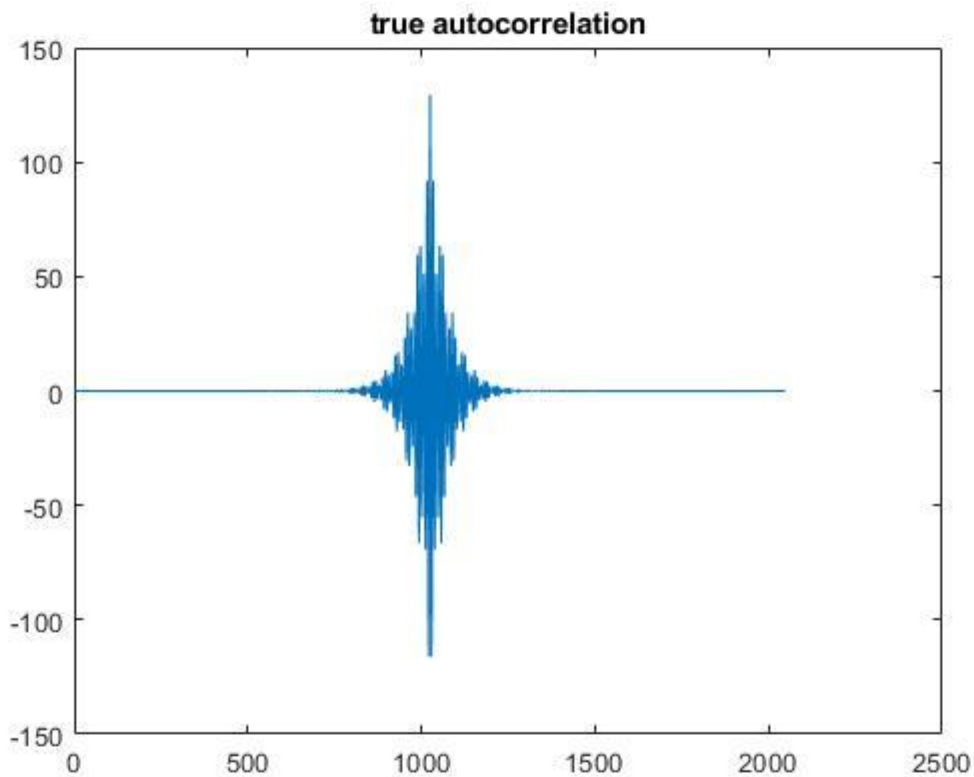
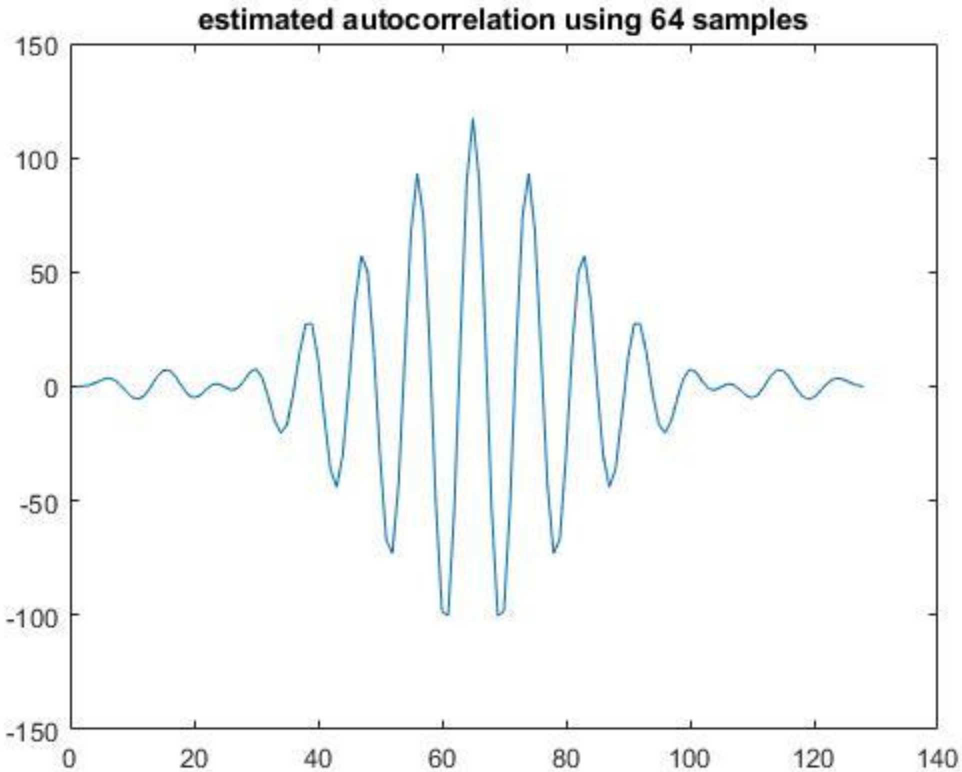
Zhaoxin Hu, z1hu, A53273948

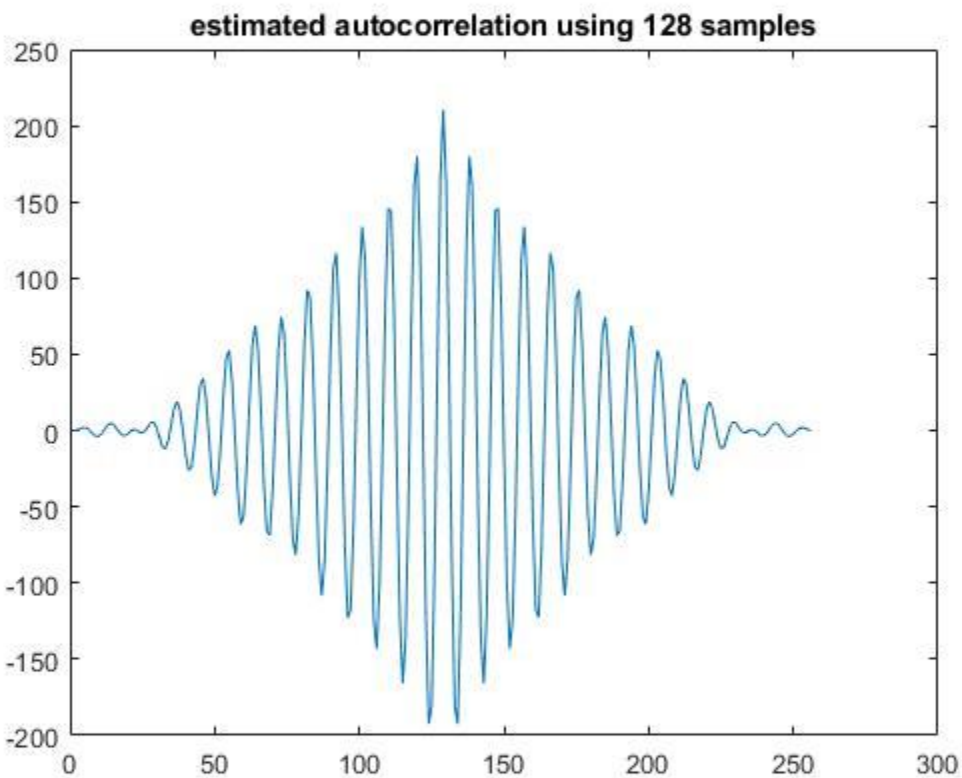
P2 (a)

