{1+b_{SL}}{1+b_{SH}} \times \frac{1+r_C'}{1+b_C'} - 1 \tag{6.117}$$

or

$$\frac{1+b_{SL}}{1+b_{SH}} \times \frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1 \tag{6.118}$$

We can then see that these factors compound to give:

$$\underbrace{\frac{1+r_L}{1+b_S}}_{\text{Stock}} \times \underbrace{\frac{1+b_{SH}}{1+b_L}}_{\text{Asset}} \times \underbrace{\frac{1+b_S}{1+b_{SH}}}_{\substack{\text{Hedging}\\ \text{cost}\\ \text{transferred}}} \times \underbrace{\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b}}_{\substack{\text{Naïve}\\ \text{currency}\\ \text{attribution}}} - 1 = \frac{1+r}{1+b} - 1 \tag{6.119}$$

The total currency effects can be broken down further to:

$$\underbrace{\frac{1+r_C}{1+r_{SC}} \times \frac{1+b_{SC}}{1+b_C}}_{\substack{\text{Currency}\\ \text{overlay}}} \times \underbrace{\frac{1+b_S}{1+b_{SH}} \times \frac{1+r_{SC}}{1+b_{SC}}}_{\substack{\text{Hedging}\\ \text{mismatch}}} \times \underbrace{\frac{1+r'_C}{1+r_C} \times \frac{1+b_C}{1+b'_C}}_{\text{Compounding}} - 1 \tag{6.120}$$

An addition ratio has been introduced, $(1+r_{SC})/(1+b_{SC})$, to ensure no residuals result from the calculation. This effect, when combined with the cost of hedging is normally close to zero. This minor adjustment is required because the cost of hedging impact implicitly included in currency overlay is applied to a slightly different denominator from that of the country allocator. All of these ratios are re-created for the standard example in Exhibit 6.78.

Exhibit 6.78 Summary of attribution factor ratios

Stock selection	$\frac{1+r_L}{1+b_S} - 1 = \frac{1.083}{1.052} - 1 = 2.95\%$
Asset allocation	$\frac{1+b_{SH}}{1+b_L} - 1 = \frac{1.051}{1.064} - 1 = -1.22\%$
Total currency effects	$\frac{1+b_S}{1+b_{SH}} \times \frac{1+C_r}{1+C_b} - 1$ or $\frac{1+b_S}{1+b_{SH}} \times \frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1$

$$\frac{1.052}{1.051} \times \frac{1.155}{1.083} \times \frac{1.064}{1.125} - 1 = 0.98\%$$

We can then see that these factors compound to give:

$$\underbrace{\frac{1+r_L}{1+b_S}}_{\text{Stock}} \times \underbrace{\frac{1+b_{SH}}{1+b_L}}_{\text{Asset}} \times \underbrace{\frac{1+b_S}{1+b_{SH}}}_{\substack{\text{Hedging}\\ \text{cost}\\ \text{transferred}}} \times \underbrace{\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b}}_{\substack{\text{Naïve}\\ \text{currency}\\ \text{attribution}}} - 1 = \frac{1+r}{1+b} - 1$$

$$\underbrace{\frac{1.083}{1.052}}_{\text{Stock}} \times \underbrace{\frac{1.051}{1064}}_{\text{Asset}} \times \underbrace{\frac{1.052}{1.051}}_{\substack{\text{Hedging}\\ \text{cost}\\ \text{transferred}}} \times \underbrace{\frac{1.155}{1083} \times \frac{1.064}{1.121}}_{\substack{\text{Naïve}\\ \text{currency}\\ \text{attribution}}} - 1 = \frac{1.155}{1.125} - 1 = 2.69\%$$

Exhibit 6.78 (*Continued*)

The total currency effects can be broken down further to:

$$\underbrace{\frac{1+r_C}{1+r_{SC}} \times \frac{1+b_{SC}}{1+b_C}}_{\text{Currency overlay}} \times \underbrace{\frac{1+b_{SH}}{1+b_S} \times \frac{1+r_{SC}}{1+b_{SC}}}_{\text{Hedging mismatch}} \times \underbrace{\frac{1+r'_C}{1+r_C} \times \frac{1+b_C}{1+b'_C}}_{\text{Compounding}} - 1$$

$$\underbrace{\frac{1.0701}{1.0678} \times \frac{1.0688}{1.0555}}_{\text{Currency overlay}} \times \underbrace{\frac{1.052}{1.051} \times \frac{1.0678}{1.0688}}_{\text{Hedging mismatch}} \times \underbrace{\frac{1.0668}{1.0701} \times \frac{1.0555}{1.0574}}_{\text{Compounding}} - 1 = 0.98\%$$

Finally, we can summarise the currency attribution effects in Table 6.40 and overall attribution effects in Table 6.39.

Other currency issues

Unlike futures contracts in which gains and losses are channelled through the margin account, unrealised gains and losses build up in forward currency contracts. The practical consequence of this is that there will be a net forward position which damps performance in the event of unrealised gains and gears (provides leverage) performance in the event of unrealised losses. This net position is an attributable factor in its own right.

The denomination of a security does not necessarily coincide with the economic exposure of a security. A classic example are Japanese warrants and convertible bonds denominated in US dollars, Swiss francs and other currencies to encourage international investors to buy. These instruments are ultimately linked to the yen price of a security. The prices of these instruments effectively adjust for the currency movements between the denomination currency and yen and therefore are economically exposed to yen. Other common examples include American depositary receipts (ADRs).

Table 6.39 Multi-currency geometric attribution

	Stock selection (%)	County allocation (%)	Currency timing (%)	Currency allocation (%)	Other effects* (%)	
UK equities	3.8	0	3.8	0.53		
Japanese equities	−0.29	−0.88	−0.03	0.16		
US equities	−0.57	−0.34	0.25	0.58		**Total excess Return** (%)
Total	**2.95**	**−1.22**	**0.22**	**1.26**	**−0.5**	**2.70**

*In real portfolios the other effects tend to be no more than one or two basis points and are reduced if measurement periods are kept as short as possible, ideally daily.

Table 6.40 Currency attribution

	Portfolio weight $w_i + \widetilde{w}_i$ (%)	Benchmark weight $W_i + \widetilde{W}_i$ (%)	Portfolio currency c'_i (%)	Benchmark currency c_i (%)	Portfolio forwards f'_i (%)	Benchmark forwards f_i (%)	Currency allocation (%)	Timing (%)
Sterling	60	70	0.0	0.0	0.0	0.0	0.53	0.0
Yen	15	10	10.2	10.0	9.5	8.9	0.15	−0.03
US dollar	25	20	20.8	20.0	17.0	17.9	0.58	0.25
Total	**100**	**100**	**7.01**	**5.55**			**1.26**	**0.22**
						Compounding	$\frac{1+r'_C}{1+r_C} \times \frac{1+b_C}{1+b'_C}$	**−0.5**
						Hedging mismatch	$\frac{1+b_S}{1+b_{SH}} \times \frac{1+r_{SC}}{1+b_{SC}}$	**0.0**

FIXED INCOME ATTRIBUTION

The investment decision process for bond managers is very different from that of equity managers; therefore, for most fixed income investment strategies, the standard Brinson model is not suitable. Bonds are simply a series of defined future cash flows which are relatively easy to price,[32] using appropriate discount rates. Fixed income performance is therefore driven by changes in the shape of the yield curve. Systematic risk in the form of duration is a key part of the investment process. No single, standardised way of defining fixed income attribution exists. Attribution for fixed income is particularly difficult because:

(1) Asset managers employ a wide range of fixed income investment decision processes.
(2) Yield curves and the change in shape of the yield curve can be defined in many different ways.
(3) Public fixed income benchmarks are less relevant.
(4) There are a large number and wide range of securities and instruments available to bond managers.
(5) Illiquid bonds and some instruments are difficult to price accurately.
(6) There is an alternative vocabulary of yield changes rather than returns.

Fixed income investing is less homogeneous than equity investing; as a result, software providers tend to develop their own brand of return attribution. Because of differences in the definition and computation of factors, different systems often provide different values for the same return factor.

Note

Fixed income attribution analysis tends to be presented arithmetically. Although less technically correct, this is probably because:

(1) Excess returns in bond portfolios are smaller (the differences between the arithmetic and geometric excess returns are less obvious).
(2) Benchmark returns are less extreme (again, the difference between the arithmetic and geometric excess returns are less obvious).
(3) Typically, bond managers have an arithmetic mindset (which is curious, since for the most part they are more numerate).

Caution

Bond managers are normally a greater challenge for performance analysts because:

(1) Excess returns are smaller (each basis point is precious).
(2) Bond managers have a better appreciation of return before it's calculated (the combination of duration and yield changes).
(3) Bond managers are numerate (they understand the numbers).

[32] See Equation 5.113.

The yield curve

The yield curve is a graph of various bonds' redemption yields against term to maturity – a snapshot of current market yields. Asset managers will seek to add value by anticipating the changing shape of the yield curve over time.

Figure 6.10 shows a typical yield curve. At the short end of the curve to the left the starting point is short-term interest rates; to the right yields gradually increase since we expect greater risk over longer periods – in this example the yield curve is sloping upwards. The actual shape of the yield curve will reflect any expected changes in short-term rates and the risk premium for holding long-term assets. An inverted yield curve may indicate that a recession is expected.

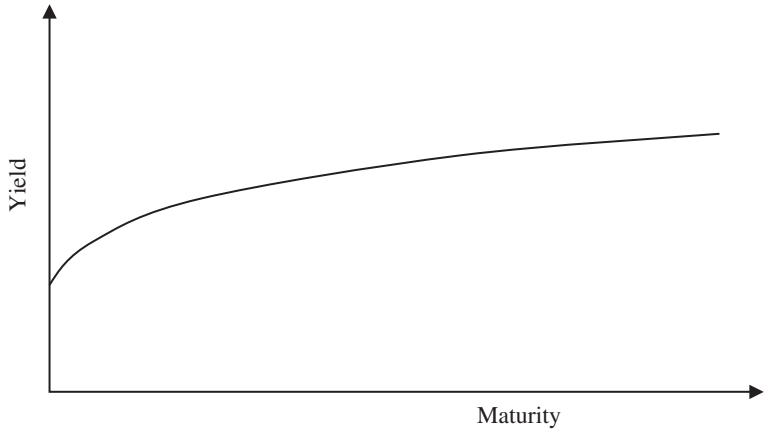

Figure 6.10 The yield curve

Yield curve analysis

The changing shape of the yield curve can be decomposed into three different movements: shift, twist (or slope) and curvature (or butterfly).

Shift

The shift effect assumes a parallel shift of yields on all maturity points on the yield curve; see Figure 6.11. Parallel shift may be measured by referencing the shift at certain maturities or using the average shift across the yield curve.

Twist (or slope)

Twist or slope measures the impact from changes in the slope or non-parallel moves of the yield curve. The yield curve may have twisted around a pivot point or moved less dramatically at one end of the yield curve than the other, resulting in a steepening or flattening; see Figure 6.12. Different asset management firms may choose different pivot points, resulting in a different perception of slope effects.

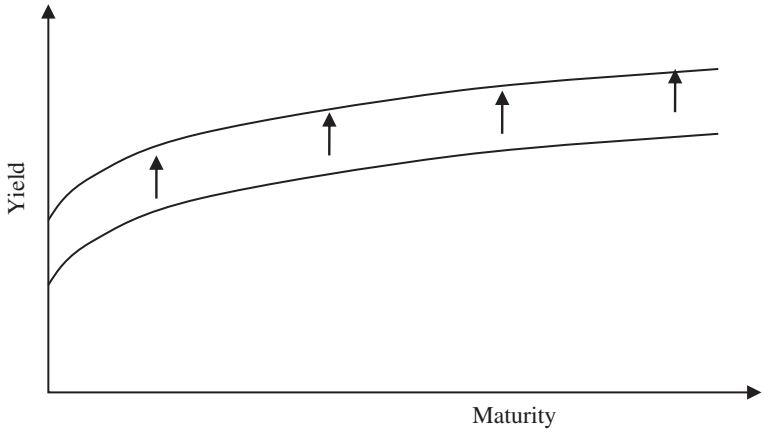

Figure 6.11 Parallel shift

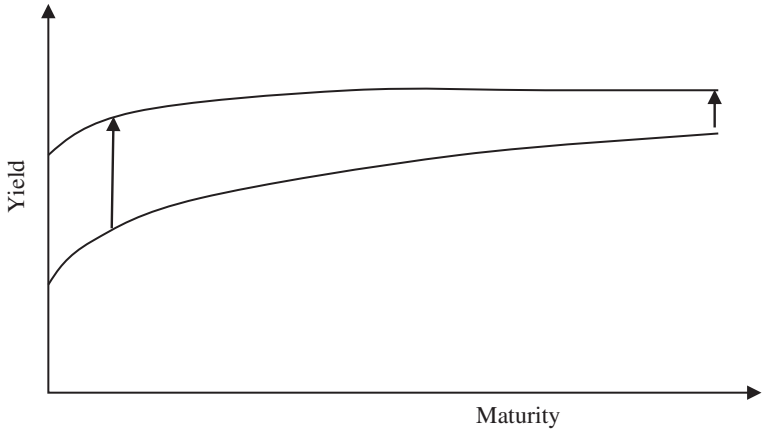

Figure 6.12 Slope (flattening yield curve)

Curvature (or butterfly)

Curvature measures the changing curvature of the yield curve. Perhaps there is a larger change in yields in the middle of the curve than either the long or short ends; see Figure 6.13.

Carry

An additional contribution to return is simply the result of time or carry. All other things being equal there will be an additional contribution from convergence or roll down as the bond approaches maturity and from the coupon return as interest is accrued and paid.

Credit (or spread)

There is an additional risk that some borrowers will default. Investors will demand higher yields for greater risk. This risk premium to compensate for uncertainty is known as spread.

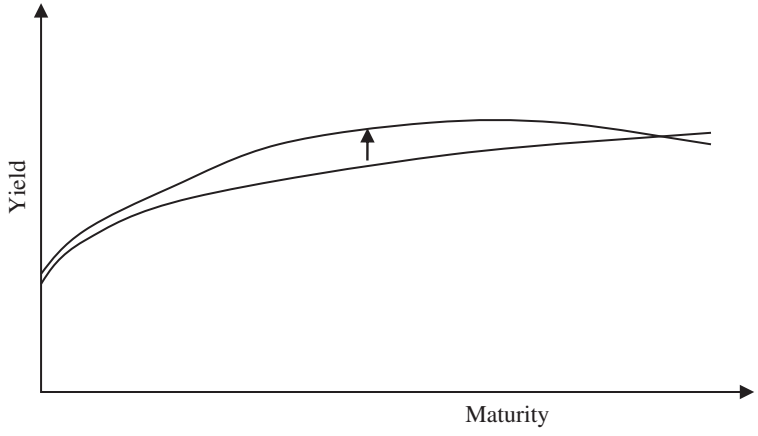

Figure 6.13 Curvature

An element of return may be due to the tightening or widening of spreads against treasuries. Spreads decline during economic expansion and widen during economic downturns and periods of high risk.

Yield curve decomposition

The total return of a bond portfolio can be decomposed into carry, movement in the yield curve (measured by shift, slope and curvature) and changing credit spreads.

Figure 6.14 illustrates the attribution effects in a typical fixed income attribution model.

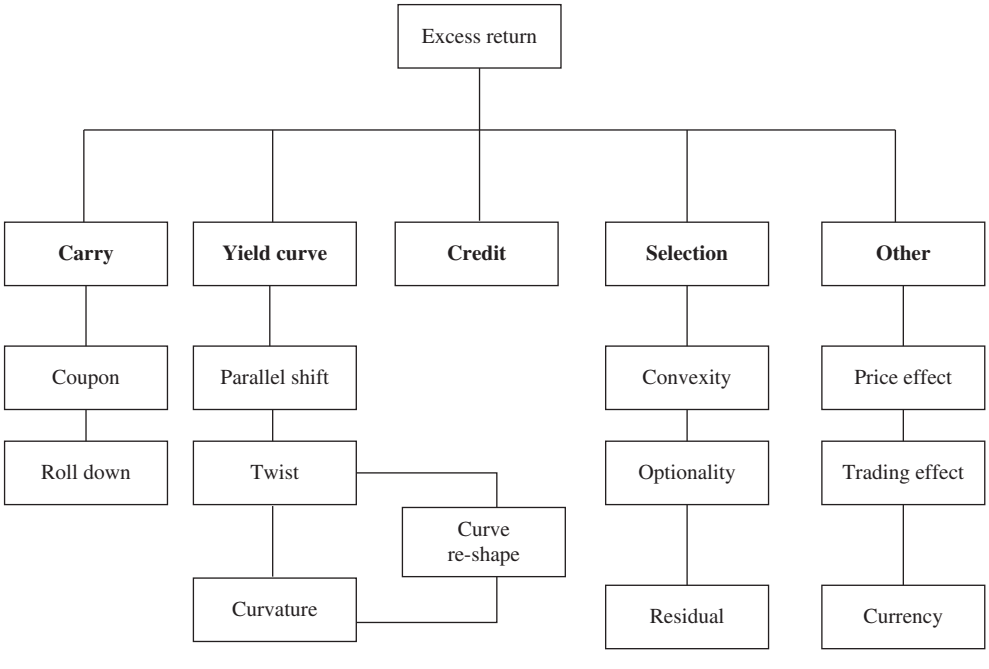

Figure 6.14 Fixed income attribution effects

Carry includes coupon payments, accrued interest and rolling down the yield curve (the pull to par effect as you approach maturity).

The **yield curve** effect represents the changing shape of the yield curve: parallel shift, twist and curvature. Often twist and curvature are combined and described as non-parallel changes in the yield curve or curve reshape.

Credit is the return due to widening or narrowing of credit spreads.

Selection effects include convexity, any optionality in the payment of future coupons and residuals. Residuals are typically caused by model or data errors and are normally quite small.

Other includes effects that are not specific fixed income effects. The price effect captures the difference in price sources between the bonds in the portfolio and bonds in the benchmark. This is a non-management effect that is a particular problem for fixed income. The trading effect measures the impact from intra-day trading. Currency will typically be managed independently.

Fixed income attribution models can be characterised as either top-down successive portfolio methods or bottom-up (yield curve decomposition) methods. Bottom-up methods calculate the impact of yield curve movements and credit spreads on individual bonds in both the portfolio and the benchmark and aggregate results.

Top-down models are perhaps easier to implement and more effective in terms of communicating to asset owners and other stakeholders.

Wagner and Tito

Wagner and Tito[33] suggested Fama-type decomposition for bonds using duration instead of beta as the measure of systematic risk; see Figure 6.15, which is similar to Figure 5.17.

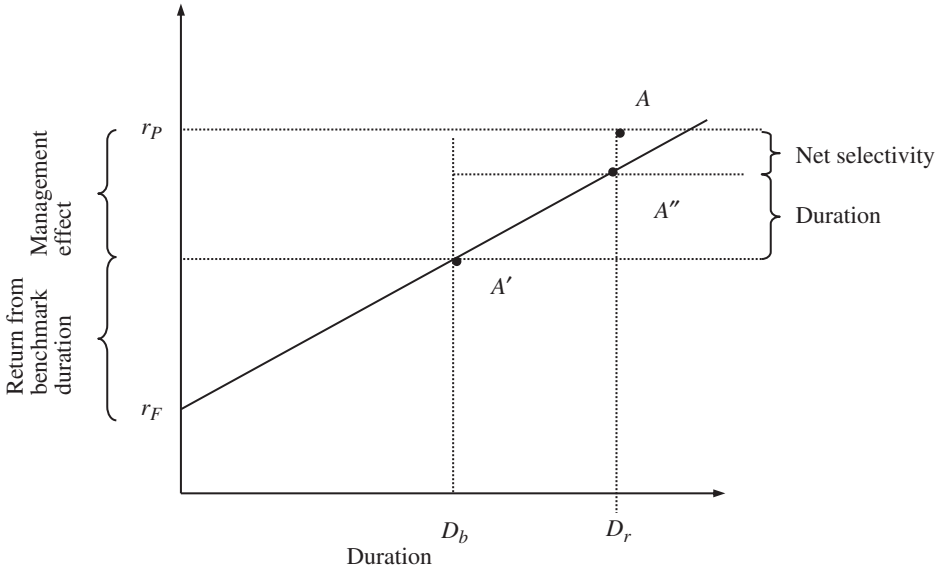

Figure 6.15 Wagner and Tito

[33] Wagner and Tito, "Definite New Measures of Bond Performance and Risk" (1977).

In many ways, Fama-type decomposition lends itself more to fixed income analysis than equity analysis since the measure of systematic risk, duration, is much more relevant in the fixed income decision process.

The difference between A'' and A' represents the added value from interest rate changes: A'' is the return expected at the duration of the portfolio and A' is the return of benchmark. The difference between A and A' represents the added value from issue selection.

Weighted duration attribution

Van Breukelen[34] suggested an approach to fixed interest attribution for top-down investment decision processes focused on the major risk factor of weighted duration which combines the approach of Wagner and Tito and the Brinson model. Essentially it is a risk-adjusted version of the Brinson model using duration as a measure of systematic risk.

Van Breukelen uses the following approximation formula for the return on bonds:

$$r_{Li} = x_i + D_i \times (-\Delta y_i) \tag{6.121}$$

where: D_i = modified duration in bond category i
Δy_i = change in yield for category i
x_i = interest rate in currency i (from Karnosky and Singer)

Note

Interest rates in Equation 6.121 are convenient for international bond portfolios with currency exposures incorporating the concepts of Karnosky and Singer. In general, and certainly for single-currency portfolios, income or coupon return is an appropriate alternative.

Using the Karnosky and Singer definition of portfolio return:

$$r = \sum_{i=1}^{i=n} w_i \times (r_{Li} - x_i) + \sum_{i=1}^{i=n} w_i \times (c_i + x_i) \tag{6.59}$$

It follows that:

$$b = \sum_{i=1}^{i=n} W_i \times (b_{Li} - x_i) + \sum_{i=1}^{i=n} W_i \times (c_i + x_i) \tag{6.122}$$

Substituting Equation 6.121 into Equation (6.59):

$$r = \sum_{i=1}^{i=n} w_i \times D_i \times (-\Delta y_i) + \sum_{i=1}^{i=n} w_i \times (c_i + x_i) \tag{6.123}$$

[34] Van Breukelen, "Fixed Income Attribution Journal" (2000).

The factor $w_i \times D_i$ is equivalent to an equity weight; the bond manager can increase exposure by either increasing weight or increasing modified duration.

It follows that the benchmark return can be described as:

$$b = \sum_{i=1}^{i=n} W_i \times D_{bi} \times (-\Delta y_{bi}) + \sum_{i=1}^{i=n} W_i \times (c_i + x_i) \tag{6.124}$$

where: D_{bi} = benchmark modified duration for category i
Δy_{bi} = change in benchmark yield for category i

Applying the standard Brinson approach to Equations 6.123 and 6.124, the excess return we wish to attribute is:

$$\begin{aligned} r - b = &\sum_{i=1}^{i=n} w_i \times D_i \times -\Delta y_i - \sum_{i=1}^{i=n} W_i \times D_{bi} \times -\Delta y_{bi} \\ &+ \sum_{i=1}^{i=n} w_i \times (c_i + x_i) - \sum_{i=1}^{i=n} W_i \times (c_i + x_i) \end{aligned} \tag{6.125}$$

Let:

$$c = \sum_{i=1}^{i=n} w_i \times (c_i + x_i) \tag{6.126}$$

$$c' = \sum_{i=1}^{i=n} W_i \times (c_i + x_i) \tag{6.127}$$

We are familiar with the last two terms which represent currency attribution in the Karnosky and Singer model (in this case without currency forwards, which can easily be added).

Van Breukelan suggests creating two reference or notional funds to measure the contribution from fixed income management, excluding currency; namely, overall duration, market selection and issue selection.

The overall duration notional fund is defined as:

$$b_D = \sum_{i=1}^{i=n} D_\beta \times D_{bi} \times W_i \times -\Delta y_{bi} + c' \tag{6.128}$$

where: $$D_\beta = \frac{D_r}{D_b} = \text{duration beta} \tag{5.127}$$

D_r = portfolio duration
D_b = benchmark duration

The duration beta is equivalent to an equity beta and can be used in the same way.

Therefore, the contribution from overall duration is:

$$\begin{aligned} b_D - b &= \sum_{i=1}^{i=n} D_\beta \times D_{bi} \times W_i \times \Delta y_{bi} - \sum_{i=1}^{i=n} D_{bi} \times W_i \times \Delta y_{bi} \\ &= \left(D_\beta \times \sum_{i=1}^{i=n} D_{bi} \times W_i - \sum_{i=1}^{i=n} D_{bi} \times W_i \right) \times \Delta y_b \\ &= \sum_{i=1}^{i=n} D_{bi} \times W_i \times (D_\beta - 1) \times \Delta y_b \end{aligned} \tag{6.129}$$

The overall duration effect should only be measured if it is part of the investment decision process.

The duration-adjusted semi-notional fund is defined as:

$$b_{DS} = \sum_{i=1}^{i=n} D_i \times w_i \times -\Delta y_{bi} + c' \tag{6.130}$$

Therefore, the contribution from weighted duration allocation is:

$$b_{DS} - b_D = \sum_{i=1}^{i=n} D_i \times w_i \times \Delta y_{bi} - \sum_{i=1}^{i=n} D_\beta \times D_{bi} \times W_i \times \Delta y_{bi} \tag{6.131}$$

Appling the same Brinson and Fachler approach as in Equation 6.18:

$$b_{DS} - b_D = \sum_{i=1}^{i=n} (D_i \times w_i - D_\beta \times D_{bi} \times W_i) \times (-\Delta y_{bi} + \Delta y_b) \tag{6.132}$$

Market allocation for category i is therefore:

$$A'_i = (D_i \times w_i - D_\beta \times D_{bi} \times W_i) \times (-\Delta y_{bi} + \Delta y_b) \tag{6.133}$$

If the overall duration is not part of the investment decision process, we can miss a step and move directly to:

$$b_{DS} - b = \sum_{i=1}^{i=n} (D_i \times w_i - D_{bi} \times W_i) \times (-\Delta y_{bi} + \Delta y_b) \tag{6.134}$$

Market allocation for category i is therefore:

$$A_i = (D_i \times w_i - D_{bi} \times W_i) \times (-\Delta y_{bi} + \Delta y_b) \tag{6.135}$$

Issue selection is calculated by:

$$r - b_{DS} - (c - c') = \sum_{i=1}^{i=n} D_i \times w_i \times (-\Delta y_{ri}) - \sum_{i=1}^{i=n} D_i \times w_i \times (-\Delta y_{bi}) - c + c' \tag{6.136}$$

Issue selection for category i is therefore:

$$S_i = D_i \times w_i \times (-\Delta y_{ri} + \Delta y_{bi}) \tag{6.137}$$

Note

Issue selection covers a range of issues: positioning on the yield curve, credit risk, optionality and convexity. Most bond managers are simply going to demand more granular information than this.

Currency allocation using Karnosky and Singer without forward contracts is:

$$C_i = (w_i - W_i) \times (c_i + x_i - c') \tag{6.70}$$

Table 6.41 provides the data for a simple numerical example of a three-category portfolio consisting of UK, Japanese and US bonds.

Table 6.41 Fixed income attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio-modified duration D_i (%)	Benchmark-modified duration D_{bi} (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Risk-free rate x_i (%)
UK bonds	50	50	7.8	5.0	5.6	3.5	1.0
Japanese bonds	20	10	1.0	2.0	0.5	0.5	0.1
US bonds	30	40	4.0	3.0	3.2	3.0	0.2
Total	100	100	5.3	3.9	3.86	3.0	0.59

Using the data in Table 6.41 the portfolio and benchmark total returns and durations are verified and the duration beta calculated in Exhibit 6.79.

Exhibit 6.79 Total returns and duration

Portfolio modified duration $D_r = \sum_{i=1}^{i=n} w_i \times D_i$

$$50\% \times 7.8 + 20\% \times 1.0 + 30\% \times 4.0 = 5.3$$

Benchmark modified duration $D_b = \sum_{i=1}^{i=n} W_i \times D_{bi}$

$$50\% \times 5.0 + 10\% \times 2.0 + 40\% \times 3.0 = 3.9$$

Duration beta $D_\beta = \dfrac{5.3}{3.9}$

Portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ $50\% \times 5.6\% + 20\% \times 0.5\% + 30\% \times 3.2\% = 3.86\%$

Benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ $50\% \times 3.5\% + 10\% \times 0.5\% + 40\% \times 3.0\% = 3.0\%$

Exhibit 6.79 (*Continued*)

Portfolio risk-free rate

$$x_r = \sum_{i=1}^{i=n} w_i \times x_i = 50\% \times 1.0\% + 20\% \times 0.1\% + 30\% \times 0.2\% = 0.58\%$$

or c *if currency returns are 0*
Benchmark risk-free rate

$$x_b = \sum_{i=1}^{i=n} W_i \times x_i = 50\% \times 1.0\% \times 10\% \times 0.1\% + 40\% \times 0.2\% = 0.59\%$$

or c′ *if currency returns are 0*

Using Equation 6.121 the implied yield changes are calculated directly from the portfolio and benchmark returns in Exhibits 6.80 and 6.81.

Exhibit 6.80 Implied portfolio yield changes

$$\text{UK bonds } \Delta y_i = \frac{-(r_i - x_i)}{D_i} = \frac{-(5.6\% - 1.0\%)}{7.8} = -0.59\%$$

$$\text{Japanese bonds} = \frac{-(0.5 - 0.1\%)}{1.0} = -0.4\%$$

$$\text{US bonds} = \frac{-(3.2\% - 0.2\%)}{4.0} = -0.75\%$$

$$\text{Total portfolio} = \frac{-(3.86\% - 0.58\%)}{5.3} = -0.62\%$$

Exhibit 6.81 Implied benchmark yield changes

$$\text{UK bonds} = \frac{-(3.5\% - 1.0\%)}{5.0} = -0.5\%$$

$$\text{Japanese bonds} = \frac{-(0.5\% - 0.1\%)}{2.0} = -0.2\%$$

$$\text{US bonds} = \frac{-(3.0\% - 0.2\%)}{3.0} = -0.93\%$$

$$\text{Total benchmark} = \frac{-(3.00\% - 0.59\%)}{3.9} = -0.62\%$$

In this particular example the overall duration is part of the decision process. The overall duration notional fund and duration-adjusted semi-notional funds are calculated in Exhibit 6.82.

Exhibit 6.82 Notional funds

Overall duration notional fund

$$b_D = \sum_{i=1}^{i=n} D_\beta \times D_{bi} \times W_i \times \Delta y_{bi} + c'$$

$$= \frac{5.3}{3.9} \times (5.0 \times 50\% \times 0.5\% + 2.0 \times 10\% \times 0.2\% + 3.0 \times 40\% \times 0.93) + 0.59\% = 3.87\%$$

Duration-adjusted semi-notional fund

$$b_{DS} = \sum_{i=1}^{i=n} D_i \times w_i \times -\Delta y_{bi} + c' = 7.8 \times 50\% \times 0.5\% + 1.0 \times 20\% \times 0.2\% + 4.0 \times 30\% \times 0.93$$

$$+ 0.59\% = 3.70\%$$

The overall duration effect is calculated by taking the difference between the duration notional fund and the benchmark return as shown in Exhibit 6.83. It is one decision and therefore one allocation number is calculated. In this example the portfolio duration is much greater than the benchmark duration; since markets are rising this is a positive effect, adding 0.87% of value.

Exhibit 6.83 Overall duration allocation

$$b_D - b = 3.87\% - 3.0\% = 0.87\%$$

Exhibit 6.84 calculates the market allocation effects for each category. Since we have adjusted for the overall duration, we must adjust the benchmark-weighted duration using the duration beta to ensure the correct effect is calculated. The portfolio is effectively overweight UK bonds which underperformed the overall index slightly losing 0.06%, underweight Japanese bonds which added 0.03% but underweight US bonds which lost 0.14% of value.

Exhibit 6.84 Market allocation

$$b_{DS} - b_D = \sum_{i=1}^{i=n} (D_i \times w_i - D_\beta \times D_{bi} \times W_i) \times (-\Delta t_{bi} + \Delta y_b) = 3.70\% - 3.87\% = 0.17\%$$

Individual market allocation	$(D_i \times w_i - D_\beta \times D_{bi} \times W_i) \times (-\Delta y_{bi} + \Delta y_b)$
UK bonds	$\left(7.8\% \times 50\% - \frac{5.3}{3.9} \times 5.0 \times 50\%\right) \times (0.5 - 0.62) = -0.06\%$
Japanese bonds	$\left(1.0 \times 20\% - \frac{5.3}{3.9} \times 2.0 \times 10\%\right) \times (0.2 - 0.63) = 0.03\%$
US bonds	$\left(4.0 \times 30\% - \frac{5.3}{3.9} \times 3.0 \times 40\%\right) \times (0.93 - 0.62) = -0.14\%$
Total market allocation	$-0.06\% + 0.03\% - 0.14\% = -0.17\%$

Exhibit 6.85 calculates the security or issue selection effects. The portfolio outperformed in UK and Japanese bonds, evidenced by yield falls greater than benchmark, but underperformed in US bonds. Currency effects are calculated in Exhibit 6.86. In this example currency returns are zero, therefore currency allocation is measuring the local interest rate allocation effects. The portfolio is overweight in low-yielding Japanese interest rates, losing value, but this is almost offset against an underweight exposure to low US interest rates.

Exhibit 6.85 Security (or issue) selection

$$r - b_{DS} = \sum_{i=1}^{i=n} D_i \times w_i \times (-\Delta y_{ri}) - \sum_{i=1}^{i=n} D_i \times w_i \times (-\Delta y_{bi}) - c + c'$$
$$= 3.86\% - 3.70\% - 0.58 + 0.59\% = 0.17\%$$

Individual selection effects	$D_i \times w_i \times (-\Delta y_{ri} + \Delta y_{bi})$
UK bonds	$7.8 \times 50\% \times (0.59\% - 0.5\%) = 0.35\%$
Japanese bonds	$1.0 \times 20\% \times (0.4\% - 0.2\%) = 0.04\%$
US bonds	$4.0 \times 30\% \times (0.75\% - 0.93\%) = -0.22\%$
Total security selection	$0.35\% + 0.04\% - 0.22\% = 0.17\%$

Exhibit 6.86 Currency allocation

$$c - c' = \sum_{i=1}^{i=n} w_i \times (c_i + x_i) - \sum_{i=1}^{i=n} W_i \times (c_i + x_i) = 0.58\% - 0.59\%$$

Individual currency allocation	$(w_i - W_i) \times (c_i + x_i - c')$
UK bonds	$(50\% - 50\%) \times (0.0\% + 1.0\% - 0.59\%) = 0.0\%$
Japanese bonds	$(20\% - 10\%) \times (0.0\% + 0.1\% - 0.59\%) = -0.05\%$
US bonds	$(30\% - 40\%) \times (0.0\% + 0.2\% - 0.59\%) = 0.04\%$
Total currency allocation	$0.0\% - 0.05\% + 0.04\% = -0.01\%$

The fixed income attribution effects are summarised in Table 6.42. This type of attribution is particularly suited for global bond portfolios and balanced portfolios. For balanced portfolios, essentially the same Brinson approach is employed but the impact of duration can be factored into the fixed income portion of the portfolio. The risk factor for equities is category weight and the risk factor for bonds is weighted duration.

Table 6.42 Weighted duration attribution

	Portfolio-weighted duration $w_i \times D_i$ (%)	Benchmark-weighted duration $W_i \times D_{bi}$ (%)	Portfolio change in yield Δy_i (%)	Benchmark change in yield Δy_{bi} (%)	Market allocation $(D_i \times w_i - D_\beta \times D_{bi} \times W_i) \times (-\Delta y_{bi} + \Delta y_b)$ (%)	Issue selection $D_i \times w_i \times (-\Delta y_{ri} + \Delta y_{bi})$ (%)	Currency allocation $(w_i - W_i) \times (c_i + x_i - c')$ (%)
UK bonds	3.9	2.5	−0.59	−0.5	−0.06	0.35	0.0
Japanese bonds	0.2	0.2	−0.4	0.2	0.03	0.04	−0.05
US bonds	1.2	1.2	−0.75	−0.93	−0.14	−0.22	0.04
Total	**100**	**100**	**−0.62**	**−0.62**	**−0.17**	**0.17**	**−0.01**
					Overall duration $b_D - b$		**0.87**

Geometric fixed income attribution

Although Van Breukelen presented an arithmetic attribution model it can be easily adapted for geometric analysis. The four-step decision process is shown for the data in Table 6.42 in Figure 6.16; the geometric difference between each step represents the total effect of that factor. Exhibit 6.87 calculates the geometric overall duration effect.

Exhibit 6.87 Overall duration allocation (geometric)

$$\frac{1+b_D}{1+b} - 1 = \frac{1.0387}{1.03} - 1 = 0.84\%$$

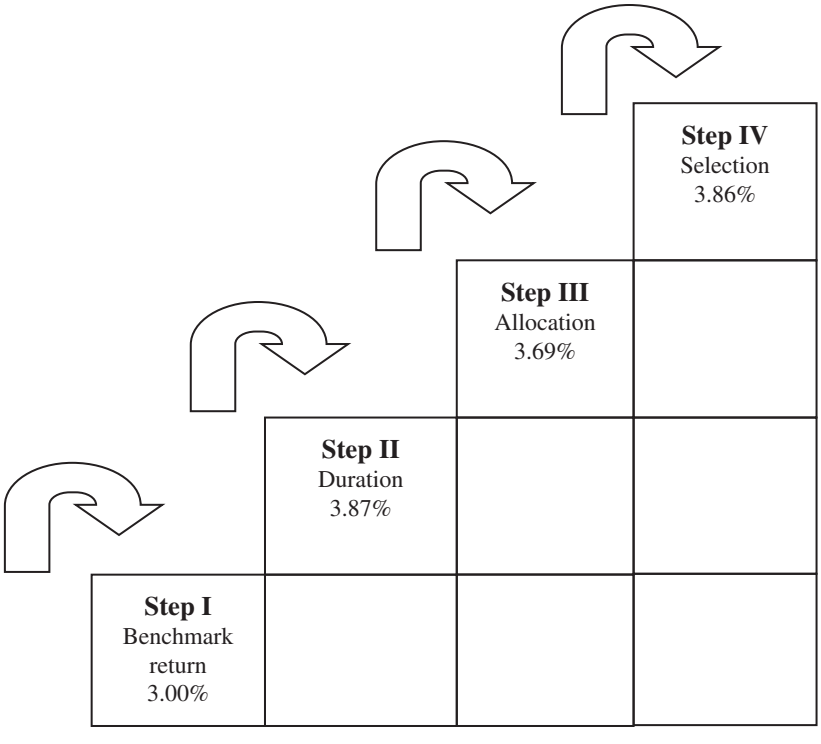

Figure 6.16 Geometric fixed income attribution

Exhibit 6.88 calculates the market allocation effect, adjusting for the overall duration effect already measured combined with the predictable contribution from allocation to the risk-free rate (currency effects in this example are ignored). Market allocation and risk-free rate allocation are combined in one step since portfolio weight (one part of the market allocation decision) determines the allocation to different interest yields.

Exhibit 6.88 Geometric market allocation

$$b'_{DS} = \sum_{i=1}^{i=n} D_i \times w_i \times -\Delta y_{bi} + c = 7.8 \times 50\% \times 0.5\% + 1.0 \times 20\% \times 0.2\% + 4.0 \times 30\% \times 0.93 + 0.58\% = 3.69\%$$

$$\frac{1 + b'_{DS}}{1 + b_D} - 1 = \frac{1.0369}{1.0387} - 1 = -0.17\%$$

Individual geometric market effects are:

$$A'^{G}_{i} = (D_i \times w_i - D_\beta \times D_{bi} \times W_i) \times \frac{(-\Delta y_{bi} + \Delta y_b)}{1 + b_D} \tag{6.136}$$

UK bonds $= \left(7.8 \times 50\% - \frac{5.3}{3.9} \times 5.0 \times 50\%\right) \times \frac{(0.5 - 0.62)}{1.0387} = -0.06\%$

Japanese bonds $= \left(1.0 \times 20\% - \frac{5.3}{3.9} \times 2.0 \times 10\%\right) \times \frac{(0.20 - 0.62)}{1.0387} = 0.03\%$

US bonds $= \left(4.0 \times 30\% - \frac{5.3}{3.9} \times 3.0 \times 40\%\right) \times \frac{(0.93 - 0.62)}{1.0387} = -0.13$

$$C^G_i = (w_i - W_i) \times \left(\frac{1 + x_i}{1 + c'} - 1\right) \times \frac{1 - c'}{1 + b_D} \tag{6.137}$$

Noting that $c_i = 0$ for all i in this example.

UK risk-free rate $= (50\% - 50\%) \times \left(\frac{1.01}{1.0059} - 1\right) \times \frac{1.0059}{1.0387} = 0.00\%$

Japanese risk-free rate $= (20\% - 10\%) \times \left(\frac{1.001}{1.0059} - 1\right) \times \frac{1.0059}{1.0387} = -0.05\%$

US risk-free rate $= (30\% - 40\%) \times \left(\frac{1.002}{1.0059} - 1\right) \times \frac{1.0059}{1.0387} = +0.04\%$

Total market allocation $= -0.06\% + 0.03\% - 0.13\% + 0.0\% - 0.05\% + 0.04\% = -0.17\%$

A revised semi-notional fund b'_{DS} reflecting the combined step is calculated. Issue selection is calculated in Exhibit 6.89. As an alternative to using the change in yield applied to weighted duration, it is possible to adjust the benchmark return to that expected at the portfolio duration in order to calculate issue selection. The interpretation of the attribution effects summarised in Table 6.43 remains identical to the arithmetic example.

Campisi framework

Using similar concepts to Van Breukelen and Wagner and Tito, Campisi[35] suggests breaking down fixed income returns into income and price return as illustrated in Figure 6.17.

In the Campisi framework the total return can be expressed as income plus price change:

$$\text{Total return} = \text{Income return} + \text{Price change} \tag{6.142}$$

[35] Campisi, "Primer on Fixed Income Performance Attribution" (2000).

Exhibit 6.89 Geometric issue selection

$$\frac{1+r}{1+r'_{DS}} - 1 = \frac{1.0386}{1.0369} - 1 = 0.16\%$$

Duration-adjusted benchmark

UK bonds $\quad b_{Di} = \frac{D_{ri}}{D_{bi}} \times (b_i - x_i) + x_i = \frac{7.8}{5.0} \times (3.5\% - 1.0\%) + 1.0\% = 4.9\%$

Japanese bonds $= \frac{1.0}{2.0} \times (0.5\% - 0.1\%) + 0.1\% = 0.3\%$

US bonds $\quad = \frac{4.0}{3.0} \times (3.0\% - 0.2\%) + 0.2\% = 3.93\%$

Individual issue selection (from Equation 6.28)

$$S^G{}_i = w_i \times \left(\frac{1+r_i}{1+b_{Di}} - 1\right) \times \frac{(1+b_{Di})}{(1+b'_{DS})} \tag{6.140}$$

UK bonds $\quad = 50\% \times \left(\frac{1.056}{1.049} - 1\right) \times \frac{1.049}{1.0369} = 0.34\%$

Japanese bonds $= 20\% \times \left(\frac{1.005}{1.003} - 1\right) \times \frac{1.003}{1.0369} = 0.04\%$

US bonds $\quad = 30\% \times \left(\frac{1.032}{1.0393} - 1\right) \times \frac{1.0393}{1.0369} = -0.20\%$

or individual selection (from Equation 6.29)

$$S_i^G = D_i \times w_i \times \frac{(-\Delta y_{ri} + \Delta y_{bi})}{1+b'_{DS}} \tag{6.141}$$

UK bonds $\quad = 7.8 \times 50\% \times \frac{(0.59\% - 0.5\%)}{1.0369} = 0.34\%$

Japanese bonds $= 1.0 \times 20\% \times \frac{(0.4\% - 0.2\%)}{1.0369} = 0.04\%$

US bonds $\quad = 4.0 \times 30\% \times \frac{(0.75\% - 0.93\%)}{1.0369} = -0.21\%$

Total $\quad = 0.34\% - 0.4\% + 0.21\% = 0.16\%$

where:

$$\text{Income return} = \frac{\text{Annual coupon rate}}{\text{Beginning market price}} \tag{6.143}$$

$$\begin{aligned}\text{Price return} &= \text{Effect of yield changes}\\ &= \text{Treasury effect} + \text{Spread effect} + \text{Selection effect}\end{aligned} \tag{6.144}$$

The Treasury effect represents the impact in the change of Treasury interest rates. The sensitivity to the change in yields of a portfolio is measured by modified duration; see Equation 5.118.

Table 6.43 Geometric weighted duration attribution

	Portfolio-weighted duration $w_i \times D_i$ (%)	Benchmark-weighted duration $W_i \times D_{bi}$ (%)	Portfolio change in yield Δy_i (%)	Benchmark change in yield Δy_{bi} (%)	Market allocation (%)	Issue selection (%)	Risk-free rate (%)
UK bonds	3.9	2.5	−0.59	−0.5	−0.06	0.34	0.0
Japanese bonds	0.2	0.2	−0.4	0.2	0.03	0.04	−0.05
US bonds	1.2	1.2	−0.75	−0.93	−0.13	−0.21	0.04
Total	**100**	**100**	**−0.62**	**−0.62**	**−0.16**	**0.16**	**−0.01**
					Overall duration $\frac{1+b_D}{1+b} - 1$		**0.84**

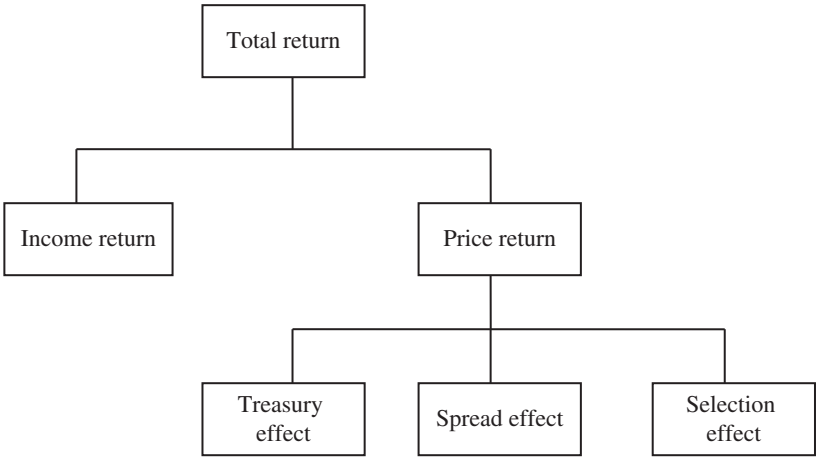

Figure 6.17 Campisi framework

In effect, Campisi adapts Equation 6.121, replacing local interest rates with income return and including spread effects as follows:

$$r_i = I_i + D_i \times (-\Delta y_i) + D_i \times (-\Delta ys_i) + \varepsilon_i \tag{6.145}$$

where: I_i = return from income in sector i
D_i = modified duration in sector i
Δy_i = change in Treasury interest rates at duration D_i
Δys_i = change in benchmark spreads in sector i
ε_i = residual return not explained by income, Treasury or spread effects

It follows that the benchmark return is:

$$b_i = I_{bi} + D_{bi} \times (-\Delta y_i) + D_{bi} \times (-\Delta ys_i) \tag{6.146}$$

where: I_{bi} = return from income in benchmark sector i
D_{bi} = benchmark-modified duration in sector i

Clearly, there should be no residual returns in the benchmark; it is assumed that any return not explained by income or Treasury effects must be spread effects.

Attribution effects using the Campisi framework are calculated using data from Table 6.44.

The structural added value from income is simply the portfolio income return minus the benchmark income return, 0.52% – 0.45%, which rounds to an added value of 0.08%. To calculate the Treasury effect, we need the interest rate changes for each level of duration shown in Exhibit 6.90.