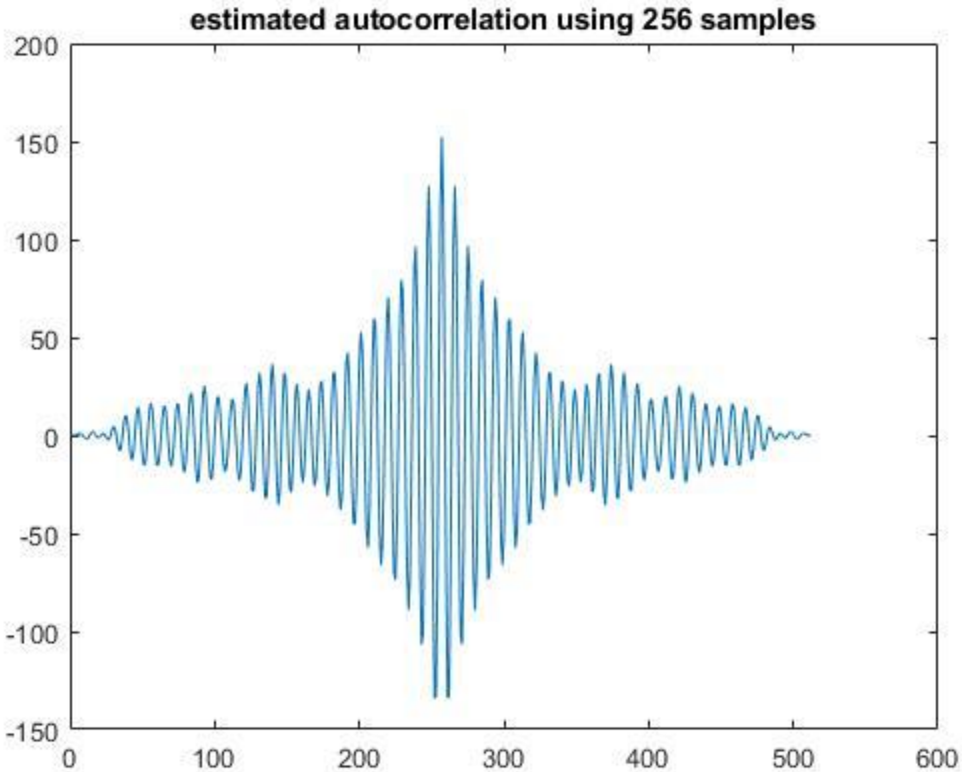


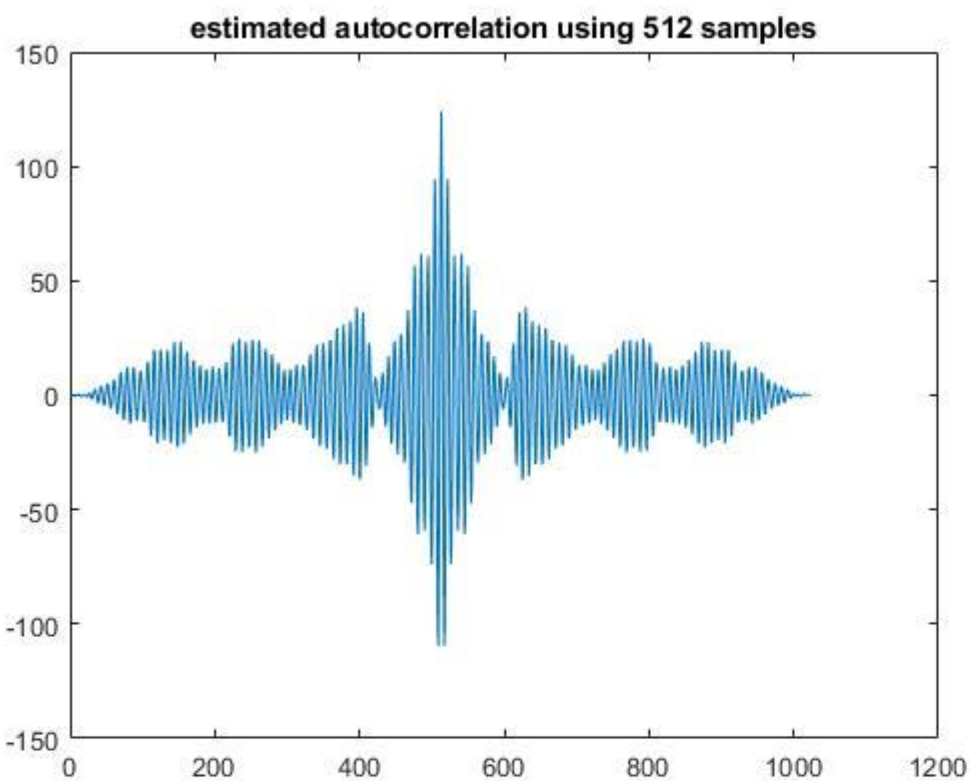
Since the input is a zero mean, unit variance Gaussian white noise (w), the true autocorrelation of the output RP (x) is rxx[m] = h[m]*h*[-m], where h[m] is the impulse response of the system. The convolution can be done with DFT and inverse DFT.