The benchmark spread effect can be calculated in each sector assuming that any return contribution that is neither income nor Treasury effect in the benchmark must be caused by spread. The implied yields calculated from the benchmark can be applied to the durations of

Table 6.44 Campisi framework

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio-modified duration D_i (%)	Benchmark-modified duration D_{bi} (%)	Portfolio return r_i (%)	Portfolio income return I_i (%)	Benchmark return b_i (%)	Benchmark income return I_{bi} (%)
Treasuries	20	50	4.75	4.75	6.0	0.3	6.0	0.3
Corporates	65	40	3.6	5.25	4.4	0.52	8.0	0.57
High yield	15	10	4.3	4.0	5.6	0.82	5.0	0.71
Total	100	100	3.94	4.88	4.9	0.52	6.7	0.45

Exhibit 6.90 Treasury effect

Duration	Interest change	Price effect
D_i	Δy_i (%)	$D_i \times (-\Delta y_i)$ (%)
3.6	−1.00	+3.60
4.0	−1.05	+4.20
4.3	−1.10	+4.73
4.75	−1.20	+5.70
5.25	−1.35	+7.09

Portfolio Treasury return is therefore:

$$20\% \times 5.7\% + 65\% \times 3.6\% + 15\% \times 4.73\% = 4.19\%$$

Benchmark Treasury return is therefore:

$$50\% \times 5.7\% + 40\% \times 7.09\% + 10\% \times 4.2\% = 6.11\%$$

Therefore, the added value from the Treasury effect is 4.19% − 6.11% = 1.92%

the portfolio sectors to calculate the contribution from spread in the portfolio; any residual return must be a selection effect. Spread and selection effects are calculated in Exhibit 6.91 and attribution effects using the Campisi framework are summarised in Table 6.45.

Exhibit 6.91 Spread effects

Benchmark spread effect:

$$\Delta y_{Si} = \frac{-(b_i - I_{bi} - D_{bi} \times (-\Delta y_i))}{D_{bi}}$$

Corporates $= \dfrac{-(8.0 - 0.57\% - 5.25 \times 1.35)}{5.25} = -0.07\%$

High yield $= \dfrac{-(5.0 - 0.71\% - 4.0 \times 1.05)}{4.0} = -0.02\%$

The contribution to return from spread in the benchmark is therefore:

Corporates: Return $= -\Delta y_{Si} \times D_{bi} = 0.07 \times 5.25 = 0.35\%$

Contribution $= W_i \times -\Delta y_{Si} \times D_{bi} = 40\% \times 0.07 \times 5.25 = 0.14\%$

High yield: Return $= 0.02 \times 4.0 = 0.09\%$

Contribution $= 10\% \times 0.02 \times 4.0 = 0.01\%$

Total contribution 0.14% + 0.01% = 0.15%

The contribution to return from spread in the portfolio using the benchmark change in spreads is:

Corporates: Return $= -\Delta y_{Si} \times D_i = 0.07 \times 3.6 = 0.24\%$

Contribution $= w_i \times -\Delta y_{Si} \times D_i = 65\% \times 0.07 \times 3.6 = 0.15\%$

High yield: Return $= 0.02 \times 4.3 = 0.10\%$

Contribution $= 15\% \times 0.02 \times 4.3 = 0.01\%$

Total contribution 0.15% + 0.01% = 0.17%

Exhibit 6.91 (*Continued*)

Therefore, the added value from spread effect is 0.17% − 0.15% = 0.02%

Any contribution to return not derived from income, Treasury effect or spread must be issue selection:

Corporates 4.4% − 0.52% − 3.6% − 0.24% = 0.04% (Contribution 65% × 0.04% = 0.03%)

High yield 5.6% − 0.82% − 4.73% − 0.10% = −0.05% (Contribution 15% × −0.05% = −0.01%)

Therefore, the added value from issue selection is 0.03% − 0.01% = 0.02%

Table 6.45 Campisi framework – attribution summary

	Portfolio (%)	Benchmark (%)	Excess return (%)
Total return	**4.90**	**6.70**	**−1.80**
Income return	0.52	0.45	0.08
Treasury effect	4.19	6.11	−1.92
Spread	0.17	0.15	0.02
Issue selection	0.02	n/a	0.02

Campisi does not suggest breaking down returns any further although it is relatively straightforward to apply a combination of both the Brinson model and Van Breukelen to obtain a more detailed analysis. Income effects are calculated in Exhibit 6.92, Treasury effects in Exhibit 6.93, the spread effect in Exhibit 6.94 and results are summarised in Table 6.46.

Interpreting the results, the portfolio was short duration and lost 1.18% for a parallel shift in interest rates and another 0.74% given that long rates fell more than short rates. Structurally the portfolio benefited from slightly higher income and a tightening in credit spreads plus a slight contribution from issue selection.

For comparison, attribution effects using the normal Brinson model are calculated in Exhibit 6.95 and summarised in Table 6.47. The misleading conclusion from the standard approach would be significantly poor selection performance in corporates.

Exhibit 6.92 Income return attribution

Using the standard Brinson model to attribute the return from income

Income allocation:

Treasuries	(20% − 50%) × (0.3% − 0.45%) = 0.04%
Corporates	(65% − 40%) × (0.57% − 0.45%) = 0.03%
High yield	(15% − 10%) × (0.71% − 0.45%) = 0.01%
Total	0.04% + 0.03% + 0.01% = 0.09%

Income selection:

Treasuries	20% × (0.3% − 0.3%) = 0.0%
Corporates	65% × (0.52% − 0.57%) = −0.03%
High yield	15% × (0.82 − 0.71%) = 0.02%
Total	0.00% − 0.03% + 0.02% = 0.01%

Exhibit 6.93 Detailed Treasury effect attribution

Interest rate change at the benchmark duration of 4.88 = –1.25%
Impact of a parallel shift in interest rates:
$= (D_r - D_b) \times -y_b = (3.94 - 4.88) \times 1.25\% = -1.18\%$
Impact of non-parallel move in yield curve:

Treasuries	$w_i \times D_i \times (-y_i + y_b) = 20\% \times 4.75 \times (1.20\% - 1.25\%) = -0.05\%$
Corporates	$= 65\% \times 3.6 \times (+1.0\% - 1.25\%) = -0.59\%$
High yield	$= 15\% \times 4.3 \times (+1.1\% - 1.25\%) = -0.10\%$
Total	$- 0.05\% - 0.59\% - 0.10\% = -0.74\%$

Exhibit 6.94 Detailed spread effect attribution

Total benchmark spread change:

$$y_{Sb} = \frac{-(6.7 - 0.45\% - 6.11)}{4.88} = -0.03\%$$

Impact of overall spread changes at portfolio duration compared to benchmark duration:

$$(D_r - D_b) \times -y_{Sb} = (3.94 - 4.88) \times 0.03\% = -0.03\%$$

Spread allocation:

Treasuries	$(w_i \times D_i - W_i \times D_{bi}) \times (-y_{Si} + y_{Sb}) = (20\% \times 4.75 - 50\% \times 4.75) \times (0.0\% - 0.03\%) = 0.04\%$
Corporates	$= (65\% \times 3.6 - 40\% \times 5.25) \times (0.07\% - 0.03\%) = 0.01\%$
High yield	$= (15\% \times 4.3 - 10\% \times 4.0) \times (0.02\% - 0.03\%) = 0.0\%$
Total	$0.04\% + 0.01\% + 0.0\% = 0.05\%$

Table 6.46 Campisi framework detailed attribution

	Income allocation (%)	Income selection (%)	Duration (%)	Other yield curve effects (%)	Spread duration (%)	Spread (%)	Issue selection (%)
Treasuries	0.04	0.00	n/a	−0.05	n/a	0.04	0.00
Corporates	0.03	−0.03	n/a	−0.59	n/a	0.01	0.03
High yield	0.01	0.02	n/a	−0.1	n/a	0.00	−0.01
Total	0.09	−0.01	−1.18	−0.74	−0.03	0.05	0.02

Yield curve decomposition

Unlike the top-down approaches of Van Breukelen and Campisi, bond managers are more inclined to the accuracy of bottom-up models that decompose the yield effects for each bond or instrument in the portfolio and the benchmark and aggregate results into attribution analysis more consistent with their investment decision process.

Exhibit 6.95 Standard Brinson model

Allocation:	
Treasuries	$(20\% - 50\%) \times (6.0\% - 6.7\%) = 0.21\%$
Corporates	$(65\% - 40\%) \times (8.0\% - 6.7\%) = 0.33\%$
High yield	$(15\% - 10\%) \times (5.0\% - 6.7\%) = -0.09\%$
Total	$0.21\% + 0.33\% - 0.09\% = 0.45\%$
Selection:	
Treasuries	$0.21\% + 0.33\% - 0.09\% = 0.45\%$
Corporates	$65\% \times (4.4\% - 8.0\%) = -2.34\%$
High yield	$15\% \times (5.6\% - 5.0\%) = 0.09\%$
Total	$0.00\% - 2.34\% + 0.09\% = -2.25\%$

Table 6.47 Attribution using the standard Brinson model

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
Treasuries	20	50	6.0	6.0	0.21	0.0
Corporates	65	40	4.4	8.0	0.33	−2.34
High yield	15	10	5.6	5.0	−0.09	0.09
Total	**100**	**100**	**4.9**	**6.7**	**0.45**	**−2.25**

Caution

Yield curve decomposition will require accurate pricing of all the instruments in both the portfolio and the benchmark and significant operational effort.

Yield curve decomposition, consistent with Figure 6.14, is demonstrated on the portfolio and benchmark data shown in Table 6.48. To keep things simple each bond pays an annual coupon one year from today's date (T). Spot yields used to construct the yield curve at the start of the period and one quarter later are shown in Table 6.49.

Table 6.48 Fixed income portfolio and benchmark data

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Portfolio modified duration (years)	Benchmark modified duration (years)
Government 3% T+3	10	0	−0.20		2.87	
Government 4.5% T+5	10	25	−0.95	−0.95	4.53	4.53
Government 5% T+7	15	0	−2.68		6.04	
Corporate 6.5% T+7	25	0	−2.78		5.86	
Government 6% T+10	10	25	−3.85	−3.85	7.93	7.93
Government 5.5% T+12	10	25	−4.53	−4.53	9.28	9.28
Government 3% T+14	10	0	−6.18		11.42	
Government 4% T+15	10	25	−6.11	−6.11	11.57	11.57
Total	**100**	**100**	**−3.28**	**−3.86**	**7.13**	**8.33**

Table 6.49 Start and end of period yields

Quarter	Start of period yield (%)	End of period yield (%)
	1.25	1.73
1 year	1.25	1.73
	1.28	1.77
	1.32	1.82
	1.35	1.85
2 years	1.38	1.89
	1.42	1.94
	1.45	1.98
	1.48	2.01
3 years	1.52	2.05
	1.55	2.09
	1.58	2.13
	1.62	2.16
4 years	1.65	2.21
	1.68	2.24
	1.71	2.29
	1.74	2.33
5 years	1.77	2.38
	1.80	2.44
	1.83	2.50
	1.85	2.56
6 years	1.88	2.62
	1.91	2.67
	1.94	2.72
	1.97	2.77
7 years	2.00	2.80
	2.03	2.83
	2.05	2.85
	2.08	2.88
8 years	2.10	2.90
	2.13	2.93
	2.15	2.95
	2.18	2.96
9 years	2.20	2.98
	2.22	3.00
	2.23	3.01
	2.25	3.01
10 years	2.26	3.02
	2.27	3.03
	2.28	3.03
	2.30	3.03
11 years	2.31	3.04
	2.33	3.04
	2.33	3.04
	2.34	3.05

Table 6.49 (*Continued*)

Quarter	Start of period yield (%)	End of period yield (%)
12 years	2.35	3.05
	2.36	3.07
	2.37	3.07
	2.38	3.08
13 years	2.39	3.09
	2.40	3.11
	2.41	3.11
	2.42	3.12
14 years	2.43	3.14
	2.44	3.14
	2.45	3.15
	2.45	3.15
15 years	2.46	3.15

Using the yields in Table 6.49 and Equation 5.113 start and end of period clean prices for one of the government bonds in Table 6.48 are calculated in Table 6.50 and Table 6.51, respectively.

The change in shape of the yield curve between the start and end of period can be described by movements in shift, slope and curvature as shown in Table 6.52. For each bond in both the portfolio and the benchmark we can calculate the change in price for simple progression in time (rolldown) and for movement in the yield curve shown in Tables 6.53 to 6.56.

Table 6.50 Start of period price 6% coupon 10-year government bond

	Cash flow	Spot yield (%)	Present value
1 year	6	1.25	5.93
2 years	6	1.38	5.84
3 years	6	1.52	5.74
4 years	6	1.65	5.62
5 years	6	1.77	5.50
6 years	6	1.88	5.36
7 years	6	2.00	5.22
8 years	6	2.10	5.08
9 years	6	2.20	4.93
10 years	106	2.26	84.78
	Price		**134.00**

Table 6.51 End of period price 6% coupon 10-year government bond

	Cash flow	Spot yield (%)	Present value
0.75 year	6	1.73	5.92
1.75 years	6	1.85	5.81
2.75 years	6	2.01	5.68
3.75 years	6	2.16	5.54
4.75 years	6	2.33	5.38
5.75 years	6	2.56	5.19
6.75 years	6	2.77	4.99
7.75 years	6	2.88	4.82
8.75 years	6	2.96	4.65
9.75 years	106	3.01	79.37
	Price		**127.34**

Table 6.52 Describing the change in shape of the yield curve

Quarter	Start of period yield (%)	Plus shift (%)	Plus slope (%)	Plus curvature (%)
0.75 year	1.25	1.75	1.75	1.73
1 year	1.25	1.75	1.75	1.73
	1.28	1.78	1.79	1.77
	1.32	1.82	1.83	1.82
	1.35	1.85	1.86	1.85
2 years	1.38	1.88	1.90	1.89
	1.42	1.92	1.94	1.94
	1.45	1.95	1.98	1.98
	1.48	1.98	2.01	2.01
3 years	1.52	2.02	2.05	2.05
	1.55	2.05	2.09	2.09
	1.58	2.08	2.13	2.13
	1.62	2.12	2.16	2.16
4 years	1.65	2.15	2.20	2.21
	1.68	2.18	2.23	2.24
	1.71	2.21	2.27	2.29
	1.74	2.24	2.30	2.33
5 years	1.77	2.27	2.33	2.38
	1.80	2.30	2.37	2.44
	1.83	2.33	2.40	2.50
	1.85	2.35	2.43	2.56
6 years	1.88	2.38	2.47	2.62
	1.91	2.41	2.50	2.67
	1.94	2.44	2.53	2.72
	1.97	2.47	2.57	2.77
7 years	2.00	2.50	2.60	2.80
	2.03	2.53	2.63	2.83
	2.05	2.55	2.66	2.85
	2.08	2.58	2.69	2.88

Table 6.52 (*Continued*)

Quarter	Start of period yield (%)	Plus shift (%)	Plus slope (%)	Plus curvature (%)
8 years	2.10	2.60	2.72	2.90
	2.13	2.63	2.75	2.93
	2.15	2.65	2.78	2.95
	2.18	2.68	2.80	2.96
9 years	2.20	2.70	2.83	2.98
	2.22	2.72	2.86	3.00
	2.23	2.73	2.88	3.01
	2.25	2.75	2.89	3.01
10 years	2.26	2.76	2.91	3.02
	2.27	2.77	2.93	3.03
	2.28	2.78	2.94	3.03
	2.30	2.80	2.96	3.03
11 years	2.31	2.81	2.98	3.04
	2.33	2.83	3.00	3.04
	2.33	2.83	3.01	3.04
	2.34	2.84	3.02	3.05
12 years	2.35	2.85	3.03	3.05
	2.36	2.86	3.05	3.07
	2.37	2.87	3.06	3.07
	2.38	2.88	3.07	3.08
13 years	2.39	2.89	3.09	3.09
	2.40	2.90	3.11	3.11
	2.41	2.91	3.12	3.11
	2.42	2.92	3.13	3.12
14 years	2.43	2.93	3.15	3.14
	2.44	2.94	3.16	3.14
	2.45	2.95	3.17	3.15
	2.45	2.95	3.18	3.15
15 years	2.46	2.96	3.20	3.15

Table 6.53 Price 6% coupon 10-year government bond including rolldown

	Cash flow	Spot yield (%)	Present value
0.75 year	6	1.25	5.94
1.75 years	6	1.35	5.86
2.75 years	6	1.48	5.76
3.75 years	6	1.62	5.65
4.75 years	6	1.74	5.53
5.75 years	6	1.85	5.40
6.75 years	6	1.97	5.26
7.75 years	6	2.08	5.12
8.75 years	6	2.18	4.97
9.75 years	106	2.25	85.36
	Price		**134.85**

Table 6.54 Price 6 coupon 10-year government bond including shift

	Cash flow	Spot yield (%)	Present value
0.75 year	6	1.75	5.92
1.75 years	6	1.85	5.81
2.75 years	6	1.98	5.68
3.75 years	6	2.12	5.55
4.75 years	6	2.24	5.40
5.75 years	6	2.35	5.25
6.75 years	6	2.47	5.09
7.75 years	6	2.58	4.93
8.75 years	6	2.68	4.76
9.75 years	106	2.75	81.40
	Price		**129.79**

Table 6.55 Price 6% coupon 10-year government bond including slope

	Cash flow	Spot yield (%)	Present value
0.75 year	6	1.75	5.92
1.75 years	6	1.86	5.81
2.75 years	6	2.01	5.68
3.75 years	6	2.16	5.54
4.75 years	6	2.30	5.39
5.75 years	6	2.43	5.23
6.75 years	6	2.57	5.06
7.75 years	6	2.69	4.89
8.75 years	6	2.80	4.71
9.75 years	106	2.89	80.28
	Price		**128.49**

Table 6.56 Price 6% coupon 10-year government bond including curvature

	Cash flow	Spot yield (%)	Present value
0.75 year	6	1.73	5.92
1.75 years	6	1.85	5.81
2.75 years	6	2.01	5.68
3.75 years	6	2.16	5.54
4.75 years	6	2.33	5.38
5.75 years	6	2.56	5.19
6.75 years	6	2.77	4.99
7.75 years	6	2.88	4.82
8.75 years	6	2.96	4.65
9.75 years	106	3.01	79.37
	Price		**127.34**

Returns for each return factor are then calculated in Exhibit 6.96.

Exhibit 6.96 Return factors 6% coupon 10-year government bond

Return from income (coupon) for the quarter:

$$\text{Coupon} = \frac{6.0\%}{134 \times 4} = 1.21\% \text{ (adapted from Equation 6.142)}$$

$$\text{Rolldown return} = \frac{\text{Price}_{Roll} - \text{Price}_S}{\text{Price}_S} \tag{6.147}$$

$$= \frac{134.85 - 134.0}{134.0} = 0.63\%$$

Therefore, the return just from the passage of time alone is $1.21\% + 0.63\% = 1.84\%$

$$\text{Shift return} = \frac{\text{Price}_{Shift} - \text{Price}_{Roll}}{\text{Price}_S} \tag{6.148}$$

$$= \frac{129.79 - 134.85}{134.0} = -3.78\%$$

$$\text{Slope return} = \frac{\text{Price}_{Slope} - \text{Price}_{Shift}}{\text{Price}_S} \tag{6.149}$$

$$= \frac{128.49 - 129.79}{134.0} = -0.97\%$$

$$\text{Curvature return} = \frac{\text{Price}_{Curve} - \text{Price}_{Slope}}{\text{Price}_S} \tag{6.150}$$

$$= \frac{127.34 - 128.49}{134.0} = -0.86\%$$

$$\text{Clean price return} = \frac{\text{Price}_E - \text{Price}_S}{\text{Price}_S} \tag{6.151}$$

$$= \frac{127.34 - 134.0}{134.0} = -4.97\%$$

Clean price return = Rolldown + Shift + Slope + Curvature

$$= 0.63\% - 3.78\% - 0.97\% - 0.86\% = -4.97\%$$

Total return = Coupon + Clean Price Return = $1.12\% - 4.97\% = -3.85\%$

Similar progressions of return can be calculated for each bond in both the portfolio and benchmark. The corporate bond in the portfolio is exposed to additional credit risk. Future cash flows are discounted at a higher rate because of the additional risk of default. In this particular example credit spreads have widened, causing additional underperformance. Contributions to portfolio return are summarised in Table 6.57 and contributions to benchmark return in Table 6.58. Attribution effects are summarised in Table 6.59.

The portfolio outperformed by 0.58%; 0.02% of this outperformance was "baked in" at the start of the period with a structurally higher income or coupon. Whatever happened to interest rates or credit spreads, this outperformance was no surprise. There was a parallel shift higher in interest rates of 0.5%, and because of the shorter duration in the portfolio this added 0.58% of outperformance. Positioning on the yield curve was mildly positive, interest rates rose more at the long end than the short end, but the portfolio was slightly more exposed in the middle with an offsetting cost of 0.19%. The portfolio lost 0.13% exposed to widening credit spreads.

Table 6.57 Portfolio contributions to return

	Weight (%)	Coupon (%)	Rolldown (%)	Shift (%)	Twist (%)	Curvature (%)	Spread (%)
Government 3% T+3	10	0.07	0.05	−0.13	−0.01	0.00	0.00
Government 4.5% T+5	10	0.10	0.06	−0.21	−0.03	−0.01	0.00
Government 5% T+7	15	0.16	0.10	−0.43	−0.08	−0.15	0.00
Corporate 6.5% T+7	25	0.32	0.17	−0.69	−0.12	−0.23	−0.13
Government 6% T+10	10	0.11	0.06	−0.38	−0.10	−0.09	0.00
Government 5.5% T+12	10	0.10	0.06	−0.44	−0.14	−0.04	0.00
Government 3% T+14	10	0.07	0.07	−0.54	−0.20	−0.01	0.00
Government 4% T+15	10	0.08	0.07	−0.55	−0.22	0.00	0.00
Total	**100**	**1.01**	**0.63**	**−3.38**	**−0.88**	**−0.53**	**−0.13**

Table 6.58 Benchmark contributions to return

	Weight (%)	Coupon (%)	Rolldown (%)	Shift (%)	Twist (%)	Curvature (%)	Spread (%)
Government 3% T+3	0	0.00	0.00	0.00	0.00	0.00	0.00
Government 4.5% T+5	25	0.25	0.14	−0.53	−0.06	−0.03	0.00
Government 5% T+7	0	0.00	0.00	0.00	0.00	0.00	0.00
Corporate 6.5% T+7	0	0.00	0.00	0.00	0.00	0.00	0.00
Government 6% T+10	25	0.28	0.16	−0.94	−0.24	−0.21	0.00
Government 5.5% T+12	25	0.26	0.16	−1.10	−0.34	−0.10	0.00
Government 3% T+14	0	0.00	0.00	0.00	0.00	0.00	0.00
Government 4% T+15	25	0.21	0.17	−1.38	−0.54	0.01	0.00
Total	**100**	**0.99**	**0.63**	**−3.96**	**−1.19**	**−0.34**	**0.00**

Table 6.59 Attribution effects

	Coupon (%)	Rolldown (%)	Shift (%)	Twist (%)	Curvature (%)	Spread (%)	Total (%)
Government 3% T+3	0.07	0.05	−0.13	−0.01	0.00	0.00	**−0.02**
Government 4.5% T+5	−0.15	−0.08	0.32	0.04	0.02	0.00	**0.14**
Government 5% T+7	0.16	0.10	−0.43	−0.08	−0.15	0.00	**−0.40**
Corporate 6.5% T+7	0.32	0.17	−0.69	−0.12	−0.23	−0.13	**−0.70**
Government 6% T+10	−0.17	−0.10	0.57	0.15	0.13	0.00	**0.58**
Government 5.5% T+12	−0.15	−0.10	0.66	0.21	0.06	0.00	**0.68**
Government 3% T+14	0.07	0.07	−0.54	−0.20	−0.01	0.00	**−0.62**
Government 4% T+15	−0.12	−0.10	0.83	0.32	−0.01	0.00	**0.92**
Total	**0.02**	**0.00**	**0.58**	**0.30**	**−0.19**	**−0.13**	**0.58**

MULTI-PERIOD ATTRIBUTION

We observed in Chapter 4 that the sum of the arithmetic excess return for each finite period does not sum to the total arithmetic excess return for the total period:

$$R - \overline{R} \neq \sum (R_t - \overline{R}_t) \tag{6.152}$$

Therefore, over multiple periods we should not expect our arithmetic attribution factors which add up over single periods to add up for the total period under analysis. Excess return through active management decisions compounds over time, a fact that should be accounted for in multi-period frameworks. Naturally the end users of attribution analysis and asset owners expect their analysis to add up without residuals. Software providers have typically found solutions, often keeping their methods more or less proprietary.

Note

Many have assumed that the software providers maintained a cloak of secrecy about their multi-period attribution methodologies to retain a commercial advantage. I suspect they may have been more embarrassed about the lack of economic justification of their methodologies – the primary justification being – it works!

Solutions can be categorised into two types, smoothing algorithms such as those described by Carino[36] and Menchero[37] and linking algorithms (sometimes described as dollar attribution) such as those in GRAP,[38] Frongello,[39] and Bonafede, Forestri and Matheos.[40]

Smoothing algorithms in effect take the resulting residual and methodically redistribute this residual across other factors, Carino employing logarithms and Menchero redistributing the residual in an optimised way that minimises the change in any single factor. GRAP, Frongello, and Bonafede et al. compound (link) attribution effects through time and achieve identical results, but, crucially, their methods are order-dependent: reverse the time periods and the attribution result will change. Frongello acknowledges order dependence but regards it as a strength, while others, including Laker,[41] would describe it as a weakness.

Davies and Laker[42] suggest an "exact" method for calculating attributes over multiple periods in the Brinson model at a portfolio, but not category, level.

[36] Carino, "Combining Attribution Effects Over Time" (1999).
[37] Menchero, "An Optimized Approach to Linking Attribution Effects Over Time" (2000).
[38] GRAP (Groupe de Recherche en Attribution de Performance), "Synthèse des modèles d'attribution de performance" (1997).
[39] Frongello, "Linking Single Period Attribution Results" (2002).
[40] Bonafede, Foresti, and Matheos, "A Multi-Period Linking Algorithm That Has Stood the Test of Time" (2002).
[41] Laker, "A View from Down Under" (2002).
[42] Davies and Laker, "Multiple-period Performance Attribution Using the Brinson Model" (2001).

Note

All of these methods are truly "black box"[43] methods; they do not need to be understood. They are a presentational convenience to ensure that numbers that should not add up, in fact do add up. If arithmetic excess returns are preferred, then a smoothing or linking algorithm must be employed; there is no economic justification other than presentational convenience. The choice of software system will most likely determine the method used. The answers will be slightly different, but in most cases the interpretation of results will not differ.

SMOOTHING ALGORITHMS

Carino

Carino suggests the results for the single period can be transformed into results that naturally cumulate over time. Continuously compounded returns may be summed as demonstrated in Equation 3.12.

Using this relationship Carino introduces the factor:

$$k_t = \frac{\ln(1 + r_t) - \ln(1 + b_t)}{r_t - b_t} \tag{6.153}$$

$$\text{if } r_t = b_t \text{ set } k_t = \frac{1}{(1 + r_t)}$$

Since from Chapter 3 for continuously compounded returns we know that:

$$\ln(1 + r) = \ln(1 + r_1) + \ln(1 + r_2) + \cdots + \ln(1 + r_n) \tag{3.12}$$

and similarly for the benchmark:

$$\ln(1 + b) = \ln(1 + b_1) + \ln(1 + b_2) + \cdots + \ln(1 + b_n) \tag{6.154}$$

Subtracting Equation 6.154 from Equation 3.12:

$$\ln(1 + r) - \ln(1 + b) = \ln(1 + r_1) - \ln(1 + b_1) \cdots + \ln(1 + r_n) - \ln(1 + b_n) \tag{6.155}$$

Substituting Equation 6.148 into Equation 6.150 and simplifying:

$$\ln(1 + r) - \ln(1 + b) = \sum_{t=1}^{T} k_t \times (r_t - b_t) \tag{6.156}$$

[43] From a flight recorder in an aircraft (which is in fact not black but bright orange!) – a complex system or device whose internal workings are hidden or not readily understood.

To transform back to the desired arithmetic difference Carino introduced a similar factor for the entire period:

$$k = \frac{\ln(1+r) - \ln(1+b)}{r-b} \tag{6.157}$$

Therefore, substituting Equation 6.157 into Equation 6.156:

$$r - b = \sum_{t=1}^{t=n} \frac{k_t}{k} \times (r_t - b_t) \tag{6.158}$$

It follows that:

$$r - b = \sum_{t=1}^{t=n} \frac{k_t}{k} \times A_t + \sum_{t=1}^{t=n} \frac{k_t}{k} \times S_t + \sum_{t=1}^{t=n} \frac{k_t}{k} \times I_t \tag{6.159}$$

Table 6.60 extends the example data in Table 6.6 over four quarters. The portfolio return for the year is now 3.86% and the benchmark return is –9.41%. The objective is to calculate annual attribution effects that add to the arithmetic excess return of 13.27%.

Table 6.60 Example data over four quarters

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
First quarter						
UK equities	40	40	20	10	0.0	4.0
Japanese equities	30	20	−5	−4	−1.04	−0.3
US equities	30	40	6	8	−0.16	−0.6
Total	**100**	**100**	**8.3**	**6.4**	**−1.2**	**3.1**
Second quarter						
UK equities	70	40	−5	−7	−0.72	1.4
Japanese equities	20	30	3	4	−0.86	−0.2
US equities	10	30	−5	−10	1.08	0.5
Total	**100**	**100**	**−3.4**	**−4.6**	**−0.5**	**1.7**
Third quarter						
UK equities	30	50	−20	−25	2.5	1.5
Japanese equities	50	40	8	5	1.75	1.5
US equities	20	10	−15	−20	−0.75	1.0
Total	**100**	**100**	**−5.0**	**−12.5**	**3.5**	**4.0**
Fourth quarter						
UK equities	30	40	10	5	−0.3	1.5
Japanese equities	50	40	−7	−5	−0.7	−1.0
US equities	20	20	25	10	0.0	3.0
Total	**100**	**100**	**4.5**	**2.0**	**−1.0**	**3.5**
Annual			Portfolio return (%)	Benchmark return (%)		
Total			**3.86**	**−9.41**		

Carino factors are calculated for each period and the overall period in Exhibit 6.97.

Exhibit 6.97 Carino factors

	$k_t = \dfrac{\ln(1 + r_t) - \ln(1 + b_t)}{r_t - b_t}$
First quarter	$k_1 = \dfrac{(\ln 1.083 - \ln 1.064)}{(8.3\% - 6.4\%)} = 0.93156$
Second quarter	$k_2 = \dfrac{(\ln 0.966 - \ln 0.954)}{(-3.4 + 4.6\%)} = 1.04168$
Third quarter	$k_3 = \dfrac{(\ln 0.95 - \ln 0.875)}{(-5.0\% + 12.5\%)} = 1.09651$
Fourth quarter	$k_4 = \dfrac{(\ln 1.045 - \ln 1.02)}{(4.5\% - 2.0\%)} = 0.96857$
Year	$k = \dfrac{(\ln 1.0386 - \ln 0.9059)}{(3.86\% + 9.41\%)} = 1.3013$

Using the Carino factors calculated in Exhibit 6.97 revised attribution effects can be calculated for the first quarter as shown in Exhibit 6.98.

Exhibit 6.98 Carino revised Q1 attribution factors

UK asset allocation	$0\% \times \dfrac{k_1}{k} = 0\% \times \dfrac{0.93156}{1.03013} = 0\%$
Japanese asset allocation	$= -1.04\% \times \dfrac{0.93156}{1.03013} = -0.94\%$
US asset allocation	$= -0.16\% \times \dfrac{0.93156}{1.03013} = -0.14\%$
UK stock selection	$= 4.0\% \times \dfrac{0.93156}{1.03013} = 3.62\%$
Japanese stock selection	$= -0.3\% \times \dfrac{0.93156}{1.03013} = -0.27\%$
US stock selection	$= -0.6\% \times \dfrac{0.93156}{1.03013} = -0.54\%$

Extending the process in Exhibit 6.98 to all four quarters we can calculate revised attribution factors that sum to the arithmetic difference for the entire four-quarter period in Table 6.61.

The objective has been achieved in Table 6.61; our attribution factors are now additive. It should be noted that our revised attribution effects in each quarter are unique for that overall period. If we lengthen the period of analysis, we will need to recalculate new revised attribution effects for each quarter. This is both counterintuitive and cumbersome, particularly if we wish to calculate attribution effects over a number of years.

Table 6.61 Revised attribution factors for the four-quarter period (Carino)

	Original arithmetic attribution		Revised Carino attribution	
	Asset allocation (%)	Stock selection (%)	Asset allocation (%)	Stock selection (%)
First quarter				
UK equities	0.0	4.0	0.0	3.62
Japanese equities	−1.04	−0.3	−0.94	−0.27
US equities	−0.16	−0.6	−0.14	−0.54
Total	**−1.2**	**3.1**	**−1.09**	**2.80**
Second quarter				
UK equities	−0.72	1.4	−0.73	1.42
Japanese equities	−0.86	−0.2	−0.87	−0.2
US equities	1.08	0.5	1.09	0.51
Total	**−0.5**	**1.7**	**−0.51**	**1.72**
Third quarter				
UK equities	2.5	1.5	2.66	1.60
Japanese equities	1.75	1.5	1.86	1.60
US equities	−0.75	1.0	−0.8	1.06
Total	**3.5**	**4.0**	**3.73**	**4.26**
Fourth quarter				
UK equities	−0.3	1.5	−0.28	1.41
Japanese equities	−0.7	−1.0	−0.66	−0.94
US equities	0.0	3.0	0.0	2.82
Total	**−1.0**	**3.5**	**−0.94**	**3.29**
Four-quarter total			Asset allocation (%)	Stock selection (%)
UK equities			1.65	8.04
Japanese equities			−0.60	0.18
US equities			0.15	3.85
Total			**1.20**	**12.07**
	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
Total	**3.86**	**−9.41**	**1.20**	**12.07**

Menchero

Menchero offers a similar if more sophisticated approach to Carino; he suggests the introduction of a constant factor M into Equation 6.152 that takes into account the characteristic scaling which arises from geometric compounding:

$$r - b \approx M \times \sum_{t=1}^{T} (r_t - b_t) \tag{6.160}$$

Logically Menchero chooses for M the ratio of the difference of the arithmetic average between the portfolio and benchmark returns with the difference of the geometric average portfolio and benchmark returns.

$$M = \frac{(r - b)/T}{\left[(1 + r)^{1/T} - (1 + b)^{1/T}\right]} \tag{6.161}$$

$$\text{If } r = b \text{ set } M = (1 + r)^{\frac{(T-1)}{T}} \tag{6.162}$$

Unfortunately, this still leaves a residual in Equation 6.155, which means we must calculate a corrective term α_t such that:

$$r - b = \sum_{t=1}^{t=T} (M + \alpha_t) \times (r_t - b_t) \tag{6.163}$$

Calculating α_t as small as possible in order that the linking coefficients $(M + \alpha_t)$ be distributed as uniformly as possible to provide an "optimal solution", Menchero uses Lagrange multipliers to calculate:

$$\alpha_t = \left(\frac{r - b - M \times \sum_{t=1}^{t=T} (r_t - b_t)}{\sum_{t=1}^{t=T} (r_t - b_t)^2} \right) \times (r_t - b_t) \tag{6.164}$$

It follows that:

$$r - b = \sum_{t=1}^{t=T} (M + \alpha_t) \times A_t + \sum_{t=1}^{t=T} (M + \alpha_t) \times S_t + \sum_{t=1}^{t=T} (M + \alpha_t) \times I_t \tag{6.165}$$

Menchero factors are calculated for each period and the overall period for the data in Table 6.60 is shown in Exhibit 6.99.

Revised Menchero attribution effects for the first quarter are calculated in Exhibit 6.100.

Extending the process in Exhibit 6.100 to all four quarters we can calculate revised attribution factors that sum to the arithmetic difference for the entire four-quarter period in Table 6.62.

The Menchero method achieves our objective but with a slightly different solution from the Carino approach. It should be noted that our revised effects are unique for the overall period; it is necessary to recalculate each individual period's effects again as we lengthen the period of analysis. With the added disadvantage of complexity, this method is also counterintuitive and cumbersome, although Menchero would claim, with some justification, that the change to any factor is optimised.

Exhibit 6.99 Menchero factors

$$\alpha_t = \left(\frac{r - b - M \times \sum_{t=1}^{t=T} (r_t - b_t)}{\sum_{t=1}^{t=T} (r_t - b_t)^2} \right) \times (r_t - b_t)$$

$$M = \frac{(+3.86\% + 9.41\%)}{\left[(1.0386)^{\frac{1}{4}} - (0.9059)^{\frac{1}{4}} \right]} = 0.97813$$

$$\sum_{t=1}^{t=T} (r_t - b_t) = (8.3\% - 6.4\% - 3.4\% + 4.6\% - 5.0\% + 12.5\% + 4.5\% + 2\%) = 13.1\%$$

$$\sum_{t=1}^{t=T} (r_t - b_t)^2 = (1.9\%)^2 + (1.2\%)^2 + (7.5\%)^2 + (2.5\%)^2 + (2.5\%)^2 = 0.6755\%$$

Therefore, for the data in Table 6.61:

$$\alpha_t = \frac{(3.86\% + 9.41\% - 0.97813 \times 13.1\%)}{0.6755\%} \times (r_t - b_t) = 0.67579 \times (r_t - b_t)$$

First quarter $\alpha_1 = 0.67579 \times (8.3\% - 6.4\%) = 0.01284$

Second quarter $\alpha_2 = 0.67579 \times (-3.2\% + 4.6\%) = 0.00946$

Third quarter $\alpha_3 = 0.67579 \times (-5.0\% + 12.5\%) = 0.05068$

Fourth quarter $\alpha_4 = 0.67579 \times (4.5\% - 2\%) = 0.01689$

Exhibit 6.100 Menchero revised attribution factors

UK asset allocation	$= 0\% \times (M + \alpha_1) = 0\% \times (0.97813 + 0.01284) = 0\%$
Japanese asset allocation	$= -1.04\% \times (0.97813 + 0.01284) = -1.03\%$
US asset allocation	$= -0.16\% \times (0.97813 + 0.01284) = -0.16\%$
UK stock selection	$= 4.0\% \times (0.97813 + 0.01284) = 3.96\%$
Japanese stock selection	$= -0.3\% \times (0.97813 + 0.01284) = -0.3\%$
US stock selection	$= -0.6\% \times (0.97813 + 0.01284) = -0.59\%$

LINKING ALGORITHMS

GRAP method

Carino and Menchero are examples of smoothing algorithms in which the natural residual of the multi-period arithmetic attribution is structurally distributed across all the contributions to performance.

GRAP (Groupe de Recherche en Attribution de Performance), a Paris-based working group of performance experts, proposed a different type of "linking" approach as follows:

Let a_t = arithmetic excess return in period t, then:

$r_1 = b_1 + a_1$ for period $t = 1$ and $r_2 = b_2 + a_2$ for period $t = 2$

Table 6.62 Revised attribution factors for the four-quarter period (Menchero)

	Original arithmetic attribution		Revised Menchero attribution	
	Asset allocation (%)	Stock selection (%)	Asset allocation (%)	Stock selection (%)
First quarter				
UK equities	0.0	4.0	0.0	3.96
Japanese equities	−1.04	−0.3	−0.94	−0.30
US equities	−0.16	−0.6	−0.14	−0.59
Total	**−1.2**	**3.1**	**−1.19**	**3.07**
Second quarter				
UK equities	−0.72	1.4	−0.71	1.38
Japanese equities	−0.86	−0.2	−0.85	−0.2
US equities	1.08	0.5	1.07	0.49
Total	**−0.5**	**1.7**	**−0.49**	**1.68**
Third quarter				
UK equities	2.5	1.5	2.57	1.54
Japanese equities	1.75	1.5	1.80	1.54
US equities	−0.75	1.0	−0.77	1.03
Total	**3.5**	**4.0**	**3.60**	**4.11**
Fourth quarter				
UK equities	−0.3	1.5	−0.30	1.49
Japanese equities	−0.7	−1.0	−0.70	−0.99
US equities	0.0	3.0	0.0	2.98
Total	**−1.0**	**3.5**	**−0.99**	**3.48**
Four-quarter total			Asset allocation (%)	Stock selection (%)
UK equities			1.56	8.38
Japanese equities			−0.78	0.05
US equities			0.14	3.91
Total			**0.92**	**12.34**
	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
Total	**3.86**	**−9.41**	**0.92**	**12.34**

The total return over the first two periods is therefore:

$$
\begin{aligned}
(1+r) &= (1+b_1+a_1)\times(1+b_2+a_2)\\
&= (1+b_1+a_1)\times(1+b_2)+(1+b_1+a_1)\times a_2\\
&= (1+b_1)\times(1+b_2)+a_1\times(1+b_2)+(1+r_1)\times a_2\\
&= (1+b)+a_1\times(1+b_2)+(1+r_1)\times a_2\\
(r-b) &= a = a_1\times(1+b_2)+(1+r_1)\times a_2 \qquad (6.166)
\end{aligned}
$$

In effect, the excess return for the first period is reinvested in the benchmark return of the second period and the excess return of the second period is compounded by the actual portfolio return in the first period.

Over n periods we can generalise:

$$a = \sum_{T=1}^{n} a_T \times \prod_{t=1}^{T-1}(1 + r_t) \times \prod_{t=T+1}^{n}(1 + b_t) \tag{6.167}$$

In effect, the excess return for period $t = T$ is compounded by the actual portfolio return up to period $t = T$ and reinvested in the benchmark thereafter.

It follows that:

$$r - b = \sum_{T=1}^{n}(A_T + S_T + I_T) \times \prod_{t=1}^{T-1}(1 + r_t) + \prod_{t=T+1}^{n}(1 + b_t) \tag{6.168}$$

GRAP revised first-quarter attribution effects are calculated in Exhibit 6.101.

Exhibit 6.101 GRAP revised first-quarter attribution factors

UK asset allocation	$= 0\% \times (1 + b_2) \times (1 + b_3) \times (1 + b_4) = 0\% \times 0.954 \times 0.875 \times 1.02 = 0\%$
Japanese asset allocation	$= -1.04\% \times 0.954 \times 0.875 \times 1.02 = -0.89\%$
US asset allocation	$= -0.16\% \times 0.954 \times 0.875 \times 1.02 = -0.14\%$
UK stock selection	$= 4.0\% \times 0.954 \times 0.875 \times 1.02 = 3.41\%$
Japanese stock selection	$= -0.3\% \times 0.954 \times 0.875 \times 1.02 = -0.26\%$
US stock selection	$= -0.6\% \times 0.954 \times 0.875 \times 1.02 = -0.51\%$

GRAP revised second-quarter attribution effects are calculated in Exhibit 6.102.

Exhibit 6.102 GRAP revised second-quarter attribution factors

UK asset allocation	$= -0.72\% \times (1 + r_1) \times (1 + b_3) \times (1 + b_4)$
	$= -0.72\% \times 1.083 \times 0.875 \times 1.02 = -0.7\%$
Japanese asset allocation	$= -0.86 \times 1.083 \times 0.875 \times 1.02 = -0.83\%$
US asset allocation	$= 1.08\% \times 1.083 \times 0.875 \times 1.02 = 1.04\%$
UK stock selection	$= 1.4\% \times 1.083 \times 0.875 \times 1.02 = 1.35\%$
Japanese stock selection	$= -0.2\% \times 1.083 \times 0.875 \times 1.02 = -0.19\%$
US stock selection	$= 0.5\% \times 1.083 \times 0.875 \times 1.02 = 0.48\%$

Extending the process in Exhibit 6.101 and Exhibit 6.102 to all four quarters, we can calculate revised attribution factors that sum to the arithmetic difference for the entire four-quarter period as shown in Table 6.63.

The objective is achieved but with a different solution from either the Carino or Menchero approaches.

Table 6.63 Revised attribution factors for the four-quarter period (GRAP)

	Original arithmetic attribution		Revised GRAP attribution	
	Asset allocation (%)	Stock selection (%)	Asset allocation (%)	Stock selection (%)
First quarter				
UK equities	0.0	4.0	0.0	3.41
Japanese equities	−1.04	−0.3	−0.89	−0.26
US equities	−0.16	−0.6	−0.14	−0.51
Total	**−1.2**	**3.1**	**−1.04**	**2.64**
Second quarter				
UK equities	−0.72	1.4	−0.70	1.35
Japanese equities	−0.86	−0.2	−0.83	−0.19
US equities	1.08	0.5	1.04	0.48
Total	**−0.5**	**1.7**	**−0.48**	**1.64**
Third quarter				
UK equities	2.5	1.5	2.67	1.60
Japanese equities	1.75	1.5	1.87	1.60
US equities	−0.75	1.0	−0.80	1.07
Total	**3.5**	**4.0**	**3.73**	**4.27**
Fourth quarter				
UK equities	−0.3	1.5	−0.30	1.49
Japanese equities	−0.7	−1.0	−0.70	−0.99
US equities	0.0	3.0	0.0	2.98
Total	**−1.0**	**3.5**	**−0.99**	**3.48**

	Four-quarter total		Asset allocation (%)	Stock selection (%)
	UK equities		1.67	7.85
	Japanese equities		−0.55	0.16
	US equities		0.11	4.02
	Total		**1.24**	**12.03**
	Portfolio return (%)	**Benchmark return** (%)	**Asset allocation** (%)	**Stock selection** (%)
Total	**3.86**	**−9.41**	**1.24**	**12.03**

Frongello

Frongello suggested a linking algorithm using the same concept as the GRAP method, with:

$$f_T = a_T \times \prod_{t=1}^{t=T-1} (1 + r_t) + b_T \times \sum_{t=1}^{t=T-1} f_t \tag{6.169}$$

where: f_t = the revised Frongello attribution effect for period t

The first part of this equation compounds the single-period arithmetic excess return a_T with the cumulative return of the actual portfolio up to the prior period, while the second part is the gain on the sum of the prior period Frongello-adjusted excess returns generated by the current benchmark return as demonstrated in Exhibit 6.103.

Exhibit 6.103 Frongello revised second-quarter attribution factors

Note that Frongello first-period attribution effects are never adjusted.	
UK asset allocation	$= -0.72\% \times (1 + r_1) + b_2 \times f_1 = -0.72\% \times 1.083 - 0.046 \times 0.0\% = -0.78\%$
Japanese asset allocation	$= -0.86\% \times 1.083 - 0.046 \times -1.04\% = -0.88\%$
US asset allocation	$= 1.08\% \times 1.083 - 0.046 \times -0.16\% = 1.18\%$
UK stock selection	$= 1.4\% \times 1.083 - 0.046 \times 4.0\% = 1.33\%$
Japanese stock selection	$= -0.2\% \times 1.083 - 0.046 \times -0.30\% = -0.2\%$
US stock selection	$= 0.5\% \times 1.083 - 0.046 \times -0.6\% = 0.57\%$

It follows that:

$$a = r - b = \sum_{T=1}^{T=n} (A_T + S_T + I_T) \times \prod_{t=1}^{t=T-1} (1 + r_t) + b_T \times \sum_{t=1}^{t=T-1} f_t \tag{6.170}$$

Frongello revised third-quarter attribution effects are calculated in Exhibit 6.104.