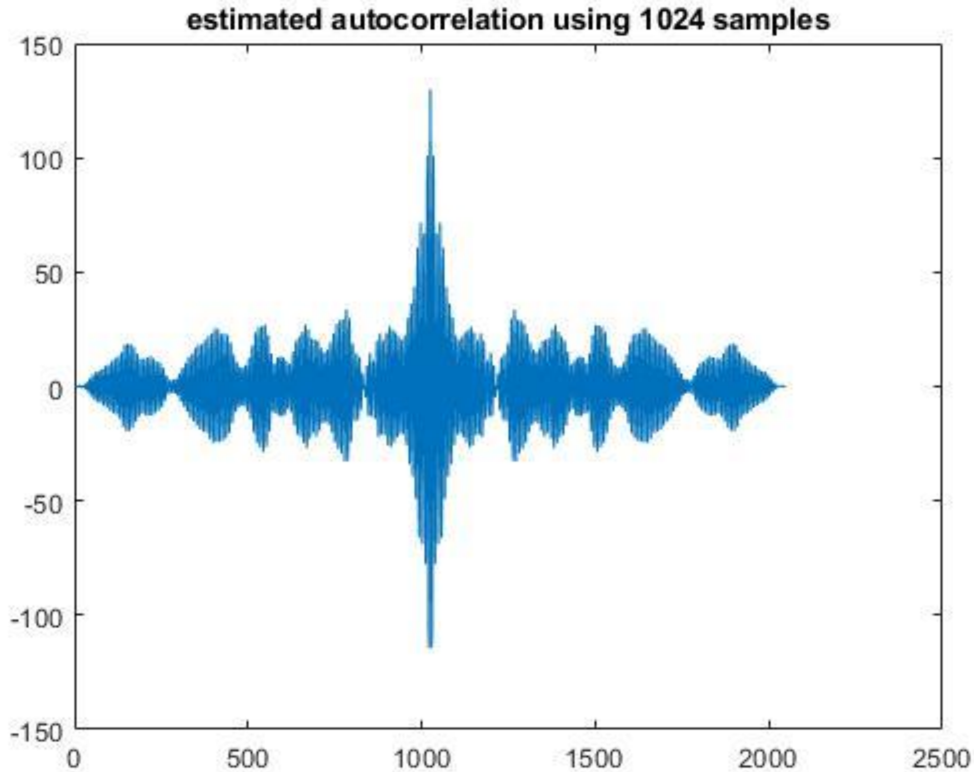
P2 (b)


estimated autocorrelation using 64 samples
150
100
50
0
-50
-100
-150
0
20
40
60
80
100
120
140


estimated autocorrelation using 128 samples
250
200
150
100
50
0
-50
-100
-150
-200
0
50
100
150
200
250
300

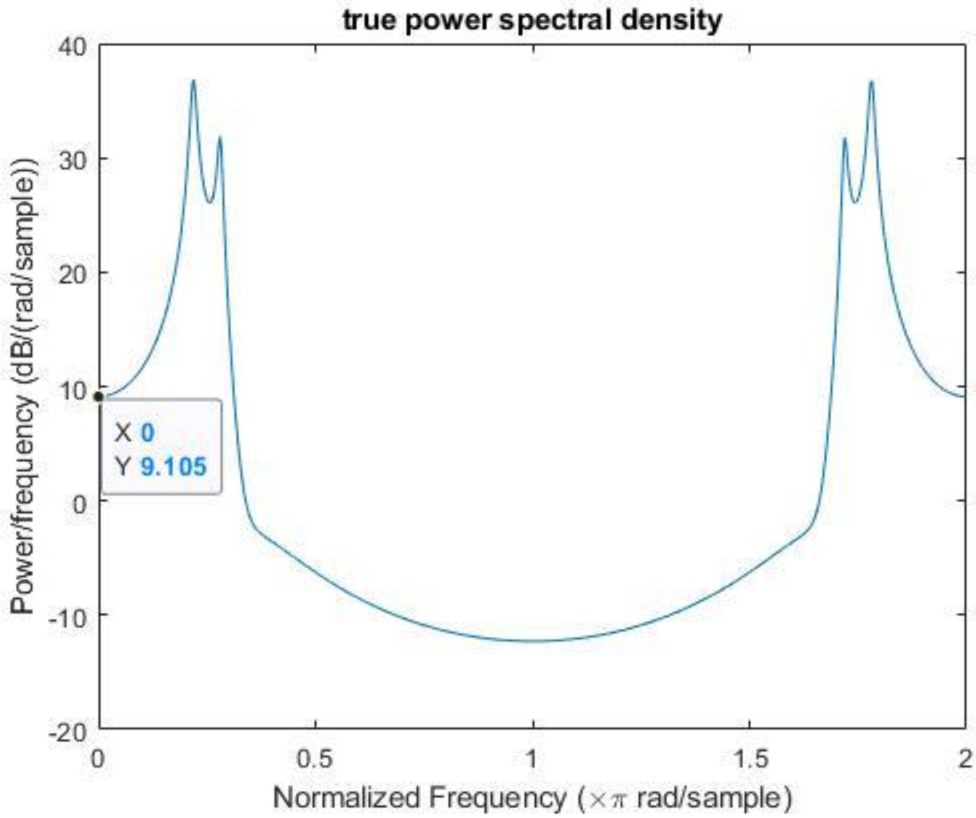

estimated autocorrelation using 256 samples