Exhibit 6.104 Frongello revised third-quarter attribution factors

UK asset allocation	$= 2.5\% \times (1 + r_1) \times (1 + r_2) + b_3 \times (f_2 + f_1)$
	$= 2.5\% \times 1.083 \times 0.966 - 0.125 \times (-0.78 + 0.0\%) = 2.71\%$
Japanese asset allocation	$= 1.75\% \times 1.083 \times 0.966 - 0.125 \times (-0.88\% - 1.04\%) = 2.07\%$
US asset allocation	$= -0.75\% \times 1.083 \times 0.966 - 0.125 \times (1.18\% - 0.16\%) = -0.91\%$
UK stock selection	$= 1.5\% \times 1.083 \times 0.966 - 0.125 \times (1.33\% + 4.0\%) = 0.9\%$
Japanese stock selection	$= 1.5\% \times 1.083 \times 0.966 - 0.125 \times (-0.2\% - 0.3\%) = 1.63\%$
US stock selection	$= 1.0\% \times 1.083 \times 0.966 - 0.125 \times (0.57\% - 0.6\%) = 1.05\%$

Table 6.64 Revised attribution factors for the four-quarter period (Frongello)

	Original arithmetic attribution		Revised Frongello attribution	
	Asset allocation (%)	Stock selection (%)	Asset allocation (%)	Stock selection (%)
First quarter				
UK equities	0.0	4.0	0.0	4.0
Japanese equities	−1.04	−0.3	−1.04	−0.3
US equities	−0.16	−0.6	−0.16	−0.6
Total	**−1.2**	**3.1**	**−1.2**	**3.1**
Second quarter				
UK equities	−0.72	1.4	−0.78	1.33
Japanese equities	−0.86	−0.2	−0.88	−0.20
US equities	1.08	0.5	1.18	0.57
Total	**−0.5**	**1.7**	**−0.49**	**1.70**
Third quarter				
UK equities	2.5	1.5	2.71	0.90
Japanese equities	1.75	1.5	2.07	1.63
US equities	−0.75	1.0	−0.91	1.05
Total	**3.5**	**4.0**	**3.87**	**3.58**
Fourth quarter				
UK equities	−0.3	1.5	−0.26	1.62
Japanese equities	−0.7	−1.0	−0.69	−0.97
US equities	0.0	3.0	0.0	3.00
Total	**−1.0**	**3.5**	**−0.95**	**3.65**

	Four-quarter total		Asset allocation **(%)**	Stock selection **(%)**
	UK equities		1.67	7.85
	Japanese equities		−0.55	0.16
	US equities		0.11	4.02
	Total		**1.24**	**12.03**
	Portfolio return (%)	**Benchmark return (%)**	**Asset allocation (%)**	**Stock selection (%)**
Total	**3.86**	**−9.41**	**1.24**	**12.03**

Extending the process in Exhibit 6.103 and Exhibit 6.104 to all four quarters, we can calculate revised attribution factors that sum to the arithmetic difference for the entire four-quarter period in Table 6.64.

The Frongello method produces the same effects as the GRAP method for the total period.

Both these methods are order-dependent – the linked results will alter if the order of quarters is rearranged.

Note

Despite being order-dependent, I prefer these methods to Carino and Menchero. They are slightly more intuitive and computationally less cumbersome, I have a natural aversion to distributing residuals implicit in the Carino and Menchero methods, and clearly recalculating attribution effects every time the period is extended is extremely inefficient.

Davies and Laker

Davies and Laker refer back to Brinson, Hood and Beebower's original article that suggested applying the Brinson model over multiple periods and also refer to the work of Kirievsky and Kirievsky.[44]

They suggest compounding each of the notional funds to derive the total attribution effects for multiple periods as follows:

Arithmetic excess return over total period:

$$r - b = \prod_{t=1}^{t=n}(1 + r_t) - \prod_{t=1}^{t=n}(1 + b_t) \tag{6.171}$$

Asset allocation:

$$\prod_{t=1}^{t=n}(1 + b_{S,t}) - \prod_{t=1}^{t=n}(1 + b_t) \tag{6.172}$$

where: $b_{S,t}$ = the semi-notional (allocation notional) fund in period t

$$b_S = \prod_{t=1}^{t=n}(1 + b_{S,t}) - 1$$

Stock selection:

$$\prod_{t=1}^{t=n}(1 + r_{S,t}) - \prod_{t=1}^{t=n}(1 + b_t) \tag{6.173}$$

where: $r_{S,t}$ = the selection notional fund in period t

$$r_S = \prod_{t=1}^{t=n}(1 + r_{S,t}) - 1$$

[44] Kirievsky and Kirievsky, "Attribution Analysis: Combining Attribution Effects Over Time Made Easy" (2000).

Interaction:

$$\prod_{t=1}^{t=n}(1+r_t)-\prod_{t=1}^{t=n}(1+r_{S,t})-\prod_{t=1}^{t=n}(1+b_{S,t})=\prod_{t=1}^{t=n}(1+b_t) \tag{6.174}$$

The Davies and Laker method establishes total contributions for allocation, selection and interaction. Their 2001 article did not address individual sector returns, although it did hint at the use of a Carino-type algorithm to calculate individual sector contributions that add up to the total for each factor.

Using the data from Table 6.60 again, the Brinson notional funds are calculated for each quarter and compounded in Exhibit 6.105.

Exhibit 6.105 Brinson notional funds

Quarter 1

The allocation notional return $b_{S,1}=\sum_{i=1}^{i=n} w_i \times b_i$ for quarter 1 is:

$b_{S,1}=40\% \times 10\% + 30\% \times -4\% + 30\% \times 8\% = 5.2\%$

The selection notional return $r_{S,1}=\sum_{i=1}^{i=n} W_i \times r_i$ for quarter 1 is:

$r_{S,1}=40\% \times 20\% + 20\% \times -5\% + 40\% \times 6\% = 9.4\%$

Quarter 2

$b_{S,2}=70\% \times -7.0\% + 20\% \times 4.0\% + 10\% \times 10.0\% = -5.1\%$

$r_{S,2}=40\% \times -5.0\% + 30\% \times 3.0\% + 30\% \times -5.0\% = -2.6\%$

Quarter 3

$b_{S,3}=50\% \times -20.0\% + 40\% \times 8.0\% + 10\% \times -15.0\% = -9.0\%$

$r_{S,3}=30\% \times -25.0\% + 50\% \times 5.0\% + 20\% \times -20.0\% = -8.3\%$

Quarter 4

$b_{S,4}=30\% \times 5.0\% + 50\% \times -5.0\% + 20\% \times 10.0\% = 1.0\%$

$r_{S,4}=40\% \times 5.0\% + 40\% \times -5.0\% + 20\% \times 10.0\% = 6.2\%$

Compounded notional funds

$$\prod_{t=1}^{t=n}(1+b_{S,t})=1.052\times 0.949\times 0.91\times 1.01=0.9176$$

$$\prod_{t=1}^{t=n}(1+r_{S,t})=1.094\times 0.974\times 0.917\times 1.062=1.0377$$

Total or "exact" attribution effects are calculated for the entire period in Exhibit 6.106.

Exhibit 6.106 Exact attribution effects

$$r = \prod_{t=1}^{t=n}(1 + r_t) - 1 = 3.86\%$$

$$b = \prod_{t=1}^{t=n}(1 + b_t) - 1 = -9.41\%\%$$

$$r_S = \prod_{t=1}^{t=n}(1 + r_{S,t}) - 1 = 3.77\%$$

$$r_S = \prod_{t=1}^{t=n}(1 + r_{S,t}) - 1 = 3.77\%$$

Excess return $3.86\% + 9.41\% = 13.27\%$

Stock selection $3.77\% + 9.41\% = 13.18\%$

Asset allocation $-8.24 + 9.41\% = 1.17\%$

Interaction $3.86\% - 3.77\% + 8.24\% - 9.41\% = -1.08\%$

$$\underbrace{13.27\%}_{\text{Excess return}} = \underbrace{13.18\%}_{\text{Stock selection}} = \underbrace{1.17\%}_{\text{Asset selection}} + \underbrace{-1.08\%}_{\text{Interaction}}$$

Only the total level attribution effect can be calculated this way. To establish the contribution from each category a smoothing algorithm must be used on each effect separately.

This method combines arithmetic and geometric concepts and is really an evolution stage between the arithmetic and full geometric methodologies.

The Davies and Laker method literally compounds the basic flaw of the Brinson model, interaction, with the result that this factor is even less meaningful. It is perhaps more sensible to combine interaction with stock selection, defining stock selection as:

$$\prod_{t=1}^{t=n}(1 + r_t) - \prod_{t=1}^{t=n}(1 + b_{St}) \tag{6.175}$$

Therefore, stock selection plus asset allocation fully explains excess return:

$$r - b = \underbrace{\prod_{t=1}^{t=n}(1 + r_t) - \prod_{t=1}^{t=n}(1 + b_{St})}_{\text{Stock selection}} + \underbrace{\prod_{t=1}^{t=n}(1 + b_{St}) - \prod_{t=1}^{t=n}(1 + b_t)}_{\text{Asset allocation}} \tag{6.176}$$

Multi-period geometric attribution

Multi-period geometric attribution does not suffer the same linking challenges as multi-period arithmetic attribution. Chapter 4 demonstrated that geometric excess returns compound over time. Geometric attribution effects also compound to provide the single-period excess return, therefore substituting Equation 6.32 into Equation 4.32:

$$\prod_{t=1}^{t=n}(1+S_t^G)\times\prod_{t=1}^{t=n}(1+A_t^G)-1=g \tag{6.177}$$

where: S_t^G = the total geometric attribution to stock selection in period t
A_t^G = the total geometric attribution to asset allocation in period t

Table 6.65 summarises the geometric attribution effects for all quarters.

Table 6.65 Geometric attribution effects for all quarters

	Asset allocation (%)	Stock selection (%)
First quarter		
UK equities	0.00	3.80
Japanese equities	−0.98	−0.29
US equities	−0.15	−0.57
Total	**−1.13**	**2.95**
Second quarter		
UK equities	−0.75	1.48
Japanese equities	−0.90	−0.21
US equities	1.13	0.53
Total	**−0.52**	**1.79**
Third quarter		
UK equities	2.86	1.65
Japanese equities	2.00	1.65
US equities	−0.86	1.10
Total	**4.00**	**4.40**
Fourth quarter		
UK equities	−0.29	1.49
Japanese equities	−0.69	−0.99
US equities	0.0	2.97
Total	**−0.98**	**3.47**
	Asset allocation	**Stock selection**
Total	**1.29**	**13.19**

Unlike for multi-period arithmetic attribution, there is no need to make any continual adjustments as the total period of measurement extends. The total attribution effects compound as demonstrated in Exhibit 6.107.

Exhibit 6.107 Multi-period geometric attribution effects

Stock selection	$1.0295 \times 1.0179 \times 1.044 \times 1.0347 - 1 = 13.19\%$
Asset allocation	$0.9887 \times 0.9948 \times 1.04 \times 0.9902 - 1 = 1.29\%$
Geometric excess return	$\frac{1.0386}{0.9059} - 1 = 1.1319 \times 1.0129 - 1 = 14.64\%$

In fact, the geometric total effects are the geometric equivalent of the Davies and Laker arithmetic effects as shown in Exhibit 6.108.

Exhibit 6.108 Geometric total effects

$$r = \prod_{t=1}^{t=n}(1 + r_t) - 1 = 3.86\%$$

$$b = \prod_{t=1}^{t=n}(1 + b_t) - 1 = -9.41\%\%$$

$$b_S = \prod_{t=1}^{t=n}(1 + b_{S,t}) - 1 = -8.24\%$$

$$\text{Excess return } \frac{1.0386}{0.9059} - 1 = 14.64\%$$

$$\text{Stock selection } \frac{1.0386}{0.9176} - 1 = 13.19\%$$

$$\text{Asset allocation } \frac{0.9176}{0.9059} - 1 = 1.29\%$$

$$\underbrace{14.64\%}_{\substack{\text{Geometric}\\ \text{excess}\\ \text{return}}} = \underbrace{1.1319\%}_{\substack{\text{Stock}\\ \text{selection}}} \times \underbrace{1.0129\%}_{\substack{\text{Asset}\\ \text{allocation}}} - 1$$

The total geometric attribution effects for each period need not be adjusted. Within each period the attribution effects for each category sum to the total geometric effect; therefore,

the unadjusted categories cannot be compounded in the same way as the totals. The individual category effects can be adjusted if desired so that they compound to the total effect by Equation 6.178.

$$\widehat{S}_i = (1 + S_i) \times \left(\frac{1 + S}{\prod_{i=1}^{i=n} (1 + S_i)} \right)^{\left(\frac{|S_I|}{\sum |S_i|} \right)} - 1 \tag{6.178}$$

where: $\widehat{S}_i$ = adjusted geometric effect for category i

Adjusted geometric effects for the first quarter are calculated for stock selection in Exhibit 6.109. The adjustments are quite small and need not be changed if the period of measurement is extended.

Exhibit 6.109 Adjusted stock selection

$$\prod_{i=1}^{i=n} (1 + S_i) = 1.038 \times 0.9971 \times 0.9943 = 1.0291$$

$$\sum |S_i| = 3.8\% + 0.29\% + 0.53\% = 4.62\%$$

UK equities $\quad 1.038 \times \left(\frac{1.0295}{1.02916} \right)^{\frac{3.8\%}{4.62\%}} - 1 = 3.83\%$

Japanese equities $\quad 0.9971 \times \left(\frac{1.0295}{1.02916} \right)^{\frac{0.29\%}{4.62\%}} - 1 = -0.28\%$

US equities $\quad 0.9943 \times \left(\frac{1.0295}{1.02916} \right)^{\frac{0.57\%}{4.62\%}} - 1 = -0.57\%$

$$\prod_{i=1}^{i=n} (1 + \widehat{S}_i) = 1.0383 \times 0.9972 \times 0.9943 = 1.0295$$

Note

For most scenarios the choice of linking method will not normally change the interpretation of results. I favour geometric excess returns and therefore I'm more comfortable with the geometric linking. For arithmetic excess returns I prefer the GRAP or Frongello method.

Annualisation of excess return

Just like portfolio returns and benchmark returns, excess returns can (and should) be annualised. Geometric excess returns lend themselves to the natural compounding process of multi-period returns; arithmetic exercise returns do not. Extending Exhibit 3.22 by adding benchmark data introduces the excess returns shown in Exhibit 6.110.

Exhibit 6.110 Annualised excess return

	Portfolio (%)	Benchmark (%)	Arithmetic excess (%)	Geometric excess (%)
2003	10.5	8.6	+1.9	+1.75
2002	−5.6	−8.1	+2.5	+2.72
2001	23.4	18.7	+4.7	+3.96
2000	−15.7	−13.2	−2.5	−2.88
1999	8.9	12.5	−3.6	−3.20
Cumulative	18.2	15.70	+2.5	+2.15
Average	4.3	3.70	+0.60	n/a
Annualised	3.4	2.96	+0.44	+0.43

There are three potential methods for calculating an annual excess return:

(1) The average of the cumulative arithmetic excess return:

$$\frac{2.5\%}{5} = 0.5\%$$

(2) The arithmetic difference of the average annual returns:

$$4.3\% - 3.7\% = 0.6\%$$

(3) The arithmetic difference of the annualised annual returns:

$$3.4\% - 2.96\% = 0.44\%$$

In my view both methods (1) and (2) are incorrect; if an annualised arithmetic excess return must be calculated, use method (3).

Geometrically all three methods give the same answer, another reason for using geometric excess returns.

(1) The geometric average of the cumulative geometric excess return:

$$(1.0216)^{\frac{1}{5}} - 1 = 0.43\%$$

(2) The geometric average of the geometric excess returns:

$$(1.0175 \times 1.0272 \times 1.0396 \times 0.9712 \times 0.968.)^{\frac{1}{5}} - 1 = 0.43\%$$

(3) The geometric difference of the annualised returns:

$$\frac{1.034}{1.0296} - 1 = 0.43\%$$

Attribution annualisation

Expanding Equation 6.177 to include currency using Equation 6.118:

$$\prod_{t=1}^{t=n}(1 + S_t^G) \times \prod_{t=1}^{t=n}(1 + A_t^G) \times \prod_{t=1}^{t=n}(1 + C_t^G) - 1 = g \tag{6.179}$$

where: C_t^G = the total geometric attribution to currency in period t

It therefore follows that the annualised contribution to stock selection, asset allocation and currency in turn compound to produce the annualised geometric excess return; see Exhibit 6.111. Geometric excess returns are internally consistent and can be annualised for

Exhibit 6.111 Annualised contribution to excess return

	Currency (%)	Portfolio (%)	Benchmark (%)	Geometric excess (%)	Stock selection (%)	Asset allocation (%)
2003	10.5	8.6	+1.75	+2.3	+1.8	−2.30
2002	−5.6	−8.1	+2.72	+1.1	+0.5	+1.10
2001	23.4	18.7	+3.96	+0.7	+1.1	+2.11
2000	−15.7	−13.2	−2.88	−2.5	−0.9	+0.51
1999	8.9	12.5	−3.20	−2.7	+1.1	−1.60
Cumulative	18.17	15.70	+2.15	−1.20	+3.63	−0.24
Annualised	3.4	2.96	+0.43	−0.24	+0.72	−0.05

Stock selection $(1.023 \times 1.011 \times 1.007 \times 0.975 \times 0.973)^{\frac{1}{5}} - 1 = -0.24\%$

Asset allocation $(1.018 \times 1.005 \times 1.011 \times 0.991 \times 1.011)^{\frac{1}{5}} - 1 = +0.72\%$

Currency $(0.977 \times 1.011 \times 1.0211 \times 1.0051 \times 0.984)^{\frac{1}{5}} - 1 = -0.05\%$

Note: $\left(\underbrace{0.988}_{\text{Stock}} \times \underbrace{1.0363}_{\text{Asset}} \times \underbrace{0.9976}_{\text{Currency}}\right) - 1 = +2.15\%$ (Cumulative)

And $\left(\underbrace{0.9976}_{\text{Stock}} \times \underbrace{1.0072}_{\text{Asset}} \times \underbrace{0.9995}_{\text{Currency}}\right) - 1 = +0.43\%$ (Annualised)

multiple periods. The logic of systematically distributing residuals in the form of Carino and Menchero becomes less appealing and geometric compounding more appealing the longer the period under analysis.

CONTRIBUTION ANALYSIS (OR ABSOLUTE RETURN ATTRIBUTION)

Portfolios without benchmarks or absolute return strategies do not readily fit into the definition of performance attribution since there is no benchmark to attribute against. Contribution analysis simply breaks down the total return of the portfolio by individual instrument or instrument types to provide some information of the sources of return in the portfolio.

Even though there is a zero or cash return benchmark, standard attribution analysis may still be applied. The portfolio manager may be employing a clear strategy to add value to an absolute return objective – that strategy may be converted to customised benchmark and hence attribution can be calculated. Market neutral-type strategies are a good example of this approach.

RISK-ADJUSTED ATTRIBUTION

Brinson, Hood and Beebower,[45] in a follow-up article to their original, suggested that systematic risk measures such as beta or duration could be used in conjunction with their standard model. This is only appropriate if the portfolio manager is using systematic risk in the investment decision process. For example, an equity portfolio manager could choose to increase the beta of a category rather than or as well as going overweight to achieve an asset allocation effect. We can measure the impact of this decision in the standard Brinson model using the CAPM regression equation to calculate systematic risk-adjusted returns.

$$\text{CAPM regression equation } r_p - r_F = \alpha + \beta \times (b - r_F) + \varepsilon \tag{5.40}$$

Rearranging Equation 5.40 and ignoring the error term we can break down the return into selectivity or Jensen's alpha and the return derived from systematic risk:

$$r_p = \underbrace{\alpha}_{\text{Selectivity}} = \underbrace{r_F + \beta \times (b - r_F)}_{\text{Systematic risk}} \tag{6.180}$$

We can define an intermediate benchmark return adjusted for systematic risk b'_i:

$$b'_i = x_i + \beta_i \times (b_i - x_i) \tag{6.181}$$

where: x_i = risk-free rate in country i
b_i = benchmark return in country i
β_i = systematic risk in country i

To identify the added value from asset allocation due to systematic risk we must calculate the return of an additional intermediate fund.

In this notional fund adjusted for systematic risk the asset allocation weights of the actual fund are applied to the risk-adjusted benchmark returns within each category.

$$\text{Systematic risk notional fund } b'_S = \sum_{i=1}^{i=n} w_i \times b'_i \tag{6.182}$$

The total excess return is now the combination of three wealth ratios:

$$\frac{1+r}{1+b} - 1 = \underbrace{\frac{1+r}{1+b'_S}}_{\text{Selectivity}} \times \underbrace{\frac{1+b'_S}{1+b_S}}_{\text{Systematic risk}} \times \underbrace{\frac{1+b_S}{1+b}}_{\text{Asset allocation}} - 1 \tag{6.183}$$

[45] Brinson, Singer and Beebower, "Determinants of Portfolio Performance II: An Update" (1991).

Selectivity

To identify the total contribution to selectivity we can use the ratio of the portfolio return compared to the systematic risk adjusted notional return:

$$\frac{(1+r)}{(1+b'_S)} - 1 \tag{6.184}$$

The contribution to selectivity in the *i*th category is now:

$$S'_i = w_i \times \left(\frac{1+r_i}{1+b'_i} - 1 \right) \times \frac{(1+b'_i)}{(1+b'_S)} \tag{6.185}$$

To calculate the contribution from systematic risk we can use the ratio of the systematic risk notional fund with the semi-notional fund:

$$\frac{(1+b'_S)}{(1+b_S)} - 1 \tag{6.186}$$

The contribution from systematic risk in the *i*th category is now:

$$R_i = w_i \times \left(\frac{1+b'_i}{1+b_i} - 1 \right) \times \frac{(1+b_i)}{(1+b_S)} \tag{6.187}$$

The asset allocation term is unchanged.

Our standard data example from Table 6.3 is extended to include the risk-free rate for each market and the beta in Table 6.66. We must make the assumption that the portfolio manager is using beta as part of the asset allocation process.

Table 6.66 Standard data example with risk-free rate and beta

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Risk-free rate (%)	β
UK equities	40	40	20	10	1.0	1.3
Japanese equities	30	20	−5	−4	0.1	1.0
US equities	30	40	6	8	0.2	0.8
Total	**100**	**100**	**8.3**	**6.4**		

Using the regression equation we can calculate risk-adjusted benchmark returns and therefore calculate the attribution effects for selectivity and systematic risk as shown in Exhibit 6.112, 6.113 and 6.114, respectively.

Exhibit 6.112 Return adjusted for systematic risk

Revised benchmark returns adjusted for systematic risk

UK equities	$x_i + \beta_i \times (b_i - x_i) = 1.0\% + 1.3 \times (10.0\% - 1.0\%) = 12.7\%$
Japanese equities	$0.1\% + 1.0 \times (-4.0\% - 0.1\%) = -4.0\%$
US equities	$0.2\% + 0.8 \times (8.0\% - 0.2\%) = 6.44\%$

Systematic risk notional fund

$b'_S = 40\% \times 12.7\% + 30\% \times -4\% + 30\% \times 6.44\% = 5.81\%$

Exhibit 6.113 Selectivity

$$\frac{(1+r)}{(1+b'_S)} - 1 = \frac{1.083}{1.0581} - 1 = 2.35\%$$

Individual country selectivity effects are:

UK equities	$40\% \times \left(\frac{1.20}{1.127} - 1\right) \times \frac{1.127}{1.0581} = 2.76\%$
Japanese equities	$30\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.96}{1.0581} = -0.28\%$
US equities	$30\% \times \left(\frac{1.06}{1.0644} - 1\right) \times \frac{1.0644}{1.0581} = -0.12\%$
Total	$2.76\% - 0.28\% - 0.12\% = 2.35\%$

Exhibit 6.114 Systematic risk

$$\frac{(1+b'_S)}{(1+b_S)} - 1 = \frac{1.0581}{1.052} - 1 = 0.58\%$$

Individual country selectivity effects are:

UK equities	$40\% \times \left(\frac{1.127}{1.10} - 1\right) \times \frac{1.10}{1.0581} = 1.03\%$
Japanese equities	$30\% \times \left(\frac{0.96}{0.96} - 1\right) \times \frac{0.96}{1.0581} = 0.0\%$
US equities	$30\% \times \left(\frac{1.0644}{1.06} - 1\right) \times \frac{1.06}{1.0581} = -0.44\%$

Asset allocation effects are unchanged from before. The revised risk-adjusted attribution summary is shown in Table 6.67. The systematic risk asset allocation and asset allocation effects can be compounded to calculate the overall asset allocation effect. Note there is now a contribution to UK equity allocation of 1.03% due to the high beta of greater than 1 in a

Table 6.67 Risk-adjusted attribution

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Systematic risk allocation (%)	Selectivity (%)
UK equities	40	40	20	10	0.0	1.03	2.76
Japanese equities	30	20	−5	−4	−0.98	0.0	−0.28
US equities	30	40	6	8	−0.15	−0.44	−0.12
Total	**100**	**100**	**8.3**	**6.4**	**−1.13**	**0.58**	**2.35**

rising market. In effect, part of the 20.0% return is due to high systematic risk and is therefore not entirely stock selection.

Since portfolios are rarely managed in this manner risk-adjusted attribution for equities is seldom used.

MULTI-LEVEL ATTRIBUTION

Figure 6.8 presented a typical three-step, top-down attribution process using the Brinson model:

Step 1. Benchmark
Step 2. Asset allocation (represented by the semi-notional fund)
Step 3. Stock selection

Figure 6.12 presents a four-step process:

Step 1. Benchmark
Step 2. Duration
Step 3. Allocation (market-adjusted for duration and risk-free rate)
Step 4. Selection

For more complex multi-level decision processes, provided we can identify each step and, importantly, the order of the decision process, we can still employ the Brinson model.

Consider the balanced (equity and fixed income) portfolio in Table 6.68.

For this portfolio there are five clear steps in the decision process, illustrated in Figure 6.18. Each intermediate step is represented by a semi-notional fund and the wealth ratio between each successive step represents the impact of that isolated decision:

$$\frac{1+{}^{1}b_S}{1+b} \times \frac{1+{}^{2}b_S}{1+{}^{1}b_S} \times \frac{1+{}^{3}b_S}{1+{}^{2}b_S} \times \frac{1+r}{1+{}^{3}b_S} = \frac{1+r}{1+b} \tag{6.188}$$

The semi-notional fund for each step d is calculated as follows using benchmark returns appropriate to the level of the decision process:

$${}^{d}b_S = \sum_{i=1}^{i=n} {}^{d}w_i \times {}^{d}b_i \tag{6.189}$$

Table 6.68 Multi-level attribution

		Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)
Level 1	Equities	58.5	55.0	8.49	8.95
Level 2	US	19.0	15.0	5.36	7.50
Level 3	Financial	*10.0*	*5.0*	8.38	9.60
Level 3	Software	*9.0*	*10.0*	2.00	6.45
Level 2	Europe	39.5	40	5.36	7.50
Level 3	Autos	*22.5*	*25.0*	16.05	15.60
Level 3	Chemicals	*17.0*	*15.0*	2.00	−0.67
Level 1	Bonds	**41.5**	**45.0**	**2.35**	**1.78**
Level 2	Govt.	12.5	10.0	2.00	1.00
Level 2	Corp.	29.0	35.0	2.50	2.00
Total		**100.0**	**100.0**	**5.94**	**5.72**

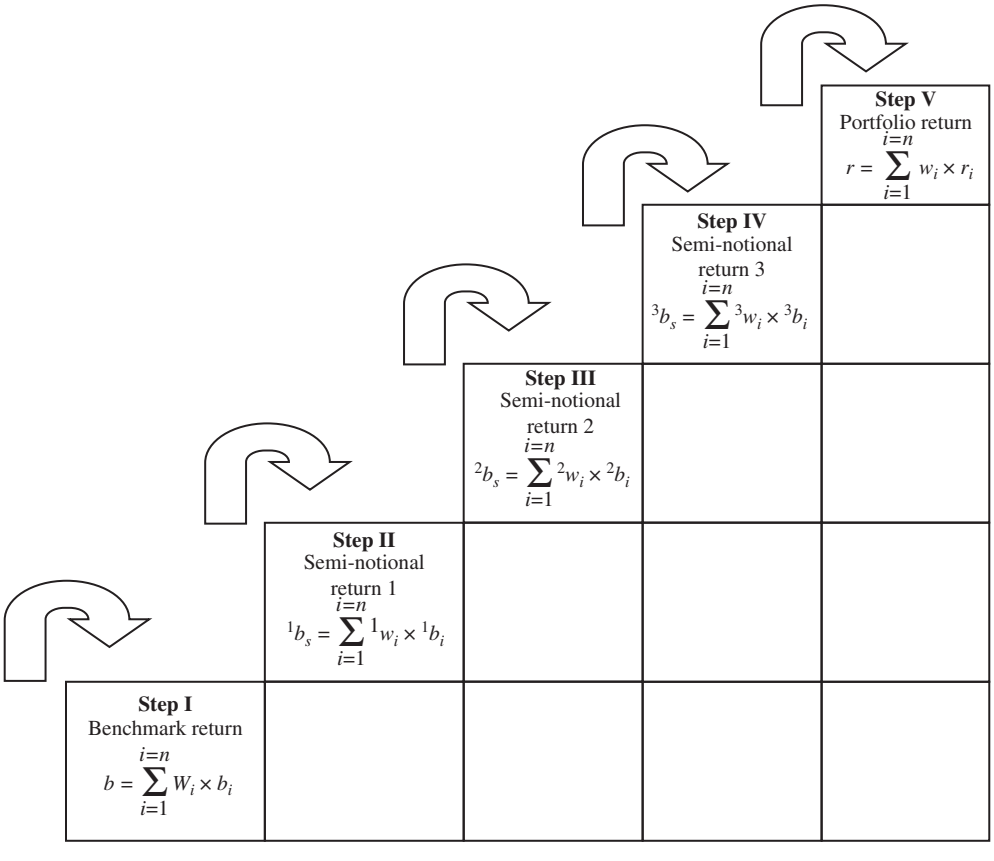

Figure 6.18 Multi-level attribution

The five-step investment decision process is as follows:

Step 1. Benchmark
Step 2. Semi-notional return 1 (representing the allocation between equities and bonds)
Step 3. Semi-notional return 2 (representing the allocation between equity countries and bond sectors)
Step 4. Semi-notional return 3 (representing the allocation between equity industrial sectors – note there is no decision process for bonds at this level in this particular process)
Step 5. The final step as always representing stock selection

The contribution to geometric asset allocation in the ith category for the dth level in the decision process is:

$$({}^{d}w_i - {}^{d}W_i) \times \left(\frac{1 + {}^{d}b_i}{1 + {}^{d-1}b_i} - 1 \right) \times \frac{1 + {}^{d-1}b_i}{1 + {}^{d-1}b_S} \tag{6.190}$$

${}^{d-1}b_S$ must be the subindex in which the category is present. At each asset allocation level, the effect is adjusted for the impact of decisions already made. Note that ${}^{0}b_S = b$, the original benchmark. For the first step, since there are no prior effects, asset allocation is:

$$({}^{1}w_i - {}^{1}W_i) \times \left(\frac{1 + {}^{1}b_i}{1 + b} - 1 \right) \tag{6.191}$$

The total attribution for each asset allocation step in the decision process is:

$$\frac{1 + {}^{d}b_S}{1 + {}^{d-1}b_S} - 1 \tag{6.192}$$

The final step, stock selection, is measured by:

$${}^{d}w_i \times \left(\frac{1 + r_i}{1 + {}^{d}b_i} - 1 \right) \times \frac{1 + {}^{d}b_i}{1 + {}^{d}b_S} \tag{6.193}$$

Portfolio, benchmark and semi-notional returns for each level of data shown in Table 6.68 are calculated in Exhibit 6.115.

Exhibit 6.115 Calculation of multi-level portfolio benchmark and semi-notional returns

Using Table 6.68 the portfolio return is:

$$r = 10\% \times 8.38\% + 9.0\% \times 2.0\% + 22.5\% \times 16.05\% + 17\% \times 2.0\%$$

$$12.5\% \times 2.0\% + 29\% \times 2.5\% = 5.94\%$$

Or

$$r = 58.5\% \times 8.49\% + 41.5\% \times 2.35\% = 5.94\%$$

Note that each subsector is the sum of its parts.

Exhibit 6.115 (*Continued*)

The benchmark return is:

$$b = 5\% \times 9.6\% + 10\% \times 6.45\% + 25\% \times 15.6\% + 15\% \times -0.67\%$$
$$10\% \times 1.0\% + 35\% \times 2.0\% = 5.72\%$$

Or

$$b = 55\% \times 8.95\% + 45\% \times 1.78\% = 5.72\%$$

The semi-notional return 1 is:

$${}^{1}b_S = 58.5\% \times 8.95\% + 41.5\% \times 1.78\% = 5.98\%$$

The semi-notional return 2 is:

$${}^{2}b_S = 19\% \times 7.50\% + 39.5\% \times 9.50\% + 12.5\% \times 1.0\% + 29\% \times 2.0\% = 5.88\%$$

The semi-notional return 3 is:

$${}^{3}b_S = 10\% \times 9.60\% + 9\% \times 6.45\% + 22.5\% \times 15.6\% + 17\% \times -0.67\%$$
$$+12.5\% \times 1.0\% + 29.0\% \times 2.0\% = 5.64\%$$

Top-level attribution effects for each step in the decision process are calculated in Exhibit 6.116.

Exhibit 6.116 Multi-level attribution effects

$$\frac{1+{}^{1}b_S}{1+b} \times \frac{1+{}^{2}b_S}{1+{}^{1}b_S} \times \frac{1+{}^{3}b_S}{1+{}^{2}b_S} \times \frac{1+r}{1+{}^{3}b_S} - 1 = \frac{1+r}{1+b} - 1$$

Level 1 allocation (step 2)	$\frac{1+{}^{1}b_S}{1+b} - 1 = \frac{1.0598}{1.0572} - 1 = 0.24\%$
Level 2 allocation (step 3)	$\frac{1+{}^{2}b_S}{1+{}^{1}b_S} - 1 = \frac{1.0588}{1.0598} - 1 = -0.09\%$
Level 3 allocation (step 4)	$\frac{1+{}^{3}b_S}{1+{}^{2}b_S} - 1 = \frac{1.0564}{1.0588} - 1 = -0.23\%$
Stock selection (step 5)	$\frac{1+r}{1+{}^{3}b_S} - 1 = \frac{1.0594}{1.0564} - 1 = 0.28\%$
Total excess return	$\frac{1+r}{1+b} - 1 = \frac{1.0594}{1.0572} - 1 = 0.21\%$

$$\frac{1.0598}{1.0572} \times \frac{1.0588}{1.0598} \times \frac{1.0564}{1.0588} \times \frac{1.0594}{1.0564} - 1 = \frac{1.0594}{1.0572} - 1$$

Individual level 1 allocation effects are calculated in Exhibit 6.117, level 2 allocation effects in Exhibit 6.118, level 3 allocation effects in Exhibit 6.119 and security selection in Exhibit 6.120.

Exhibit 6.117 Level 1 multi-level attribution effects

$$\frac{1+{}^1b_S}{1+b} - 1 = \frac{1.0598}{1.0572} - 1 = 0.24\%$$

Equities $\quad ({}^1w_i - {}^1W_i) \times \left(\frac{1+{}^1b_i}{1+b} - 1\right) = (58.5\% - 55\%) \times \left(\frac{1.0895}{1.0572} - 1\right) = 0$

Bonds $\quad = (41.5\% - 45\%) \times \left(\frac{1.0178}{1.0572} - 1\right) = 0.13\%$

Total $\quad = 0.11\% + 0.13\% = 0.24\%$

Exhibit 6.118 Level 2 multi-level attribution effects

$$\frac{1+{}^2b_S}{1+{}^1b_S} - 1 = \frac{1.0588}{1.0598} - 1 = -0.09\%$$

Individual level 2 attribution effects:

$$= ({}^2w_i - {}^2W_i) \times \left(\frac{1+{}^2b_i}{1+{}^1b_i} - 1\right) \times \frac{1+{}^1b_S}{1+{}^1b_S}$$

US equities $\quad = (19\% - 15\%) \times \left(\frac{1.075}{1.0895} - 1\right) \times \frac{1.0895}{1.0598} = -0.05\%$

European equities $\quad = (39.5\% - 40\%) \times \left(\frac{1.095}{1.0895} - 1\right) \times \frac{1.0895}{1.0598} = -0.00\%$

Government bonds $\quad = (12.5\% - 10\%) \times \left(\frac{1.01}{1.0178} - 1\right) \times \frac{1.0178}{1.0598} = -0.02\%$

Corporate bonds $\quad = (29.0\% - 35\%) \times \left(\frac{1.02}{1.0178} - 1\right) \times \frac{1.0178}{1.0598} = -0.01\%$

Total $\quad = -0.05\% + 0.0\% - 0.02\% - 0.01\% = -0.09\%$

Exhibit 6.119 Level 3 multi-level attribution effects

$$\frac{1+{}^3b_S}{1+{}^2b_S} - 1 = \frac{1.0564}{1.0588} - 1 = -0.23\%$$

Individual level 3 attribution effects:

$$= ({}^3w_i - {}^3W_i) \times \left(\frac{1+{}^3b_i}{1+{}^2b_i} - 1\right) \times \frac{1+{}^2b_i}{1+{}^2b_S}$$

Financials $\quad = (10\% - 5\%) \times \left(\frac{1.096}{1.075} - 1\right) \times \frac{1.075}{1.0588} = 0.10\%$

Software $\quad = (9\% - 10\%) \times \left(\frac{1.0645}{1.075} - 1\right) \times \frac{1.075}{1.0588} = 0.01\%$

Automobiles $\quad = (22.5\% - 25\%) \times \left(\frac{1.156}{1.095} - 1\right) \times \frac{1.095}{1.0588} = -0.14\%$

Chemicals $\quad = (17.0\% - 15\%) \times \left(\frac{0.9933}{1.095} - 1\right) \times \frac{1.095}{1.0588} = -0.19\%$

Total $\quad = 0.1\% + 0.01\% - 0.14\% - 0.19\% = -0.23\%$

Exhibit 6.120 Security/issue selection multi-level attribution effects

$$\frac{1+r}{1+{}^3b_S} - 1 = \frac{1.0594}{1.0564} - 1 = 0.28\%$$

Security/issue selection effects:

$${}^3w_i \times \left(\frac{1+r_i}{1+{}^3b_i} - 1\right) \times \frac{1+{}^3b_i}{1+{}^3b_S}$$

Financials $= 10\% \times \left(\frac{1.0838}{1.096} - 1\right) \times \frac{1.096}{1.0564} = -0.12\%$

Software $= 9\% \times \left(\frac{1.02}{1.0645} - 1\right) \times \frac{1.0645}{1.0564} = -0.38\%$

Automobiles $= 22.5\% \times \left(\frac{1.1605}{1.156} - 1\right) \times \frac{1.156}{1.0564} = 0.09\%$

Chemicals $= 17\% \times \left(\frac{1.02}{0.9933} - 1\right) \times \frac{0.9933}{1.0564} = 0.43\%$

Government bonds $= 12.5\% \times \left(\frac{1.02}{1.01} - 1\right) \times \frac{1.01}{1.0564} = 0.12\%$

Corporate bonds $= 29\% \times \left(\frac{1.025}{1.02} - 1\right) \times \frac{1.02}{1.0564} = 0.14\%$

Total $= -0.12\% - 0.38\% + 0.09\% + 0.43\% + 0.12\% + 0.14\% = 0.28\%$

Attribution effects are summarised in Table 6.69. Interpretation of the attribution effects is fairly straightforward. The first allocation decision overweighting equities and underweighting bonds is positive, adding 0.24% of value. The level 2 decisions, overweighting US equities and government bonds and underweighting corporates, lost 0.09%. The level 3 decisions, underweighting automobiles and overweighting chemicals, only partially offset by overweighting financials, lost 0.23%.

Table 6.69 Multi-level attribution effects

		Level 1 (%)	Level 2 (%)	Level 3 (%)	Selection (%)
Level 1	Equities	**0.11**	**−0.06**	**−0.23**	**0.03**
Level 2	US		−0.05	0.11	−0.50
Level 3	Financial			0.10	−0.12
Level 3	Software			0.01	−0.38
Level 2	Europe		−0.00	−0.34	0.52
Level 3	Autos			−0.14	0.09
Level 3	Chemicals			−0.19	0.43
Level 1	Bonds	**0.13**	**−0.03**		**0.26**
Level 2	Govt.		−0.02		0.12
Level 2	Corp.		−0.01		0.14
Total		**0.24**	**−0.09**	**−0.23**	**0.28**

Positive selection effects in automobiles, chemicals, government and corporate bonds outweighed negative selection effects in financials and software to add 0.28% of geometric excess return. Overall, the portfolio outperformed by 0.21% with positive contributions from level 1 allocation and selection and negative contribution from both level 2 and level 3 decisions.

Balanced attribution

A balanced investment strategy aims to balance risk and return by allocating liquid asset types such as equities, bonds and cash and illiquid assets such as real estate, private assets and infrastructure. Balanced portfolios might consist of just equities and bonds but with longer time horizons in mind, might in addition include other "alternative assets",[46] exploiting illiquid risk premiums to create broader multi-asset portfolios.

As previously discussed, money-weighted methodologies are typically used to measure the return of illiquid assets and time-weighted methodologies are used to measure the return of liquid assets. To facilitate accurate attribution analysis, without residuals, a consistent return methodology must be used. In most cases, the most appropriate consistent methodology will be time-weighted because:

(1) The majority of assets in balanced portfolios are liquid.
(2) Time-weighted returns can be attributed.
(3) The impact of timing is naturally captured in attribution effects.

Because internal rates of return assume a single constant force of return throughout the period of measurement, this return methodology is not suitable for attribution, although Illmer and Marty[47] and Illmer[48] offer potential solutions. Attribution using a modified Dietz method (a good approximation to the internal rate of return) is easy to disaggregate and in fact is suggested as a single-period solution for private equity by O'Shea and Jeet.[49]

⚠ Caution

Modified Dietz is less useful as an approximation to internal rate of return over longer time periods.

Benchmark construction for multi-asset portfolios can be difficult. Identifying benchmarks for illiquid assets is difficult, resulting in less transparency and little visibility of underlying risk exposures. Security-level analysis may truly only be able for liquid asset classes.

Balanced and multi-level attribution are naturally closely associated. The balanced attribution approach must reflect the investment decision process, and the wide range of asset classes and order of decision lead to multiple approaches. Figures 6.19 to 6.25 illustrate a number of different approaches. Figure 6.19 demonstrates a four-level, five-step decision process. As always, the benchmark is the first step, and the second step is a level 1 allocation decision between liquid and illiquid assets. The level 2 decision allocates the liquid assets between equities and bonds and the illiquid assets between real estate, private assets and infrastructure. The level 3 decision allocates by region or country. The level 4 decision is at security,

[46] Hence the label "alternative assets", including real estate, private assets (equity and debt), infrastructure, and hedge funds.
[47] Illmer and Marty, "Decomposing the Money-weighted Rate of Return" (2003).
[48] Illmer, "Decomposing the Money-weighted Rate of Return" (2009).
[49] O'Shea and Jeet, "Single-Period Brinson-Style Performance Attribution for Private Capital" (2018).

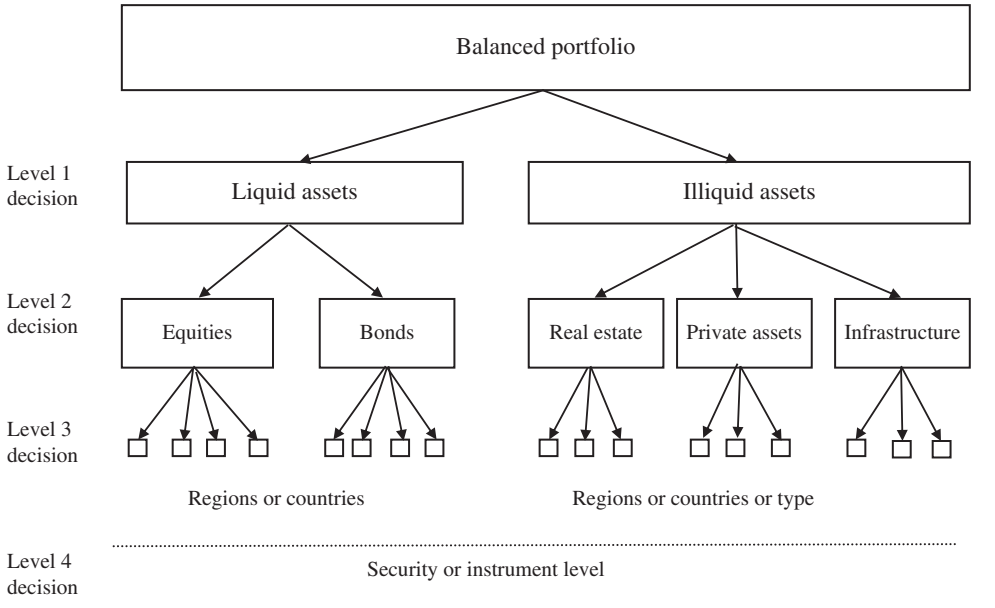

Figure 6.19 Four-level, five-step decision process

instrument or individual asset level. Attribution effects should be calculated at each level of the decision process. Attribution effects at each level may be calculated in the context of the whole portfolio or "drilled down" in the context of each decision level depending on the preference of the end user.

Figure 6.20 is an example of a three-level, four-step process. Step 1 is of course the strategic allocation of the asset owner embedded in the strategic benchmark. Step 2, or the level 1 decision, is the allocation across the whole range of assets, equities, bonds, real estate, private assets and infrastructure. Cash might be a separate asset class but is typically included in the allocation of each asset category. Currency is typically not regarded as a separate level in the decision process but if managed independently might be treated as a separate internal

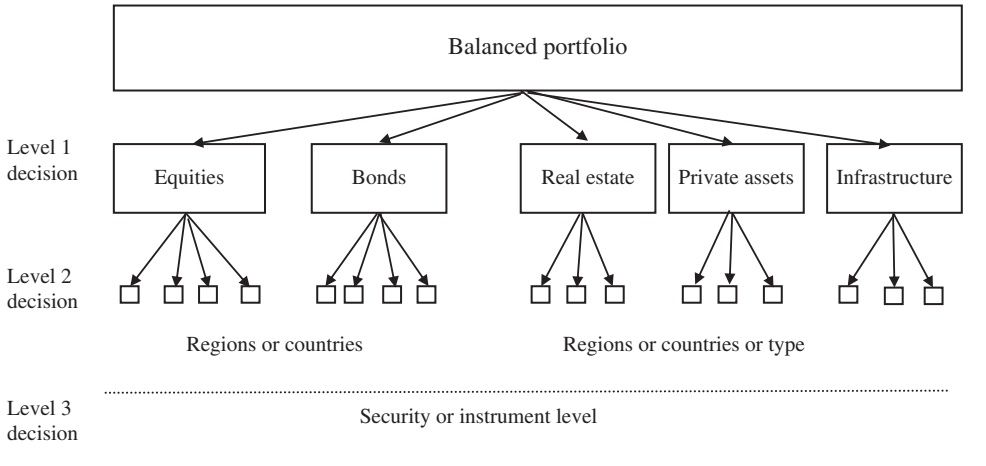

Figure 6.20 Three-level, four-step decision process

or external overlay strategy. Any asset might generate currency exposure; the independent currency overlay manager will use a currency derivative to manage currency exposures in line with the investment mandate and strategic currency benchmark. Illiquid assets cause currency issues for the overlay manager. Major changes in the valuation of illiquid assets may cause unexpected currency exposures, of which the currency overlay manager is unaware, resulting in unmanaged currency effects. The level 2 decision allocates by region or country.

The level 3 decision is at security, instrument or individual asset level.

Figure 6.21 represents a quite different two-level, three-step decision process, in which the level 1 allocation decision is a country decision and the second-level decision is within countries from the bottom up. This approach is very similar to the standard approach described in Figure 6.8.

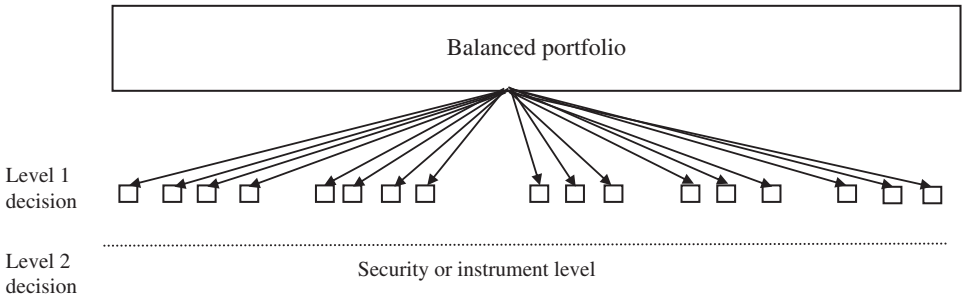

Figure 6.21 Two-level, three-step decision process

The approach in Figure 6.22 is a four-level, five-step regional decision process in which the level 1 decision is regional – North America, Europe, Asia, emerging markets, for example. The level 2 decision initiates an investment process similar to the three-level, four-step decision

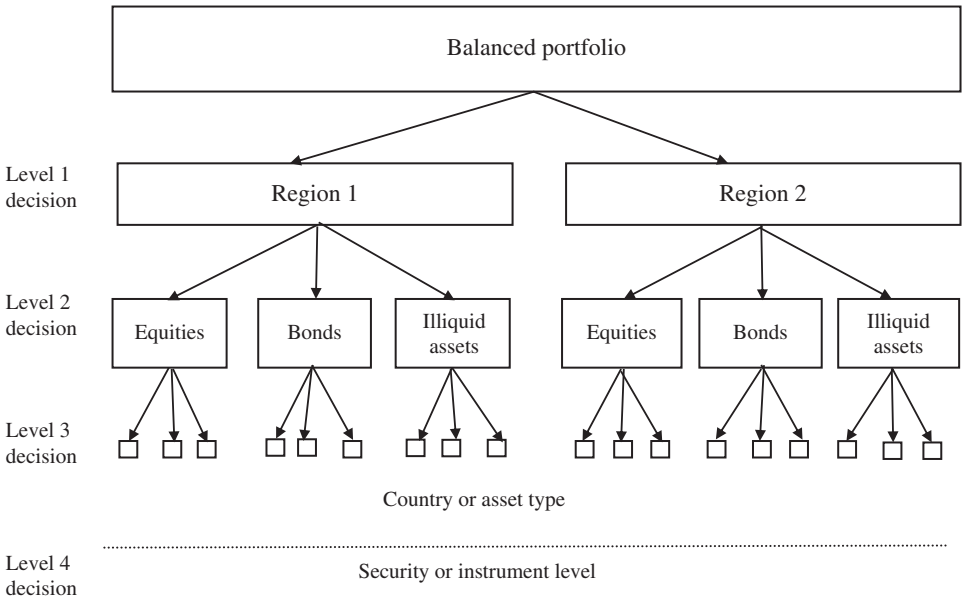

Figure 6.22 Four-level, five-step regional decision process

process described in Figure 6.19 within each region. The level 3 decision allocates by country. The level 3 decision is at security, instrument or individual asset level.

Figure 6.23 shows a three-level, four-step decision process for the liquid assets, bonds and equities only. The level 1 decision represents an allocation between bonds and equities. The level 2 decision allocates by region or country. The level 3 decision is at the security, instrument or individual asset level for equities or perhaps credit risk, positioning on the yield curve and other factors for fixed income. For balanced portfolios consisting of liquid assets only, the investment process for each asset type may be completely different, requiring substantially different attribution approaches that may be difficult to combine and that are often required at different levels of analysis. For certain investment decision processes, it might be appropriate to utilise a weighted duration methodology for the fixed income portion of the portfolio. Because the weighted duration approached is a duration-adjusted version of the Brinson model it works well in a balanced context. A weighted duration balanced attribution approach is demonstrated in Exhibits 6.121 to 6.126 using the data in Tables 6.70 and 6.71. In a weighted duration approach, asset allocation at all levels involving fixed income is a combination of weight and duration – hence fixed income benchmarks must be adjusted to take account of duration as shown in Exhibit 6.121. Portfolio, benchmark, semi-notional returns and semi-notional returns adjusted for duration are calculated in Exhibit 6.122. Multi-level attribution effects are calculated in Exhibit 6.123. Level 1 attribution effects are calculated in Exhibit 6.124; an overweight equities position lost 0.21% and an underweight fixed income position lost 9 basis points. The shorter-duration position in fixed income in effect compounded the underweighted physical position and lost a further 0.04%, a combined loss of 0.35% of geometric excess return for level 1 decisions. Level 2 decisions calculated in Exhibit 6.125 also lost value, 0.15% overweighting US equities and 0.13% underweighting European equities. Fixed income effects were largely neutral, but note the use of weighted duration and the separation of carry. Selection effects are calculated in Exhibit 6.126. Both US and European equities underperformed, US bonds outperformed and European bonds underperformed. Two alternative methods of calculating the same selection numbers are shown, using weighted duration and duration-adjusted benchmarks, respectively. Selection effects are adjusted for the duration decision already measured in the level 1 and level 2 decisions. Attribution effects are summarised in Table 6.72.

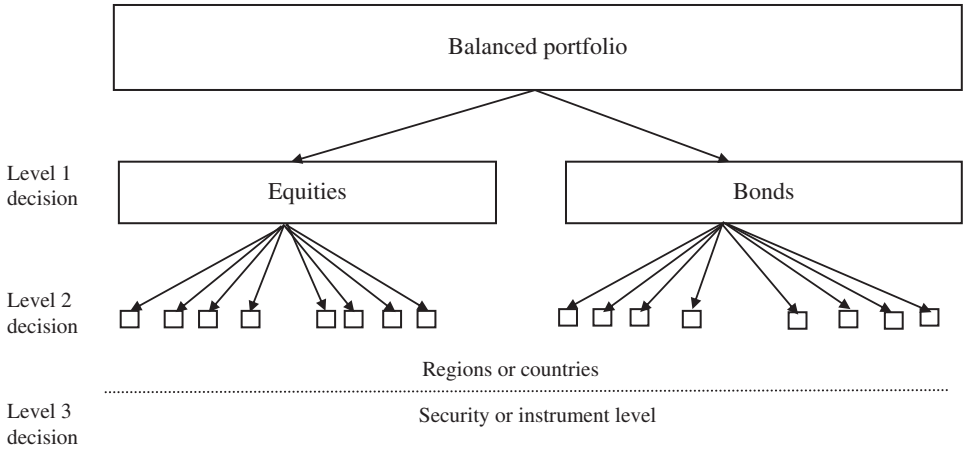

Figure 6.23 Three-level, four-step liquid assets decision process

Table 6.70 Weighted duration balanced attribution

		Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)
Level 1	Equities	**34.8**	**30**	**−6.40**	**−5.05**
Level 2	US	25.5	15	−6.80	−6.20
Level 2	Europe	9.3	15	−5.30	−3.90
Level 1	Bonds	**65.2**	**70**	**1.41**	**1.23**
Level 2	$ denominated	24.7	30	2.4	1.40
Level 2	€ denominated	40.5	40	0.80	1.10
Total		**100**	**100**	**−1.31**	**−0.66**

Table 6.71 Weighted duration balanced attribution – duration and implied yield changes

	Portfolio duration (%)	Benchmark duration (%)	Carry (%)	Implied portfolio yield change (%)	Implied benchmark yield change (%)
$ denominated	4.1	3.5	0.5	−0.46	−0.26
€ denominated	4.0	5.0	0.2	−0.15	−0.18
Total	**4.04**	**4.36**	**0.33**	**−0.27**	**−0.21**

Exhibit 6.121 Benchmarks adjusted for portfolio duration

Using the data in Tables 6.70 and 6.71 and Equation 6.121
The duration-adjusted benchmark adapting Equation 6.121 is:

$$b_{Di} = \frac{D_{ri}}{D_{bi}} \times D_{bi} \times -\Delta y_{bi} + x_i \tag{6.194}$$

$$\$ \text{ denominated} = \frac{4.1}{3.5} \times 3.5 \times 0.26\% + 0.5\% = 1.55\%$$

or

$$\$ \text{ denominated} = \frac{4.1}{3.5} \times (1.4\% - 0.5\%) + 0.5\% = 1.55\%$$

$$€ \text{ denominated} = \frac{4.0}{5.0} \times 5.0 \times 0.18\% + 0.2\% = 0.92\%$$

or

$$€ \text{ denominated} = \frac{4.0}{5.0} \times (1.1\% - 0.2\%) + 0.2\% = 0.92\%$$

$$\text{Bonds overall} = \frac{4.04}{4.36} \times 4.36 \times 0.21\% + 0.33\% = 1.16\%$$

Or

$$\text{Bonds overall} = \frac{4.04}{4.36} \times (1.23\% - 0.33\%) + 0.33\% = 1.16\%$$

Exhibit 6.122 Calculation of portfolio, benchmark and semi-notional returns

Using Table 6.59 the portfolio return is:

$$r = 25.2\% \times -6.8\% + 9.3\% \times -5.3\% + 24.7\% \times 2.4\% + 40.5\% \times 0.8\% = -1.31\%$$

Or

$$r = 34.8\% \times -6.40\% + 65.2\% \times 1.41\% = -1.31\%$$

The benchmark return is:

$$b = 15\% \times -6.2\% + 15\% \times -3.9\% + 30\% \times 1.4\% + 40\% \times 1.1\% = -0.66\%$$

Or

$$b = 30\% \times -5.05\% + 70\% \times 1.23\% = -0.66\%$$

The semi-notional return 1 is:

$${}^{1}b_S = 34.8\% \times -5.05\% + 65.2\% \times 1.23\% = -0.96\%$$

The semi-notional return 1 adjusted for duration is:

$${}^{1}b_{DS} = 34.8\% \times -5.05\% + 65.2\% \times 1.16\% = -1.00\%$$

The semi-notional return 2 is:

$${}^{2}b'_{DS} = 25.5\% \times -6.20\% + 9.3\% \times -3.90\% + 24.7\% \times 1.55\% + 40.5\% \times 0.92\% = -1.19\%$$

Exhibit 6.123 Multi-level balanced duration-adjusted attribution

$$\frac{1+{}^{1}b_S}{1+b} \times \frac{1+{}^{1}b_{DS}}{1+{}^{1}b_S} \times \frac{1+{}^{2}b'_{DS}}{1+{}^{1}b_{DS}} \times \frac{1+r}{1+{}^{2}b'_{DS}} - 1 = \frac{1+r}{1+b} - 1$$

Level 1 allocation (step 2) $\frac{1+{}^{1}b_S}{1+b} - 1 = \frac{0.9904}{0.9934} - 1 = -0.30\%$

Level 1 allocation (including duration) $\frac{1+{}^{1}b_{DS}}{1+{}^{1}b_S} - 1 = \frac{0.99}{0.9904} - 1 = -0.04\%$

Level 2 allocation (step 3) (including carry) $\frac{1+{}^{2}b'_{DS}}{1+{}^{1}b_{DS}} - 1 = \frac{0.9881}{0.99} - 1 = -0.19\%$

Selection (step 4) $\frac{1+r}{1+{}^{2}b'_{DS}} - 1 = \frac{0.9869}{0.9881} - 1 = -0.12\%$

Total excess return $\frac{1+r}{1+b} - 1 = \frac{0.9869}{0.9934} - 1 = -0.66\%$

$$\frac{0.9904}{0.9934} \times \frac{0.99}{0.9904} \times \frac{0.9881}{0.99} \times \frac{0.9869}{0.9881} - 1 = \frac{0.9869}{0.9934} - 1$$

Exhibit 6.124 Level 1 balanced duration-adjusted attribution

$$\frac{1+{}^1b_S}{1+b} - 1 = \frac{0.9904}{0.9934} - 1 = 0.30\%$$

Equities $\quad ({}^1w_i - {}^1W_i) \times \left(\frac{1+{}^1b_i}{1+b} - 1\right) = (34.8\% - 30\%) \times \left(\frac{0.9495}{0.9934} - 1\right) = -0.21\%$

Bonds $\quad = (65.2\% - 70\%) \times \left(\frac{1.0123}{0.9934} - 1\right) = -0.09\%$

Total $\quad = -0.21\% - 0.09\% = -0.30\%$

Adjusting for duration:

$$\frac{1+{}^1b_S}{1+b} \times \frac{1+{}^1b_{DS}}{1+{}^1b_S} \times -1 = \frac{1+{}^1b_{DS}}{1+b} - 1$$

$$\frac{0.9904}{0.9934} \times \frac{0.99}{0.9904} - 1 = \frac{0.99}{0.9934} - 1$$

$$0.997 \times 0.9996 - 1 = -0.35\%$$

Exhibit 6.125 Level II balanced duration-adjusted attribution

$$\frac{1+{}^2b'_{DS}}{1+{}^1b_{DS}} - 1 = \frac{0.9881}{0.99} - 1 = -0.19\%$$

Individual level 2 attribution effects for equities:

$$= ({}^2w_i - {}^2W_i) \times \left(\frac{1+{}^2b_i}{1+{}^1b_i} - 1\right) \times \frac{1+{}^1b_i}{1+{}^1b_{DS}}$$

US equities $= (25.5\% - 15\%) \times \left(\frac{0.938}{0.9495} - 1\right) \times \frac{0.9495}{0.99} = -0.12\%$

European equities $= (9.3\% - 15\%) \times \left(\frac{0.961}{0.9495} - 1\right) \times \frac{0.9495}{0.99} = -0.07\%$

Individual level 2 attribution effects for bonds: $= (D_i \times {}^2w_i - D_{bi} \times {}^2W_i) \times \frac{(-\Delta y_{bi} + \Delta y_b)}{1 + {}^1b_{DS}}$ (6.195)

\$ denominated bonds $= (4.1 \times 24.7\% - 4.0 \times 30\%) \times \frac{(-0.26\% + 0.21\%)}{0.99} = 0.00\%$

€ denominated bonds $= (4.0 \times 40.5\% - 5.0 \times 40\%) \times \frac{(-0.18\% + 0.21\%)}{0.99} = 0.01\%$

Individual level 2 attribution effects for carry:

\$ denominated bonds $= (24.7\% - 30\%) \times \left(\frac{1.005}{1.0033} - 1\right) \times \frac{1.0033}{0.99} = -0.01\%$

€ denominated bonds $= (40.5\% - 40\%) \times \left(\frac{1.002}{1.0033} - 1\right) \times \frac{1.0033}{0.99} = 0.00\%$

Total $= -0.12\% - 0.07\% + 0.00\% + 0.01\% - 0.01\% + 0.00\% = -0.19\%$

Exhibit 6.126 Security/issue selection balanced duration-adjusted attribution effects

$$\frac{1+r}{1+{}^{2}b'_{DS}} - 1 = \frac{0.9869}{0.9881} - 1 = -0.12\%$$

Individual selection effects for equities are:

$$= {}^{2}w_i \times \left(\frac{1+r_i}{1+{}^{2}b_i} - 1\right) \times \frac{1+{}^{2}b_i}{1+{}^{2}b_S}$$

US equities $= 25.5\% \times \left(\frac{0.932}{0.938} - 1\right) \times \frac{0.938}{0.9881} = -0.15\%$

European equities $= 9.3\% \times \left(\frac{0.967}{0.961} - 1\right) \times \frac{0.961}{0.9881} = -0.13\%$

Individual selection effects for bonds are:

$$= {}^{2}w_i \times \left(\frac{1+r_i}{1+{}^{2}b_{Di}} - 1\right) \times \frac{1+{}^{2}b_{Di}}{1+{}^{2}b_S}$$

\$ denominated bonds $= 24.7\% \times \left(\frac{1.024}{1.0155} - 1\right) \times \frac{1.0155}{0.9881} = 0.21\%$

€ denominated bonds $= 40.5\% \times \left(\frac{1.008}{1.0092} - 1\right) \times \frac{1.0092}{0.9881} = -0.05\%$

Or

$$= (D_i \times {}^{2}w_i) \times \frac{(-\Delta y_{ri} + \Delta y_{bi})}{1+{}^{2}b_s}$$

\$ denominated bonds $= (4.1 \times 24.7\%) \times \frac{(-0.46\% + 0.26\%)}{0.9881} = 0.21\%$

€ denominated bonds $= (4.0 \times 40.5\%) \times \frac{(-0.15\% + 0.18\%)}{0.9881} = -0.05\%$

Total $= -0.15\% - 0.13\% + 0.21\% - 0.05\% = -0.12\%$

Table 6.72 Balanced duration-adjusted attribution effects

		Level 1 allocation (%)	Level 1 duration (%)	Level 2 allocation (%)	Level 2 carry (%)	Selection
Level 1	Equities	**−0.21**		**−0.19**		**−0.29**
Level 2	US			−0.12		−0.15
Level 2	Europe			−0.07		−0.14
Level 1	Bonds	**−0.09**	**−0.04**	**0.00**		**0.16**
Level 2	\$ denominated			0.00	−0.01	0.21
Level 2	€ denominated			001	0.00	−0.05
Total		**−0.30**	**−0.04**	**−0.19**		**−0.12**

Figure 6.24 illustrates an asymmetric decision process in which there are fewer steps in the decision process for different parts of the portfolio. The level 1 decision is an allocation between equities and bonds. The level 2 decision allocates by region or country for equities only; there is no level 2 decision for the bond part of the portfolio. The level 3 decision is at security, instrument or individual asset level.

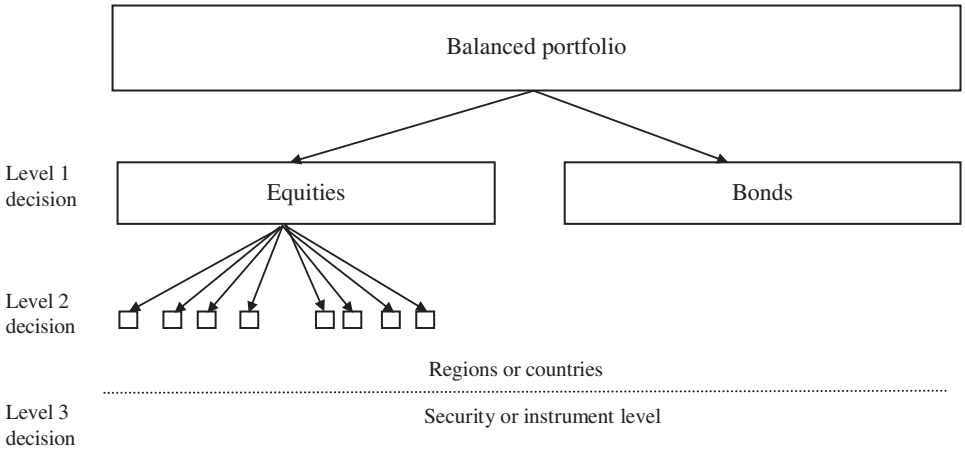

Figure 6.24 Three-level, four-step asymmetric decision process

Figure 6.25 replicates the multi-level decision process shown in Figure 6.18 and summarised in Table 6.69. The level 1 decision represents an allocation between bonds and equities. The level 2 decision allocates by region for equities and by government bonds or corporate bonds for fixed income. The level 3 decision allocates by industrial sector for equities only. The level 4 decision is at security, instrument or individual asset level.

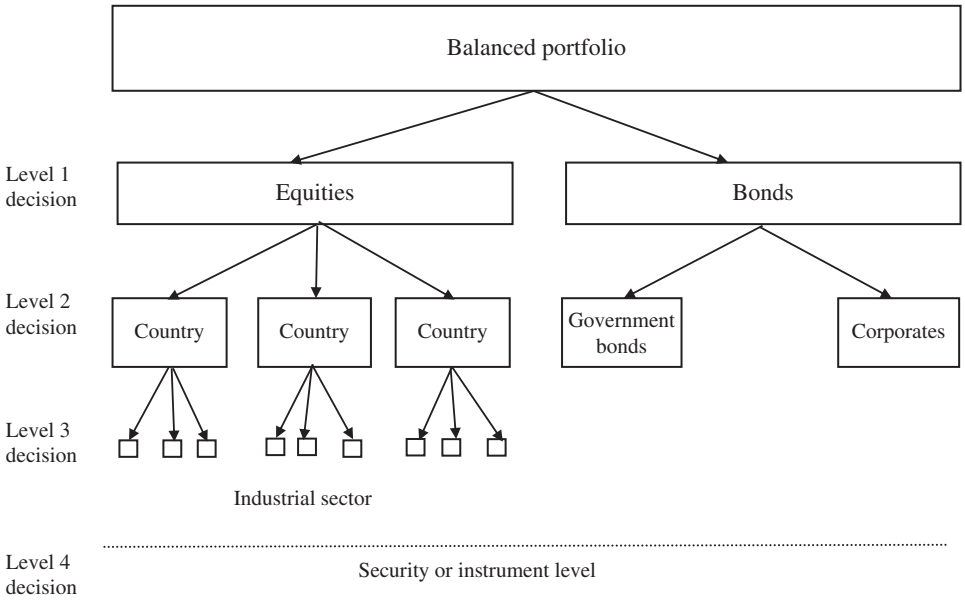

Figure 6.25 Four-level, five-step asymmetric decision process

ABOR, IBOR OR PBOR

Where should the data for attribution analysis be sourced? There are three options:

(1) ABOR – the accounting book of record. ABOR is the official book of record, aligned with regulatory and asset owner requirements. Associated with the back office, data is accurate, frequently reconciled, more rigorous but slow to deliver and somewhat restricted.
(2) IBOR – the investment book of record. IBOR is aligned with the investment decision process and the requirements of the front office. IBOR is flexible, perhaps real time, more complex but less rigorous.
(3) PBOR – the performance book of record. PBOR is aligned with the investment decision process and the requirements of the middle office. PBOR combines the accuracy and rigour of ABOR but the flexibility and suitability of IBOR.

Attribution analysis must reflect the investment decision process to add value. Although accurate, ABOR does not necessarily provide the data to support appropriate attribution analysis. IBOR is suitable for the internal and immediate requirements of the front office, but is less suitable for external use. PBOR is the best of both worlds, connecting and reconciling the official ABOR with the flexibility and appropriateness of IBOR.

EVOLUTION OF PERFORMANCE ATTRIBUTION METHODOLOGIES

The evolution of return attribution analysis is illustrated in Figure 6.26.

Comparative performance measurement starts with the BAI study in 1968, but return attribution analysis itself starts with Fama in 1972. From Fama there is a divergence between asset grouping or variance-type attribution, with the UK Society of Investment Analysts introducing the concept of selection, allocation and notional portfolios in 1972; and returns-based attribution developed a little later by Fama and French, Carhart, Sharpe, and many others. Fixed income attribution naturally veers in a different direction via Wagner and Tito in 1977 (replacing beta with duration in the Fama decomposition) and Lord in 1997 responding to the very different investment decision processes of bond managers.

There is a very early split between the dominant arithmetic Brinson model and the total-level geometric approach from Holbrook. Smoothing and linking algorithms were soon needed for multi-period arithmetic models to add up, for the most part designed and implemented by software firms to meet client requirements. Algorithms started to be published several years after initial use with Carino, soon followed by Menchero, GRAP, Frongello, and Bonafede, Foresti and Matheos. Allen is a direct link from Holbrook but addresses multi-currency issues. Burnie, Knowles and Teder, along with Bain and Bacon, develop geometric selection and allocation calculations that are missing from Holbrook and from Allen. Ankrim and Hensel and, with more success, Karnosky and Singer adapt the basic Brinson model for multi-currency attribution including, importantly, taking into account interest rate differentials. Van Breukelen also adapts the Brinson model for successive portfolio fixed income attribution.

Figure 6.26 firmly cements Brinson and Fachler at the centre of developments in attribution analysis. In more recent years we have seen many variants of yield curve decomposition models reflecting the diversity of fixed income investment decision processes.

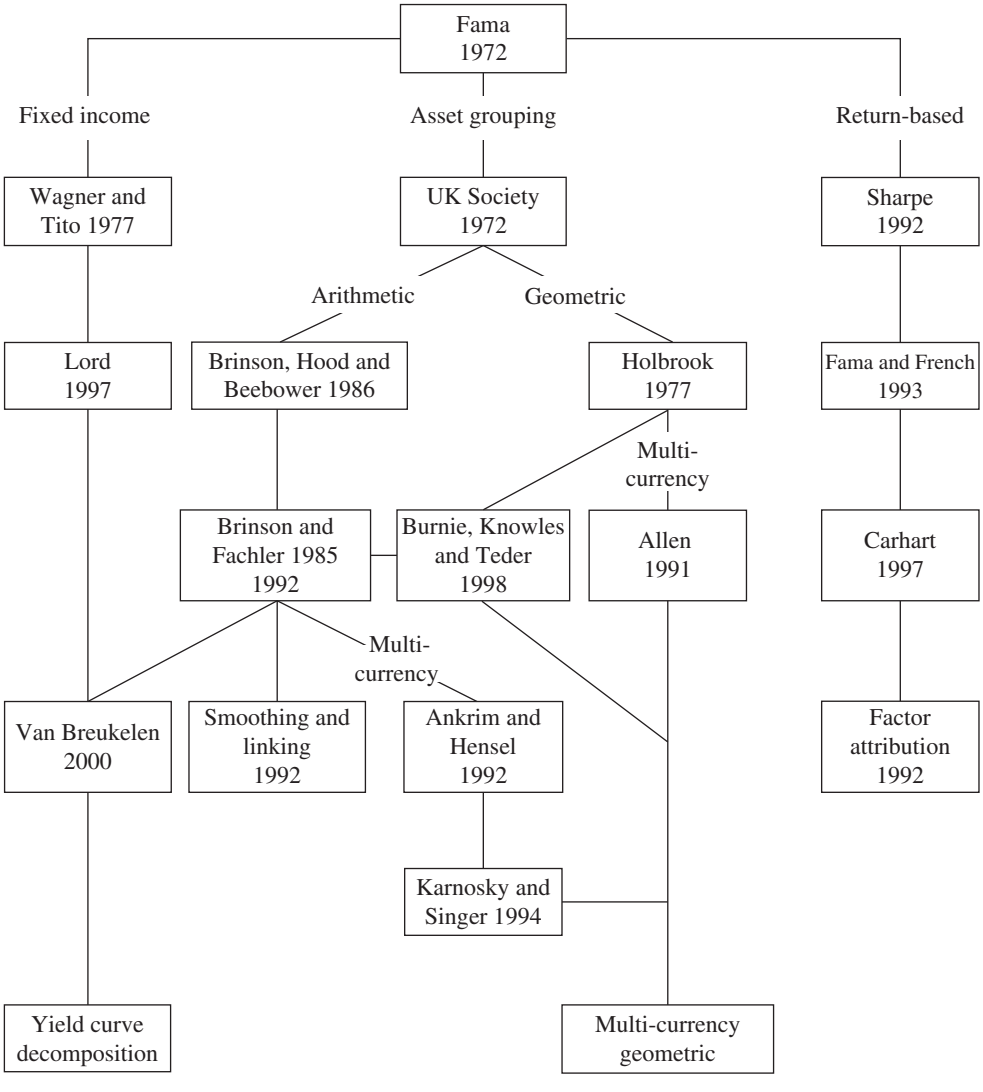

Figure 6.26 The evolution of attribution methodologies

7

Performance Presentation Standards

Without standards, there can be no improvement.

Taiichi Ohno (1912–1990)

A lot of the evil in the world is actually not intentional. A lot of people in the financial system did a lot of damage without intending to.

George Soros (1930–)

WHY DO WE NEED PERFORMANCE PRESENTATION STANDARDS?

The CFA Institute (formerly known as the Association for Investment Management and Research or AIMR) sponsored the creation of the Global Investment Performance Standards (GIPS®) to provide an ethical framework for the calculation and presentation of the performance history (or track record) of asset management firms. GIPS are voluntary standards based on the fundamental principles of full disclosure and fair representation of performance returns.

The need for the standards first became apparent in the United States in the mid-1980s. Asset owners, in seeking firms to manage their assets, would obviously see a large number of presentations from asset managers, the overwhelming majority of whom presented above average performance, begging the question, "Where are the below average managers?" The answer, unfortunately, was that some of the below average managers were presenting above average returns.

Asset managers were very selective about the investment track records they presented to prospective clients. Most marketing managers would be well aware over which time period they performed best and would consequently "cherry pick" the best period to show performance. Often single representative accounts would be used to calculate the firm's track record. Invariably the representative account would be one of the better performing accounts for that investment strategy. If the representative account performed poorly, a rationale would be found to choose a new one. Asset managers might also be selective in the choice of calculation methodology as demonstrated in Chapter 3. Other abuses include misrepresenting theoretical, model or hypothetical performance, using an inappropriate or retrospectively changing a benchmark and presenting information with inadequate or misleading disclosures. Often the performance track records presented by asset managers were not a fair and honest representation of the performance they had delivered to their existing clients.

The result was the creation of the AIMR Performance Presentation Standards[1] (AIMR-PPS), voluntary performance guidelines for the North American market. In the United Kingdom the National Association of Pension Funds (NAPF) produced its own

[1] Financial Analysts Federation, "Performance Presentation Standards" (1987).

guidelines for balanced pension funds[2] encouraged by one of the principal recommendations of a Committee of Enquiry into Investment Performance Measurement[3] also initiated by the NAPF.

Note

My interest in the standards developed at this time. I wished to bring my firm, a London-based subsidiary of a large US bank, into compliance with both the AIMR-PPS and the NAPF guidelines, and it proved impossible to achieve both. The AIMR-PPS standards and the NAPF guidelines shared the same ethical objectives yet in certain regards they were contradictory. With responsibility for a variety of European offices I could envisage the nightmare scenario of a separate set of contradictory standards in each European office, and it was clear a set of global standards was required.

GLOBAL INVESTMENT PERFORMANCE STANDARDS (GIPS®) – A HISTORY

In 1995 the CFA Institute sponsored and funded the GIPS Committee to develop a single standard for presenting investment performance. The objectives the committee set were:

(1) To obtain worldwide acceptance of a standard for the calculation and presentation of investment performance in a fair, comparable format that provides full disclosure.
(2) To ensure accurate and consistent investment data for reporting, record keeping, marketing and presentation.
(3) To promote fair, global competition among investment firms for all markets without creating barriers to entry for new firms.
(4) To foster the notion of industry self-regulation on a global basis.

In a relatively short period this committee finalised the GIPS standards in February 1999 and, having met its objectives, disbanded.

The GIPS Committee was replaced by the Investment Performance Council (IPC), whose principal goal was to have all countries adopt the GIPS standard.

To facilitate convergence the IPC promoted a transition from existing local standards to GIPS by approving Country Versions of GIPS (CVGs) while updating the GIPS standards to incorporate local best practice. The revised single standard was issued in February 2005 with an effective start date of 1 January 2006.

This version of GIPS included a number of improvements plus new sections on real estate and private equity that effectively removed the need for CVGs.

In 2006 the IPC was transformed into the GIPS Executive Committee, the effective decision-making authority for the standards and the GIPS Council, which facilitated the

[2] NAPF, "Pension Fund Investment Performance Code" (1992).
[3] NAPF, "Committee of Enquiry Report into Investment Performance Measurement" (1990).

involvement of all GIPS Standards Sponsors in the ongoing development and promotion of the GIPS standards.

The GIPS Council consisted of country sponsors that had adopted the standards and provided four members of the GIPS executive committee.

In 2008, the GIPS Executive Committee began a review of the GIPS standards to further improve the standards, resulting in the 2010 edition of the GIPS standards[4] with an effective date of 1 January 2011.

The mission of the GIPS Executive Committee was to "promote ethics and integrity and instill trust through the use of the GIPS standards by achieving universal demand for compliance by asset owners, adoption by asset managers and support from regulators for the ultimate benefit of the global investment community".

In 2017, the GIPS Executive Committee concluded that the GIPS standards in their current form couldn't achieve their stated mission without a number of improvements. The GIPS standards had been successful in terms of traditional asset managers: at that point 24 of the top 25 asset managers, 74 of the top 100, claimed compliance with GIPS for all or some of their business, representing over 60% of the world's assets under management,[5] 41 countries endorsed the GIPS standards and crucially there were no competing standards. However, the GIPS standards had not penetrated other parts of the asset management industry. The GIPS standards needed to better accommodate the needs of asset managers of alternative assets (private equity, private debt, infrastructure etc.), asset managers of pooled funds and asset managers of high-net-worth clients; they needed to address the technical requirements of overlay asset strategies, re-engage with asset owners and consolidate a complex array of guidance statements and formal answers to technical questions.

The GIPS Executive Committee started the process of a major update by issuing a Consultation Paper for public comment, which described the proposed key concepts of the GIPS standards and requested feedback on specific issues. Following this public comment an exposure draft was issued on 31 August 2018 and the final version of the 2020 edition of the GIPS standards was issued on 30 June 2019.[6]

As at 31 December 2021, 41 countries, including all the major capital markets, have adopted GIPS. A full list is included in Table 7.1.

Table 7.1 Country GIPS adoption

Country	Sponsoring organisation
Australia	Financial Services Council – Performance Analyst Group
Brazil	CFA Society Brazil
Canada	Canadian Investment Performance Council
Chile	CFA Society Chile
China	CFA Society Beijing
Czech Republic	CFA Society Czech Republic Czech Capital Market Association

(Continued)

[4] CFA Institute, "Global Investment Performance Standards (GIPS®)" (2010).

[5] Cerulli Associates (2015).

[6] CFA Institute, "Global Investment Performance Standards, for Firms, for Asset Owners, for Verifiers" (2019).

Table 7.1 (*Continued*)

Country	Sponsoring organisation
Denmark	CFA Society Denmark
France	CFA Society France
Germany	BVI – Federal Investment and Asset Management e.V. CFA Society Germany
Ghana	Ghana Securities Industry Association
Greece	CFA Society Greece
India	CFA Society India
Indonesia	Indonesia Association of Mutual Fund Managers,
	CFA Society Indonesia
Ireland	Irish Association of Investment Managers
Italy	ABI Italian Banking Association
	The Italian Association of Financial Analysts, Assogestioni Association
	Market Development of Pension Funds, Assirevi & CFA Society Italy
Japan	The Securities Analysts Association of Japan
Kazakhstan	Association of Financial & Investment Analysts
Liechtenstein	Liechtenstein Investment Fund Association
Mexico	CFA Society Mexico
Micronesia	Asia Pacific Association for Fiduciary Studies
Netherlands	CFA Society VBA Netherlands
New Zealand	CFA Society New Zealand
Nigeria	CFA Society Nigeria, Pensions Operators Association of Nigeria
	Fund Managers Association of Nigeria
Norway	The Norwegian Society of Financial Analysts CFA Society Norway
Pakistan	CFA Society Pakistan
Peru	CFA Society Peru Association of Promoting Companies of the Capital Market
Philippines	CFA Society Philippines
	Trust Officers Association of the Philippines
	Fund Managers Association of the Philippines
Poland	CFA Society Poland
Portugal	Portuguese Association of Financial Analysts
Russia	CFA Association Russia
Saudi Arabia	CFA Society Saudi Arabia
Singapore	Investment Management Association of Singapore
South Africa	Association for Savings Investment, South Africa
Spain	CFA Society Spain, Association of Collective Investment Institutions and Pension Funds
	Spanish Institute of Financial Analysts
Sri Lanka	CFA Society Sri Lanka
Sweden	CFA Society Sweden
Switzerland	Asset Management Association Switzerland
Thailand	CFA Society Thailand
Ukraine	The Ukrainian Association of Investment Business
United Kingdom	The Investment Association, Pensions and Lifetime Savings Association
	The Association of Consulting Actuaries, The Society of Pension Professionals
	The Investment Property Forum, The Alternative Investment Management Association
	The Personal Investment Management and Financial Advice Association
	The Association of Professional Pension Trustees
United States	United States Investment Performance Committee

ADVANTAGES FOR ASSET MANAGERS

The advantages of a global standard for asset owners are obvious; they can select asset managers based on good quality information with a certain level of confidence that the numbers presented are a fair and honest representation of that firm's track record.

⚠ Caution

For their own protection asset owners should only hire asset managers who are compliant with the standards.

Non-compliance with the standards may suggest a weaker commitment to ethical standards or weak internal controls insufficient to claim compliance. If performance measurement controls are not best practice, that may be an indication that other controls within the firm are weak.

For asset managers the advantages are less obvious and of course there is the cost of compliance to be offset; however, in my view the following advantages significantly outweigh the costs of compliance:

(1) *Marketing advantage*

Clearly in the early stages of a standard an asset manager can gain a marketing advantage by claiming compliance with a good quality standard. In the United States it has now become a marketing disadvantage not to be compliant, a situation that will arise in Europe and other regions at some stage.

Many asset owners will not welcome firms into the selection process if they are not compliant and, in some circumstances, if their claim of compliance is not independently verified. Any trustee is taking the risk of future criticism or legal action by selecting an asset manager who does not comply to performance presentation standards if subsequently things go wrong and it is established that the original presentation was misleading.

(2) *Level playing field – international passport*

The standards are designed to encourage global competition and eliminate barriers to entry. In effect, GIPS allows asset managers to market their track record worldwide with the knowledge they are presenting to the same standards as local competitors.

The pressure to present misrepresentative performance is not so great if asset managers are confident their competitors are operating to the same standard.

(3) *Increased professionalism*

To achieve compliance a firm must have good quality performance measurement processes and procedures in place and a commitment to the ethical presentation of performance track records. This naturally increases the profile and importance of performance analysts in the firm.

(4) *Risk control*

The standards require a basic level of risk control. Asset managers are required to investigate outliers in their track record to ensure that those portfolios are being managed within the composite guidelines. This is good business practice, identifying poor performing portfolios early, ensuring good performance is real and ensuring the entire firm is aware and understands the investment objectives and guidelines for each portfolio.

(5) *Business efficiency and data quality*

Establishing effective procedures and improving performance measurement systems obviously requires investment; however, doing things right the first time is obviously more

efficient than calculating performance incorrectly and wasting time and resources investigating and correcting errors. Clearly, it's very inefficient for individual portfolio managers to be wasting their time ensuring returns are calculated correctly.

Note

In my view the improvements in risk control, business efficiency and data quality alone justify the costs of compliance with the GIPS standards. I strongly recommend compliance for all asset management firms, even if the external benefits of compliance are not apparent.

THE GIPS STANDARDS

GIPS are ethical standards for investment performance presentation to ensure fair representation and full disclosure of a firm's performance track record.

The 2020 edition of the GIPS standards has three chapters:

(1) GIPS Standards for Firms
(2) GIPS Standards for Asset Owners
(3) GIPS Standards for Verifiers

In this chapter I will focus on the GIPS standards for firms. I will list the required provisions together with some explanation of the requirement and additional commentary if appropriate. Readers are of course encouraged to refer to the standards themselves and in particular the wonderful and thorough support material provided by the CFA Institute. The GIPS standards for asset owners are, of course, very similar. They are for asset owners that do not compete for business but report to an oversight body, and are fundamentally designed to encourage asset owners to consider GIPS compliance. The view is that if asset owners are considering their own compliance with the GIPS standards, they are more likely to demand GIPS compliance of their asset managers. The GIPS standards for verifiers focuses on the independent verification process:

(1) Purpose of verification
(2) Verifier qualification requirements
(3) Scope of verification
(4) Agreeing on the terms of engagement
(5) Required verification procedures
(6) Maintenance of verifier documentation
(7) Representation letter
(8) Verification report
(9) Recommendation letter
(10) Performance examination

Caution

The standards are complex, flexible by design and at times subjective. Claiming compliance without independent verification is highly risky. I strongly encourage all asset managers to be independently verified.

Organisations that compete for investment management business must comply with the GIPS standards for firms,[7] which is divided into the following eight sections:

(1) Fundamentals of compliance
(2) Input data and calculation methodology
(3) Composite and pooled fund maintenance
(4) Composite time-weighted return report
(5) Composite money-weighted return report
(6) Pooled fund time-weighted return report
(7) Pooled fund money-weighted return report
(8) GIPS advertising guidelines

The GIPS standards consist of requirements, which *must* be followed in order for a firm to claim compliance, and recommendations, which are optional but *should* be followed because they represent good practice in performance presentation. Firms must comply with all applicable requirements of the GIPS standards, including any Guidance Statements, interpretations, and Questions & Answers (Q&As) published by CFA Institute and the GIPS standards governing bodies.

A standard can only be successful in the long run if it is promoted and if it has the ability to respond to changes in market practice, correct errors in the standards and to provide interpretation where and when required. CFA Institute uses Guidance Statements, formal Questions & Answers (Q&As) and Handbooks to provide new guidance, interpretation and correct errors and misunderstandings. Asset managers and verifiers are required to understand their content and keep up to date with the standards. Information on GIPS, guidance statements and how to subscribe for the GIPS Newsletter can be found on the GIPS website at www.gipsstandards.org.

FUNDAMENTALS OF COMPLIANCE

Section 1 of the GIPS standards for firms outline the fundamentals of compliance with the GIPS standards. There are 39 required provisions in Section 1 covering issues, including:

(1) Definition of the firm
(2) Maintaining policies and procedures
(3) Providing GIPS Reports
(4) Benchmark selection
(5) Correcting errors in GIPS Reports
(6) Composite descriptions
(7) Recordkeeping
(8) Linking of theoretical and actual performance
(9) Portability
(10) Use of time-weighted and money-weighted rates of returns

[7] In the context of the GIPS standards asset managers are referred to as firms.

Definition of the firm

The definition of the firm is the foundation for firm-wide compliance and creates defined boundaries whereby total firm assets can be determined. The GIPS standards must be applied on a firm-wide basis; firms cannot claim to be in compliance with the GIPS standards with regard only to certain of the asset classes, investment strategies, products or pooled funds that they manage.

Relevant Provisions

1.A.1 The GIPS standards MUST be applied on a firm-wide basis. Compliance MUST be met on firm-wide basis and cannot be met on a composite, pooled fund, or portfolio basis.

1.A.2 The firm MUST be defined as an investment firm, subsidiary, or division held out to the public as a distinct business entity.

1.A.3 To initially claim compliance with the GIPS standards, the firm MUST attain compliance for a minimum of five years or for the period since the firm inception if the firm has been in existence for less than five years.

1.A.4 The firm MUST comply with all applicable requirements of the GIPS standards, including any Guidance Statements, interpretations, and Questions & Answers (Q&As) published by CFA Institute and the GIPS standards governing bodies.

A firm cannot claim compliance with the GIPS standards until it meets the applicable requirements of the GIPS standards for a least a five-year period, or since inception of the firm if the firm has been in existence for less than five years.

To comply with the GIPS standards, a firm must be an investment firm, subsidiary or division held out to clients or potential clients as a distinct business entity. A distinct business entity is a unit, division, department or office that is organizationally and functionally segregated from other units, divisions, departments or offices and that retains discretion over the assets it manages and that should have autonomy over the investment decision-making process. Possible criteria for identifying a distinct business entity are the organisation being a legal entity, having a distinct market or client type, or using a separate and distinct investment process.

An investment organisation might include multiple GIPS-defined firms if these defined firms are genuinely distinct business entities, such as a private client unit and a hedge fund division of a larger firm. Even two entities within the same parent company offering substantially similar investment products but in separate geographic jurisdictions might be considered distinct business entities. The GIPS standards encourage the broadest, most meaningful definition of the firm. The scope of this meaningful definition should include all geographical (country, regional, etc.) offices operating under the same brand name, regardless of the actual name of the investment management firm. The GIPS standards are not explicit on this point, but firm definitions should not overlap.

Caution

The way in which the investment management organisation is held to the public is a key factor in defining the firm. How is the firm portraying itself?

The standards require firm-wide compliance to ensure that poor-performing accounts have not been excluded from the performance track record. Once the firm has been defined the exercise of allocating accounts to composites can begin – it determines the universe of portfolios to be allocated.

The firm definition must be meaningful, rational and fair. The definition cannot be used narrowly to exclude poor performing product areas.

Although the broadest definition is recommended it is acceptable to define a number of firms within the same organisation, provided they meet the above criteria, with a view of combining into one firm at a later date. Not all firms within a single organisation need be compliant simultaneously, allowing part of the organisation time to work on bringing its firm into compliance. This flexibility is often used geographically, although it should never be used to exclude an underperforming part of the business.

Corporate restructuring or merger and acquisition activity may well result in changes in the firm definition.

Maintaining policies and procedures

Firms must document the policies and procedures used in establishing and maintaining compliance with the GIPS standards. Policies and procedures must be applied consistently.

Relevant Provisions

1.A.5 The firm MUST:

(a) Document its policies and procedures used in establishing and maintaining compliance with the requirements of the GIPS standards, as well as any recommendations it has chosen to adopt, and apply them consistently.

(b) Create policies and procedures to monitor and identify changes and additions to all of the Guidance Statements, interpretations, and Q&As published by CFA Institute and the GIPS standards governing bodies.

1.A.6 The firm MUST:

(a) Comply with all applicable laws and regulations regarding the calculation and presentation of performance.

(b) Create policies and procedures to monitor and identify changes and additions to laws and regulations regarding the calculation and presentation of performance.

Completing and updating the firm's documentation of performance-related policies and procedures may prove to be one of the biggest contributions the GIPS implementation effort makes to effective operational management. Because preparing proper documentation is typically treated as less urgent than other business matters, it is common for refinements in policies and processes to be documented only in e-mails and memoranda. If these changes are not integrated into manuals or handbooks, over time the existing documentation can become inadequate or inaccurate. Implementing the GIPS standards may help managers assemble resources and allocate time to upgrading their existing documentation. The effort to document longstanding policies and procedures leads to actions taken which improve internal controls and/or achieve operational efficiencies.

Providing GIPS Reports

Firms must "make every reasonable effort" to provide a GIPS Report to all prospective clients and to all limited distribution pooled fund prospective investors. A prospective client is any person or entity that has expressed interest in one of the firm's composite strategies and qualifies to invest in the composite. A prospective investor is any person or entity that has expressed interest in one of the firm's pooled funds and qualifies to invest in the pooled fund. Existing clients and investors may also qualify as prospective clients and investors for any strategy that is different from their current investment strategy or pooled fund. Investment consultants and other third parties are included as prospective clients and investors if they represent individuals or entities that qualify as prospective clients or investors.

Relevant Provisions

1.A.11 The firm MUST make every reasonable effort to provide a GIPS Composite Report to all prospective clients when they initially become prospective clients. The firm MUST NOT choose to which prospective clients it presents a GIPS Composite Report.

1.A.12 Once the firm has provided a GIPS Composite Report to a prospective client, the firm MUST provide an updated GIPS Composite Report at least every 12 months if the prospective client is still a prospective client.

1.A.13 The firm MUST make every reasonable effort to provide a GIPS Report to all limited distribution pooled fund prospective investors when they initially become prospective investors. The GIPS Report may be either:

(a) A GIPS Polled Fund Report, or

(b) A GIPS Composite Report. A GIPS Composite Report may be provided only if the limited distribution pooled fund is included in the respective composite.

The firm MUST NOT choose to which limited distribution pooled fund prospective investors it presents a GIPS Report.

1.A.14 Once the firm has provided a GIPS Pooled Fund Report or a GIPS Composite Report to a limited distribution pooled fund prospective investor, the firm MUST provide an updated GIPS Pooled Fund Report or GIPS Composite Report at least once every 12 months if the limited distribution pooled fund prospective investor is still a limited distribution pooled fund prospective investor.

1.A.15 The firm may provide a GIPS Pooled Fund Report or a GIPS Composite Report that includes the broad distribution pooled fund to broad distribution pooled fund prospective investors but is not required to do so.

1.A.16 When providing GIPS Reports to prospective clients and prospective investors, the firm MUST update these reports to include information through the most recent annual period-end within 12 months of that annual period-end.

1.A.17 The firm MUST be able to demonstrate how it made every reasonable effort to provide:

(a) A GIPS Composite Report to those prospective clients required to receive a GIPS Composite Report.

(b) A GIPS Pooled Report or GIPS Composite Report to those limited distribution pooled fund prospective investors required to receive a GIPS Pooled Report or GIPS Composite Report.