200
150
100
50
0
-50
-100
-150
0
100
200
300
400
500
600


estimated autocorrelation using 512 samples
150
100
50
0
-50
-100
-150
0
200
400
600
800
1000
1200



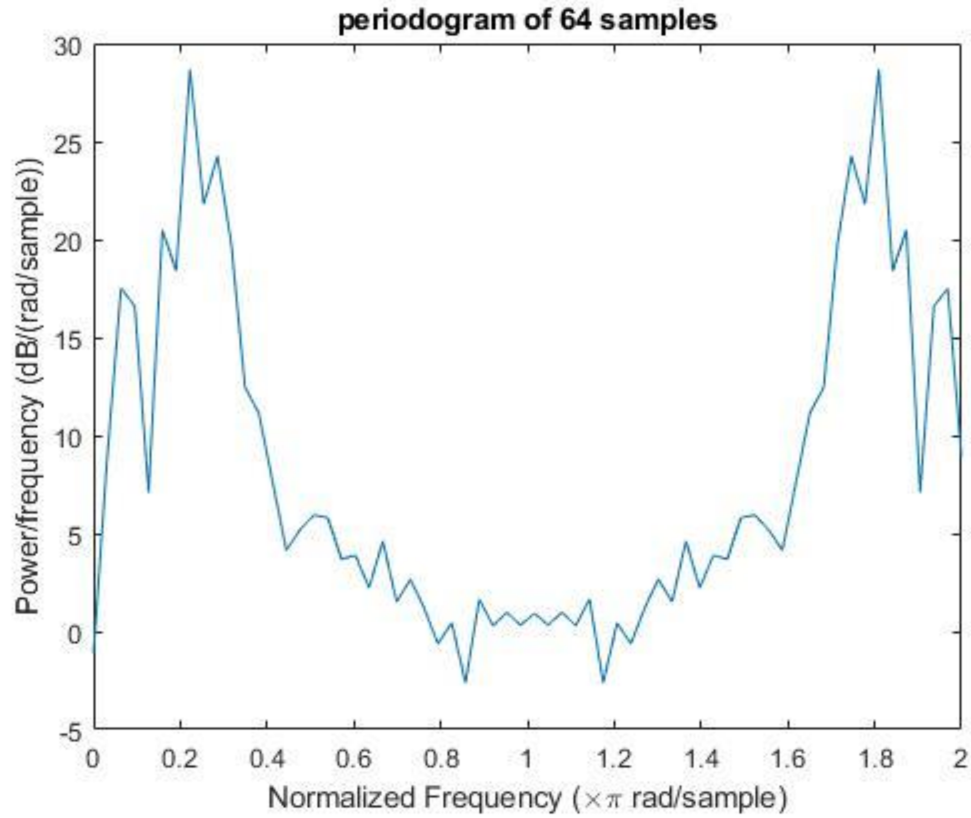
As we can see, as more and more samples are used to estimate the autocorrelation, the estimate tends to be better.