A GIPS Report is either a GIPS Pooled Fund Report or a GIPS Composite Report. A GIPS Composite Report may be provided to the prospective investor if the pooled fund is included in the composite. The requirement to make every reasonable effort to provide a GIPS Report is met if a prospective client or investor has received a GIPS Report within the previous 12 months.

A broad distribution pooled fund is a pooled fund that is regulated under a framework that would permit the general public to purchase or hold the pooled fund's shares and is not exclusively offered in one-on-one presentations. In most jurisdictions, broad distribution pooled funds are well regulated. Because of both the complexity and diversity of local laws and regulations applying to broad distribution pooled funds, which typically specify what must be included in the materials for these funds, as well as what must not be included, it is not required that a GIPS Report be provided to broad distribution prospective investors. If a firm wishes to provide a GIPS Pooled Fund Report or GIPS Composite Report that includes the broad distribution pooled fund to a broad distribution pooled fund prospective investor, however, it may do so. The GIPS standards consider any pooled fund that is not a broad distribution pooled fund to be a limited distribution pooled fund.

The GIPS standards require that firms comply with all applicable laws and regulations regarding the calculation and presentation of performance, irrespective of whether the performance is presented in a GIPS Report or a report mandated by the laws of the jurisdiction. In the rare cases when laws and regulations conflict with the GIPS standards, firms are required to comply with the laws and regulations and disclose the manner in which the laws or regulations conflict with the GIPS standards. The GIPS standards also require firms to create policies and procedures to monitor and identify changes and additions to laws and regulations regarding the calculation and presentation of performance.

Benchmark selection

Benchmarks (see Chapter 4) are essential tools for articulating asset owner requirements and evaluation of asset manager performance.

Relevant Provisions

1.A.18 A composite benchmark used in a GIPS Composite Report MUST reflect the investment mandate, objective or strategy of the composite. The firm MUST NOT use a price-only benchmark in a GIPS Composite Report.

1.A.19 A pooled fund benchmark used in a GIPS Pooled Report MUST reflect the investment mandate, objective or strategy of the pooled fund. The firm MUST NOT use a price-only benchmark in a GIPS Pooled Fund Report.

Firms are required to present benchmark total returns and standard deviations that reflect the composite's investment mandate, objective or strategy. The benchmark published in the GIPS Composite Report may differ from the benchmark of constituent portfolios within the composite, particularly if portfolios within the composite are measured against a different benchmark in a broadly defined composite. Also, more than one benchmark may be shown; the firm must determine the appropriate benchmark for each composite.

The firm must not use a price-only benchmark – it is clearly misleading to present composite and pooled fund returns against a benchmark that excludes income. Price-only indexes are published by index providers and even if a total return option is not available it remains inappropriate to show a price-only benchmark. Some benchmarks appear to be price-only because the underlying asset does not generate income; these should be considered total return benchmarks – in effect, the income element is zero.

Caution

In my view it is never appropriate to use a price-only benchmark inside or outside a GIPS Report in any circumstances.

Correcting errors in GIPS Reports

Errors will occur. There are many types, including:

(1) Valuation errors
(2) Missed trades
(3) Incorrectly timed cash flows
(4) Incorrect allocation of portfolios to composites
(5) Missed or incorrect disclosures
(6) Failures to follow procedures
(7) Inappropriate methodologies
(8) Policies and procedures not allowed within the standards
(9) Errors in externally published information, such as benchmarks
(10) Misprints

Relevant Provisions

1.A.20 The firm MUST correct material errors in GIPS Composite Reports and MUST:

(a) Provide the corrected GIPS Composite Report to the current verifier.
(b) Provide the corrected GIPS Composite Report to current clients and any former verifiers that received the GIPS Composite Report that had the material error.
(c) Make every reasonable effort to provide the corrected GIPS Composite Report to all current prospective clients and prospective investors that received the GIPS Composite Report that had the material error. The firm is not required to provide the correct GIPS Composite Report to former clients, former investors, former prospective clients, or former prospective investors.

1.A.21 The firm must correct material errors in GIPS Pooled Fund Reports and MUST:

(a) Provide the corrected GIPS Pooled Fund Report to the current verifier.
(b) Provide the corrected GIPS Pooled Fund Report to current investors and any former verifiers that received the GIPS Pooled Fund Report that had the material error.

(c) Make every reasonable effort to provide the corrected GIPS Pooled Fund Report to all current prospective investors that received the GIPS Pooled Fund Report that had the material error. The firm is not required to provide the correct GIPS Composite Report to former investors or former prospective investors.

Errors do not necessarily threaten the firm's claim of compliance, but they must be handled correctly. Firms must decide whether previously reported returns should be recalculated and whether or not clients should be informed of the error.

Well-managed firms will have:

(1) A written policy and procedures for handling errors
(2) A definition of materiality
(3) An error correction process

Clearly, materiality is an issue; no value is gained by reissuing presentations if the error amounts to one or two basis points, but one or two whole percentage points might be a different matter. Firms are free to define their own levels of materiality but are required to make available their policies and procedures on error correction on request. An error is material if it is probable that the judgement of a reasonable person relying on the presentation would have changed.

The error correction process should:

(1) Recalculate returns.
(2) Determine if the error is material.
(3) Decide what action to take.
(4) Document the original presentation, the corrected presentation and the action taken.

For material errors firms must provide the corrected GIPS Report to the current verifier, current clients and investors who received the erroneous report and previous verifiers who received the erroneous report. Firms must also make reasonable efforts to provide a corrected GIPS Report to current prospective clients and investors who received the erroneous reports. Firms must have procedures to identify clients and investors who may have received erroneous GIPS Reports in the past. Clearly, material errors have significant consequences, but setting high thresholds of materiality is not necessarily the solution – clients and investors will make their own assessment of the firms' levels of materiality.

Actions are also required for non-material errors. Based on the magnitude of the error, actions potentially include:

(1) Correcting the GIPS Report and disclosing the error in the report
(2) Correcting the GIPS Report with no disclosure
(3) Taking no action

The firm is not required to provide the corrected GIPS Report to former clients, former investors, former prospective clients or former prospective investors. Material errors may also apply to benchmarks (whether or not the error is caused by the firm) and incorrect or missing disclosures and other requirements.

Note

Having effective and appropriate error correction procedures is a key requirement for asset managers. As part of their selection due diligence asset owners should always request and review asset manager error correction policies and procedures.

Composite descriptions

Firms must provide a list of composite descriptions to any prospective client asking for such information, and they must provide upon request a compliant presentation for any composite listed. Discontinued composites must remain on the list for at least five years after their termination date. Firms must also provide a complete list of pooled fund descriptions for limited distribution funds to any limited distribution pooled fund prospective investor that makes such a request, although the list need only include funds for which the prospective investor is eligible.

Relevant Provisions

1.A.22 The firm MUST maintain:

(a) A complete list of composite descriptions. The firm MUST include terminated composites on this list for at least five years after the composite termination date.

(b) A complete list of pooled fund descriptions for limited distribution pooled funds. The firm is not required to include terminated limited distribution pooled funds on the list.

(c) A complete list of broad distribution pooled funds. The firm is not required to include terminated broad distribution pooled funds on this list.

1.A.23 The firm MUST provide:

(a) The complete list of composite descriptions to any prospective client that makes such a request.

(b) The complete list of pooled fund descriptions for limited distribution pooled funds to any limited distribution pooled fund prospective investor that makes such a request. This list may include only the limited distribution pooled funds for which the prospective investor is eligible.

(c) The complete list of broad distribution pooled funds to any broad distribution pooled fund prospective investor that makes such a request. This list may include only the broad distribution pooled funds for which the prospective investor is eligible. If the firm maintains a complete list of broad distribution pooled funds on its website, the firm may instead direct the prospective investor to the firm's website.

(d) The pooled fund description for any broad distribution pooled fund to any broad distribution pooled fund prospective investor that makes such a request.

1.A.24 The firm MUST provide:

(a) A GIPS Composite Report for any composite listed on the firm's list of composite descriptions to any prospective client that makes such a request.

(b) A GIPS Pooled Fund Report or GIPS Composite Report, provided the limited distribution pooled fund is included in the respective composite, for any limited distribution pooled fund on the firm's list of pooled fund descriptions to any limited distribution pooled fund prospective investor that makes such a request.

Note

The availability of composite and pooled fund lists is intended to discourage firms from cherry-picking strategies to present to prospective clients and investors. Asset owners should always request these lists. The speedy and informed delivery of these lists will be a good test of the asset manager's effectiveness.

Terminated composites are included on the composite list to ensure firms don't hide the underperformance of failed strategies by closing composites and perhaps opening new composites with very slightly different descriptions. Terminated pooled funds are not required to be on this list simply because investors are unable to invest in them. Firms must also provide a complete list of broad distribution pooled fund names to any prospective investor upon request. While this list is not required to include broad distribution pooled fund descriptions, a firm must provide a description of a specific broad distribution pooled fund to any broad distribution pooled fund investor who makes such a request.

Record keeping

It is a fundamental requirement of the GIPS standards that all information in a GIPS Report must be supported. Firms must maintain records to be able to recalculate their performance history as well as substantiate all other information included within or associated with a GIPS Report.

Relevant Provisions

1.A.25 All data and information necessary to support all items included in GIPS Composite Reports, GIPS Pooled Fund Reports and GIPS Advertisements MUST be captured, maintained, and available within a reasonable time frame for all periods presented in these reports and advertisements.

1.A.26 The firm is responsible for its claim of compliance with the GIPS standards and MUST ensure that the records and information provided by any third party on which the firm relies meet the requirements of the GIPS standards.

Data does not have to be immediately accessible; it might be held offsite or with a third-party service provider. Data must be retrievable within a reasonable time frame; service level agreements with third parties should accommodate rapid retrieval of such data. Records stored in a system that is not operable will not satisfy recordkeeping requirements. All firms are expected to have disaster recovery plans sufficient to mitigate the loss of records for any reason. In the event of a catastrophic event (destruction of all records, electronic and other backups) firms should attempt to reconstruct GIPS Reports using information from custodians, consultants, verifiers, clients and any other third party with appropriate records. If this is not possible, firms may continue to claim compliance and disclose that records are not available for the impacted period(s). Disclosure of the lack of records is not open to firms that have not yet claimed compliance.

The GIPS Handbook[8] provides a comprehensive list of the records that may be required to support a claim of compliance:

Records to Support Portfolio-Level (Segregated Account and Pooled Fund) Returns

(1) Portfolio statements, including positions and valuations, as well as information supporting the determination of fair value,
(2) Information to prove that performance is based on actual assets, including bank/custodial statements and reconciliations,
(3) Portfolio transactions reports,
(4) Outstanding trades reports,
(5) Corporation action reports,
(6) Income received/earned reports,
(7) Accrued income reports,
(8) Foreign or withholding tax reclaim reports,
(9) Cash flow/weighted cash flow reports,
(10) Foreign exchange rates,
(11) Information on calculation methodology used,
(12) Information provided by a third party (e.g., a subadvisor or custodian) for which a firm may need to take additional steps to ensure the information can be relied on to meet the requirements of the GIPS standards,
(13) Investment management fee information, and
(14) Information specific to pooled funds:
    (a) Pooled fund net asset value (NAV) reports,
    (b) Pooled fund dividend and gain distributions,
    (c) Supporting documentation for a pooled fund's NAV (e.g., trail balances),
    (d) Fee data, including custody and administration fees,
    (e) Pooled fund expense ratio information,
    (f) Shareholder/owner information and cash flow activity, and
    (g) Profit and loss allocation reports for limited distribution pooled funds (LDPFs).

[8] CFA Institute, *GIPS Standards Handbook for Firms* (November 2020).

Reports to Support Composite-Level Returns and Other Composite-Level Data

(1) Portfolios include in the composite:
   (a) When each portfolio entered (and exited, if applicable) the composite,
   (b) Each portfolio's return for each period,
   (c) The value used to weight each portfolio (beginning value or beginning value plus weighted external cash flows) for each period.
(2) The number of portfolios in the composite and the composite assets as of each annual period-end and any other period for which this information is presented in GIPS Reports,
(3) Internal dispersion data,
(4) Investment management fee information, if composite returns are calculated using model investment management fees,
(5) Support for the three-year annualised ex-post standard deviation calculation and any additional risk measures.
(6) The exchange rates used to convert composite returns into different currencies.

Records to Support the Inclusion of a Portfolio in a Specific Composite

(1) Composite definition, particularly related to the composite criteria, including the definition of discretion.
(2) Portfolios excluded from all composites and reasons for exclusion.
(3) Investment management agreements and investment guidelines, as well as amendments thereto.
(4) Reports provided to clients and investors if used to help determine composite assignment.
(5) Analytics used to support composite assignment or composite exclusion.
(6) E-mail/other correspondence with clients regarding documented changes to a portfolio's investment mandate, objective, or strategy.

Records to Support Other Information Included in a GIPS Report

(1) Advisory-only assets,
(2) Uncalled committed capital,
(3) Firm and composite overlay exposure,
(4) Composite investment management fee schedules,
(5) Pooled fund expense ratio and investment management fee schedules,
(6) Performance-based fee calculations,
(7) Benchmark returns, including custom benchmark calculations,
(8) Estimated transaction costs, and
(9) Supplemental information.

Records to Support a Firm's Claim of Compliance

(1) GIPS standards policies and procedures, covering all periods for which the firm claims compliance with the GIPS standards,
(2) Definition of the firm, historical and current,
(3) Supporting calculation for total firm assets as of each annual period-end and any other period for which total firm assets are presented in GIPS Reports,
(4) Composite inception and creation dates,

(5) Determination of whether each pooled fund is a broad distribution pooled fund (BDPF) or LDPF,
(6) List of composite descriptions,
(7) List of LDPF descriptions, including inception date,
(8) List of BDPFs, if not maintained on the firm's website,
(9) GIPS Composite Reports for all composites, and
(10) GIPS Pooled Fund Reports for all pooled funds for which a GIPS Pooled Fund Report was created.

Any Additional Records Necessary to Support a Claim of Compliance

(1) Marketing output/request for proposal (RFP) responses,
(2) System and control reports from independent accountants or other third parties (e.g., accounting reports, other internal controls/compliance reports for the client and/or custodians),
(3) Third-party (e.g., subadvisory, custodial, performance data provider) agreements,
(4) Minutes of relevant decision-making committees (e.g., a board, an investment committee, a GIPS compliance committee),
(5) Client and investor fee schedules/agreements,
(6) Systems manuals, especially for the systems that generate the portfolio, pooled fund, and composite returns, as well as GIPS Reports (including both numerical information and disclosures),
(7) Documentation of efforts made to provide all prospective clients and prospective investors with GIPS Reports,
(8) Documentation that the firm followed its error correction policy, including efforts made to provide, in the case of a material error, a corrected GIPS Report, including disclosure of the error, to all appropriate parties in accordance with the firm's error correction policy,
(9) Underlying benchmark data (if not publicly available), and
(10) Documentation providing the following to any prospective client or prospective investor who requested:
    (a) A list of composite description or LDPF description,
    (b) A list of BDPFs, if this information is not maintained on the firm's website,
    (c) A BDPF description, if this information is not maintained on the firm's website,
    (d) A GIPS Report,
    (e) Policies for valuing portfolios (segregated accounts and pooled funds),
    (f) Policies for calculation performance,
    (g) Policies for preparing GIPS Reports,
    (h) Verification report(s), and
    (i) Performance examination report(s).

Linking of theoretical and actual performance

Firms must not link actual performance with theoretical performance.

Relevant Provisions

1.A.27 The firm MUST NOT link actual performance to historical theoretical performance.

1.A.28 Changes in the firm's organization MUST NOT lead to alteration of historical performance.

1.A.29 For time-weighted returns presented in GIPS Reports, the firm MUST NOT link non-GIPS-compliant performance for periods beginning on or after the minimum effective compliance date to GIPS-compliant performance. The firm may link non-GIPS-compliant performance to GIPS-compliant performance in GIPS Reports provided that only GIPS-compliant performance is presented for periods beginning on or after the minimum effective compliance date.

1.A.30 For money-weighted returns presented in GIPS Reports, the firm MUST NOT present non-GIPS-compliant performance for periods ending on or after the minimum effective compliance date. The firm may present non-GIPS-compliant performance in GIPS Reports for periods prior to the minimum effective compliance date.

Theoretical performance is performance that is not derived from a portfolio or composite with actual assets invested in the strategy presented. In the process of developing, testing, and refining new investment strategies, firms frequently construct model portfolios and use historical security prices to simulate theoretical performance in past measurement periods. Composites cannot include simulated, back-tested, hypothetical, indicative, ex-ante, forward-looking or model portfolios. The firm must not combine different composites, pooled funds, or carve-outs to create a simulated strategy and present it as a composite. Even though consisting of actual assets the combination of composites, portfolios, pooled funds or carve-outs requires theoretical investment decisions and is therefore a model. A hiatus in performance or missing history cannot be completed with theoretical performance. Theoretical performance may be included in a GIPS Report if it is not linked with actual performance, if it is clearly labelled as supplemental information and only if it is presented to clients and investors who are sufficiently knowledgeable to assess the relevance of the information.

Caution

Back-testing and model performance certainly have their uses from an internal control perspective. But I would always ignore them from an external perspective – the self-selection issues alone are so significant – even for the most rigorous of internal processes. Poor theoretical performance is simply never made available externally.

A GIPS-defined firm may very well undergo subsequent changes in its corporate structure or organisational design. However, changes in a firm's organisation are not permitted to lead

to alteration of historical composite results. In fact, apart from correcting errors, historical composite results may not be altered.

The minimum effective compliance date is largely academic at this point and relates to the ability to link performance compiled prior to the GIPS standards being published in 2000 with GIPS-compliant performance thereafter and for real estate and private equity strategies from 1 January 2006.

Portability

The performance track record belongs to the asset management firm, not an individual portfolio manager. The GIPS standards take the view that performance is generated by many factors (e.g., the support and guidance of senior management, the research function, the dealing department, feedback from colleagues, the performance team, effective risk control, the asset allocation committee, etc.) and therefore not all the drivers of performance are necessarily portable. The senior portfolio manager may be the major contributor, but could the same performance have been delivered by that portfolio manager in a different environment?

Relevant Provisions

1.A.31 When the firm jointly markets with other firms, the firm claiming compliance with the GIPS standards MUST ensure that the firm is clearly defined and separate relative to other firms being marketed and also that it is clear which firm is claiming compliance.

1.A.32 Performance from a past firm or affiliation may be used to represent the historical performance of the new or acquiring firm and linked to the performance of the new or acquiring firm if the new or acquiring firm meets the following requirements on a composite-specific or pooled fund-specific basis:

(a) Substantially all of the investment decision makers MUST be employed by the new or acquiring firm (e.g., research department staff, portfolio managers, and other relevant staff);

(b) The decision-making process MUST remain substantially intact and independent within the new or acquiring firm.

(c) The new or acquiring firm must have records to support the performance; and

(d) There MUST be no break in the track record between the past firm or affiliation and the new or acquiring firm.

If any of the above requirements are not met, the performance from a past firm or affiliation MUST NOT be linked to the ongoing performance record of the new or acquiring firm.

1.A.33 Performance from a past firm or affiliation may be used to represent the historical performance of the new or acquiring firm when there is a break in the track record between the past firm or affiliation and the new or acquiring firm if the new or acquiring firm meets the following requirements on a composite-specific or pooled fund-specific basis:

(a) Substantially all of the investment decision makers MUST be employed by the new or acquiring firm (e.g., research department staff, portfolio managers, and other relevant staff);
(b) The decision-making process MUST remain substantially intact and independent within the new or acquiring firm;
(c) The new or acquiring firm MUST have records to support performance;
(d) The new or acquiring firm MUST separately present the performance before the break and after the break; and
(e) The new or acquiring firm MUST NOT link performance prior to the break in the track record to the performance after the break in the track record.

1.A.34 If the firm acquires another firm or affiliation, the firm has one year to bring any non-compliant assets into compliance. Assets of the acquired non-compliant firm or affiliation MUST meet all the requirements of the GIPS standards within one year of the acquisition date, on a going forward basis.

The GIPS standards do, however, allow performance from a past firm to represent the historical performance of a new or acquiring firm and for that performance to be linked to the performance of the new or acquiring firm, on a specific composite or pooled fund basis, if certain requirements are met, namely:

- Substantially all the investment decision makers are employed by the new or acquiring firm.
- The decision-making process remains substantially intact and independent within the new or acquiring firm.
- The new or acquiring firm has records that document and support the reported performance.
- There is no break in the track record.

A break may occur in a track record if an investment team transfers from one firm to another and for a period of time were not able or did not have discretion to make investment decisions. The track record of the past firm may be available with the records to support the performance; thus, it is portable, but the performance of the portfolio managers while at the first firm may not be linked to their performance at the new firm because of the gap or break in the track record. The performance prior to the break must be presented separately.

If a GIPS-compliant firm acquires another firm or affiliation, the firm is given one year to bring any non-compliant assets into compliance.

⚠ Caution

The "portability" of past performance is a complex and sometimes contentious subject – one year might appear to be a long grace period, but portability is a complex area that frequently requires the whole year to resolve adequately.

The GIPS standards are written to ensure that portability is difficult to achieve and is most likely to occur in the event of merger or acquisition. Lift-outs of a team from one asset manager to another are possible, provided that substantially all the investment decision makers move and the decision-making process substantially remains intact, but if this lift-out is unfriendly it is most unlikely that the new firm will have sufficient documentation to support the track

record, and it may also be difficult to avoid a break in the track record. That said, a firm does not have to link all strategies of the acquired firm; performance may be linked on a specific composite or pooled fund basis, a relaxation when compared to the previous standard.

Note

The 2020 version of the GIPS standards is notably weaker than previous versions of the GIPS standards. Portability is now available on a composite-by-composite basis, which allows a certain element of self-selection at the time of acquisition or merger.

Use of time-weighted or money-weighted returns

Firms must present time-weighted returns unless certain criteria are met, in which case the firm may present money-weighted returns.

Relevant Provisions

1.A.35 The firm MUST present time-weighted returns unless certain criteria are met, in which case the firm may present money-weighted returns. The firm may present money-weighted returns only if the firm has control over the external cash flows into the portfolios in the composite or pooled fund and the portfolios in the composite have or the pooled fund has at least one of the following characteristics:

(a) Closed-end
(b) Fixed life
(c) Fixed commitment
(d) Illiquid investments as a significant part of the investment strategy

1.A.36 The firm MUST choose if it will present time-weighted returns, money-weighted returns, or both for each composite or pooled fund, and it must consistently present the selected returns for each composite or pooled fund.

A time-weighted return is defined as a method of calculating period-by-period returns that reflects the change in value and negates the effects of external cash flows. Typical time-weighted methodologies used in a GIPS context include true time-weighted returns and hybrid methods such as linked modified Dietz (see Chapter 3).

A money-weighted return is defined as the return for a period that reflects the change in value and the timing and size of external cash flows. The most common money-weighted return in the context of GIPS is the internal rate or return (IRR). Firms may present money-weighted returns only if the firm has control over the external cash flows of the portfolios in the composite or the pooled fund *and:*

(1) The portfolio is a closed-end, fixed life, or fixed commitment portfolio
or
(2) Illiquid investments are a significant part of the investment strategy.

Time-weighted returns are intended to neutralise the timing impact of external cash flows and allow fair comparison of performance track records of asset managers with different cash flow experience. Fair comparison of investment management performance is the core of the GIPS standards and therefore favours time-weighted returns; however, the GIPS standards recognise that time-weighted returns are not appropriate for all circumstances, in particular for illiquid assets such as private equity, infrastructure and real estate and for specific fund structures such as closed-end, fixed life and fixed commitment funds for which the presentation of money-weighted returns remains dominant within the industry. It should be noted that control of external cash flows is a necessary condition to use money-weighted rates of return, but this alone is not sufficient – the portfolio must also meet one of the other requirements specified by the GIPS standards.

Caution

Please review Chapter 3 to understand the various merits of time-weighted and money-weighted returns. The moment firms step away from time-weighted returns, comparability is compromised. The GIPS standards are explicitly designed to avoid firms taking the opportunity of the control of cash flow to switch to money-weighted returns and the potential opportunity to "game" returns or self-select methodologies. In my view money-weighted returns should not be used for liquid assets under any circumstances, with or without control of external cash flow.

Claiming compliance with the GIPS standards

A firm cannot represent that it is in compliance with the GIPS standards "except for" or make any other statements that might indicate partial compliance – no exceptions to the GIPS standards are permitted.

Relevant Provisions

1.A.37 If the firm chooses to include a GIPS Composite Report or GIPS Pooled Report in marketing materials, the firm MUST indicate this fact in the marketing materials.

1.A.38 The firm must notify CFA Institute of its claim of compliance by submitting the GIPS Compliance Notification Form. This form:

(a) MUST be filed when the firm initially claims compliance with the GIPS standards.

(b) MUST be updated annually with information as of the most recent 31 December, with the exception of firm contact information, which MUST be current as of the form submission date.

(c) MUST be filed annually thereafter by 30 June.

1.A.39 If the firm chooses to be verified, it MUST gain an understanding of the verifier's policies for maintaining independence and MUST consider the verifier's assessment of independence.

Statements characterising the calculation methodology used in a composite presentation as being in accordance with, in compliance with, or consistent with the GIPS standards, or similar statements, are prohibited.

Statements referring to the performance of a current portfolio as being "calculated in accordance with the Global Investment Performance Standards" are also prohibited except when a GIPS-compliant firm reports the performance of a segregated account to current clients or a pooled fund to current investors.

Note

The GIPS standards represent more than just return calculations. They are ethical standards to ensure fair representation and full disclosure; they rely on the integrity of input data, correctly developed, consistently applied policies and procedures, and appropriate presentation of performance.

Firms must notify CFA Institute of its claim of compliance by submitting the GIPS Compliance Notification Form. This form must be filed when the firm initially claims compliance with the GIPS standards, must be updated annually with information as of the most recent 31 December and must be filed annually by 30 June. The firm must not claim compliance with the GIPS standards unless the GIPS Compliance Notification Form has been submitted to CFA Institute. The GIPS Compliance Notification Form must reflect the definition of the firm used to determine firm-wide compliance and not the legal entity of the firm.

INPUT DATA AND CALCULATION METHODOLOGY

Section 2 of the GIPS Standards for Firms cover input data and calculation methodology. The standards provide a blueprint for the consistency of input data crucial for effective compliance and full and fair comparisons of investment performance. There are 50 required provisions in Section 2, covering:

(1) Firm assets, composite assets and pooled fund assets
(2) Overlay exposure
(3) Returns
(4) Valuation
(5) Time-weighted returns
(6) Money-weighted returns
(7) Net returns
(8) Composite returns
(9) Private market investments
(10) Real estate
(11) Net-of-fees carve-out returns
(12) Wrap fees, side pockets and subscription lines of credit

Firm assets, composite assets and pooled fund assets

Assets under management (total firm assets or firm-wide assets) is not strictly performance-related information but is often associated with performance presentations as part of the process of demonstrating competency managing a particular asset class.

Relevant Provisions

2.A.1 Total Firm Assets:

(a) MUST be the aggregate fair value of all discretionary and non-discretionary assets managed by the firm. This includes both fee-paying and non-fee-paying portfolios.
(b) Must include assets assigned to a subadvisor provided the firm has discretion over the selection of the subadvisor.
(c) MUST NOT include advisory-only assets.
(d) MUST NOT include uncalled committed capital.

2.A.2 Total Firm Assets, Composite Assets, and Pooled Fund Assets MUST:

(a) Include only actual assets managed by the firm.
(b) Be calculated net of discretionary leverage and not grossed up as if leverage did not exist.

2.A.3 The firm MUST NOT double-count assets when calculating total firm assets, composite assets, or pooled fund assets.

2.A.4 Composite and pooled fund performance MUST be calculated using only actual, assets managed by the firm.

Total firm assets represent the aggregate fair value of assets for which the firm has investment responsibility. The definition of the firm determines the boundaries of which portfolios should or should not be included. Total firm assets include the assets of discretionary and non-discretionary portfolios and fee-paying and non-fee-paying portfolios but excludes advisory-only assets and uncalled committed capital.

Advisory-only assets are assets for which the firm provides investment recommendations, and the firm has no control over implementation of investment decisions and does not have trading authority. Advisory-only assets may not be included in total firm assets, but a firm may wish to report firm-wide advisory-only assets either as a separate value or in addition to reporting total firm assets, total firm assets plus firm-wide advisory-only assets. For non-discretionary assets, typically the firm implements investment decisions and has authority to trade but does not have sufficient flexibility to fully implement the investment strategy. Without sufficient flexibility to implement strategy, asset managers should not be held fully responsible for the resulting performance.

Uncalled committed capital is the amount of capital pledged to an investment vehicle that remains undrawn, and in fact may never be drawn down. Uncalled committed capital may not be included in total firm assets, but a firm may wish to report firm-wide uncalled committed capital either as a separate value or in addition to reporting total firm assets, total firm assets plus uncalled committed capital.

Overlay exposure

An overlay strategy is a strategy in which management of part of an investment strategy is carried out separately from the underlying portfolio.

Relevant Provisions

2.A.5 Total firm overlay exposure MUST include all discretionary and non-discretionary overlay strategy portfolios for which the firm has investment management responsibility.

2.A.6 When calculating overlay exposure, the firm MUST:

(a) Use the notional exposure of the overlay strategy portfolios, the value of the underlying portfolios being overlaid, or a specified target exposure.

(b) Use the same method for all portfolios within a composite.

2.A.7 When calculating overlay strategy portfolio returns, the firm MUST:

(a) Use as the denominator the notional exposure of the overlay strategy portfolio, the value of the underlying portfolio being overlaid, or a specified target exposure.

(b) Use the same method for all portfolios within a composite.

Overlay strategies typically consist of derivative instruments used to change the risk profile of the underlying assets. Overlay exposure is the notional value of the overlay strategy, the value of the underlying assets being overlaid or a target exposure.

Overlay exposure may not be included in total firm assets, but firms may present firm-wide overlay exposure. For overlay exposure composites only, firms may present firm-wide overlay exposure rather than total firm assets. Overlay exposure is a better representation of the relative importance of the overlay investment strategy than assets under management.

Returns

Firms must follow general accounting and performance measurement principles in order to calculate returns that meet the requirements of the GIPS standards.

Relevant Provisions

2.A.8 Total returns MUST be used.

2.A.9 Trade date accounting MUST BE used.

2.A.10 Accrual accounting MUST be used for fixed-income securities and all other investment that earn interest income, except that interest income on cash and cash equivalents may be recognized on a cash basis. Any accrued income MUST be included in the beginning and ending portfolio values when performance is calculated.

2.A.11 Returns from cash and cash equivalents MUST be included in all return calculations, even if the firm does not control the specific cash investment(s).

2.A.12 Returns for periods of less than one year MUST NOT be annualized.

2.A.13 All returns MUST be calculated after the deduction of transaction costs incurred during the period. The firm may use estimated transaction costs only for those portfolios for which actual transaction costs are not known.

2.A.14 For portfolios with bundled fees, if the firm cannot estimate transaction costs or if actual transaction costs cannot be segregated from a bundled fee:

(a) When calculating gross-of-fees returns, returns MUST be reduced by the entire bundled fee or the portion of the bundled fee that includes the transaction costs.

(b) When calculating net-of-fees returns, returns MUST be reduced by the entire bundled fee or the portion of the bundled fee that includes the transaction costs and the investment management fee.

2.A.15 All required returns MUST be calculated net of discretionary leverage unless otherwise specified.

2.A.15 All required returns MUST be calculated net of discretionary leverage unless otherwise specified.

2.A.16 The firm MUST calculate performance in accordance with its composite-specific or pooled fund-specific calculation policies.

2.A.17 For portfolios invested in underlying pooled funds, all returns MUST reflect the deduction of all fees and expenses charged at the underlying pooled fund level, unless the firm controls the investment management fees of the underlying pooled funds. When the firm controls the investment management fees of the underlying pooled funds, the firm may calculate gross-of-fees returns that do not reflect the deduction of the underlying pooled fund investment management fees.

2.A.18 When calculating additional risk measures:

(a) The periodicity of the composite or pooled fund returns and the benchmark MUST be the same.

(b) The risk measure calculation methodology of the composite or pooled fund and the benchmark MUST be the same.

Return calculations must reflect realised gains and losses, unrealised gains and losses and income.

Transactions should be trade date, not settlement date. Trades may not be settled for some time; hence settlement date may lag trade date by a few days (this will vary between different market and asset types). The moment a trade is confirmed the portfolio is exposed to price movements; this is particularly important if the gap between the trade date and settlement date straddles a period-end date.

In order to reflect the true economic value of the portfolio, accrual accounting reflecting the impact of income when earned and expenses when incurred, rather than when income is received and when expenses are paid, is more appropriate. The GIPS standards require that accrual accounting is utilised for fixed income securities and all investments that earn interest (except cash) and only recommend accrual accounting for dividends. Accrual accounting should be employed for both beginning period and end period values in order to provide the correct denominator in the return calculation. Accrued income is in effect capital available to the asset manager; excluding accrued income at the start of the period inappropriately leverages return.

Note

The GIPS standards do not require accrual accounting for equity dividends. This is a pragmatic reflection that many accounting systems do not accrue for equity dividends, but it is **not** best practice. My very strong recommendation is to always accrue for equity dividends. Most commercial equity indexes accrue dividends.

Returns from cash, near cash and cash equivalents must be included in return calculations. The allocation to cash in a portfolio, rather than the return on cash in the portfolio, is a significant contributor to relative return. The return on cash itself may be the responsibility of a third party (i.e., custodian or money market fund) but it is important to include all elements in the return of the portfolio.

The GIPS standards do not permit the bad practice of annualising returns for periods of less than one year. It is reasonable to annualise down for periods longer than one year but annualising up makes the implicit assumption that the return achieved so far in the year can be repeated for the remainder of the year – which is not a valid assumption (with the possible exception of cash returns).

All return calculations (gross and net of fees) must reflect transaction costs. Transaction costs include brokerage commission, exchange fees and taxes (stamp duty) but do not include custodial fees. If permitted by local regulation, firms may estimate transaction costs if actual transaction costs are not known.

Note

Gross-of-fees return is intended to reflect the underlying return of the asset manager before deducting their fees. Transaction costs are part of the underlying return caused by asset manager decisions and therefore should always be included in the gross return.

Net-of-fees return is intended to reflect the underlying return of the asset manager after deducting their fees. Custodial fees (transaction related or otherwise) represent a different relationship between custodian and asset owner and do not necessarily reflect the underlying performance of the asset manager.

Bundled fees include multiple fee types "bundled" into a single fee. If firms are unable to identify the portion of a bundled fee that includes transaction costs, then the entire bundled fee must be deducted when calculating gross-of-fee returns. This might seem unfair, but it would certainly be unfair to allow the entire bundled fee to be grossed-up.

Returns must be calculated net of discretionary leverage. Net of leverage in this context means after borrowings, or in other words, the denominator of the return calculation. Take, for example, a portfolio of £5,000,000 that borrows a further £1,000,000. The gross assets are £6,000,000 but the net assets remain £5,000,000.

Firms must create composite-specific policies and procedures for calculating the returns of portfolios in the composite and calculating the return of the composite itself. These

policies must be applied consistently and apply to outsourced service providers. Different methodologies for different types of portfolios (i.e., pooled funds and segregated accounts) may be included in the same composite but must be applied consistently. The GIPS Standards Handbook provides examples of items firms should address in their policies and procedures, including:

- How the firm ensures that the information from third-party service providers meets the requirements of the GIPS standards and can be used, as necessary, to produce returns that comply with the GIPS standards.
- The policies for estimating transaction costs, if estimated transaction costs are used.
- The fees and expenses deducted when calculating gross-of-fees returns and net-of-fees returns.
- The methodology for calculating a time-weighted return (TWR) for portfolios and composites for which the firm presents a TWR.
- The methodology for calculating a money-weighted return (MWR) for portfolios and composites for which the firm presents an MWR.
- The calculation methodology for portfolios with external cash flows.
- The treatment of reclaimable withholding taxes when recording interest income and dividends.
- How model investment management fees are calculated, if used in calculating net-of-fee returns.
- The treatment of performance-based fee clawbacks, if any.
- The treatment of side pockets, if any.

When investing in underlying pooled funds, all returns must reflect the deduction of all fees and expenses charged by the underlying pooled fund, unless the firm controls the investment management fees of the underlying pooled funds, in which case these fees may be grossed up for gross-of-fees returns. Clearly, if the firm does not control the investment management fees of the pooled fund, then the fees charged are part of the investment decision process to purchase the pooled fund. In a fund-of-funds vehicle the fees charged might reflect the investment management fees of the underlying funds and therefore it is perfectly reasonable to gross-up the fees of the underlying funds to represent gross-of-fees returns.

For all the reasons described in detail in Chapter 5, the periodicity specifically, and the methodology in general, of the composite and the benchmark risk measures must be the same.

Valuation

Accurate return calculations require accurate valuations. Portfolio valuations must be based on fair value.

Note

Fair value

Fair value is defined as the amount at which an investment could be sold in an arm's-length transaction between willing parties in an orderly transaction.

Relevant Provisions

2.A.19 Portfolios MUST be valued in accordance with the definition of fair value.

2.A.20 The firm MUST value portfolios in accordance with the composite-specific or pooled fund-specific valuation policy.

2.A.21 If the firm uses the last available historical price or preliminary value as fair value the firm MUST:

(a) Consider it to be the best approximation of the current fair value.

(b) Assess the difference between the approximation and final value and the effect on composite or pooled fund assets, total firm assets, and performance, and also make any adjustments when the value is received.

2.A.22 Composites and pooled funds MUST have consistent beginning and ending annual valuation dates. Unless the composite or pooled fund is reporting on a non-calendar fiscal year, the beginning and ending valuation dates MUST be at calendar year-end or on the last business day of the year.

Fair value must reflect the market value of the security – the objective, observable, unadjusted quoted price if available. If not available, the firm's policies and procedures must address valuation. The GIPS standards provide a recommended valuation hierarchy in recommendation 2.B.6:

Recommended Valuation Hierarchy

(1) Investments must be valued using objective, observable, unadjusted quoted market prices for identical investments in active markets on the measurement date, if available (i.e., market value). If such prices are not available, then investments should be valued using:

(2) Objective, observable quoted market prices for similar investments in active markets (e.g., market prices for investments within the same asset class with similar characteristics to those of the investment being valued). If such prices are not available or appropriate, then investments should be valued using:

(3) Quoted prices for identical or similar investments in markets that are not active (markets in which there are few transactions for the investment, the prices are not current, or price quotations vary substantially over time and/or between market makers). If such prices are not available or appropriate, then investments should be valued based on:

(4) Market-based inputs, other than quoted prices, that are observable for the investment (e.g., reported earnings). If such inputs are not available or appropriate, then investments should be valued based on:

(5) Subjective, unobservable inputs for the investment where markets are not active at the measurement date. Unobservable inputs should be used to measure fair value only when observable inputs and prices are not available or appropriate. Unobservable inputs reflect the firm's own assumptions about the assumptions that market participants use in pricing the investment and should be developed based on the best information available under the circumstances. Unobservable inputs may be based on the reporting company's own data, adjusted using other available information, such as default rates or discounts for lack of liquidity.

Market value and fair value will be the same for actively traded securities. Subjective, unobservable inputs represent the last rung of the valuation hierarchy. Typically, firms use proprietary pricing models together with a pricing committee or other oversight group. If the firm uses a preliminary or estimated value as the fair value, the firm is making the assumption that the value used is the best approximation of fair value.

To aid comparability, valuation dates must be at calendar year-end or on the last business day of the year. Unfortunately, public holidays vary across the world and from time to time last business days are not directly comparable.

The GIPS standards recommend that valuations be obtained from a qualified independent third party at least once every 12 months for private market investments such as private equity, infrastructure and similar illiquid assets.

Time-weighted returns

The real core of the GIPS standards (together with consistency and transparency) is comparability; therefore, it is no surprise that time-weighted returns are at the core of return calculation. True time-weighting and linked modified Dietz are the two most common acceptable methodologies. Approximations using benchmarks such as analyst's test, index substitution and the regression method are not acceptable.

Relevant Provisions

2.A.23 When calculating time-weighted returns for portfolios that are included in composites, all portfolios except private market investment portfolios (see 2.A.40) must be valued:

(a) At least monthly.
(b) As of calendar month-end or the last business day of the month.
(c) On the date of all large cash flows. The firm MUST define large cash flow for each composite to determine when portfolios in that composite MUST be valued.

2.A.24 When calculating time-weighted returns for all portfolios except private market investment portfolios (see 2.A.41) included in composites, the firm MUST:

(a) Calculate returns at least monthly.
(b) Calculate monthly returns through the calendar month-end or the last business day of the month.
(c) Calculate subperiod returns at the time of all large cash flows, if daily returns are not calculated.
(d) For external cash flows that are not large cash flows, calculate portfolio returns that adjust for daily weighted external cash flows, if daily returns are not calculated.
(e) Treat external cash flows according to the firm's composite-specific policy.
(f) Geometrically link period and subperiod returns.
(g) Consistently apply the calculation methodology used for an individual portfolio.

(*continued*)

(*continued*)

2.A.25 When calculating time-weighted returns for pooled funds that are not included in a composite, pooled funds MUST be valued:

(a) At least annually.
(b) As of the calendar or fiscal year-end.
(c) Whenever there are subscriptions to or redemptions from the pooled fund.
(d) As of the period-end for any period for which performance is calculated.

2.A.26 When calculating time-weighted returns for pooled funds that are not included in a composite, the firm MUST:

(a) Calculate returns at least annually.
(b) Calculate annual returns through the calendar or fiscal year-end or the last business day of the year.
(c) Calculate subperiod returns at the time of all subscriptions and redemptions.
(d) Geometrically link periodic and subperiod returns.
(e) When calculating pooled fund net returns, calculate pooled fund net returns that are net of total pooled fund fees.

2.A.27 The firm MUST establish a pooled fund inception date for each pooled fund to determine when the pooled fund's track record begins.

Time-weighted returns are favoured in GIPS because of the need for comparability. For fair comparison the impact of cash flows must be removed. Requiring valuations at the point of cash flow increases the theoretical level of accuracy and removes the opportunity to game returns by self-selecting the approximate methodology most advantageously impacted by cash flow.

When calculating time-weighted returns the GIPS standards require that portfolios are valued at least monthly as of the calendar month-end or the last business day of the month to facilitate comparability of information. Firms are also required to value portfolios on the date of all large cash flows (see Chapter 3). Large cash flows are defined by the firm for each composite at a level at which the firm determines that an external cash flow may distort returns. The level must be determined and documented in advance, may be a single value or aggregated over a finite period, consists of cash and/or assets and may be described as an absolute value or a percentage of portfolio or composite value.

⚠ Caution

Firms must not define a large cash flow at a high level solely for the purpose of avoiding the need to recalculate returns.

The requirement to adjust for daily weighted cash flows is, in effect, a direction to use the modified Dietz method. The requirement to link period and subperiod returns is the essence of time-weighting, although not necessarily true time weighting. Revaluing for a single large external cash flow may result in two subperiods consisting of modified Dietz calculations; these sub-periods and, indeed, monthly periods are geometrically linked resulting in a hybrid time-weighted methodology.

The inception date for a broad distribution pooled fund is typically when the fund first calculates a net asset value unit price. For a limited distribution fund, the inception date may be when:

(1) Investment management fees are first charged;
(2) The first investment-related cash flow takes place;
(3) The first capital call is made; or
(4) The first committed capital is closed and legally binding.

Money-weighted returns

Time-weighted returns may be the core of the GIPS standards; however, the standards recognise that time-weighted returns are not always appropriate and in certain restricted circumstances allow the option of a money-weighted return.

Relevant Provisions

2.A.28 When calculating money-weighted returns, the firm MUST value portfolios at least annually and as of the period-end for any period for which performance is calculated.

2.A.29 When calculating money-weighted returns, the firm MUST:

(a) Calculate annualized since-inception money-weighted returns.
(b) Calculate money-weighted returns using daily external cash flows.
(c) Include stock distributions as external cash flows and value stock distributions at the time of distribution.
(d) When calculating pooled fund net returns, calculate pooled fund net returns that are net of total pooled fund fees.

In the context of the GIPS standards, a money-weighted return implies an internal rate of return methodology (IRR) (see Chapter 3). For internal rate of return, valuations are only needed at the end of the period of measurement; therefore, the GIPS standards only require valuations at least annually.

Note

Frequency of valuation

Although for money-weighted returns firms must only value portfolios at least annually, more frequent valuations are generally required for client reporting. Typically, money-weighted returns imply illiquid assets. I appreciate the difficulties, costs and timing issues of more frequent valuations; however, I strongly recommend a minimum of quarterly valuations.

Since-inception money-weighted rates of return cover the entire investment period since inception and do not involve linking of any subperiods, simply a single calculation over the entire period. Since-inception internal rates of return are the standard calculation methodology for most illiquid assets.

Net returns

Simply put, net-of-fees returns are gross-of-fees returns reduced by the asset manager's fee.

Relevant Provisions

2.A.30 When calculating composite net-of-fees returns, investment management fees used in the calculation MUST be either:

(a) Actual investment management fees incurred by each portfolio in the composite, or

(b) A model investment management fee appropriate to prospective clients.

2.A.31 If the firm uses model investment management fees to calculate composite net-of-fees returns, the returns calculated MUST be equal to or lower than those that would have been calculated using investment management fees.

2.A.32 When calculating pooled fund net returns, total pooled fund fees used in the calculation MUST be either:

(a) Actual total pooled fund fees, or

(b) A model total pooled fund fee appropriate to prospective investors.

2.A.33 If the firm uses model total pooled fund fees to calculate pooled fund net returns, the returns calculated MUST be equal to or lower than those that would have been calculated using actual total pooled fund fees.

2.A.34 When calculating composite net-of-fees returns and pooled fund net returns, the firm MUST reflect any performance-based fee clawback in the period in which it is repaid.

Fees are typically "ad valorem" based on asset values and include performance fees. The net-of fees calculation must use actual fees or a model fee appropriate to the prospective client receiving the GIPS Report. Model fee returns must be lower or equal to the net-of-fee return that would have been calculated if actual fees had been used. Performance-based fee clawback, the repayment of previously earned performance fees resulting from subsequent underperformance, must be reflected in the period that it is repaid.

Note

The unwinding of accrued performance fees and/or actual clawback may in rare circumstances result in net-of-fees returns greater than gross-of-fees returns for finite periods – not intuitive, but correct, nevertheless.

Composite returns

A composite is an aggregation of a number of portfolios into a single group that represents a particular investment strategy or objective. Composites represent the asset manager's fair and honest representation of performance, "warts and all".

Relevant Provisions

2.A.35 Composite time-weighted returns except private market investment composites (see 2.A.42) MUST be calculated at least monthly.

2.A.36 Composite time-weighted returns MUST be calculated by using one of the following approaches:

(a) Asset-weighting the individual portfolio returns using beginning-of-period values;

(b) Asset-weighting the individual portfolio returns using a method that reflects both beginning-of-period values and external cash flows; or

(c) Using the aggregate method.

2.A.37 Composite gross-of-fees returns must reflect the deduction of transaction costs.

2.A.38 Composite net-of-fees returns must reflect the deduction of transaction costs and investment management fees.

2.A.39 When calculating composite money-weighted returns, the firm MUST calculate composite returns by aggregating the portfolio-level information for those portfolios included in the composite.

To create composites returns for use in presentations, individual portfolio returns are combined using one of three asset-weighted methodologies (see Chapter 3):

(1) Beginning period weights
(2) Beginning period weight plus cash flows (in effect the denominator of the modified Dietz method)
(3) Total aggregation (basically combining the assets of all the qualifying portfolios and calculating the return as a single portfolio).

Note

Composite asset weighting

Of course, different asset-weighting methodologies will lead to different results but there is no structural bias, and all are acceptable. For simplicity I would recommend beginning-period weights.

Clearly, larger portfolios have a more significant influence on composite returns. Equal-weighted composites were initially considered by the standard setters but declined because of the potential to "game" results in small portfolios.