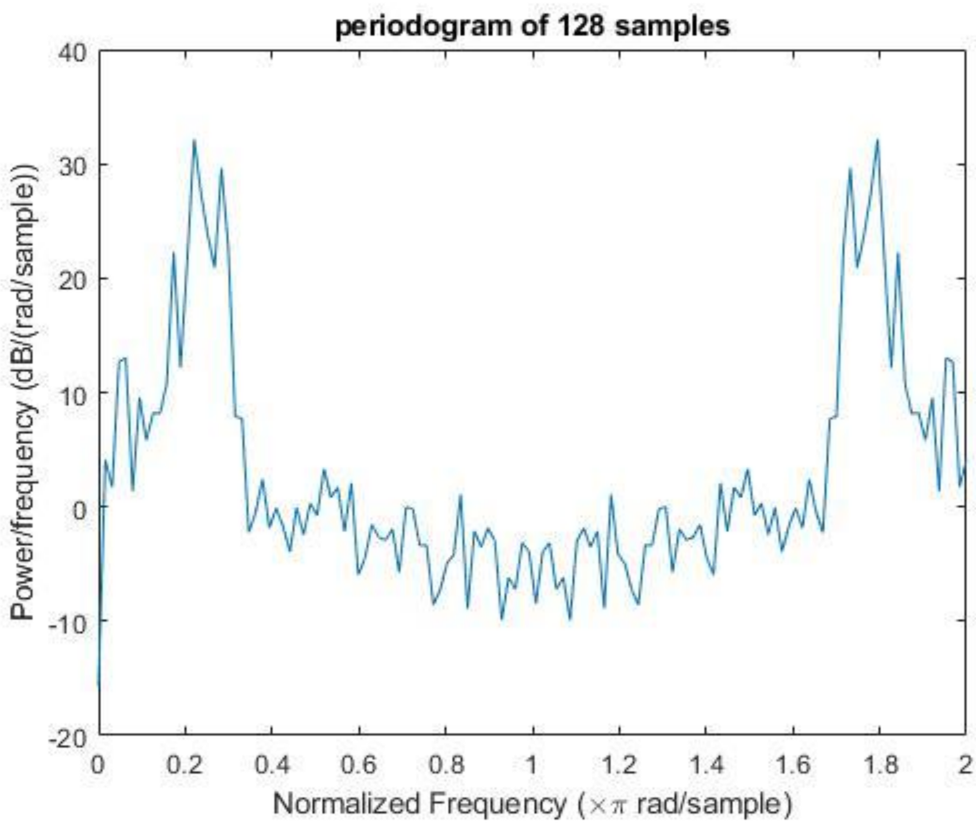
P2 (c)

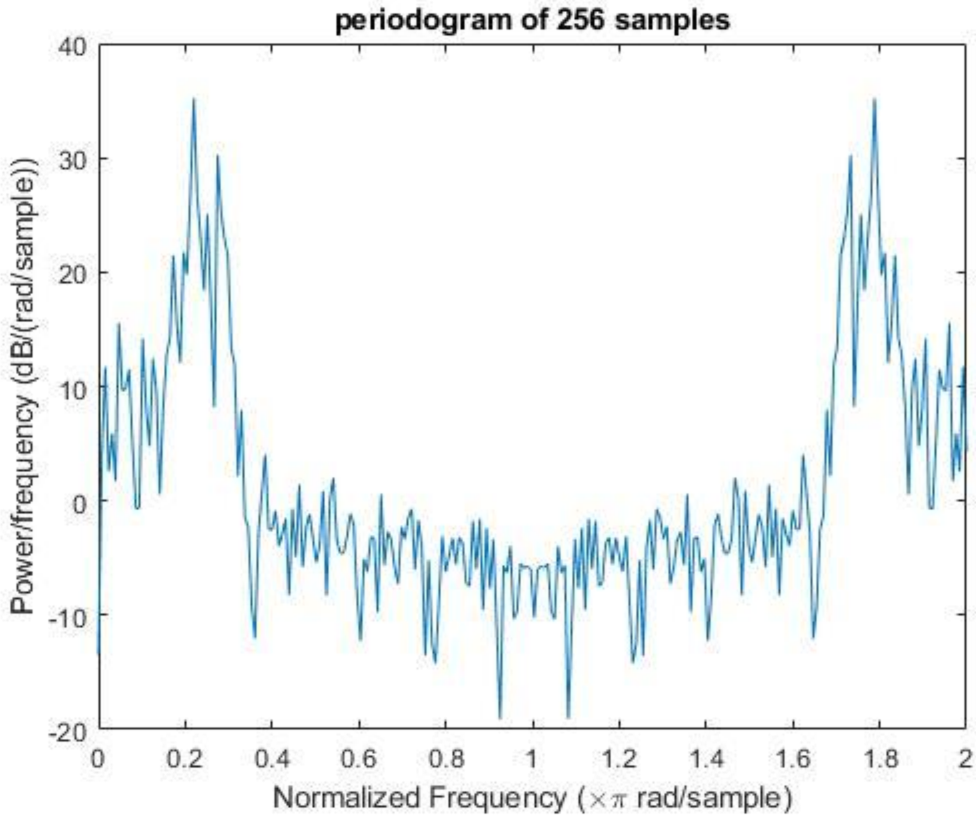


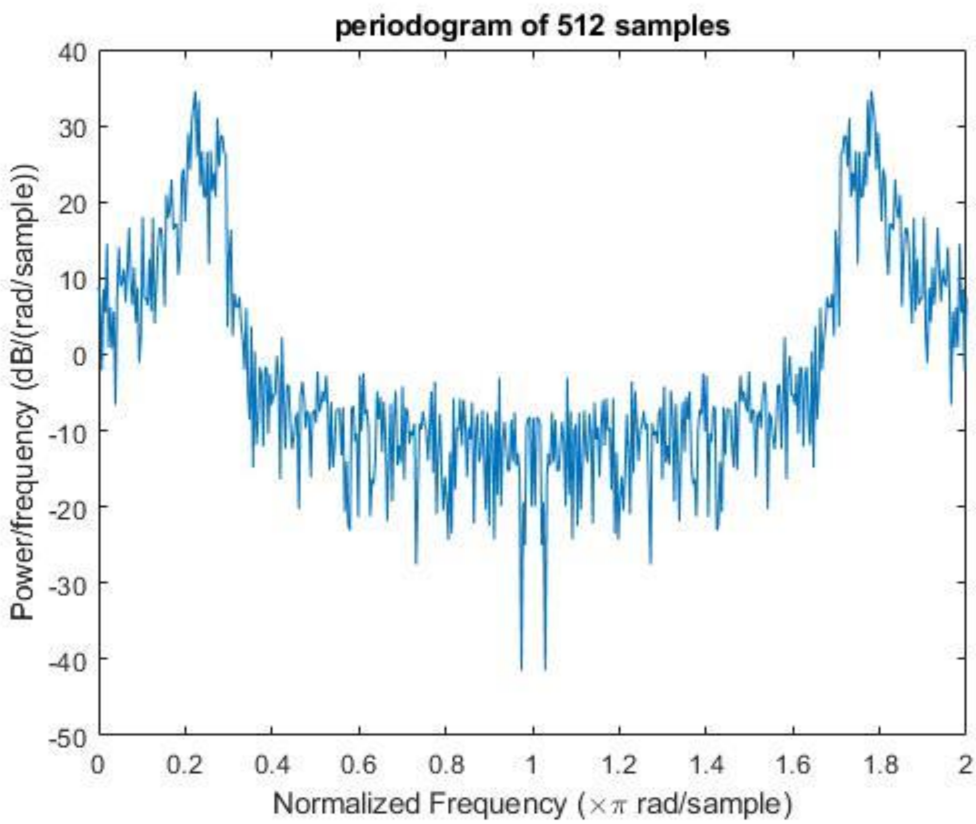