Private market investments

Private market investments include illiquid assets such as private equity, private debt, infrastructure and real estate.

Relevant Provisions

2.A.40 When calculating time-weighted returns for private market investment portfolios that are included in composites, private market investment portfolios must be valued:

(a) At least quarterly.
(b) As of each quarter-end or the last business day of the quarter.

2.A.41 When calculating time-weighted returns for private market investment portfolios that are included in composites, the firm MUST:

(a) Calculate returns quarterly.
(b) Calculate quarterly returns through the calendar quarter-end or the last business day of the quarter.
(c) Calculate portfolio returns that adjust for daily weighted external cash flows.
(d) Treat external cash flows according to the firm's composite-specific policy.
(e) Geometrically link period and subperiod returns.
(f) Consistently apply the calculation methodology used for an individual portfolio.

2.A.42 Composite time-weighted returns for private market investment composites must be calculated at least quarterly.

When calculating time-weighted returns for private market investment, assets must be valued at least quarterly, and quarterly modified Dietz calculations may be geometrically linked.

Real estate

Unless client agreements state otherwise, real estate investments must be externally valued by a qualified independent third party.

Relevant Provisions

2.A.43 Real estate investments in a real estate open-end fund MUST have an external valuation at least once every 12 months.

2.A.44 Real estate investments that are not in a real estate open-end fund MUST:

(a) Have an external valuation at least once every 12 months unless client agreements stipulate otherwise, in which case real estate investments MUST have an external valuation at least every 36 months or per the client agreement if the client agreement requires external valuations more frequently than every 36 months; or
(b) Be subject to an annual financial statement audit performed by an independent public accounting firm. The real estate investments MUST be accounted for at fair value, and the most recent audited financial statements available MUST contain an unmodified opinion issued by an independent public accounting firm.

2.A.45 External valuations for real estate investments MUST be performed by an independent third party who is a professionally designated or certified commercial property valuer or appraiser. In markets where these professionals are not available, the firm MUST take necessary steps to ensure that only qualified independent property valuers or appraisers are used.

2.A.46 The firm MUST NOT use external valuations for real estate investments when the valuer's or appraiser's fee is contingent upon the investment's appraised value.

If client agreements state otherwise, external valuations may be at longer frequencies up to 36 months, mainly to reduce costs. Annual financial statement audits may substitute external valuations.

Net-of-fees carve-out returns

Carve-outs in the context of the GIPS standards are covered in more detail later in this chapter.

Relevant Provisions

2.A.47 When calculating net-of-fees returns of composites containing carve-outs, the investment management fees for carve-outs MUST be representative of the investment management fees charged or that would be charged to the prospective client:

(a) When presenting performance to a prospective client for a standalone portfolio, the investment management fee MUST be representative of the investment management fees for a standalone portfolio managed according to that strategy.

(b) When presenting performance to a prospective client for a multi-asset strategy portfolio, the investment management fee must be representative of the investment management fees for a multi-asset strategy portfolio managed according to that strategy.

A carve-out is a subset of a multi-strategy portfolio's assets. By their very nature, carve-out return calculations do not normally include fees. In most cases it will be necessary to choose a model fee in order to calculate net-of-fees carve-out returns. The investment management fee of the broader multi-strategy portfolio may not be appropriate for the narrower carve-out element – a fee rate appropriate for carved-out strategy must be used.

Wrap fee, side pockets and subscription lines of credit

Wrap fees, side pockets and subscription lines of credit are highly technical elements of the standards that are not relevant for most firms.

Relevant Provisions

2.A.48 When calculating returns to be presented to a wrap fee prospective client, returns MUST be calculated net of the entire wrap fee. This is applicable to all wrap fee portfolios in the composite as well as any non-wrap fee portfolios in the composite.

2.A.49 All composite and pooled fund returns MUST include the effect of any discretionary side pockets held by portfolios in the composite or the pooled fund.

2.A.50 When calculating money-weighted returns for composites and pooled funds without the subscription lines of credit, the firm MUST include the cash flows from the subscription lines of credit.

Side pockets are segregated investments mainly used by alternative investment pooled funds to separate distressed or illiquid assets from more liquid assets. Side pockets are typically not available to new investors. Although not available to new investors, they represent the true performance of the pooled fund and must be included in all return calculations.

Caution

Side pockets clearly have a legitimate purpose. Distressed, illiquid and hard-to-value assets can be "parked" aside. They allow existing investors to redeem units while still retaining an exposure to illiquid or low-value assets and avoid new investors buying into the fund at an artificially low value and diluting future gains or buying in at an artificially high value. They do muddy the waters, though; the performance experience is different for different classes of investors and there are other issues related to ongoing fees, performance fees and fairness. Care needs to be taken to ensure that the performance of side pockets is appropriately reflected in the asset manager's track record and is not excluded.

A subscription line of credit is a credit facility secured against uncalled committed capital used to facilitate the management of cash flow, initially for short periods (just a few days) but now increasingly used for longer periods.

Caution

Some critics[9] might argue that subscription lines of credit distort performance measures, generating leverage and altering the timing of external cash flows. Should the return calculation be based on the investor's experience of cash flow or the strategy's experience of cash flow? The GIPS standards facilitate understanding the impact of subscription lines of credit on the return calculation.

[9] Kazemi, Subscription Line of Credit: Benefits, Risks and Distortions, CAIA Association, September 2020.

COMPOSITE AND POOLED FUND MAINTENANCE

Composites are the key tool that allows asset managers to provide to asset owners a fair and honest representation of investment strategy performance, and as such, constitute an essential element of the GIPS standards. Section 3 of the GIPS standards for firms includes 19 provisions related to composite maintenance, including carve-outs.

Composite maintenance

A composite is an aggregation of one or more portfolios managed according to a similar investment mandate, objective or strategy. A portfolio may be a segregated account owned by a single asset owner or a pooled fund owned by multiple asset owners. Creating meaningful composites is essential to the fair representation, consistency and comparability of performance over time and among firms. Pooled funds must be included in composites if they meet a composite definition. The firm is not required to create a composite that only includes one or more pooled funds unless the firm offers the strategy as a segregated account.

Note

Typically, pooled funds are well regulated in most markets. The GIPS standards are written from the perspective of encouraging asset managers of pooled funds to become compliant without introducing onerous requirements that are not necessary. Requiring composites consisting exclusively of pooled funds, where those pooled funds are exclusively offered to prospective investors (not clients), is unnecessary. However, if the exact same strategy is offered as a segregated account to a prospective client, then the composite must exist and include the pooled fund.

Note

According to the GIPS standards, pooled funds must be included in composites if they meet a composite definition. While this is a clear provision, I would suggest that in most cases pooled funds don't meet the typical composite definitions designed for segregated accounts. Pooled funds return profiles are different from segregated accounts: they suffer/enjoy constant cash flow, typically they retain cash levels to meet redemption requirements or take time to process new allocations. Segregated accounts and closed-end funds offer more flexibility to invest in less liquid assets than open-ended pooled funds.

Relevant Provisions

3.A.1 The firm MUST create composites for the firm's strategies that are managed for or offered as a segregated account.

3.A.2 All actual, fee-paying, discretionary segregated accounts MUST be included in at least one composite. Non-discretionary portfolios MUST NOT be included in composites.

3.A.3 All actual, fee-paying, discretionary pooled funds MUST be included in at least one composite if they meet a composite definition. The firm is not required to create a composite that only includes one or more pooled funds unless the firm offers the strategy as a segregated account. The firm may terminate any composite that was created solely to include one or more pooled funds if the composite is not representative of the firm's strategy offered as a segregated account.

3.A.4 Non-fee-paying discretionary portfolios may be included in a composite. If the firm includes non-fee-paying discretionary portfolios in a composite, those portfolios MUST be subject to the same policies and procedures as fee-paying portfolios.

3.A.5 Composites MUST be defined according to investment mandate, objective or strategy. Composites MUST include all portfolios, including segregated accounts and pooled funds that meet the composite definition. The firm MUST NOT exclude portfolios from composites based solely on legal structure differences.

3.A.6 Any change to composite definition MUST NOT be applied retroactively.

3.A.7 Composites MUST include new portfolios on a timely and consistent composite-specific basis after each portfolio comes under management.

3.A.8 Composites MUST include only those portfolios that are managed for the full performance measurement period for which the composite return is calculated. Portfolios that are not managed for the full measurement period MUST NOT be included in the composite.

3.A.9 Terminated portfolios MUST be included in the historical performance of the composite up to the last full measurement period that each portfolio was under management and for which the firm has discretion.

3.A.10 Portfolios MUST NOT be moved from one composite to another unless documented client-directed changes to a portfolio's investment mandate, objective, or strategy or the redefinition of the composite make it appropriate. The historical performance of the portfolio MUST remain with the original composite. Portfolios MUST NOT be moved into or out of composites as a result of the firm's tactical changes.

3.A.11 If a firm sets a minimum asset level for portfolios to be included in a composite, the firm:

(a) MUST NOT include portfolios below the composite-specific minimum asset level in that composite.

(b) MUST NOT apply retroactively any changes to that composite-specific minimum asset level.

3.A.12 A firm that removes portfolios from composites because of significant cash flows MUST define "significant" on an ex-ante, composite-specific basis and MUST consistently follow the composite-specific policy.

3.A.13 A firm that uses temporary new accounts to remove the effect of a significant cash flow MUST establish policies on an ex-ante, composite-specific basis. Temporary new accounts MUST NOT be included in composite performance.

3.A.14 The firm MUST include the performance record of actual wrap fee portfolios in appropriate composites in accordance with the firm's established portfolio inclusion policies. Once established, these composites (containing actual wrap fee portfolios) MUST be used when presenting GIPS Composite Reports to wrap fee prospective clients.

In order to promote fair representations of performance, the GIPS standards require firms to create and maintain composites for all strategies for which the firm manages segregated accounts or markets as a segregated account. Firms must include all actual fee-paying, discretionary, segregated accounts in at least one composite defined by investment mandate, objective, or strategy. Firms must maintain and make available information about all of the strategies they manage using composites or pooled funds. All accounts managed to that strategy, including lost accounts, must be included, thus eliminating the practice of "cherry-picking" good performing accounts. Every portfolio must belong to at least one composite (to avoid the performance record of a poor performing account being lost), therefore composite definitions may overlap. Non-fee-paying portfolios may be included in a composite provided that they are discretionary and subject to the same policies and procedures as fee-paying portfolios, but non-discretionary portfolios must not be included in a firm's composites.

The firm's definition of discretion is based on the firm's freedom to implement its investment strategies. A portfolio is discretionary if the firm is able to implement the intended investment strategy.

If the asset owner imposes restrictions on the firm's freedom to make investment decisions to buy, hold and sell securities, or requires approval to be sought before initiating a transaction, in the course of implementing the investment strategy, then the firm must consider whether the portfolio is, in fact, discretionary. In general, restrictions that impede the investment process to such an extent that the strategy cannot be implemented as intended may be presumed to render the portfolio non-discretionary.

In some cases, the asset owner's investment constraints may inhibit the asset manager's flexibility. An asset owner might prohibit investment in securities issued by companies operating in industries they consider socially unacceptable, such as alcohol, armaments, fossil fuels, tobacco or gambling. An asset owner might prohibit the sale of company stock, or a foundation might similarly ban the sale of "sentimental holdings", securities issued by the company in which its founder made a fortune. Additionally, legal restrictions may apply. For instance, a public fund might be statutorily precluded from investing in nondomestic securities. None of these constraints automatically renders a portfolio non-discretionary. Rather, in these and other cases, the asset manager must determine whether or not these restrictions materially impact the asset manager's ability to execute the composite strategy. In some cases, where there are multiple portfolios with similar constraints it may be appropriate to classify a portfolio as discretionary and include it in a composite with other, similarly constrained portfolios.

Recognising that degrees of discretion exist, the firm must consider the interactions among asset owner-directed constraints, the portfolio's strategy or style, and the investment process,

notably including the financial instruments employed. For example, an asset owner's investment policy might prohibit the use of derivative securities such as futures, swaps and options. In this case, the firm must consider whether the restriction is pertinent. The fact that the asset owner prohibits the use of derivatives may be irrelevant if the asset manager simply buys, holds and sells common stocks. If the use of derivative securities is central to the firm's implementation of the investment mandate, however, then the asset owner's policy may render the portfolio non-discretionary.

In some cases, the pattern of external cash flows might make a portfolio non-discretionary. For example, if an asset owner frequently makes large withdrawals, perhaps on a regular schedule, the asset manager might have to maintain such a high level of liquidity that they cannot truly implement the investment strategy as they do for other portfolios with a similar stated investment mandate, objective or strategy.

The firm's definition of discretion establishes criteria by which the firm determines which portfolios must be included in a composite. If an actual, fee-paying segregated account is discretionary, it must be included in at least one composite; if it is not discretionary, it must not be included in any composite. What is discretionary to one firm might be non-discretionary to another. Likewise, what is considered discretionary to the asset owner might be considered non-discretionary to the firm and vice versa. The firm must make the determination of discretionary status consistent with their definition of discretion. This determination must be documented and consistently applied over time.

⚠ Caution

Discretion may be defined at the composite or portfolio level which, frankly, in my view is a step too far. Discretion should be defined and applied consistently across the firm. Defining discretion at a lower level opens the firm to the charge of defining portfolios as non-discretionary in order to lose a poor performance track record. Such action, of course, is not permitted.

Composites must be defined according to investment mandate, objective or strategy; composites must include all segregated accounts that meet the composite definition as documented in the firm's policies and procedures; and the composite description, including detailed criteria that determine the assignment of portfolios to the composite, must be made available upon request. The firm must not exclude portfolios from composites based solely on legal structure differences. Well-defined composites will be objectively representative of the firm's products and consistent with the firm's marketing strategy.

The composite definition provides the detailed criteria that determine the assignment of portfolios to composites. Criteria may include, but are not limited to, investment mandate, style or strategy, asset class, the use of derivatives, leverage and/or hedging, targeted risk metrics, investment constraints or restrictions, and/or portfolio type.

Firms may define composites based on the portfolios' benchmarks, as long as the benchmarks reflect the investment strategy and the firm has no other composites with the same characteristics.

Composite definitions must be documented in the firm's policies and procedures.

If a firm manages portfolios with overlay strategies, the firm is required to create an overlay composite for an overlay strategy when the overlay strategy is managed separately from the underlying portfolio and the firm offers the overlay strategy as a segregated account. Firms are not required to create an overlay strategy composite when the overlay strategy is implemented as part of a broader strategy but may do so.

Reporting currency must not be used as a criterion for composite definition unless it affects the investment strategy. Portfolios with different base currencies may be included in the same composite, but their assets and returns must be expressed in the same currency as the composite. A hedging strategy will most likely require establishing additional composites. The return of a hedged to US $ strategy is very different in return profile to a hedged to Swiss franc strategy – these do not belong in the same composite.

Note

Composite returns are typically calculated in a base currency, but these base currency returns can be converted for any currency for presentation using the currency conversion techniques described in Chapter 3. However, converting to a hedged return is in effect a model return and hence not allowed.

New portfolios must be included in a composite on a timely and consistent basis after the portfolio comes under management. Firms are required to establish, document and consistently apply a policy of including new portfolios in the appropriate composites on a timely basis. It may take time to invest the assets of a new portfolio in accordance with the desired investment strategy, however, particularly when the portfolio is funded in kind (that is, with securities other than cash and cash equivalents) and the portfolio has to be repositioned to align with the firm's strategy, or when the securities to be purchased are relatively illiquid (e.g., in emerging markets). Accordingly, the GIPS standards give firms some discretion to determine when to add the new portfolio to a composite. In such cases, the firm must establish a policy on a composite-by-composite basis and apply it consistently to all new portfolios. A firm must include a terminated portfolio in the historical performance of the appropriate composite through the last full measurement period that the portfolio was under management.

A firm might decide that a particular composite will not include any portfolios whose value is below a specified level because the investment strategy can only be fully implemented for portfolios above a certain amount. If a firm sets a minimum asset level for portfolios to be included in a composite, the firm must not include portfolios below that asset level in that composite. The composite-specific minimum asset level can be changed prospectively but must not be applied retroactively.

Caution

Minimum assets levels should not be used solely to exclude portfolios from composites. Assuming there is no appropriate investment strategy applicable below a certain minimum level means the firm is implicitly assuming that portfolios below this level are in effect non-discretionary. In any event GIPS Reports should not be presented to prospective clients who don't have sufficient assets above the minimum asset level.

Portfolios cannot be switched from one composite to another unless documented changes in the portfolio's investment mandate, objective or strategy or the redefinition of the composite make it appropriate. The historical performance of the portfolio must remain with the original composite.

Note

If the GIPS standards permitted firms to transfer portfolios from one composite to another at will, an unethical firm might identify and exploit opportunities to improve the reported performance of selected composites by repopulating them with the portfolios whose investment results were most advantageous during the measurement period. Only a material error can cause the historic track record to change.

A portfolio can be switched from one composite to another if the asset owner revises the mandate, objective or strategy governing the investment of portfolio assets and the guideline changes are documented. Appropriate documentation such as the investment management agreement or other communication with the client must support the inclusion of any portfolio in a composite.

Alternatively, a portfolio can be reassigned to another composite if the original composite is redefined in such a way that the portfolio no longer fits it. The GIPS standards are clear that changes to a composite definition must not be applied retroactively.

Firms are allowed to temporarily remove a portfolio from a composite due to a significant cash flow, provided a policy to do so is established in advance. A significant cash flow is the level at which the firm determines that one or more client-directed external cash flows may temporarily prevent the firm from implementing the composite strategy. Basically, a significant cash flow means that, for a short period, the portfolio cannot follow the composite strategy while the cash flow (in or out) is being absorbed or processed; therefore, any performance during this period is not representative of the composite strategy.

Caution

Firms should be aware that if they have a single-portfolio composite, and that portfolio experiences a significant cash flow, temporarily removing that portfolio from that composite will create a hiatus or gap in the track record of the composite.

Note

Significant cash flows should not be confused with large cash flows. Significant cash flows are cash flows significant enough to impact the implementation of the investment strategy, resulting in the temporary removal of the portfolio from the composite. Large cash flows are cash flows large enough to distort the calculation of return. In most cases large cash flows will actually be smaller than significant cash flows, but they are separate issues and should be defined and operated independently.

Alternatively, firms may use temporary new accounts to remove the effect of a significant cash flow, again, provided an appropriate policy is established in advance. In a temporary new account, the cash flow is placed in a separate account, and transactions are undertaken and merged with the existing account at the end of a measurement period. This effectively removes the "cash drag" effect while the new money is invested. It also removes the transaction costs of investing the new money. Clearly the transaction costs are important for the asset owner in this case; they will want to know if the asset manager has controlled costs and timed investing the new assets successfully. However, it is arguable whether including the costs of investing new money is useful for judging the asset manager's ongoing investment strategy.

Note

In practice temporary new accounts are rarely used because of the operational difficulties and system capabilities, let alone the requirements of an ex-ante policy.

The composite history of portfolio can become quite complex. Take the timeline history of a portfolio in Figure 7.1. The firm receives the initial cash inflow at point 1 with an investment strategy that qualifies for entry in composite A. The portfolio only enters composite A at point 2, consistent with the composite's new portfolio policy at the start of the next full measurement period. The portfolio remains in composite A for a while but at point 3, consistent with the firm's policy of exiting the composite at the last full measurement period, exits composite A, after receiving notification of a change of investment strategy qualifying the portfolio in enter composite B. The portfolio only enters composite B at point 4 consistent with the firm's new portfolio policy for composite B. The portfolio is effectively non-discretionary between points 3 and 4 while the asset manager rearranges the portfolio consistent with the investment strategy of composite B. The portfolio exits composite B at point 5, being the end of the last full measurement period prior to notice of termination to cancel the management contract. The portfolio is finally removed from the firm after funds are returned according to the asset owner's instructions at point 6.

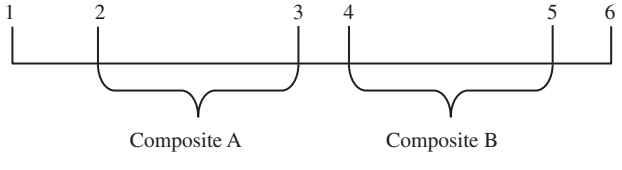

Figure 7.1 Composite maintenance

Initially firms have complete flexibility to define their own composites. This flexibility allows firms to differentiate their product offering and encourages the development of new products.

The firm must decide between narrowly or widely defined composites. Widely defined composites will include minor variations in strategy (for example, the firm might conclude that the return series of a global equity account with a restriction disallowing investment in Australia is very similar to an unrestricted global equity account and therefore both accounts could co-exist

in a widely defined global equity composite). Widely defined composites are easier to administer and allow the asset manager to present composites with larger assets under management. Narrowly defined composites are smaller, need to be monitored more closely because minor changes to investment strategy might require moving a portfolio to another composite, and are more difficult to administer, but the dispersion of returns within the composite will be narrower, indicating tighter investment controls.

Carve-outs

A carve-out is a subset of a multi-strategy portfolio's assets used to create a track record for a narrower mandate from a portfolio managed to a broader mandate. Typically, carve-outs are used by asset managers to demonstrate a competency to manage a strategy where standalone portfolios do not exist or are low in number with a short track record, but for which portions of larger portfolios do exist.

Relevant Provisions

3.A.15 Any carve-out included in a composite MUST include cash and any related income. Cash may be:

(a) Accounted for separately, or

(b) Allocated synthetically to the carve-out on a timely and consistent basis.

3.A.16 Any carve-out included in a composite MUST be representative of a standalone portfolio managed or intended to be managed according to that strategy.

3.A.17 When the firm creates a carve-out of a particular strategy, allocates cash to the carve-out and includes the carve-out in a composite, the firm MUST create carve-outs with allocated cash from all portfolios and portfolio segments within the firm managed to that strategy and MUST include those carve-outs with allocated cash in the composite.

3.A.18 When the firm has or obtains standalone portfolios managed in the same strategy as the carve-outs with allocated cash, the firm MUST create separate composite for the standalone portfolios.

3.A.19 The firm MUST NOT combine different composites, pooled funds or carve-outs to create a simulated strategy and present it as a composite.

By their very nature carve-out returns offer greater potential to mislead than standalone portfolios. Carve-outs by definition are portions of a larger portfolio, the criteria for which are determined by the firm. Because cash tends to act as a drag on performance (over the long term we would expect markets to outperform cash), if cash is not included in the calculation of the carve-out, the return may not be representative of what would have been achieved by a standalone portfolio.

Cash is one problem. There are others, such as:

(1) Concentration

Because carve-outs are parts of larger portfolios they tend to contain a smaller number of securities than a standalone portfolio and consequentially are potentially riskier.

(2) Currency
If the larger portfolio contains a currency overlay strategy it is very difficult, if not impossible, to isolate the currency allocation of the carve-out.

(3) Asset allocation
Within the larger portfolio, asset allocation "bets" are taken within the context of the overall benchmark, not the implied benchmark of the carve-out. In other words, the carve-out may not be managed in the same way as a standalone portfolio.

(4) Bet size
Bet sizes are exaggerated with carve-outs – particularly if the carve-out is a small percentage of the overall strategy.

(5) Composite administration
Allocation of portfolios to composites becomes significantly more difficult and expensive. For standalone portfolios it is easy to identify the number of accounts and changes to investment guidelines. However, if carve-outs are used then all carve-outs managed to that strategy must be allocated to that composite – the firm must demonstrate that all carve-out strategies are included and that procedures are in place to identify changes to carve-out strategies within larger portfolios.

The GIPS standards state that any carve-out included in a composite must include cash and any related income. Cash may be accounted for separately (often these are described as subportfolios) or allocated synthetically to the carve-out on a timely and consistent basis. Carve-outs that are managed, including their own cash allocation within a broader portfolio, are considered to be portfolios in their own right in the context of the GIPS standards. Crucially, the GIPS standards require that any carve-out included in a composite must be representative of a standalone portfolio managed according to that strategy. Standalone portfolios typically include some degree of cash which is, in effect, an overweight allocation to a non-correlated asset. This allocation can have a significant impact over time. If the carve-out doesn't include cash, it won't suffer the "cash drag" effect which surfaces for typical portfolios over long periods of time given generally rising markets.

When the firm obtains standalone portfolios managed in the same strategy as the carve-outs with allocated cash, the firm must create a separate composite for the standalone portfolios, which gives any prospective client access to the "true" performance of the strategy and the ability to compare the performance of the composite based on carve-outs to the performance of the composite based on standalone portfolios managed to the same strategy.

⚠ Caution

I would strongly recommend that firms avoid the use of carve-outs and if absolutely required only use them if standalone portfolios are not available. I believe it is very difficult to demonstrate that the performance of a carve-out is representative of standalone performance.

PRESENTATION AND REPORTING

There are four reporting sections in the GIPS standards that detail the requirements and recommendations for reporting using the following types of returns:

- Composite time-weighted return
- Composite money-weighted return

- Pooled fund time-weighted return
- Pooled fund money-weighted return

Each of the four reporting sections is self-contained and includes requirements and recommendations relevant to that particular report. For the most part, the requirements for composite and pooled fund time-weighted returns are replicated, as are the requirements for composite and pooled fund money-weighted returns. With a few notable differences, disclosures are consistent across all four reports. Therefore, I will provide commentary on Section 4 Composite time-weighted return reports and notable differences in Section 5 Composite money-weighted return reports only.

Composite time-weighted return report

There are 18 required presenting and reporting provisions in Section 4, including:

(1) Returns, dispersion and risk
(2) Unobservable inputs, gross- or net-of-fees, multiple benchmarks, breaks in performance, carve-outs and non-fee-paying portfolios
(3) Committed capital and advisory assets
(4) Reporting currency, carve-outs, overlay strategies, wrap fees and supplemental information

Returns, dispersion and risk

The GIPS standards require at least five years of performance history initially, presented annually, increasing to 10 years as the data becomes available. This avoids the "cherry-picking" of time periods and provides some information about the consistency of performance.

Relevant Provisions

4.A.1 The firm MUST present in each GIPS Composite report:

(a) At least five years of performance (or for the period since the composite inception date if the composite has been in existence less than five years) that meets the requirements of the GIPS standards. After the firm presents a minimum of five years of GIPS-compliant performance (or for the period since the composite inception date if the composite has been in existence less than five years), the firm MUST present an additional year of performance each year, building up to a minimum of 10 years of GIPS-compliant performance.
(b) Composite returns for each annual period.
(c) When the initial period is less than a full year, the return from the composite inception date through the initial period end.
(d) When the composite terminates, the return from the last annual period-end through the composite termination date.
(e) The total return for the benchmark for each annual period and for all other periods for which composite returns are presented, unless the firm determines there is no appropriate benchmark.

(f) The number of portfolios in the composite as of each annual period end. If the composite contains five or fewer portfolios at period-end, the number of portfolios is not required.
(g) Composite assets as of each annual period-end.
(h) Total firm assets as of each annual period-end.
(i) A measure of internal dispersion of individual portfolio annual returns for each annual period. If the composite contains five or fewer portfolios for the full year, a measure of internal dispersion is not required.
(j) For composites for which monthly composite returns are available, the three-year annualised ex-post standard deviation (using monthly returns) of the composite and the benchmark as of each annual period-end.

Ten years is an appropriate period to assess performance. The initial five years required by the standards is set to encourage new entrants for whom 10 years of compliance history might prove to be too tough a barrier.

Note

Quarterly reporting

Although the GIPS standards only require annual periods, I would recommend presenting quarterly data and updating composite reports on a quarterly basis. Asset owners should request quarterly data.

Firms must present the number of portfolios and a measure of the internal dispersion of the returns of the individual portfolios within a composite. The number of portfolios over time will convey to the asset owner the growth of portfolios over time and how many other portfolios have been successfully implemented. In order to avoid the risk of identifying a single client, firms are not required to identify the number of portfolios if less than six.

In reviewing the performance of a composite, it's not only the asset-weighted returns that are of interest but also the spread of returns within the composite, otherwise known as dispersion or internal risk. A wide dispersion of returns might indicate weak investment controls within the firm or that the composite is defined too broadly to provide meaningful information.

The standards do not mandate a particular measure of dispersion and quite a few are available:

(1) Equal-weighted standard deviation

$$S_D = \sqrt{\frac{\sum \left(R_i - \overline{R}\right)^2}{n}} \tag{7.1}$$

where: R_i = the return on the ith portfolio
n = the number of portfolios

(2) Asset-weighted dispersion

$$D = \sqrt{\sum W_i \times (R_i - R)^2} \tag{7.2}$$

where:

W_i = weight of the ith portfolio in the composite
R = composite return

(3) High–low

High–low is simply the highest and lowest returns within the composite for the period under measurement. GIPS require a measure of dispersion for each year, therefore only the returns of portfolios for the entire annual period should be included.

(4) Interquartile range

Interquartile range is the difference between the first and third quartiles (25th and 75th percentiles).

Dispersion measures are only relevant if there are sufficient portfolios in the composite for the full year; for five or fewer portfolios, a dispersion measure need not be calculated.

Note

Both high–low and equal-weighted standard deviation are common dispersion measures. High–low is too aggressive for me; either of the returns, high or low, might be an outlier and hence not provide a true measure of dispersion; interquartile range is a good alternative to high–low. Asset-weighted standard deviation is too complicated for little or no added insight. Equal-weighted standard deviation is my preferred dispersion measure, particularly for composites with a large number of constituents.

The firm must present the three-year annualised ex-post standard deviation (using monthly returns) or variability (see Equations 5.5 and 5.7) of the composite or pooled fund and of the benchmark as of each annual period-end. The rationale is to give prospective clients a comparable indication of the risk of an investment strategy.

Note

Some asset managers might argue, because of the return profile of their investment strategy, that standard deviation is not an appropriate risk measure. However, because all GIPS-compliant presentations use the same measure based on historical experience rather than subjective inputs, the GIPS standards allow for some degree of comparability. Variability of absolute return compared to benchmarks and competitors is useful to most asset owners.

Unobservable inputs, gross- or net-of-fees, multiple benchmarks, breaks in performance, carve-outs and non-fee-paying portfolios

Unobservable inputs reflect the firm's judgment of fair value. A large percentage value should cause concern for asset owners.

Firm's may present gross-of-fees returns, net-of-fees returns or both, but must be clearly labelled. Net-of-fees returns in certain circumstances will be required by regulation.

Note

Views vary, but I take the view that gross-of-fees returns are the more useful since they represent the underlying performance of the asset manager and allow for fairer comparison across a range of different fee structures. Fees are negotiable, and should be excluded when determining the skill of the asset manager. Obviously if fees are not negotiable, in a pooled fund, for example, it makes sense to present net-of-fees returns since that is the actual experience of the investor.

Relevant Provisions

4.A.2 The firm MUST present the percentage of the total fair value of composite assets that were valued using subjective unobservable inputs (as described in provision 2.B.6.e) as of the most recent annual period-end, if such investments represent a material amount of composite assets.

4.A.3 The firm MUST clearly label or identify:

(a) The periods that are presented.

(b) If composite returns are gross-of-fees or net-of-fees.

4.A.4 If the firm includes more than one benchmark in the GIPS Composite Report, the firm must present and disclose all required information for all benchmarks presented.

4.A.5 If the composite loses all of its member portfolios, the composite track record must end. If portfolios are later added to the composite, the composite track record MUST restart. The periods both before and after the break in track record MUST be presented, with the break in performance clearly shown. The firm MUST NOT link performance prior to the break in track record to the performance after the break in track record.

4.A.6 If the composite includes carve-outs with allocated cash, the firm MUST present the percentage of composite assets represented by carve-outs with allocated cash as of each period end.

4.A.7 If the composite includes non-fee-paying portfolios, the firm MUST present the percentage of composite assets represented by non-fee-paying portfolios as of each period-end when net-of-fees returns are presented and are calculated using actual investment management fees.

More than one benchmark may be shown in a GIPS Composite Report, and indeed portfolios within the composite may have different assigned benchmarks, provided the investment strategies qualify for composite inclusion.

⚠ Caution

You may be allowed to include multiple benchmarks in GIPS Composite Reports, but for me this indicates confusion or composite definitions that are too broad – one benchmark should suffice. In the management of single portfolios, it is certainly bad practice to allocate two or more benchmarks – which benchmark are you asking the asset manager to outperform exactly?

It's worth noting that the composite, and its history prior to any break in performance, continues to exist even if there are no constituent portfolios. Composites are not automatically terminated if there are no member portfolios; asset managers may continue to offer that investment strategy and are of course obliged to present the historic performance to prospective clients. Firms must not geometrically link returns across any gap or break in performance. A zero return, benchmark or any model return is not allowed; the returns before and after the break must be clearly shown.

Because of all the issues associated with carve-outs with allocated cash, described earlier, it is important to understand what percentage of the track record is impacted by carve-out returns, which might be less reliable.

⚠ Caution

Carve-outs with allocated cash – percentage of composite

This information is valuable. If 100%, asset owners are made aware that currently this asset manager does not operate any standalone portfolios in this strategy. If less than 100%, then standalone portfolios exist and these should be used to assess the performance capabilities of the asset manager.

Non-fee-paying portfolios could include proprietary assets or seed capital (established in order to create a track record). These track records are valid, but clearly, if net-of-fees returns are presented it is important to understand the proportion of the portfolio that consists of non-fee-paying portfolios.

Committed capital and advisory assets

Total firm assets include assets managed by subadvisors that the firm has authority to select but must not include advisory-only assets, uncalled committed capital or overlay exposure. Advisory-only assets will be an issue for a very small minority of specialist firms. Uncalled committed capital may be an issue for some alternative asset managers.

Relevant Provisions

4.A.8 If the firm chooses to present uncalled committed capital, or a combination of composite assets and composite uncalled committed capital, the firm MUST:

(a) Present composite uncalled committed capital for the same periods for which the combination of composite assets and composite uncalled committed capital is presented.
(b) Clearly label composite uncalled committed capital as such.
(c) Clearly label the combination of composite assets and composite uncalled committed capital as such.

4.A.9 If the firm chooses to present firm-wide uncalled committed capital or a combination of total firm assets and firm-wide uncalled committed capital, the firm MUST:

(a) Present firm-wide uncalled committed capital for the same periods for which the combination of total firm assets and firm-wide uncalled committee capital is presented.
(b) Clearly label firm-wide uncalled committed capital as such.
(c) Clearly label the combination of total firm assets and firm-wide uncalled committed capital as such.

4.A.10 If the firm chooses to present advisory-only assets that reflect the composite's investment mandate, objective, or strategy, or a combination of composite assets and advisory-only assets that reflect the composites investment mandate, objective, or strategy, the firm MUST:

(a) Present advisory-only assets that reflect the composite's mandate, objective, or strategy for the same periods for which the combination of composite assets and advisory-only assets that reflect the composite's investment mandate, objective, or strategy is presented.
(b) Clearly label advisory-only assets that reflect the composite's investment mandate, objective, or strategy as such.
(c) Clearly label the combination of composite assets and advisory-only assets that reflect the composite's investment mandate, objective, or strategy as such.

4.A.11 If the firm chooses to present firm-wide advisory-only assets or a combination of total firm assets and firm-wide advisory-only assets, the firm MUST:

(a) Present firm-wide advisory-only assets for the same periods for which the combination of total firm assets and firm-wide advisory-only assets is presented.
(b) Clearly label firm-wide advisory assets as such.
(c) Clearly label the combination of total firm assets and firm-wide advisory-only assets as such.

Advisory-only assets are assets for which the firm provides investment recommendations but has no control over implementation of investment decisions and no trading authority. There is an important distinction between advisory-only assets which are excluded from total firm assets and non-discretionary portfolio assets which are included. In the case of advisory-only

assets, the firm has no direct authority to manage or trade the account. In the case of non-discretionary assets, firms are unable to fully implement the intended investment strategy.

If a firm wishes to report firm-wide advisory-only assets it may present a separate value, or in addition to total firm assets, a combination of total firm assets and firm-wide advisory-only assets.

Committed capital is pledges of capital to an investment vehicle by investors (limited partners and the general partner) or the firm. Committed capital is typically drawn down over a period of time. Uncalled committed capital is the amount of committed capital that has yet to be drawn into the investment vehicle.

If a firm wishes to report firm-wide uncalled committed capital, it may present it as a separate value or present a combination of total firm assets and firm-wide uncalled committed capital as a supplement to the required disclosure of total firm assets.

Reporting currency, carve-outs, overlay strategies, wrap fees and supplemental information

All required and recommended information in a GIPS Report must be presented in the same currency. This includes returns, benchmarks, assets and risk measures.

It is acceptable to present composite returns in different currencies using the type of geometric conversion methodologies described in Chapter 3. The base currency of the composite may be US dollars, but it would be perfectly reasonable to convert the GIPS Composite Report information to euros to present to a European client.

Caution

It seems intuitive that changing the reporting currency should not change the underlying variability or standard deviation of the strategy. This is not the case; the underlying variability of the strategy will change between reporting currencies.

Caution

While it is acceptable to change the reporting currency of a GIPS Report, it is not acceptable to calculate a hedged return. In effect that would be a model result. Perfectly hedged returns are extremely difficult to create in practice.

Relevant Provisions

4.A.12 All required and recommended information in the GIPS Composite Report MUST be presented in the same currency.

4.A.13 When the firm presents the performance of a composite that includes carve-outs with allocated cash and also has a composite of standalone portfolios managed

according to the same strategy, the firm MUST present for the composite of standalone portfolios:

(a) The composite returns for each annual period for which the composite of standalone portfolios exists, and
(b) The composite assets as of each annual period-end for which the composite of standalone portfolios exists.

This information MUST be included in the GIPS Composite Report of the composite that includes carve-outs with allocated cash.

4.A.14 For overlay strategy composites, the firm must present composite overlay exposure as of each annual period-end. For those periods for which the firm presents composite overlay exposure, the firm may choose not to present composite assets.

4.A.15 For overlay composites, the firm is not required to present total firm assets and may instead choose to present total firm overlay exposure as of each annual period-end.

4.A.16 For wrap fee composites, when the firm presents performance to a wrap fee prospective client, the firm MUST present:

(a) The composite that includes the performance of all actual wrap fee portfolios, if any, managed according to the composite investment mandate, objective, or strategy, regardless of wrap fee sponsor.
(b) Composite performance that is net of the entire wrap fee.
(c) The percentage of composite assets represented by wrap fee portfolios as of each annual period-end.

4.A.17 For wrap fee composites, when the firm presents pure gross-of-fees returns, the firm MUST:

(a) Clearly label returns as pure gross-of-fees.
(b) Identify pure gross-of-fees returns as supplemental information.

4.A.18 Any supplemental information included in the GIPS Composite Report:

(a) MUST relate directly to the composite.
(b) MUST NOT contradict or conflict with the required or recommended information in the GIPS Composite Report.
(c) MUST be clearly labelled as supplemental information.

Firms may wish to provide firm-wide overlay exposure, and for overlay strategy composites the firm is not required to report total firm assets and may instead report firm-wide overlay exposure. Overlay strategies are implemented using derivatives, which often results in exceedingly small or even negative asset values. Overlay exposure is much more meaningful in terms of assessing the volume of assets managed by the firm.

Wrap fees are associated with a particular investment service provided in the United States. Wrap fees are typically expensive, all inclusive, asset-based fees including a combination of investment management fees, transaction costs, custody fees and administration fees.

Basically, there are four types of information to be found on a GIPS Report:

(1) Additional information
(2) Supplemental information
(3) Non-performance-related information
(4) Misleading information

Additional information is performance-related information that is derived from information required or recommended by the GIPS standards and needs no special identification.

Supplemental information is defined as any performance-related information included as part of a compliant performance presentation that supplements or enhances the required/or recommended disclosure and presentation provisions of GIPS.

Non-performance-related information is basically narrative, and includes information about the firm and its employees, a discussion about economic conditions and impact on the investment strategy and perhaps more detailed discussion about the investment strategy.

Misleading information is information in conflict with the standards and of course must never by used by a GIPS-compliant firm.

Supplemental information is a powerful aid for firms that want to enhance the quality of their presentation by providing more information. Supplemental information must satisfy the spirit and principles of GIPS; it must not contradict a compliant presentation and must be clearly labelled as supplemental since it is not covered by verification.

Examples of supplemental information include attribution, ex-ante risk analysis of a representative account, segment returns, composite or portfolio-level holdings and peer group rankings.

Caution

Supplemental information must not be used to bypass the GIPS presentation standards, although the standards in no way restrict any information being presented that is specifically requested by the client. Given the opportunity to present potentially misleading supplemental information, asset managers should take great care with its use.

Composite money-weighted reports

There are 15 required reporting and presentation provisions in Section 5, many of which are duplicated or analogous to Section 4. Specific money-weighted reporting and presentation provisions include annualised-since-inception money-weighted returns, subscription lines of credit, and additional performance metrics suitable for private equity-type strategies.

Relevant Provisions

5.A.1 The firm MUST present in each GIPS Composite Report:

(a) The annualized composite since-inception money-weighted return through the most recent annual period-end.

(b) When the composite has a track record that is less than a full year, the non-annualized composite since-inception money-weighted return through the initial annual period-end.

(c) When the composite terminates, the annualized composite since-inception money-weighted return through the composite termination date.

(d) The since-inception money-weighted return for the benchmark for the same period as presented for the composite, unless the firm determines there is no appropriate benchmark.
(e) See 4.A.1.f.
(f) See 4.A.1.g.
(g) See 4.A.1.h.

5.A.2 If a subscription line of credit is used, the firm MUST present the composite since-inception money-weighted return both with and without the subscription line of credit through the most recent annual period-end. The firm is not required to present returns without the subscription line of credit when the subscription line of credit has all of the following characteristics:
(a) The principal was repaid within 120 days using committed capital drawn down through a capital call.
(b) No principal was used to fund distributions.

5.A.3 See 4.A.2.

5.A.4 If portfolios in the composite have committed capital, the firm MUST present the following items as of the most recent annual period-end:
(a) Composite since-inception paid-in capital.
(b) Composite since-inception distributions.
(c) Composite cumulative committed capital.
(d) Total value to since-inception paid-in capital (investment multiple or TVPI).
(e) Since-inception distributions to since-inception paid-in capital (realisation multiple or DPI).
(f) Since-inception paid-in capital to cumulative committed capital (PIC multiple).
(g) Residual value to since-inception paid-in capital (unrealized multiple or RVPI).

5.A.5 The firm MUST clearly label or identify:
(a) The periods that are presented.
(b) If composite returns are gross-of-fees or net-of-fees.
(c) If composite returns do or do not reflect the subscription line of credit. This information is required only if the firm presents both with and without the subscription line of credit.

5.A.6 See 4.A.5.
5.A.7 See 4.A.6.
5.A.8 See 4.A.7.
5.A.9 See 4.A.8.
5.A.10 See 4.A.9.
5.A.11 See 4.A.10.
5.A.12 See 4.A.11.
5.A.13 See 4.A.12.
5.A.14 See 4.A.13.
5.A.15 See 4.A.18.

If a firm meets the requirements for presenting money-weighted returns, and chooses to do so, the firm must present the annualised since-inception money-weighted return through

the most recent annual period-end. Although the use of money-weighted returns compromise comparability, the GIPS standards recognise that there are circumstances and asset categories for which since-inception money-weighted returns may be more appropriate.

Firms must not compare a time-weighted benchmark with a composite's since-inception money-weighted return. Money-weighted benchmarks can be easily calculated using public market index data reflecting the external cash flow experience of the composite.

Note

The public market equivalent method (see Chapter 4) is an example of a money-weighted benchmark.

Subscription lines of credit have the potential to distort returns, generating leverage and altering the timing of external cash flows. By requiring returns, both with and without the subscription line of credit, the GIPS standards facilitate understanding the impact of the subscription lines of credit on the return calculation. Money-weighted benchmarks must also be calculated with and without subscription lines of credit. Reflecting the more legitimate use of a subscription line of credit for short-term operational reasons, firms are not required to calculate a since-inception money-weighted return if the principal was repaid within 120 days using committed capital drawn down through a capital call and no principal was used to fund distributions.

Since-inception money-weighted return is a useful measure to assess private equity performance, but because of difficulties in comparison (benchmarks and competitors) and potential manipulation of the measure and illiquidity, it is not sufficient and additional measures provide further insight. Some of these additional measures are required in GIPS Composite Reports.

Composite since-inception paid-in capital

Since-inception paid-in capital includes all capital inflows (including distributions that are subsequently recalled) to all portfolios in a composite.

Composite since-inception distributions

Since-inception distributions includes all cash and stock distributed to clients or investors.

Composite cumulative committed capital

Cumulative committed capital represents the total pledges of capital portfolios in a composite. Committed capital can be either drawn (paid-in) or undrawn (dry powder).

Total value to since-inception paid-in capital (TVPI) or multiple of investment capital (MOIC) or investment multiple

TVPI or MOIC is a multiple that indicates how many times the investment is worth compared with the original investment.

$$\text{TVPI or MOIC} = \frac{\text{Since-inception distributions} + \text{Residual value}}{\text{Since-inception paid-in capital}} \tag{7.3}$$

Note

TVPI is the key measure, in addition to since-inception money-weighted return, in assessing the performance of private equity assets. High since-inception money-weighted returns and high TVPIs are considered attractive.

Since-inception distributions to since-inception paid-in capital (realisation multiple or DPI)

The realisation multiple (or DPI) is the ratio of capital distributed to total paid-in capital.

$$\text{DPI} = \frac{\text{Since-inception disributions}}{\text{Since-inception paid-in capital}} \tag{7.4}$$

Since-inception paid-in capital to cumulative committed capital (PIC multiple)

The PIC multiple, also known as the dry-powder ratio, calculates how much capital is left to invest compared to total committed capital.

$$\text{PIC} = \frac{\text{Since-inception paid-in capital}}{\text{Cummulative committed capital}} \tag{7.5}$$

Residual value to since-inception paid-in capital (unrealised multiple or RVPI)

The unrealised multiple or (RVPI) is the ratio of the composite's current fair value (or residual value) at the end of the period divided by total paid-in capital.

$$\text{RVPI} = \frac{\text{Residual value}}{\text{Since-inception paid-in capital}} \tag{7.6}$$

Note:

$$\text{TVPI} = \text{DPI} + \text{RVPI} \tag{7.7}$$

DISCLOSURES

Disclosures allow firms to provide more information relevant to the performance presentation. The standards include both required and recommended disclosures. If in doubt the asset manager should add disclosures to assist asset owners in interpreting the performance presentation. The required GIPS disclosures typically provide useful information that puts other performance data into context, encourages asset owners to seek additional information by advertising its availability, and on occasion discourages asset managers from adopting a particular approach by being forced to disclose the approach.

Meeting the objectives of fair representation and full disclosure may call for providing more information than the GIPS standards minimally require. Asset managers are well advised to prepare compliance checklists to ensure that the disclosure requirements and, where feasible, the recommendations of the GIPS standards are met for the firm as a whole, and for each composite or pooled fund presented.

There are 48 required disclosure provisions for composite time-weighted reports and 44 required disclosure provisions for composite money-weighted reports, mostly duplicated but including:

(1) Claim of compliance
(2) Firm, composite and benchmark definitions
(3) Fee disclosures
(4) Inception date, creation date, availability of lists of composite policies and procedures, leverage and estimated transaction costs
(5) Significant events, redefinition, minimum asset levels and withholding tax
(6) Conflicts with regulation, carve-out disclosures and subadvisors
(7) Benchmark disclosures
(8) Significant cash flow disclosure and material errors
(9) Risk measures, overlay strategy, real estate valuation and theoretical performance disclosures

Claim of compliance

Once a firm has met all of the required elements of GIPS, the firm may claim compliance. It is the firm that claims compliance; the claim is not specific to an individual presentation, composite or client type. The GIPS standards specify the precise wording of the claim of compliance to be used only in a GIPS Report. The wording differs depending upon whether or not a firm has been verified and additionally examined.

Caution

Firms cannot pick and choose to whom they present compliant presentations. GIPS are ethical standards, and in making a claim of compliance the firm is stating that all its performance presentations are a fair and honest representation of historic performance.

Relevant Provisions

4.C.1 Once the firm has met all the applicable requirements of the GIPS standards, the firm MUST disclose its compliance with the GIPS standards using one of the following compliance statements. The compliance statement for a composite MUST only be used in a GIPS Composite Report.

(a) For a firm that is verified:

> [Insert name of firm] claims compliance with the Global Investment Performance Standards (GIPS®) and has prepared and presented this report in compliance with the GIPS standards. [Insert name of firm] has been independently verified for the periods [insert dates]. The verification report(s) is/are available upon request.
>
> A firm that claims compliance with the GIPS standards must establish policies and procedures for complying with all the applicable requirements of the

GIPS standards. Verification provides assurance on whether the firm's policies and procedures related to composite and pooled fund maintenance, as well as the calculation, presentation, and distribution of performance, have been designed in compliance with the GIPS standards and have been implemented on a firm-wide basis. Verification does not provide assurance on the accuracy of any specific performance report.

(b) For composites of a verified firm that have also had a performance examination:

[Insert name of firm] claims compliance with the Global Investment Performance Standards (GIPS®) and has prepared and presented this report in compliance with the GIPS standards. [Insert name of firm] has been independently verified for the periods [insert dates].

A firm that claims compliance with the GIPS standards must establish policies and procedures for complying with all the applicable requirements of the GIPS standards. Verification provides assurance on whether the firm's policies and procedures related to composite and pooled fund maintenance, as well as the calculation, presentation, and distribution of performance, have been designed in compliance with the GIPS standards and have been implemented on a firm-wide basis. The [insert name of composite] has had a performance examination for the periods [insert dates]. The verification and performance examination reports are available upon request.

(c) For a firm that has not been verified:

[Insert name of firm] claims compliance with the Global Investment Performance Standards (GIPS®) and has prepared and presented this report in compliance with the GIPS standards. [Insert name of firm] has not been independently verified.

The firm MUST NOT exclude any portion of the respective compliance statement. Any modifications to the compliance statement MUST be additive.

4.C.2 The firm MUST disclose the following: "GIPS® is a registered trademark of CFA Institute. CFA Institute does not endorse or promote this organization, nor does it warrant the accuracy or quality of the content herein."

In addition to the claim of compliance the firm must also include the following disclosure note: "GIPS® is a registered trademark of CFA Institute. CFA Institute does not endorse or promote this organization, nor does it warrant the accuracy or quality of the content herein", although this need not be attached or linked to the claim of compliance.

Firm, composite and benchmark definitions

The definition of the "firm" used to determine the firm's total assets and firm-wide compliance is a required disclosure. A clear explanation of the way in which the firm is defined enables the prospective client to understand precisely which investment organisation (or unit of a larger entity) is presenting results, is claiming compliance, and will be responsible for managing the client's assets if hired.

Relevant Provisions

4.C.3 The firm MUST disclose the definition of the firm used to determine total firm assets and firm-wide compliance.
4.C.4 The firm MUST disclose the composite description.
4.C.5 The firm MUST disclose:
(a) The benchmark description, which MUST include the key features of the benchmark or the name of the benchmark for a readily recognized index or other point of reference.
(b) The periodicity of the benchmark if benchmark returns are calculated less frequently than monthly.

The composite description provides general information regarding the investment mandate, objective, or strategy of the composite. The composite description may be more abbreviated than the composite definition but must include all key features of the composite and must include enough information to allow a prospective client to understand the key characteristics of the composite's investment mandate, objective or strategy, including:

- The material risks of the composite's strategy.
- How leverage, derivatives and short positions may be used, if they are a material part of the strategy.
- If illiquid investments are a material part of the strategy.

To differentiate between a composite definition and a composite description, it might be helpful to think of a composite description as focused on the description of the strategy represented by the composite. In contrast, a composite definition includes not only the composite description, but also the detailed criteria that determine whether and when a portfolio is included in a composite.

Firms must disclose the benchmark description, which must include the key features of the benchmark or the name of the benchmark for a readily recognised index. If returns are calculated less frequently than monthly, firms must disclose the periodicity of the benchmark.

Fee disclosures

Gross-of-fees returns, net-of-fees returns, or both may be presented. When presenting gross-of-fees returns, firms must disclose if they deduct any other fees in addition to transaction costs. Similarly, when presenting net-of-fees returns, firms must disclose if any other fees are deducted in addition to the transaction costs and the investment management fee and if the net of fees returns are net of any performance-based fees or carried interest. Also, if presenting net-of-fees returns, the firm must disclose if model or actual investment management fees are used. If model fees are used and gross-of-fees returns are not presented, the model fee used and the methodology used to calculate the model fees must be disclosed.

Relevant Provisions

4.C.6 When presenting gross-of-fees returns, the firm MUST disclose if any other fees are deducted in addition to transaction costs.

4.C.7 When presenting net-of-fee returns, the firm MUST disclose:

(a) If any other fees are deducted in addition to investment management fees and transaction costs.

(b) If net-of-fees returns are net of any performance-based fees or carried interest.

(c) If model or actual investment management fees are used.

(d) If model investment management fees are used, and composite gross-of-fees returns are not presented, the model investment management fee used to calculate net-of-fees return.

(e) If model investment management fees are used, the methodology used to calculate net-of-fees returns.

4.C.8 The firm MUST disclose which fees and expenses other than investment management fees (e.g., research costs) are separately charged by the firm to clients, if material.

4.C.9 The firm MUST disclose or otherwise indicate the reporting currency.

4.C.10 The firm MUST disclose which measure of internal dispersion is presented.

4.C.11 The firm MUST disclose the current fee schedule appropriate to prospective clients or prospective investors.

(a) When presenting performance to a prospective client for a standalone portfolio, the fee schedule MUST reflect the fee schedule for a standalone portfolio managed according to that strategy.

(b) When presenting performance of a composite that includes carve-outs to a prospective client for a multi-asset strategy portfolio, the fee schedule MUST reflect the fee schedule for a multi-asset strategy portfolio managed according to that strategy.

(c) When presenting a wrap fee composite to a wrap fee prospective client, the fee schedule MUST reflect the total wrap fee.

(d) When presenting a GIPS Composite Report to a prospective investor for a pooled fund included in the composite, the firm MUST disclose the pooled fund's current fee schedule and expense ratio.

4.C.12 If the fee schedule includes performance-based fees or carried interest, the firm MUST disclose the performance-based fee description or carried interest description.

The firm must disclose the fee schedule appropriate to the prospective clients or prospective investors. The term "fee schedule" refers to the firm's current schedule of investment management fees or bundled fees appropriate to prospective clients or prospective investors. If the fee schedule includes performance-based fees or carried interest, the firm must disclose the performance-based fee description or carried interest description.

Note

The fee schedule is one of the more awkward requirements of the GIPS standards in that it is client-specific, which may mean that you can't pull out an off-the-shelf GIPS Composite Report to present to your client. The solution is to make the fee schedule itself as generic as possible.

If estimated transaction costs are used the firm must disclose that estimated transaction costs were used, the estimated costs and how they were determined.

Inception date, creation date, availability of lists of composite policies and procedures, leverage and estimated transaction costs

Firms must also disclose the composite creation date, the date on which the firm first grouped one or more portfolios to form the composite. This is not necessarily the earliest date for which composite performance is reported. This information allows the prospective client to determine whether this is a long-established strategy or perhaps one that was created recently, perhaps for the purposes of the current presentation. Firms must also disclose the composite inception date, which is the initial date of the composite's track record, a quite different concept from that of the composite creation date.

Relevant Provisions

4.C.13 The firm MUST disclose the composite inception date.
4.C.14 The firm MUST disclose the composite creation date.
4.C.15 The firm MUST disclose that the following lists are available upon request, if applicable:
(a) List of composite descriptions.
(b) List of pooled fund descriptions for limited distribution pooled funds.
(c) List of broad distribution pooled funds.
4.C.16 The firm MUST disclose that policies for valuing investments, calculating performance, and preparing GIPS Reports are available upon request.
4.C.17 The firm MUST disclose how leverage, derivatives, and short positions have been used historically, if material.
4.C.18 If estimated transaction costs are used, the firm MUST disclose:
(a) That estimated transaction costs were used.
(b) The estimated transaction costs used and how they were determined.

Firms must disclose the composite or pooled fund description and must indicate the availability, upon request, of the firm's complete list of composite descriptions, limited pooled fund descriptions and broad distribution pooled funds. This information enables prospective clients

to determine whether the GIPS Report they have been shown is the most appropriate for their needs and to request GIPS Reports of any other composites or pooled funds of interest. The list must include not only all of the firm's current composites but also any that have been discontinued within the past five years, thus poor performance in terminated composites or unsuccessful strategy launches cannot be hidden. Discontinued pooled funds are not available for investment and do not have to be shown.

Note

Asset owners should always request these lists. In practice, these requests are quite rare.

Firms must disclose the presence, use and extent of leverage, derivatives and short positions, if material. The disclosure must include a description of the frequency of use and characteristics of the instruments sufficient to identify risks. A clear explanation will help prospective clients or prospective investors interpret the historical performance and evaluate the additional risk resulting from the use of leverage or derivatives.

Significant events, redefinition, minimum asset levels and withholding tax

Firms are required to disclose all significant events that would help a prospective client or prospective investor interpret the GIPS Report. For example, any change of ownership or significant merger involving an asset manager is likely to be very disruptive internally, and is almost always a significant event. Firms must also advise prospective clients if past results in a given strategy were achieved substantially by a "star" portfolio manager who has since left the firm, or if all key members of the research team supporting the strategy have resigned. On the other hand, if a portfolio manager leaves and they were simply implementing team decisions, that would not constitute a significant event.

Relevant Provisions

4.C.19 The firm MUST disclose all significant events that would help a prospective client interpret the GIPS Composite Report. This disclosure MUST be included for a minimum of one year and for as long as it is relevant to interpreting the track record.

4.C.20 For any performance presented for periods prior to the minimum effective compliance date that does not comply with the GIPS standards, the firm MUST disclose the periods of non-compliance.

4.C.21 If the firm is redefined, the firm MUST disclose the date and description of the redefinition.

4.C.22 If the composite is redefined, the firm MUST disclose the date and description of the redefinition.

4.C.23 The firm MUST disclose changes to the name of the composite. This disclosure MUST be included for a minimum of one year and for as long as it is relevant to interpreting the track record.

(*continued*)

(*continued*)

4.C.24 The firm MUST disclose:

(a) The minimum asset level, if any, below which portfolios are not included in the composite.

(b) Any changes to the minimum asset level.

4.C.25 The firm MUST disclose if composite returns are gross or net of withholding taxes, if material.

4.C.26 The firm MUST disclose if benchmark returns are net of withholding taxes if this information is available.

If the firm imposes a minimum asset level for a portfolio to be included in the composite, this minimum asset level must be disclosed. This alerts prospective clients to the asset manager's assumption that the strategy cannot be implemented in smaller portfolios.

If a firm is redefined, it must disclose the date and describe the redefinition. If a parent company contains multiple firms, the GIPS standards recommend that each firm within the parent company disclose a list of the other firms contained within the parent company. Firm redefinitions are likely to occur in the event of merger and acquisition activity. Organisations might initially choose to define multiple firms within a broader organisation, some compliant, some not. At a later stage, perhaps when all defined firms are compliant, it might be appropriate to merge the GIPS-compliant firms into a single GIPS firm requiring redefinition.

If a firm has redefined a composite, the firm must disclose the date and description of the redefinition. Similarly, firms must disclose any changes to a composite's name. Changes to the description or name of the composite may or may not imply a change of strategy or approach, which will certainly be of interest to prospective clients.

The GIPS standards do not require firms to reflect withholding taxes, reclaimable or non-reclaimable. Asset owners clearly suffer different withholding taxes in different countries, which makes comparison difficult.

Note

Notwithstanding difficulties in comparison the GIPS standards recommend that returns are calculated net of non-reclaimable withholding taxes, and I certainly strongly encourage that. Reclaimable withholding taxes should be accrued, and in fact may take many years to be reclaimed.

The GIPS standards require disclosure if benchmark returns are net of withholding taxes. Withholding tax experience is different for different assets owners in different countries; however, some index providers calculate benchmarks assuming the worst possible experience of withholding tax.

Note

Given the difficulties of comparison, and relatively minor impact, and only if portfolio returns are net of withholding tax, I think it is appropriate to use net of withholding tax benchmarks with disclosure.

Conflicts with regulation, carve-out disclosures and subadvisors

Firms must comply with all applicable laws and regulations regarding the calculation and presentation of performance. Where laws and regulations conflict with the GIPS standards, firms are required to comply with the laws and regulations and disclose the conflict.

Note

Local laws and regulations always override the GIPS standards. In fact, prospective client requests also override the GIPS standards – there is no provision that restricts the firm from providing any information specifically requested by the client.

Relevant Provisions

4.C.27 If the GIPS Composite Report conforms with laws and/or regulations that conflict with the requirements of the GIPS standards, the firm MUST disclose this fact and disclose the manner in which the laws and/or regulations conflict with the GIPS standards.

4.C.28 If carve-outs with allocated cash are included in the composite, the firm MUST:

(a) Indicate carve-out in the composite name.
(b) Disclose that the composite includes carve-outs with allocated cash.
(c) Disclose the policy used to allocate cash to carve-outs.
(d) Disclose that the GIPS Composite Report for the composite of standalone portfolios is available upon request, if the composite of standalone portfolios exists.

4.C.29 The firm MUST disclose the use of a subadvisor and the periods a subadvisor was used.

4.C.30 The firm MUST disclose if the composite's valuation hierarchy materially differs from the recommended valuation hierarchy. (See Provision 2.B.6 for the recommended valuation hierarchy.)

If carve-outs with allocated cash are included in the composite, the firm must include the carve-out in the composite name, disclose that the composite includes carve-outs with allocated cash, disclose the policy to allocate cash to carve-outs, present the percentage of composite assets represented by carve-outs with allocated cash as of each annual period end and

disclose that the GIPS Composite Report for the composite of standalone portfolios is available upon request if it exists. In addition, the composite returns for each annual period and the composite assets as of each annual period-end for which the composite of the standalone portfolio exists must be included in the GIPS Composite Report of the composite that includes carve-outs with allocated cash.

Caution

Collectively these disclosures are designed to discourage the misleading use of carve-outs with allocated cash where more appropriate standalone composites are available. Comparison of carve-out returns against the standalone equivalent mostly demonstrate the misrepresentative nature of carve-outs and I strongly discourage their use.

Firms may use a subadvisor to manage part or all of a particular strategy. The performance of subadvised assets must be included in the composite if the firm retains hiring and firing responsibilities of the subadvisor. The firm must disclose the existence of the subadvisor and the periods for which the subadvisor was used but is not required to disclose the name of the subadvisor.

Firms must establish policies and procedures for determining portfolio valuations. The standards do not require a specific valuation hierarchy – firms can establish their own, provided it supports the definition of fair value. However, the GIPS standards effectively encourage a specific valuation hierarchy by publishing their own recommended hierarchy and requiring that the firm must disclose the fact that their own valuation hierarchy differs.

Benchmark disclosures

If the firm determines that no appropriate benchmark for the composite exists, the firm must disclose why no benchmark is presented.

Note

Instances of there genuinely being no appropriate benchmark are extremely rare, therefore requiring an explanatory disclosure is appropriate. Even for absolute return strategies, a target return or risk-free rate makes for a suitable benchmark, despite the deficiencies of this type of benchmark.

If the firm changes the benchmark, the firm must disclose the date and description of the change.

Note

Changing a benchmark retrospectively to improve relative performance is the oldest trick in the book. Benchmarks should never be retrospectively changed – if the strategy changes, the new benchmark can be spliced with the old benchmark (see Chapter 4). Very occasionally a new index may be produced with history that more accurately represents the asset manager's investment strategy than an index previously available. In such circumstances the benchmark may be changed but disclosure is essential.

The GIPS standards recommend that firms disclose material differences between the composite investment mandate, objective or strategy and the strategy represented by the benchmark. If the firm includes more than one benchmark, the firm must present and disclose all required information for all benchmarks presented. When using benchmarks that have limitations, such as peer groups, the firm should disclose these limitations (see Chapter 4).

Relevant Provisions

4.C.31 If the firm determines no appropriate benchmark for the composite exits, the firm MUST disclose why no benchmark is presented.

4.C.32 If the firm changes the benchmark, the firm MUST disclose:

(a) For a prospective benchmark change, the date and description of the change. Changes MUST be disclosed for as long as returns for the prior benchmark are included in the GIPS Composite Report.

(b) For a retroactive benchmark change, the date and description of the change. Changes MUST be disclosed for a minimum of one year and for as long as they are relevant to interpreting the track record.

4.C.33 If a custom benchmark or combination of multiple benchmarks is used, the firm MUST:

(a) Disclose the benchmark components, weights, and rebalancing process, if applicable.

(b) Disclose the calculation methodology.

(c) Clearly label the benchmark to indicate that it is a custom benchmark.

4.C.34 If a portfolio-weighted custom benchmark is used, the firm MUST disclose:

(a) That the benchmark is rebalanced using the weighted average returns of the benchmarks of all of the portfolios included in the composite.

(b) The frequency of the rebalancing.

(c) The components that constitute the portfolio-weighted custom benchmark, including the weights that each component represents, as of the most recent annual period-end.

(d) That the components that constitute the portfolio-weighted custom benchmark, including the weights that each component represents, are available for prior periods upon request.

If a custom benchmark or a combination of multiple benchmarks is used, the firm must disclose the benchmark components, weights, rebalancing process, calculation methodology and clearly label the benchmark to indicate that it is a custom benchmark.

Caution

Portfolio-weighted benchmarks are a type of custom-weighted benchmark for composites that contain portfolios managed against different benchmarks. In effect, the composite relative against this custom benchmark is a more accurate representation of the composite's excess return. Although this is true, for me this represents muddled thinking – even in a broadly defined composite the individual portfolios should be managed in such a way that the differences between the allocated benchmarks is not that great. No single portfolio will be managed against this artificially engineered portfolio-weighted benchmark; hence it should not be used.

Significant cash flow disclosure and material errors

If a firm has adopted a significant cash flow policy for a specific composite, the firm must disclose how the firm defines a significant cash flow for that composite and for which periods.

Caution

It is possible to change a significant cash flow policy ex-ante. If the composite contains only a few portfolios it might be worth reconsidering whether a significant cash flow policy is still appropriate. It will be too late to change the policy ex-post if the policy results in having no portfolios in the composites and a break in the track record.

Relevant Provisions

4.C.35 If the firm adopted a significant cash flow policy for the composite, the firm MUST disclose how the firm defines a significant cash flow for the composite and for which periods.

4.C.36 For composites with at least three annual periods of performance, the firm MUST disclose if the three-year annualized ex-post standard deviation of the composite and/or benchmark is not presented, because 36 monthly returns are not available.

4.C.37 The firm MUST disclose if performance from a past firm or affiliation is presented, and for which periods.

4.C.38 The firm MUST disclose any change to the GIPS Composite Report resulting from the correction of a material error. Following the correction of the GIPS Composite Report, this disclosure MUST be included for a minimum of one year and for as long as it is relevant to interpreting the track record. The disclosure is not required

to be included in a GIPS Composite Report that is provided to a prospective client or prospective investor that did not receive the GIPS Composite Report containing the material error.

4.C.39 If the firm chooses to not present the number of portfolios in the composite because there are five or fewer portfolios in the composite, the firm MUST disclose that the composite contains five or fewer portfolios or use similar language.

4.C.40 If the firm chooses to not present the internal dispersion of individual portfolio returns because there are five or fewer portfolios in the composite for the full year, the firm MUST disclose that the internal dispersion measure is not applicable or use similar language.

4.C.41 The firm MUST disclose if preliminary, estimated values are used to determine fair value.

4.C.42 If the firm changes the type of return(s) presented for the composite (e.g., changes from money-weighted returns to time-weighted returns), the firm MUST disclose the change and date of the change. This disclosure MUST be included for a minimum of one year and for as long as it is relevant to interpreting the track record.

A few benchmarks are only calculated, or available, on a quarterly basis. Therefore, it is impossible to calculate a three-year standard deviation using monthly data – this should be disclosed.

Material errors in a GIPS Report, according to the firm's error policy, must be corrected and disclosed in a revised GIPS Composite Report, but only for the benefit of those who received the erroneous report. Disclosure of the error is not required in GIPS Reports for new prospective clients.

⚠ Caution

Firms will clearly seek to avoid the penalty of disclosing material errors and reissuing corrected GIPS Reports and may be tempted to set a very high tolerance for material errors. Asset owners should seek to review the error correction procedures of asset managers and make their own assessment of the tolerance levels set by firms.

The number of portfolios in the composite provides a sense to the asset owner of the number of client mandates won by the asset owner and growth over the years. To avoid highlighting the existence of a single portfolio composite and hence potentially revealing the performance of an individual client, the firm need only disclose "five or fewer portfolios in the composite", rather than the specific number.

If there are five or fewer portfolios in the composite for the full year, a measure of internal dispersion is not required. Such a measure would be of little value with so few portfolios in the composite.

Preliminary, estimated values used to establish fair value are relatively common for assets, including underlying funds, which are slow or late to value. This must be disclosed and if performance is subsequently changed in response to final valuations, the firm's error correction procedures must be assessed.

Risk measures, overlay strategy, real estate valuation and theoretical performance disclosures

Performance is, at the very least, two-dimensional: risk and return. Firms are recommended to provide additional risk measures that better describe performance than the mandated measure of variability if appropriate. If included, firms must provide a description of the additional risk measure used, including the name of any risk-free rate.

Caution

The disclosure of the name of the risk-free rate used in the calculation of an additional risk measure is important because of the wide range of options (see Chapter 5) and the potential impact on any risk measure.

Relevant Provisions

4.C.43 If the firm presents additional risk measures, the firm MUST:
- (a) Describe any additional measure.
- (b) Disclose the name of the risk-free rate if a risk-free rate is used in the calculation of the additional risk measure.

4.C.44 The firm MUST disclose if gross-of-fees or net-of-fees returns are used to calculate presented risk measures.

4.C.45 For overlay strategy composites, the firm MUST disclose:
- (a) The methodology used to calculate composite overlay exposure.
- (b) If collateral and collateral income are reflected in the composite returns.

4.C.46 For real estate investments that are not in a real estate open-ended fund, the firm MUST disclose that:
- (a) External valuations are obtained, and the frequency with which they are obtained, or
- (b) The firm relies on valuations from financial statement audits.

4.C.47 For warp fee composites, when the firm presents pure gross-of-fees returns, the firm MUST disclose that pure gross-of-fees returns do not reflect the deduction of transaction costs.

4.C.48 When the GIPS Composite Report includes theoretical performance as supplemental information, the firm MUST:
- (a) Disclose that the results are theoretical, are not based on the performance of actual assets, and if the theoretical performance was derived from the retroactive or prospective application of a model.
- (b) Disclose a basic description of the methodology and assumptions used to calculate the theoretical performance sufficient for the prospective client or prospective investor to interpret the theoretical performance, including if it is based on model performance, back-tested performance, or hypothetical performance.

(c) Disclose whether the theoretical performance reflects the deduction of actual or estimated investment management fees, transaction costs, or other fees and charges that an actual client portfolio would have paid or will pay.
(d) Clearly label the theoretical performance as supplemental information.

Overlay strategies generate a series of different return calculation challenges. The return denominator is not allocated capital, but rather overlay exposure. Overlay exposure may be:

(1) The notional exposure of the overlay strategy being managed,
(2) The value of the underlaid portfolio being overlaid, or
(3) A specific target exposure.

Firms must disclose which method they are using.

Collateral required to implement the overlay strategy may be managed in different ways. Firms must establish a policy for the treatment of collateral and disclose whether collateral and collateral income are included in the composite returns.

When theoretical performance is included as supplemental information in a GIPS Report, firms are required to include a number of disclosures to ensure the recipients of the report fully understand the information presented.

Caution

If is very difficult to avoid self-selection when producing theoretical performance. I strongly encourage asset managers to avoid using theoretical performance externally and for asset owners to simply ignore it.

Relevant Provisions

5.C.36 If a subscription line of credit is used, and the firm is required to present returns both with and without the subscription line of credit, the firm MUST disclose:
(a) The purpose for using the subscription line of credit.
(b) The size of the subscription line of credit as of the most recent annual period-end.
(c) The subscription line of credit amount outstanding as of the most recent annual period-end.

Subscription lines of credit have the potential to distort returns, generating leverage and altering the timing of external cash flows. Therefore, it is extremely important that asset owners understand the context of use, the extent and the impact of any subscription lines of credit.

SAMPLE GIPS COMPOSITE REPORT

Exhibit 7.1 shows the basic minimum requirements of a GIPS Composite Report.

Exhibit 7.1 Sample GIPS Composite Report

Year	Composite Return (%)	Benchmark Return (%)	Composite Standard Deviation (%)	Benchmark Standard Deviation (%)	Internal Dispersion (%)	Number of Portfolios	Composite Assets (£M)	Firm Assets (£M)
2021	1.4	−2.1	23.4	26.0	0.9	189	1,800	16,798
2020	13.7	15.2	20.9	22.7	1.5	193	1,757	14,596
2019	12.5	14.1	22.1	24.5	1.4	225	1,438	12,758
2018	−6.7	−9.5	18.7	19.9	1.1	176	1,056	14,352
2017	24.9	25.2	23.5	20.7	2.5	183	1,126	15,147

Disclosures

1. Compliance statement

ABC Asset Management claims compliance with the Global Investment Performance Standards (GIPS®) and has prepared and presented this report in compliance with the GIPS standards. ABC Asset Management has been independently verified for the periods 1 January 2017 to 31 December 2021. The verification report is available upon request. A firm that claims compliance with the GIPS standards must establish policies and procedures for complying with all the applicable requirements of the GIPS standards. Verification provides assurance on whether the firm's policies and procedures related to composite and pooled fund maintenance, as well as the calculation, presentation, and distribution of performance, have been designed in compliance with the GIPS standards and have been implemented on a firm-wide basis. Verification does not provide assurance on the accuracy of any specific performance report.

2. Definition of the firm

ABC Asset Management, a specialist emerging markets equity manager managing institutional and private client portfolios.

3. Composite description

The Global Emerging Markets Composite includes all institutional portfolios that invest in global emerging markets equities with no more than 10% in global emerging market debt. Portfolios with more than 5% exposure to developed markets are excluded. The account minimum for the composite is £10 million.

4. Creation date

The Composite was created in February 2022 and the inception date is 31 December 2016.

5. Benchmark

The benchmark is the Freedom Global Emerging Market Index.

Exhibit 7.1 (*Continued*)

6. Fee schedule

Management fees are 0.6% per annum.

7. Return methodology

Returns are calculated gross of fees using a true time-weighted methodology. Valuations are calculated and returns reported in pounds sterling.

8. List of composites

A list of composite descriptions is available upon request.

9. Risk

Portfolio and benchmark standard deviation is three-year annualised using 36 months of monthly gross returns.

10. Internal dispersion

Internal dispersion is calculated using equal-weighted standard deviation.

11. Availability of information

ABC Asset Management's policies for valuing investments, calculating performance and preparing GIPS reports are available upon request

12. Trademark

GIPS is a registered trademark of CFA Institute. CFA Institute does not endorse or promote this organisation, nor does it warrant their accuracy or quality of the content herein

GIPS ADVERTISING GUIDELINES

The GIPS Advertising Guidelines provide firms with options for advertising when mentioning the firm's claim of compliance.

A GIPS advertisement is an advertisement by a GIPS-compliant firm that adheres to the requirements of the GIPS Advertising Guidelines.

Advertisements include any materials that are distributed in newspapers, firm brochures, fact sheets, websites, offering documents or any written or electronic material distributed to more than one party. One-on-one presentations and individual client reporting are not advertisements.

A firm that claims compliance with the GIPS standards has three options with respect to advertising:

(1) Prepare the advertisement in line with the GIPS Advertising Guidelines.
(2) Include a GIPS Report in the advertisement.
(3) Do not mention GIPS in the advertisement.

Firms are not required to claim compliance with the GIPS standards in advertisements but if they choose to claim compliance in the advertisement, they must either meet all the requirements of the GIPS Advertising Guidelines or include a GIPS Report. In cases where laws or regulations conflict with the requirements of the GIPS standards or the GIPS Advertising Guidelines, firms are required to comply with the laws or regulations.

The GIPS Advertising Guidelines include:

(1) Fundamental requirements of the GIPS Advertising Guidelines.
(2) GIPS advertisements that do not include performance.
(3) GIPS advertisements for composites.
(4) GIPS advertisements for limited distributions funds.
(5) GIPS advertisements for broad distribution funds.

Fundamental requirements of the GIPS Advertising Guidelines

There are 14 provisions in the fundamental requirements section of the GIPS Advertising Guidelines.

Relevant Provisions

8.A.1 The GIPS Advertising Guidelines apply only to firms that already claim compliance with the GIPS standards.

8.A.2 A firm that chooses to claim compliance in a GIPS advertisement MUST comply with all applicable requirements of the GIPS Advertising Guidelines.

8.A.3 The firm MUST maintain all data and information necessary to support all items included in a GIPS advertisement.

8.A.4 Returns for periods of less than one year included in a GIPS advertisement MUST NOT be annualised.

8.A.5 When time-weighted returns are presented in a GIPS advertisement, the firm MUST NOT link non-GIPS-compliant performance for periods beginning on or after the minimum effective compliance date to GIPS-compliant performance. The firm may link non-GIPS-compliant performance to GIPS-compliant performance in a GIPS advertisement provided that only GIPS-compliant performance is presented for periods beginning on or after the minimum effective compliance date.

8.A.6 When money-weighted returns are presented in a GIPS advertisement, the firm MUST NOT present non-GIPS-compliant performance for periods ending on or after the minimum effective compliance date. The firm may present non-GIPS-compliant performance in a GIPS advertisement provided that only GIPS-compliant performance is presented for periods ending on or after the minimum effective compliance date.

Of course, the GIPS Advertising Guidelines only apply to firms already in compliance. Just like in a GIPS Report, firms must maintain sufficient data and information to support everything included in the GIPS advertisement.

Relevant Provisions

8.A.7 Composite returns included in a GIPS advertisement MUST be derived from the returns included in or that will be included in the corresponding GIPS Composite Report.

8.A.8 Disclosures included in a GIPS advertisement for a composite MUST be consistent with the related disclosure in the corresponding GIPS Composite Report, unless the disclosure included in the GIPS advertisement is more current and has not yet been reflected in the corresponding GIPS Composite Report.

8.A.9 Limited distribution pooled fund returns included in a GIPS advertisement MUST be derived from the returns included in or that will be included in the corresponding GIPS Pooled Fund Report.

8.A.10 Disclosures included in a GIPS advertisement for a limited distribution fund MUST be consistent with the related disclosure in the corresponding GIPS Pooled Fund Report, unless the disclosure included in the GIPS advertisement is more current and has not yet been reflected in the corresponding GIPS Pooled Fund Report.

8.A.11 Benchmark returns included in a GIPS advertisement MUST be total returns.

8.A.12 The firm must clearly label or identify:

(a) The name of the composite or pooled fund for which the GIPS advertisement is prepared.

(b) The name of any benchmark included in the GIPS advertisement.

(c) The periods that are presented in the GIPS advertisement.

8.A.13 Other information beyond what is required or recommended under the GIPS Advertising Guidelines (e.g., composite or pooled fund returns for additional periods) MUST be presented with equal or lesser prominence relative to the information required or recommended by the GIPS Advertising Guidelines. This information MUST NOT conflict with the requirements or recommendations of the GIPS standards or the GIPS Advertising Guidelines.

8.A.14 All required and recommended information in a GIPS advertisement MUST be presented in the same currency.

All composite returns included in a GIPS advertisement must be derived from the returns that have been included in or that will be included in the GIPS Composite Report. It is permissible to include updated, more recent performance in the advertisements.

Note

Performance information should be as up to date as possible, as long as it is accurate. Therefore, of course it is permissible to include updated performance. In general firms are far too slow updating GIPS reports, particularly waiting many months for

(*continued*)

(*continued*)

the completion of independent verification. It is acceptable, and I would say desirable, to update GIPS reports with unverified shorter period performance. Of course, there is an element of self-selection, following a period of good performance, in the decision to advertise – but good due diligence will establish the value of the long-term track record.

A GIPS advertisement may include information beyond what is required or recommended; however, this information must:

(1) Be shown with equal or less prominence (in a smaller font, for example),
(2) Not conflict with the GIPS standards or advertising guidelines,
(3) Not be false or misleading, and
(4) Adhere to the principles of fair representation and full disclosure.

GIPS advertisements that do not include performance

Clearly, firms should be able to advertise the fact they are GIPS-compliant without providing a GIPS report. The GIPS Advertising Guidelines specifically allow advertising the claim of compliance without presenting performance information.

Note

In previous versions of the GIPS standards, it was not clear whether firms could advertise the fact they were GIPS-compliant without actually providing a GIPS report at the same time. Firms make a significant investment achieving compliance and it is good to see clarified in the 2020 GIPS Advertising Guidelines that firms can advertise the fact that they claim compliance.

Relevant Provisions

8.B.1 The firm MUST disclose the GIPS Advertising Guidelines compliance statement: "[Insert name of firm] claims compliance with the Global Investment Performance Standards (GIPS®)."

8.B.2 The firm MUST disclose the following: "GIPS® is a registered trademark of CFA Institute. CFA Institute does not endorse or promote this organization, nor does it warrant the accuracy or quality of the content contained herein."

8.B.3 The firm MUST disclose how to obtain GIPS-compliant performance information for the firm's strategies and products.

The firm must include the following compliance statement in any GIPS advertisement, without performance, for composites, for limited distribution pooled funds and for broad distribution pooled funds:

> "[Insert name of firm] claims compliance with the Global Investment Performance Standards (GIPS®)."

The firm must also disclose that GIPS is a registered trademark of CFA Institute. Use of the registered trademark in this way gives CFA Institute teeth to take appropriate action against any firm that misuses the term "GIPS", including any false claims of compliance.

Caution

It should be noted that CFA Institute members, CFA Program charter holders, CFA candidates, CIPM Program certificants, and CIPM candidates who misuse the term "GIPS" or misrepresent their performance history or the performance history of their firm are subject to disciplinary sanction under the CFA Institute Code of Ethics and Standards of Professional Conduct.

GIPS advertisements for composites

The GIPS Advertising Guidelines include nine provisions for advertising composites.

Relevant Provisions

8.C.1 If time-weighted returns are presented in the corresponding GIPS Composite Report, the firm MUST present composite total returns according to one of the following:

(a) One-, three-, and five-year annualised composite returns through the most recent period. If the composite has been in existence for less than five years, the firm MUST also present the annualised return since the composite inception date.

(b) The period-to-date composite return in addition to one-, three-, and five-year annualised composite returns through the same period as presented in the corresponding GIPS Composite Report. If the composite has been in existence for less than five years, the firm MUST also present the annualised return since the composite inception date.

(c) The period-to-date composite return in addition to five years of annual composite returns (or for each annual period since the composite inception date if the composite has been in existence for less than five years). The annual returns MUST be calculated through the same period as presented in the corresponding GIPS composite report.

(d) The annualised composite return for the total period that includes all periods presented in the corresponding GIPS Composite Report, through either:

(*continued*)

(*continued*)

(e) The most recent period-end, or
(f) The most recent annual period-end.

8.C.2 If money-weighted returns are presented in the corresponding GIPS Composite Report, the firm MUST present the annualized (for periods longer than one year) or non-annualised (for periods less than one year) composite since-inception money-weighted return through either:

(a) The most recent period-end, or
(b) The most recent annual period-end.

8.C.3 The firm MUST clearly label composite returns as gross-of-fees or net-of fees.

8.C.4 The firm MUST present benchmark returns for the same benchmark as presented in the corresponding GIPS Composite Report, if the corresponding GIPS Composite Report includes benchmark returns. Benchmark returns MUST be of the same return type (time-weighted returns or money-weighted returns), in the same currency, and for the same periods for which the composite returns are presented.

8.C.5 The firm MUST disclose or otherwise indicate the reporting currency.

Although all composite returns included in a GIPS advertisement must be derived from the returns in the corresponding GIPS Composite Report, they do not have to be presented in the same format and alternative options are provided.

Relevant Provisions

8.C.6 See 8.B.1.
8.C.7 See 8.B.2.
8.C.8 See 8.B.3.
8.C.9 The firm MUST disclose if the GIPS advertisement conforms with laws or regulations that conflict with the requirements or recommendations of the GIPS standards or the GIPS Advertising Guidelines, as well as the manner in which the laws or regulations conflict with the GIPS standards or the GIPS Advertising Guidelines.

The GIPS Advertising Guidelines for limited distribution pooled funds are virtually identical to those for composites.

GIPS advertisements for a broad distribution pooled fund

The GIPS Advertising Guidelines include 15 provisions specific to broad distribution pooled funds. Broad distribution pooled funds are well regulated and prospective investors are heavily protected in most, but not all, markets. If laws or regulations mandate a specific approach, then returns must be presented consistent to that approach. In less regulated, less well-protected markets, firms must instead follow the requirements of the GIPS Advertising Guidelines.

Relevant Provisions

8.G.1 If laws or regulations mandate specific pooled fund returns, the firm MUST present pooled fund returns according to the methodology and for the periods required by laws or regulations.

8.G.2 If specific periods are not mandated by laws or regulations, pooled fund returns MUST be presented consistent with one of the following options:

(a) One-, three-, and five-year annualised returns through the most recent period. If the pooled fund has been in existence for less than five years, the firm MUST also present the annualised return since the pooled fund inception date.

(b) The period-to-date return in addition to one-, three-, and five-year annualised returns through the most recent period. If the pooled fund has been in existence for less than five years, the firm MUST also present the annualised return since the pooled fund inception date.

(c) The period-to-date return in addition to five years of annual returns (or for each annual period since the pooled fund inception date if the pooled fund has been in existence for less than five years).

(d) The annualised pooled fund return since the pooled fund inception date through the most recent period.

8.G.3 If the GIPS advertisement is created for a specific pooled fund share class, and pooled fund net returns are presented, pooled fund net returns must reflect the fees and expenses of that specific share class.

8.G.4 If the GIPS advertisement is not created for a specific share class and pooled fund net returns are presented, pooled fund net returns MUST reflect the fees and expenses of:

(a) The share class with the maximum fee that is available for general distribution, or

(b) All share classes.

8.G.5 The firm MUST clearly label pooled fund returns as gross or net of total pooled fund fees.

Broad distribution pooled funds often have many share classes with different fee rates. The GIPS advertisement must reflect the fees of a specific share class if created for that share class, or the maximum fee for all share classes.

Caution

The GIPS standards allow for a weighted average net return that reflects all share classes with appropriate disclosure. It is difficult to see how this may not be misleading for at least some prospective investors. It is far better to use maximum fees or specific share classes.

Relevant Provisions

8.G.6 The firm MUST present benchmark total returns for the same periods for which the pooled fund is presented, unless the firm determines there is no appropriate benchmark.
8.G.7 The firm MUST disclose the current expense ratio and which fees and expenses are included in the expense ratio. The firm MUST disclose if performance-based fees are not reflected in the expense ratio, if applicable.
8.G.8 The firm MUST disclose or otherwise indicate the reporting currency.
8.G.9 The firm MUST disclose the pooled fund description.
8.G.10 If laws or regulations mandate specific information about the pooled fund's risk, as either a qualitative narrative or quantitative metric, the firm MUST disclose this information.
8.G.11 If laws or regulations do not mandate specific information about the pooled fund's risk the firm MUST choose and present an appropriate risk measure or qualitative disclosure that a prospective investor is likely to understand.
8.G.12 The firm MUST disclose the benchmark description, which MUST include the key features of the benchmark or the name of the benchmark for a readily recognized index or other point of reference.
8.G.13 See 8.B.1.
8.G.14 See 8.B.2.
8.G.15 See 8.C.9.

Firms must disclose the current annualised pooled fund expense ratio. The expense ratio is the ratio of total pooled fund expenses to average net assets. Expense ratios exclude transaction costs but facilitate a good comparison of the other fees and costs of pooled fund managers.

VERIFICATION

Verification is the review of the firm's performance measurement processes and procedures by an independent third party or "verifier". Verifiers conduct testing of a firm on a firm-wide basis, in accordance with the required verification procedures of the GIPS standards.

Verification provides assurance on:

(1) Whether the firm has complied with all the composite construction requirements firm-wide.
(2) Whether the firm's processes and procedures are designed to calculate and present performance in compliance with the GIPS standards.

Verification is intended to provide the firm and its prospective and current clients and pooled fund investors additional confidence in its claim of compliance with the GIPS standards. Verification does not ensure the accuracy of any particular composite presentation. Verification will almost certainly improve the expertise of performance analysts, improve internal process and procedures, improve the quality of internal documentation and provide confidence to senior management that the firm is actually compliant with the GIPS standards. There will of course

be marketing advantages and firms will be able to respond positively to request about independent verification, which should help in the selection process. The GIPS standards recommend that firms and asset owners are verified.

Note

The verification process also provides some leverage to the performance measurement team or those responsible for GIPS compliance. If they are asked to provide information that is ethically dubious, they can refer to both the GIPS standards and independent verifiers – a very effective deterrent, in my experience.

At the completion of the verification process, the firm must provide a management representation letter to the verifier confirming compliance with the GIPS standards. Based on the procedures performed, the verifier will express an opinion on whether the firm's policies and procedures for complying with the GIPS standards related to composite and pooled fund maintenance, as well as calculation, presentation, and distribution of performance, have been, in all material respects, designed in compliance with the GIPS standards and implemented on a firm-wide basis. If the verifier concludes that the firm is not in compliance, they must not issue a verification report. The firm must not state that it has been verified unless a verification report has been issued. After the verification is complete, the verifier should issue a recommendation letter to the firm describing specific findings, recommendations and other areas for improvement arising from the verification.

Note

GIPS compliance is not a time-specific project. It is not sufficient for firms to be compliant at a point in time – firms must be constantly compliant. Firms may achieve compliance, but there is always room for improvement. Verifiers will see many firms; they recognise best practice. The verifier's recommendation letter is useful for driving future improvements.

Caution

Compliance is non-trivial; claiming compliance without verification is highly risky. An erroneous claim of compliance could cause both significant reputational damage and major problems with regulators. I would recommend that verification be undertaken at least annually.

The cost of verification will be determined not only by the number of composites and portfolios but also the complexity of the business, the perceived quality of controls and the quality

of performance measurement systems. Verification can cost from as little as a few thousand dollars to hundreds of thousands of dollars for large, complex businesses.

Verifiers need only be independent of the asset manager, have a good understanding of the standards and relevant practical experience. The asset manager must consider the quality of the verifier not only in terms of attaching their name to performance presentations but in the assurance given to the firm of the accuracy of the claim and the effectiveness of the verification process.

Note

I see no conflict of interest in verification firms providing pre-compliance consultancy; I would certainly recommend that firms bring verifiers into the initial compliance process at a very early stage.

Independent verification provides assurance on whether the firm's policies and procedures for composite maintenance, as well as the calculation, presentation and distribution of performance information, have been designed in compliance with the GIPS standards and have been implemented consistently on a firm-wide basis. Crucial to the verification process is the assumption by all interested parties that the verifier performs its service in an unbiased manner and is not testing its own work. Firms are not responsible for a verifier's independence assessment; however, they must understand the verifier's policies for maintaining independence and make a judgement of their adequacy. If a firm becomes aware of an issue that might impact independence, they must inform the verifier.

Performance examination

In addition to independent verification, a firm may choose to also have a performance examination of a specific composite or pooled fund to provide additional assurance. A performance examination tests, for a specific composite, whether the firm has constructed the composite and calculated the composite performance in compliance with the GIPS standards, and whether the firm has prepared and presented the GIPS Composite Report in compliance with the GIPS standards. A performance examination tests, for a specific pooled fund, whether the firm has calculated the pooled fund performance in compliance with the GIPS standards, and whether the firm has prepared and presented the GIPS Pooled Fund Report in compliance with the GIPS standards. A performance examination may only be performed either concurrently with, or subsequent to, the completion of a verification.

The GIPS standards recommend that firms undergo verification, but performance examinations are neither required nor recommended as part of the GIPS standards. Although voluntary, most firms choose to be independently verified.

Note

I strongly encourage independent verification – I believe it adds significant value. However, I do not believe performance examinations add sufficient value to cover costs. Performance examinations are rare outside the United States.

ACHIEVING COMPLIANCE

Achieving compliance is a non-trivial exercise. Performance measurers alone cannot achieve compliance; senior management must buy in to the exercise from the start.

To increase the chance of success I would recommend establishing a steering committee chaired by the project sponsor and tasked with monitoring progress, allocating resources, ensuring co-operation within the firm and addressing specific issues.

A sound project plan is absolutely essential. GIPS compliance projects can easily drift, and for a relatively complex business at least one year should be allowed. A new relatively simple business may achieve compliance in a short period, but six months would be a very aggressive target for most businesses. Allow plenty of contingency in the project plan, which should also include time to educate the entire firm about what it means to be compliant. Many firms achieve compliance and verification but ultimately fail because client-facing individuals within the firm are unaware of their new responsibilities.

The most common issues firm struggle with are:

(1) *Definition of firm*

The firm definition determines the boundaries of the firm for establishing total firm assets – discretionary and non-discretionary.

The firm will have to justify its definition in terms of how it presents itself to the public. The firm will also have to demonstrate that all accounts that fall within the firm definition have been identified. Fee income is a useful indicator that an account exists and is particularly useful for demonstrating to verifiers that all accounts have been identified.

(2) *Definition of composite*

The firm will have almost complete flexibility to define composite guidelines initially, but once defined it is difficult to make changes.

My recommendation would be to start with narrow definitions – it is a relatively easy process to define a new composite later that encompasses a number of narrow composites. Firms are required to disclose the composite creation date to illustrate that the composite may have been created retrospectively.

(3) *Lack of data*

The performance claim must be supported by relevant data, which at a minimum will require periodic valuations and cash flows. Often one of the toughest barriers to compliance is the lack of data, particularly from accounts that are now closed.

(4) *Valuation of illiquid assets*

Clearly, valuation of assets is a key part of any performance return. The processes for valuing illiquid assets should be well documented, applied consistently and provide a fair estimate of value. Asset managers should be cognisant of their requirements to provide a fair and honest representation of performance.

MAINTAINING COMPLIANCE

Having achieved compliance, maintaining it is not that straightforward. Compliance is not only calculating the correct returns and maintaining the correct composites but also, crucially, presenting information in the correct format to prospective clients and asset owners. In essence, the claim of compliance by an asset manager means that performance presented by that asset manager is a fair and honest representation of performance. This requires that the entire firm is educated about the meaning of the claim of compliance.

Note

To help maintain compliance I would certainly appoint an individual responsible for ensuring the integrity of the firm's performance presentations and establish a quarterly review process in addition to an annual verification.

The environments in asset manager firms are so different and the flexibility built into the standards so great that it is very difficult to provide generic advice or rules that are suitable for all. However, if asset managers take as their touchstone the requirement to provide a fair and honest representation of the firm's track record, they won't go far wrong. The GIPS standards help to lay down good practice and point the way to best practice. The foundations established by adopting the standards and establishing strong procedures and good quality controls will not only enhance the quality of performance measurement analysis within the firm but strengthen the firm itself.

GIPS STANDARDS FOR ASSET OWNERS

Asset owners who manage assets and market to prospective clients should claim compliance with GIPS standards for firms. The GIPS standards for asset owners are a separate chapter for asset owners who manage their own assets directly or select asset management firms. The separate chapter for asset owners is really a reminder that the GIPS standards are primarily developed for the benefit of asset owners – this separate chapter should re-engage asset owners and hopefully increase the number of asset owners demanding compliance of the asset management firms they seek to hire, for all asset classes and investment strategies.

Asset owners claiming compliance with GIPS for asset owners have different objectives. They are demonstrating their commitment to the highest ethical standards to a wider audience, claiming compliance for their own internal reports to the oversight committee or ultimate owner and accessing verifiable, better internal controls and processes.

Asset owners generally do not have multiple clients; typically they are managing an individual, dedicated portfolio of assets for a pension fund, endowment, foundation, family office or sovereign wealth fund. Composites and pooled funds are therefore less relevant; asset owners are typically reporting the results of the total fund. Therefore, asset owners are not required to present composites in compliance with the GIPS standards and instead are required to report performance for the total fund. In the context of fewer incentives to game results and encouraging asset owners to claim compliance, the minimum initial period of compliance is one year, not five.

8

Bringing It All Together

When performance is measured, performance improves. When performance is measured and reported, the rate of improvement accelerates.

Thomas S. Monson (1927–2018)

If you can't describe what you're doing as a process, you don't know what you are doing.

W. E. Deming (1900–1993)

Though it's cold and lonely in the deep dark night, I can see paradise by the dashboard light.

Jim Steinman (1947–2021), *Paradise by the Dashboard Light*, 1977

Performance analysts are required to process large volumes of data. They must analyse data quickly and communicate effectively. Excellent data visualisation skills are essential. Data visualisation illuminates complex relationships within masses of data. Most people are more comfortable with visual representation and process information quicker if it's presented in graphical format. Well-designed performance dashboards facilitate quick analysis and communicate at a glance what has happened to, and in, the portfolio of assets. Within a dashboard, utilising size, colour, font, positioning and graphical tools, complex relationships can be identified, explained and presented.

La Grouw defines a dashboard as:

A means to provide "at a glance" visibility into key performance using simple visual elements displayed within a single digital screen.[1]

EFFECTIVE DASHBOARDS

Effective dashboards should:

(1) Focus on the key measures.

It is all too easy to drown in information. The key skill set of any performance analyst or risk controller is not, surprisingly, the accurate calculation and distribution of numbers. Too much data is nearly as bad as inaccurate data. Rather, the key skill set is to determine with the end user what is relevant and make sure that information is communicated efficiently and effectively. The goal of dashboards is to alert the user to potential issues and allow action to be taken.

[1] La Grouw, *Effective Dashboard Design* (2012).

(2) Be available.

Dashboards should be available to multiple stakeholders (portfolio managers, senior management, risk controllers, marketing and sales, compliance and, not least, asset owners) through multiple devices in and out of the office. Different stakeholders, no doubt, will require different views and some access may well need to be restricted.

(3) Be interactive.

Dashboards should be interactive to allow users the ability to drill down to root causes of issues.

DATA VISUALISATION TOOLS

The humble table of numbers can still be effective in communicating data but may be enhanced if accompanied with appropriate graphical presentation. Remember, simpler is better. All dashboards should be to the point, uncluttered, focused on the key information, internally consistent, accurate, available to relevant stakeholders, labelled coherently and should not mislead the user. All too often analysts are seduced to present the "cool" graphic that looks good but frankly fails in its primary role of communicating information. Effective data visualisation graphical tools include:

(1) *Scatter graphs*

Great for demonstrating correlations and relationships between variables (portfolio and benchmark returns, for example).

(2) *Line graphs*

Great for showing the trends of a variable over a period of time (ex-ante and ex-post tracking error against limits, for example).

(3) *Histograms*

As discussed in Chapter 5, a histogram is an excellent tool for demonstrating the distribution of returns – an essential visual tool for performance analysts.

(4) *Pie chart*

The familiar pie chart is a circular graph which is divided into slices to illustrate numerical proportion. Pie charts, bar charts and line graphs were all invented by William Playfair.[2]

(5) *Heat map*

Essentially, heat maps use colour and size to illustrate complex and large data relationships. They can be useful for identifying and focusing on particular issues. They look great but need to be used with great care, as they are sometimes difficult to interpret – looking good is not sufficient; they need to meet a purpose.

(6) *Speed dials*

A speed dial uses the metaphor of a speedometer to visually compare risk levels and limits, often colour-coded: **green** = good, **red** = bad. Keep in mind that about 8% of males of Northern European descent are colour-blind[3] (only 0.5% of females are afflicted) and that there is a higher occurrence of colour-blindness in quantitative people than in the general population. Good design avoids using colour contrast alone to express information.

[2] Bar charts and line charts in his *Commercial and Political* atlas published in 1786 and pie charts in *Statistical Breviary* published in 1801.

[3] National Eye Institute, "Facts About Colour Blindness" (February 2015).

(7) *Risk triangles*
Risk triangles can be used to represent the expected total risk of a portfolio should an additional asset or portfolio be added to an existing portfolio or asset manager mix.

(8) *Snail trail diagrams*
Snail trails are useful for showing the progression of an appraisal ratio consisting of two variables through time (reward in the vertical access, risk in the horizontal). Normally more recent time periods are shown by a thicker, darker line (or *trail*).

(9) *Spider chart (radar, web or star chart)*
A spider chart consists of a number of equi-angular spokes, each spoke representing a variable. The date length of each spoke is proportional to the size of each variable. A line can be drawn connecting all points. It's used to identify outliers.

(10) *Floating bar charts*
Floating bar charts (or box plots) are useful for showing the distribution of a variable (return, risk measure or appraisal ratio) over a number of portfolios and time periods, showing the relationship between the median, various percentiles (quartiles, quintiles or deciles) and outliers.

MANAGER SELECTION

Performance measurement, risk analysis, return attribution and adherence to ethical standards are key parts of the asset manager selection process. Asset manager selection deserves a whole book in its own right, such as *Manager Selection* by Stewart.[4]

Asset manager selection

Asset management is a negative sum game. Before costs, the average of all asset managers, in the broadest definition, can only at best match the total market. Even before management fees, but after transaction costs, asset managers as a group must underperform the market. However, for some asset managers to outperform, others must underperform. Most academic research concludes that the average manager will underperform – that is clearly true. But do superior (skilled) managers exist? If they do, can you identify them? – and if you can identify them, can you build a portfolio of asset managers that meets the objectives of the asset owner?

Caution

My experience would suggest that skilled managers do exist, but they are far outnumbered by managers who lack skill. I would suggest that persistence in skilled managers also exists but is difficult to find and statistically the period required to be certain that skill exists is probably shorter than changes in asset manager firms, investment conditions and business cycles.[5]

[4] Stewart, *Manager Selection* (2013).
[5] Harris (2002), "Performance Evaluation & Prediction," Chapter 22, pp. 442–481.

Waring, Whitney, Pirone and Castille[6] and Waring and Siegel[7] suggest that selecting a portfolio of asset managers is equivalent to selecting a portfolio of assets or securities – it is a portfolio optimisation issue. The inputs are expected alpha, expected relative risk and the expected correlation of alpha.

Performance analysis in the context of asset manager selection is really a three-step process:

(1) *Manager evaluation*
Quantitative and qualitative analysis of individual asset managers, including appropriate due diligence.

(2) *Portfolio evaluation*
The "best" asset manager in isolation need not be the best fit with the existing portfolio of asset managers. Selection diversification is a key part of the process. Asset owners should choose the asset manager that best fits the existing portfolio.

(3) *Monitoring and control*
Continuous monitoring is essential.

Manager evaluation

Manager evaluation processes will vary from asset owner to asset owner, but due diligence is essential.

Manager due diligence is commonly described as a consideration of "Four Ps": people, process, performance and potential:

(1) *People*
Asset management is a people business. The asset owner should examine the ethics, experience and availability of the key people managing the account. Is senior management accessible not just in the sales process but ongoing? What have been the past mistakes (a good manager has nothing to hide)?

(2) *Process*
The asset owner needs to understand the asset manager's investment process. Is it well articulated? Is it supported by appropriate documentation? Is it repeatable and sustainable?
What are the asset manager's internal risk controls? Is the risk control function independent? Has the strategy been stress-tested?

(3) *Performance*
Quantitative and qualitative analysis of the asset manager's track record is essential.
- Have the returns been ethically produced?
- Are they GIPS compliant?
- Are they consistent?
- Are the returns and associated attribution analysis in line with the investment process?
- What do the appraisal measures say about the risk-adjusted performance of the asset manager?
- Why have specific appraisal measures been chosen?
- Has the appraisal measure been data-mined?
- Do the appraisal measures tie in with the investment process – is there more than one? More than one appraisal measure is a red flag!

[6] Waring, Whitney, Pirone and Castille, "Optimizing Manager Structure and Budgeting Manager Risk" (2000).
[7] Waring and Siegel, "The Dimensions of Active Management" (2003).

- Is the investment process designed to maximise a particular performance measure?
- Are performance returns based on model calculations or on actual managed assets? Back-tested and model calculations might be of value to verify internal processes, but are of little value for external use. No one will publish back-test numbers if the result is negative.

(4) *Potential*

Most asset managers have some sort of capacity limit – a size beyond which the strategy does not perform in the same way. Does this strategy continue to work in the future, and will the alpha be diluted? What is the asset manager's assessment of their own capacity? What is the total market capacity for this type of strategy?

Portfolio evaluation

Asset managers should not be selected in isolation: a good fit within the existing portfolio of asset managers is essential.

Grinold's[8] Fundamental Law of Active Management provides for an estimate of the information ratio as follows:

$$IR = IC \times \sqrt{Br} \tag{8.1}$$

where: IC = information coefficient, defined as the correlation of ex-ante forecasts with ex-post return
Br = breadth or the number of independent bets

Applied to a portfolio of asset managers Grinold's fundamental law implies that increasing the number of asset managers, hence increasing breadth, should be beneficial; however, the transfer costs of firing one asset manager and hiring another, together with the ongoing costs of monitoring, should be considered. Risk triangles are particularly useful for assessing the added value of introducing a new asset manager to the existing portfolio of asset managers. A good asset manager in isolation might add value but not reduce risk if highly correlated with existing asset managers – we should seek asset managers with investment styles that are negatively correlated with existing asset managers. A slightly inferior asset manager might add a little less value but could well reduce risk, resulting in a significantly improved information ratio overall.

Monitoring and control

Asset managers who have been appointed should be reviewed on a regular basis and a judgement made about the continuing appropriateness within the asset owner's wider portfolio. A variety of issues might provoke a decision to terminate, such as:

(1) Underperformance below benchmark
(2) Style drift
(3) Identification of more skilled asset managers
(4) Reputational damage

[8] Grinold, "The Fundamental Law of Active Management" (1989).

(5) Operational issues
(6) Changes in asset and risk allocations
(7) Changes in personnel, confidence in individuals
(8) Increased correlations with other asset manager styles

Appropriate actions will range from discussing problems with asset managers to putting asset managers on alert or on a watchlist and of course terminating the relationship. Care should be taken, though; Goyal and Wahal[9] demonstrated that asset owners often overreact to near-term underperformance by "selling low", terminating existing asset managers and "buying high", hiring asset managers with good near-term outperformance. Also, Stewart, Heisler, Knittel and Neumann[10] estimated that more than $170 billion was lost during the period 1984–2007 by asset owners making rebalancing or reallocation decisions. In their study investment products receiving contributions (hired asset managers) subsequently underperform products experiencing withdrawals (fired asset managers) over one, three- and five-year periods.

Caution

The transaction costs of switching asset managers is not trivial, in terms of actual costs rebalancing the portfolio (bid/offer spread, commission and taxes), the physical cost of the search exercise and due diligence, and of course the risk of hiring a subsequently underperforming manager and firing a subsequently outperforming manager. My rule of thumb over the years has been that this cost might be as great as 3%.

THE FOUR DIMENSIONS OF PERFORMANCE

Chapter 3 focuses on ex-post return and Chapter 5 focuses on ex-post risk, but these are just two dimensions of performance. As illustrated in Figure 8.1 it is helpful to view performance measurement or performance analysis in four dimensions:

First dimension: Ex-post return
Second dimension: Ex-post risk
Third dimension: Ex-ante return
Fourth dimension: Ex-ante risk

Successful asset managers should manage, analyse and communicate in all four dimensions.

Ex-post return (the traditional dimension)

Ex-post return is clearly the most well-established dimension and the starting point of any analysis; fundamentally, all asset owners are looking for positive ex-post returns – the traditional or foundation dimension. Ex-post returns have been calculated for decades: standard policies and methodologies are well understood, codified and globally consistent. There might be a debate about whether or not, to use time-weighted or money-weighted returns, but in

[9] Goyal and Wahal, "The Selection and Termination of Investment Management Firms by Plan Sponsors" (2008).
[10] Stewart, Heisler, Knittel and Neumann, "Absence of Value: An Analysis of Investment Allocation Decisions by Institutional Plan Sponsors" (2009).

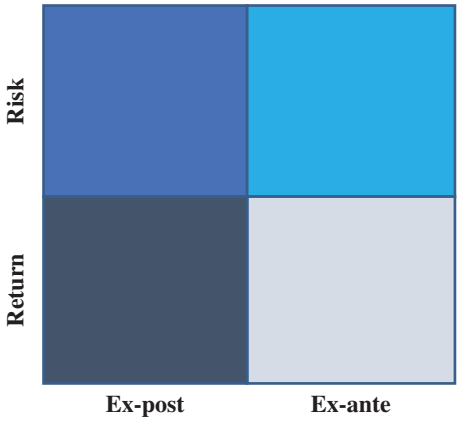

Figure 8.1 The four dimensions of performance

general the issues are well known. Policies for timing of external cash flow, income accrual and valuation of assets are universally followed. Most asset owners fully understand ex-post return and expect consistent reporting of both absolute and relative return. Global Investment Performance Standards (GIPS®) are primarily focused on the consistent calculation and fair representation of these ex-post returns.

Ex-post risk (the neglected dimension)

Ex-post risk is also well established and reasonably well understood, but perhaps a little neglected. Most investment professionals understand that risk must be taken into account, but frequently they choose just to report ex-post return in isolation, particularly in relation to performance fees. Ever since Sharpe's seminal work in 1966 (and perhaps before), it has been accepted that performance is at least two-dimensional. Ex-post return and ex-post risk are most commonly linked in the reward-to-risk ratios extensively covered in this book – performance appraisal:

$$\text{Performance appraisal} = \frac{\text{Reward}}{\text{Risk}} \tag{8.2}$$

Ex-ante return (the unknown dimension)

I describe ex-ante return as the unknown dimension, but that's not strictly true. Ex-ante returns are of course unknown, but they can, and should, be forecast. Asset managers should have an alpha thesis: they should be able to describe how they expect to add value and by how much. They should be able to demonstrate that alpha is sustainable, and they should make specific returns forecasts. These ex-ante return forecasts should be reviewed and compared directly to the realised ex-post returns once available. How close to the realised return is the forecast return? Do realised returns exceed the forecast? This comparison is described as return efficiency.

$$\text{Return efficiency} = \frac{\text{Ex-post return}}{\text{Ex-ante return}} \tag{8.3}$$

Asset managers should publish a range of return scenarios.

Ex-ante risk (the "sexy" dimension)

I've described ex-ante risk as the "sexy" dimension because it attracts the most heat and attention. Professional risk managers are a curious combination of artist, scientist and practitioner, simultaneously employing intuition, complex mathematical analysis and hard-earned experience. It's no wonder, then, that the field of financial risk management attracts the best and brightest, luring some from the most challenging fields in academia. Forecasting risk and protecting the portfolio from future risk is much more interesting and rewarding than reporting on risk ex-post and determining whether the reward was worth the risk taken after the fact. Ex-ante return should be compared to ex-ante risk to ensure the forecast is reasonable and achievable. Why forecast an information ratio of greater than 1 when past experience would suggest that it is very difficult to achieve?

$$\text{Performance forecast} = \frac{\text{Ex-ante return}}{\text{Ex-ante risk}} \tag{8.4}$$

Risk efficiency ratio

Ex-post risk measures are lagging indicators that typically do not change dramatically through time. However, for effective risk control it is essential to compare the predictive ex-ante risk calculated by internal systems with the actual realised ex-post risk of portfolios.

It is important to monitor changes over time in both the ex-ante and ex-post risk measures. It is also important to compare forecast risk with realised risk to gauge how close the forecast is to reality.

The risk efficiency ratio compares realised risk with forecast risk; ideally, we would like this ratio to be 1, indicating that our forecasting tools were efficient. If the ratio is much greater than 1, then we are aware that our forecasting tool is underestimating risk. Similarly, if the ratio is much less than 1, then we are aware that our forecasting tool is overpredicting risk. Either situation is undesirable.

$$\text{Risk efficiency} = \frac{\text{Ex-post risk}}{\text{Ex-ante risk}} = \frac{\text{Realised risk}}{\text{Forecast risk}} \tag{8.5}$$

Risk efficiency is most often calculated using tracking error but could just as easily be used with variability, VaR or conditional VaR.

Clearly this measure compares the prospective risk of a current snapshot of the portfolio with the historic ex-post risk of the portfolio after the fact, including transactional history over an extended period of typically three years. There will be differences reflecting past transactions and potential window dressing (the process of reducing bets immediately before measurement, which will of course reduce prospective risk) but the real value of this statistic is the simple comparison of ex-post with ex-ante risk statistics and the questions this will raise.

⚠ Caution

All too often ex-post statistics are calculated by performance measurers and ex-ante statistics forecast by risk managers with very little communication between the two.

Performance efficiency

Finally, asset managers should compare their chosen realised performance appraisal measure to the forecast appraisal measure.

$$\text{Performance efficiency} = \frac{\text{Performance appraisal}}{\text{Performance forecast}} \tag{8.6}$$

RISK CONTROL STRUCTURE

Risk control is of course about controlling future risk, not historical risk; however, historical, ex-post risk does play a significant role in the total risk control environment of any firm.

Control of risk is a matter of common sense, good competent management, appropriate incentives, robust structures, connected systems, simplicity, transparency and awareness.

Whichever appraisal measure you decide to focus on it is important to monitor the change in that measure over time. Risk controllers can never assume the measure is accurate; it is far better to analyse the change over time and investigate any sudden changes.

The change may result from data errors, system or model errors, change in model assumptions, or an intentional or unintentional change in actual portfolio risk. Whatever the reason for the change, it should be discussed with the asset manager and fully understood.

Performance analysts have a key role to play in the risk control environment of asset managers. In the ideal asset management organisation, I would have performance measurement, risk control, the legal department, and the internal audit function reporting to the head of the middle office, as shown in Figure 8.2. Performance analysts should never report to the front office or the marketing department.

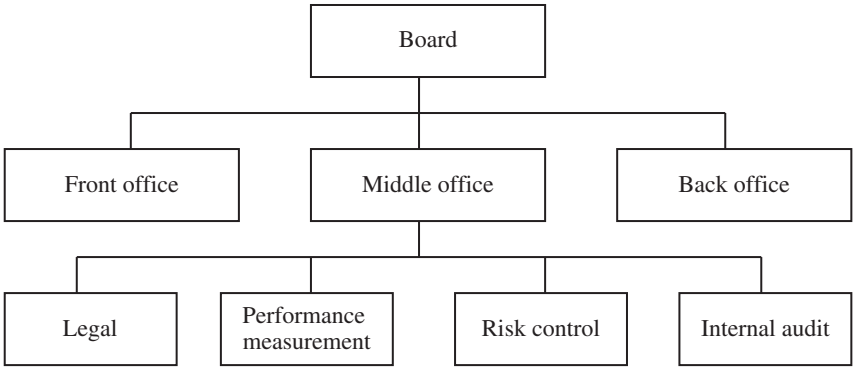

Figure 8.2 Middle office infrastructure

RISK MANAGEMENT

For effective risk management in an asset management firm, the following should be in place:

(1) *Documented risk management process*

To provide a framework for the risk control environment, every asset management firm should have a risk management process policy document that clearly articulates the firm's

attitude, management and reporting of risk. Risk controllers cannot establish a risk framework in the absence of a risk policy. The entire board must take ownership and articulate clearly the firm's policy for risk and their risk appetite. The availability of a written risk policy is a regulatory requirement in many instances, for example, for managers of mutual funds.

(2) *Independence*

For effective risk control it is essential that there are appropriate checks and balances within the firm with a clear front-, middle- and back-office structure with clear areas of responsibility and reporting lines. The middle office, which neither takes investment decisions nor processes transactions, should be independent of both the front and back office.

(3) *Risk awareness*

Risk awareness should be embedded within the firm. Every employee will own some aspect of risk; it is essential that they understand the risks they own and are constantly self-assessing their own risks. Risk literacy, at all levels, in all functions, should be encouraged.

Once identified, there are four possible responses to risk:

(a) Accept

A risk although identified may be accepted if the cost (including opportunity cost) of controlling it is greater than the potential cost of risk failure.

(b) Mitigate or transfer

Arranging appropriate insurance cover or hedging may mitigate the cost of risk failure or transfer risk to another party.

(c) Control

Risk may be controlled by establishing risk limits and establishing monitoring procedures.

(d) Eliminate

It may be appropriate to eliminate an identified risk by ceasing that type of activity.

(4) *Clear risk limits*

Risk limits should be clear, unambiguous, quantifiable and agreed by the asset owner and asset manager in the investment management agreement.

(5) *Risk and performance attribution*

The sources of risk and return should be identified and monitored independently using return attribution and other techniques. Accurate return and risk attribution can be used to identify the consistency of added value across the firm and consistency with the agreed investment objectives.

(6) *Appropriate risk-adjusted measures*

Risk and reward should be combined in risk-adjusted performance measures appropriate to the investment management strategy.

(7) *Stress and scenario tests*

Portfolios should be stressed under extreme scenarios to understand vulnerabilities in turbulent, abnormal markets.

(8) *Review process for new products, instruments and strategies*

All new products, instruments and strategies should be rigorously reviewed. For example, a new derivative instrument may meet the needs of the portfolio manager but could generate significant operational and counterparty risks that must be assessed and approved.

(9) *Monitoring of risk efficiency*
Ex-ante and ex-post risk measures should be monitored through time and consistently compared to ensure that the forecast of risk is close to the realised risk or, if different, that there is a good understanding of the possible reasons that explain the difference.

An appropriate risk control infrastructure for an asset management firm would include:

(1) *Risk management committee*
Reporting directly to the board, coordinating all risk control activity, including senior representatives from the front, middle and back office. Responsible for portfolio risk, counterparty risk, operational risk, compliance risk, review of insurance arrangements, disaster recovery, contingency planning and systems change control. Many companies now recognise the need for coordinated thinking and behaviours in relation to risk so have created the role of chief risk officer to oversee all the firm's risk activities.
(2) *Portfolio risk committee*
Reporting to the risk management committee, responsibilities include portfolio and liquidity risk, ensuring that portfolios are managed within client expectations and mandating restrictions and the approval of new products, strategies and instruments.
(3) *Counterparty risk committee*
Reporting to the risk management committee, responsible for approving counterparties and their credit limits.
(4) *Operational risk committee*
Reporting to the risk management committee, responsibilities include error monitoring, information quality and change control.

All risk measures ex-post and ex-ante to some extent rely on historical information which may not include an extreme event; therefore, it is essential in risk management to understand the impact on portfolios of potential extreme events using stress-testing (another term borrowed from engineering) and scenario analysis.

Many risk management systems will offer as standard historical scenarios as precanned stress tests but given recent events risk managers would be strongly advised to additionally develop their own tests unless they believe that financial market history tends to repeat itself. Stress tests are a useful supplement to traditional risk measures that may well reveal vulnerabilities in turbulent, abnormal markets.

⚠ Caution

There is a paradox: recent history better reflects the current relationships between securities and instruments but is less likely to contain an extreme event. In turbulent markets all bets are off: normal relationships break down and all risk measures to varying degrees lose their value.

Risk controllers should guard against complacency; it is a well-rehearsed issue that the calculation of sophisticated risk measures may lead to a false sense of security. Sadly, risk controllers must be permanently cynical, trusting no one or any calculations, particularly their own. It's worth remembering that a performance analyst tells you what happened yesterday, a trader tells you what is happening today, and a risk manager fails to forecast accurately what will happen tomorrow.

Glossary of Key Terms

Abnormal return	A return that appears to misrepresent the true economic performance of the asset being measured.
ABOR	The accounting book of record. The official book of record aligned with regulatory and asset owner requirements.
Active risk	Risk relative to the benchmark. The standard deviation of excess return, otherwise known as relative risk, tracking error or tracking risk.
Active share	A measure of the percentage difference of security holdings in the portfolio as opposed to the benchmark.
Adjusted M^2	Return adjusted for risk, skewness and kurtosis.
Adjusted Sharpe ratio	Sharpe ratio adjusted for skewness and kurtosis.
Allocation	The effect of weight allocations between asset classes, regions, countries, sectors and investment categories.
Alpha	Excess return adjusted for systematic risk.
Annualised return	The annual return which when compounded with itself will generate the cumulative return over multiple years.
Asset class	A class of investments with distinct risk/return properties, homogeneous, mutually exclusive and diversifying.
Asset manager	An investment advisory firm or financial institution making investment decisions on behalf of, and managing the assets of, asset owners.
Asset owner	Investors, typically pension funds, endowments, sovereign wealth funds, boards of investment trusts and high-net-wealth individuals.
Balanced strategy	An investment strategy containing two or more asset classes (not counting cash).
Benchmark	A point of reference against which portfolios, returns or risk are compared.

Beta	Systematic risk.
Calmar ratio	The ratio of excess return above the risk-free rate to maximum drawdown.
Carve-out	A portion of a portfolio that is by itself representative of a distinct investment strategy.
Composite	An aggregation of one or more portfolios managed according to a similar investment mandate, objective or strategy.
Continuous compounded return	The natural logarithm of simple returns.
Correlation	A standardised covariance which measures the tendency of portfolio and benchmark returns to move together.
Dispersion	A measure of the spread of returns within a composite. Sometimes described as internal risk.
Downside risk	Variability of underperformance below a minimum target rate.
Drawdown	Any continuous period of negative returns or peak-to-valley fall in performance.
Duration	Systematic risk for fixed income instruments. The average life of the present values of all future cash flows.
Ex-ante risk	Prospective or forward-looking risk, based on a snapshot of the current securities and instruments within the portfolio and their historical relationship with each other. An estimate or forecast of the future risk of the portfolio.
Ex-post risk	Historical or risk after the event.
Excess kurtosis	Kurtosis in excess of 3, the kurtosis of a normal distribution of returns.
Excess return	The excess return between the portfolio and benchmark returns. Not to be confused with the excess return above the risk-free rate. Can be calculated either arithmetically or geometrically.
Expense ratio	The ratio of total pooled fund expenses to average net assets.
Extremely large cash flow	Cash flows extreme enough to dominate the return calculation.
GIPS	Global Investment Performance Standards. Voluntary, ethical standards for investment performance presentations to ensure fair representation and full disclosure of investment performance.
Gross-of-fees	The return on investments reduced by any transaction costs, before deducting investment management fees.
Holdings-based attribution	Return attribution calculated from holdings-based data only.

IBOR	The investment book of record. Records aligned with the investment decision process and the requirements of the front office.
Information ratio	The ratio of annualised excess return compared to annualised tracking error.
Interaction	The combined effect of allocation and selection effects.
Large cash flow	A cash flow large enough to potentially distort the return calculation.
Kurtosis	The fourth moment of the return distribution, a measure of the weight of returns in the tails of the distribution relative to standard deviation. More often associated as a measure of flatness or peakedness of the return distribution.
M^2	Return adjusted for risk.
Maximum drawdown	The maximum potential loss over a specific time period, buying at the high point in the return series and selling at the lowest.
Mean return	The first moment of the return distribution, a simple arithmetic average of returns.
Money-weighted return (MWR)	Money-weighted returns weight each dollar (or pound or euro) equally, irrespective of when it is invested.
Net-of-fees	The gross-of-fees return after (or reduced by) investment management fees.
PBOR	The performance book of record. Records reconciled with ABOR but aligned with the investment decision process and the requirements of the middle office.
Performance examination	A process by which an independent verifier conducts testing of a specific composite or pooled fund in accordance with the required performance examination procedures of the GIPS standards.
Pooled fund	A collective fund with multiple asset owners.
Portfolio manager	An individual, normally employed by an asset manager or asset owner, managing a portfolio of assets.
Potential upside	The best expected gain over a given time interval under normal market conditions at a given confidence level.
Relative risk	Risk relative to the benchmark. The standard deviation of excess return, otherwise known as active risk, tracking error or tracking risk.
Return attribution	A technique used to quantify the excess return of a portfolio against its benchmark into the active decisions of the investment decision process.
Returns-based attribution	Return attribution calculated using regression analysis based on returns only.
Return efficiency	The comparison of ex-post, realised return with ex-ante, forecast return.

Risk efficiency	The comparison of ex-post, realised risk with ex-ante, forecast risk.
Risk-free rate	The rate of return an investor can expect from a theoretically risk-free investment.
Risk manager	Risk managers take risk in order to achieve higher return.
Risk controller	Risk controllers measure and monitor risk.
Selection	The selection effect of securities within an asset class, sector or investment category.
Self-selection	The process of calculating multiple returns using different methodologies and selecting the best return.
Sharpe ratio	The grandfather of composite risk/return measures. The ratio of annualised excess return above the risk-free rate divided by the variability of return.
Significant cash flow	Cash flow significant enough to impact the asset manager's investment strategy for a period of time.
Skewness	The third moment of the return distribution, a measure of asymmetry.
Sortino ratio	The ratio of annualised excess return above the minimum acceptable return divided by downside risk.
Subportfolio	A smaller part of a larger portfolio which is nevertheless managed as a standalone portfolio.
Time-weighted return (TWR)	Time-weighted returns weight each time period equally.
Total return	The rate of return that includes realised and unrealised gains and losses plus income for the measurement period.
Transaction-based attribution	Return attribution calculated directly from transaction and holdings-based data
Upside potential	Average sum of returns above target.
Upside risk	Variability of portfolio returns that exceed a given target.
Value at risk	The worst expected loss over a given time interval under normal market conditions at a given confidence level.
Variability	The standard deviation of returns.
Variance	The second moment of the return distribution. The square of standard deviation.
Verification	A process by which an independent verifier conducts testing of a firm on a firm-wide basis, in accordance with the required verification procedures of the GIPS standards.
Volatility	Sometimes used to describe the systematic risk of a portfolio but more commonly the variability.

Appendix A

Simple Attribution

A.1 ATTRIBUTION METHODOLOGY

The following methodology has been developed for use with single-currency portfolios.

A.1.1 Scenario

Suppose we have a portfolio invested in n asset classes or industrial sectors. Then, suppose that the performance of this portfolio is measured against a benchmark.

A.1.2 Portfolio returns

Let the weight of the portfolio in the ith asset class be w_i, where $\Sigma w_i = 1$.

Let the return of the portfolio assets in the ith asset class be r_i.

Now the total portfolio return is:

$$r = \sum w_i r_i$$

A.1.3 Benchmark returns

Let the weight of the benchmark in the ith asset class be W_i, where $\Sigma W_i = 1$.

Let the return of the benchmark for the ith asset class in the base currency of the portfolio be b_i.

Now the total benchmark return and in the base currency of the portfolio is $b = \Sigma W_i\, bi$.

A.1.4 Semi-notional returns

We define the semi-notional return of the ith asset class as:

$$w_i b_i$$

Now the total semi-notional return is:

$$b_S = \Sigma w_i b_i$$

A.1.5 Relative performance

We define the performance of the portfolio relative to the benchmark as:

$$\frac{1+r}{1+b} - 1$$

and it is this relative performance that we attribute in this methodology. We attribute it to two factors, stock selection and country allocation.

A.2 STOCK SELECTION

The term "stock selection" is used to describe the relative performance of the portfolio to the benchmark within a particular asset class. Intuitively this seems to be the portfolio total return, r, relative to the semi-notional total return, b_S (remembering that b_S is the sum product of portfolio weights with the benchmark returns, so any difference between b_S and r is by definition due to stock selection).

We attribute relative performance to stock selection in the ith asset class as follows:

$$w_i\left(\frac{1+r_i}{1+b_i}-1\right)\left(\frac{1+b_i}{1+b_S}\right)$$

So, the total stock selection is:

$$\begin{aligned}\sum_{i=1}^{n} w_i\left(\frac{1+r_i}{1+b_i}-1\right)\left(\frac{1+b_i}{1+b_S}\right) &= \sum \frac{w_i[(1+r_i)-(1+b_i)]}{1+b_S}\\ &= \sum \frac{w_i r_i - w_i b_i}{1+b_S}\\ &= \frac{\sum w_i r_i - b_S}{1+b_S}\\ &= \frac{1+\sum w_i r_i - (1+b_S)}{1+b_S}\\ &= \frac{1+r}{1+b_S}-1\end{aligned}$$

A.3 ASSET ALLOCATION

The term "asset allocation" is used to describe the effect of the relative weighting of the portfolio to the benchmark (or "bet") within a particular asset class.

We attribute relative performance to asset allocation in the ith asset class as follows:

$$(w_i - w_i)\left(\frac{1+b_i}{1+b}-1\right)$$

So, the total asset allocation is:

$$\begin{aligned}\sum_{i=1}^{n}(w_i - W_i)\left(\frac{1+b_i}{1+b}-1\right) &= \sum(w_i - W_i)\left(\frac{1+b_i-1-b}{1+b}\right)\\ &= \sum \frac{w_i b_i - W_i b_i - w_i b + W_i b}{1+b}\\ &= \frac{\sum w_i b_i - \sum W_i b_i}{1+b}\\ &\quad \left(\text{because } \sum w_i = \sum W_i = 1 \Rightarrow \sum(W_i - w_i)b = 0\right)\\ &= \frac{\sum w_i b_i - \sum W_i b + \sum W_i b_i - b}{1+b}\end{aligned}$$

$$= \frac{1 + \sum \left[(w_i - W_i)b_i + \sum W_i b_i\right] - (1 + b)}{1 + b}$$

$$= \frac{1 + b_s}{1 + b} - 1$$

A.4 SUMMARY

We now have attributed relative performance to the following factors:

Stock selection	$\frac{1 + r}{1 + b_S} - 1$
Asset allocation	$\frac{1 + b_S}{1 + b} - 1$

We can then see that these factors compound to give:

$$\left(\frac{1 + b_S}{1 + b}\right)\left(\frac{1 + r}{1 + b_S}\right) - 1 = \frac{1 + r}{1 + b} - 1$$

Thus, we have now accounted for all of our relative performance with no residuals (i.e., no "other" term). Once more, because these terms are geometric the relationship holds true over time.

Appendix B

Multi-Currency Attribution Methodology

The following methodology has been developed for use with multi-currency portfolios.

B.1 SCENARIO

Suppose we have a portfolio invested in n asset classes with some asset classes having currencies other than the base currency of the portfolio. Then, suppose that the performance of this portfolio is measured against a benchmark.

B.1.1 Portfolio returns

Let the weight of the portfolio in the ith asset class be w_i, where $\Sigma w_i = 1$.

Let the return of the portfolio assets in the ith asset class:

in local currency be r_{Li}
and in the base currency of the portfolio be r_i

Now the total portfolio return is:

in local currency $r_L = \Sigma w_i r_{Li}$ (or weighted average local return)
and in the base currency of the portfolio $r = \Sigma w_i r_i$

B.1.2 Benchmark returns

Let the weight of the benchmark in the ith asset class be W_i, where $\Sigma W_i = 1$.

Let the return of the benchmark for the ith asset class:

in local currency be b_{Li}
in the base currency of the portfolio be b_i
and hedged into the base currency be b_{Hi}

Now the total benchmark return is:

in local currency $b_L = \Sigma W_i\, b_{Li}$
and in the base currency of the portfolio $b = \Sigma W_i\, b_i$

B.1.3 Semi-notional returns

We define the semi-notional return of the ith asset class:

in the local currency $w_i b_{Li}$
and with any deviation from the index weightings ("bet") hedged into the base currency be $b_{SHi} = (w_i - W_i)b_{Hi} + W_i b_{Li}$

Now the total semi-notional return is:

in the local currency $b_S = \Sigma w_i b_{Li}$
and with any deviation from the index weighting ("bet") hedged into the base currency $b_{SH} = \sum\{(w_i - W_i)b_{Hi} + W_i b_{Li}\}$

B.1.4 Relative performance

We define the performance of the portfolio relative to the benchmark as:

$$\frac{1+r}{1+b} - 1$$

and it is this relative performance that we attribute in this methodology. We attribute it to three main factors, stock selection, country allocation and currency effects.

B.2 STOCK SELECTION

The term "stock selection" is used to describe the relative performance of the portfolio to the benchmark within a particular asset class. Intuitively this seems to be the portfolio total local return, r_L, relative to the semi-notional total local return, b_S (remembering that b_S is the sum product of portfolio weights with the benchmark returns, so any difference between b_S and r_L is by definition due to stock selection).

We attribute relative performance to stock selection in the ith asset class as follows:

$$w_i \left(\frac{1+r_{Li}}{1+b_{Li}} - 1 \right) \left(\frac{1+b_{Li}}{1+b_S} \right)$$

So, the total stock selection is:

$$\begin{aligned}
\sum_{i=1}^{n} w_i \left(\frac{1+r_{Li}}{1+b_{Li}} - 1 \right) \left(\frac{1+b_{Li}}{1+b_S} \right) &= \sum \frac{w_i[(1+r_{Li}) - (1+b_{Li})]}{1+b_S} \\
&= \sum \frac{w_i r_{Li} - w_i b_{Li}}{1+b_S} \\
&= \frac{\sum w_i r_{Li} - b_S}{1+b_S} \\
&= \frac{1 + \sum w_i r_{Li} - (1+b_S)}{1+b_S} \\
&= \frac{1+r_L}{1+b_S} - 1
\end{aligned}$$

B.3 ASSET ALLOCATION

The term "asset allocation" is used to describe the effect of the relative weighting of the portfolio to the benchmark (or "bet") within a particular asset class. Within this effect we include the cost of hedging the bet back to base currency, reasoning that this is a cost that should be

borne by the decision-maker making the bet. In effect we are saying that if the asset allocator causes a currency position in the portfolio, then that position must be notionally hedged back to the neutral benchmark exposure. The cost (or benefit) is reflected in the asset allocation calculation by using a fully hedged index to measure the impact.

We attribute relative performance to asset allocation in the ith asset class as follows:

$$(w_i - W_i)\left(\frac{1+b_{Hi}}{1+b_L} - 1\right)$$

So, the total asset allocation is:

$$\begin{aligned}
\sum_{i=1}^{n}(w_i - W_i)\left(\frac{1+b_{Hi}}{1+b_L} - 1\right) &= \sum(w_i - W_i)\left(\frac{1+b_{Hi}-1-b_L}{1+b_L}\right) \\
&= \sum \frac{w_i b_i - W_i b_{Hi} - w_i b_L + W_i b_L}{1+b_L} \\
&= \frac{\sum w_i b_{Hi} - \sum W_i b_{Hi}}{1+b_L} \\
&\quad \left(\text{because } \sum w_i = \sum W_i = 1 \Rightarrow \sum(W_i - w_i)b_L = 0\right) \\
&= \frac{\sum w_i b_{Hi} - \sum W_i b_{Hi} + \sum W_i b_{Li} - b_L}{1+b_L} \\
&= \frac{1 + \sum\left[(w_i - W_i)b_{Hi} + \sum W_i b_{Li}\right] - (1+b_L)}{1+b_L} \\
&= \frac{1+b_{SH}}{1+b_L} - 1
\end{aligned}$$

B.4 CURRENCY EFFECTS

B.4.1 Naïve currency performance

The difference between the base currency return of the portfolio and the weighted average local return must by definition be the total currency effect. Therefore, the currency return of the portfolio r'_C is:

$$r'_C = \frac{1+r}{1+r_L} - 1$$

Similarly, the currency return of the benchmark b'_C is:

$$b'_C = \frac{1+b}{1+b_L} - 1$$

Therefore, the naïve currency attribution within the portfolio is the difference between the portfolio currency and the benchmark currency.

$$\frac{1+r'_C}{1+b'_C} - 1 = \left(\frac{1+r}{1+r_L} \Big/ \frac{1+b}{1+b_L}\right) - 1$$

or

$$\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1$$

This is defined as naïve because it makes no allowance for the transfer of the cost of hedging discussed in asset allocation above or compounding effects between market or currency returns. It does not reflect the currency effect from the perspective of the "currency overlay manager".

B.4.2 Measured currency returns

We can also derive the "measured" currency return from the bottom up using currency exposures and returns.

We define the currency return of the benchmark in the *i*th currency as:

$$c_i = \frac{1+b_i}{1+b_{Li}} - 1$$

Because most commercial international indexes use the WM Reuters 4 o'clock closing exchange rates, this currency return can be derived from spot rates:

$$c_i = \frac{S_i^{t+1}}{S_{Si}^t} - 1$$

where: S_i^t = the spot rate of currency i at time t

Defining the benchmark forward rate of a forward currency contract as:

$$f_i = \frac{S_i^{t+1}}{F_i^{t+1}} - 1$$

where: F_i^{t+1} = the forward exchange rate of currency i at time t for conversion through a forward contract at time $t+1$

Note the interest rate differential in currency i:

$$d_i = \frac{F_i^{t+1}}{S_i^t} - 1$$

The currency return is therefore:

$$\frac{S_i^{t+1}}{S_i^t} = \frac{S_i^{t+1}}{F_{Si}^{t+1}} \times \frac{F_i^{t+1}}{S_i^t} = (1+f_i) \times (1+d_i)$$

Note that the hedged index return is the combined effect of local return with interest rate differential:

$$b_{Hi} = (1+b_{Li}) \times (1+d_i) - 1$$

or

$$b_{Hi} = \frac{1+b_i}{1+b_{Fi}} - 1$$

We define the total measured currency return of the benchmark as:

$$b_c = \sum W_i b_{Ci} + \sum \widetilde{W}_i f_i$$

where: $\widetilde{W}$ = is the benchmark weight of forward currency contracts in currency i.

The slight difference between b_C and b'_C is caused by compounding between market returns and currency.

This difference is measured by:

$$= \frac{1 + b_C}{1 + b'_C} - 1$$

We define the currency return of the portfolio in the ith currency as:

$$c'_i = \frac{1 + r_i}{1 + r_{Li}} - 1$$

Currency returns in portfolios differ from benchmark currency returns because transactions naturally occur at spot rates different from the closing spot rates.

Forward currency returns in portfolios also differ from benchmark forward currency returns.

Let the forward currency return of the portfolio in the ith currency be: f'_i

We define the total measured currency return of the portfolio as:

$$b_{SC} = \sum w_i c'_i + \sum \widetilde{w}_i f'_i$$

where: $\widetilde{w}_i$ = is the benchmark weight of forward currency contracts in currency i.

Similarly, there is a slight difference between b_{SC} and r'_C measured by:

$$= \frac{1 + r'_C}{1 + b_{SC}} - 1$$

B.4.3 Compounding effects

Comparing the impact of compounding in the portfolio with that of the benchmark the combined impact is measured by:

$$\left(\frac{1 + r'_C}{1 + b_{SC}} \Big/ \frac{1 + b'_C}{1 + b_C} \right) - 1$$

or

$$\frac{1 + r'_C}{1 + b_{SC}} \times \frac{1 + b_C}{1 + b'_C} - 1$$

This factor measures the impact of currency and market compounding invisible to the currency overlay manager but nevertheless an effect within the total portfolio.

This factor may be shown separately or more commonly combined with the currency effect, particularly if the currency overlay manager is not independent of the investment decision process.

B.4.4 Currency attribution

We define the semi-notional currency return of the portfolio as:

$$r_{SC} = \sum w_i c_i + \sum \widetilde{w}_i f_i$$

Note that the semi-notional currency return applies benchmark currency or spot returns to actual physical portfolio weights and benchmark currency forward returns to the actual portfolio forward currency weights.

We define the semi-notional currency return of the portfolio including the cost of hedging as:

$$b_{SC} = \sum W_i c_i + \sum [(\widetilde{w}_i + w_i - W_i) f_i]$$

Currency timing

Currency timing is used to describe the difference between the real portfolio currency returns and benchmark currency returns caused by intra-day FX and FFX trades at spot and forward rates different from those assumed in the benchmark.

Currency timing is analogous to stock selection.

$$w_i \left(\frac{1 + c_i'}{1 + c_i} - 1 \right) \left(\frac{1 + c_i}{1 + r_{SC}} \right)$$

and including forward contracts:

$$\widetilde{w}_i \left(\frac{1 + f_i'}{1 + f_i} - 1 \right) \left(\frac{1 + f_i}{1 + r_{SC}} \right)$$

Total currency timing effect:

$$= \frac{1 + r_C}{1 + r_{SC}}$$

Currency allocation

Currency managers generate currency exposure by using currency forward contracts or currency options whose price is derived from these forward currency contracts. They are priced by reference to spot rates and interest rate differentials between the two currencies. A forward currency contract will therefore generate two exposures, one long and one short.

It follows therefore that the currency manager can only generate a currency position by use of forward currency contracts and is therefore always exposed to interest rate differentials. To measure the impact of any currency bet we must use currency forward rates, not spot rates, to determine the impact of that currency bet.

Currency allocation is analogous to asset allocation selection:

$$(w_i + \widetilde{w}_i - W_i - \widetilde{W}_i) \left(\frac{1 + f_i}{1 + b_i} - 1 \right)$$

So, the total currency allocation performance is:

$$\frac{1 + b_{SC}}{1 + b_C} - 1$$

The total currency effects from the currency overlay perspective are:

$$\frac{1+r_C}{1+r_{SC}} \times \frac{1+b_{SC}}{1+b_C} - 1$$

B.4.5 Cost of hedging

The "cost of hedging" represents the cost or benefit of hedging the asset allocator's decisions back to the "neutral" currency benchmark.

The cost of hedging from the currency overlay perspective is:

$$\frac{1+r_{SC}}{1+b_{SC}} - 1$$

The cost of hedging from the asset allocator's perspective is:

$$\frac{1+b_{SH}}{1+b_S} - 1$$

The asset allocator's perspective includes the compounding with market returns, which causes a very slight mismatch, and is measured by:

$$\left(\frac{1+r_{SC}}{1+b_{SC}} \Big/ \frac{1+b_{SH}}{1+b_S} \right) - 1$$

or

$$\frac{1+r_{SC}}{1+b_{SC}} \times \frac{1+b_S}{1+b_{SH}} - 1$$

This impact is so small it can be ignored, unless you prefer to avoid all residuals.

B.4.6 Total currency effects

Combining all the currency effects in the portfolio we get:

$$\underbrace{\frac{1+r_C}{1+r_{SC}} \times \frac{1+b_{SC}}{1+b_C}}_{\text{Currency overlay}} \underbrace{\frac{1+b_S}{1+b_{SH}} \times \frac{1+r_{SC}}{1+b_{SC}}}_{\text{Hedging mismatch}} \underbrace{\frac{1+r'_C}{1+r_C} \times \frac{1+b_C}{1+b'_C}}_{\text{Compounding}} - 1$$

Which simplifies to:

$$\frac{1+b_S}{1+b_{SH}} \times \frac{1+r'_C}{1+b'_C} - 1$$

The naïve currency effect adjusted for the cost of hedging.

B.5 SUMMARY

We now have attributed relative performance to the following factors:

$$\text{Stock selection} \qquad \frac{1+r_L}{1+b_{SL}} - 1$$

$$\text{Asset allocaion} \qquad \frac{1+b_{SH}}{1+b_L} - 1$$

$$\text{Total currency effects} \quad \frac{1+b_S}{1+b_{SH}} \times \frac{1+r'_C}{1+b'_C} - 1 \qquad \text{or} \quad \frac{1+b_S}{1+b_{SH}} \times \frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1$$

We can then see that these factors compound to give:

$$\underbrace{\frac{1+r_L}{1+b_S}}_{\text{Stock}} \times \underbrace{\frac{1+b_{SH}}{1+b_L}}_{\text{Asset}} \times \underbrace{\frac{1+b_S}{1+b_{SH}}}_{\substack{\text{Hedging cost}\\ \text{transferred}}} \times \underbrace{\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b}}_{\text{Naïve currency attribution}} - 1 = \frac{1+r}{1+b} - 1$$

Thus, we have now accounted for all of our relative performance with no residuals (i.e., no "other" term). Once more, because these terms are geometric, the relationship holds true over time.

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