Since the input is a zero mean, unit variance Gaussian white noise (w), the true power spectral density of the output RP (x) is $Rxx(e^{jw}) = H(e^{jw})H^*(e^{jw})$, where $H(e^{jw})$ is the frequency response of the system, which is found via 'freqz'.

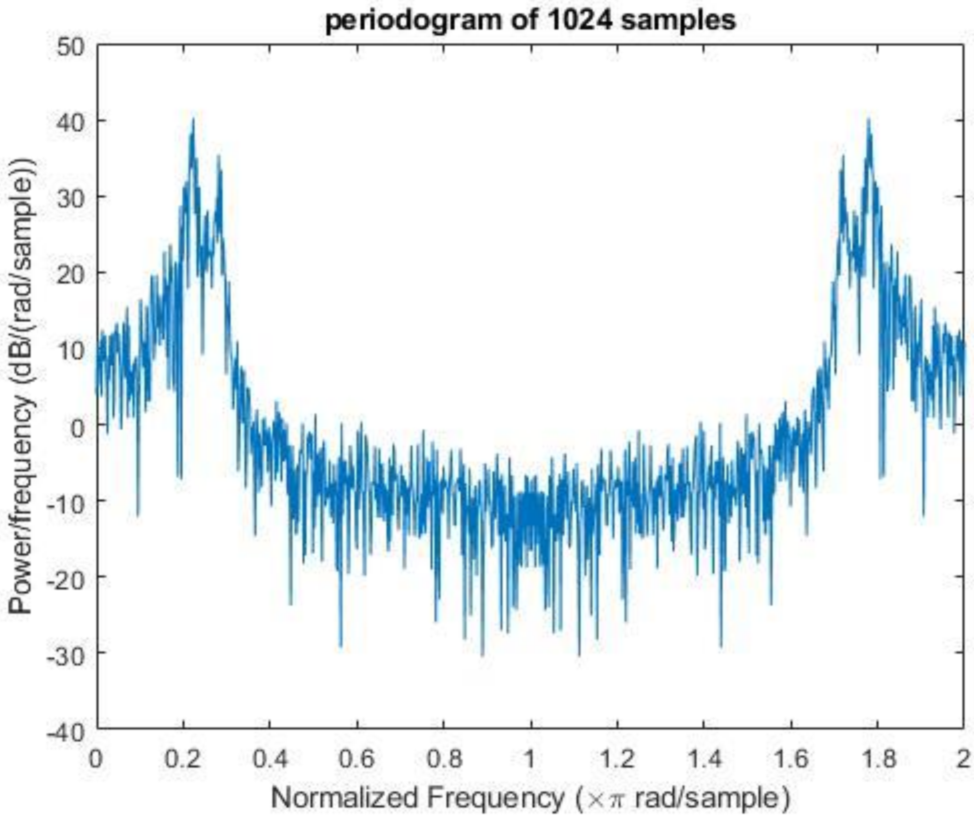
P2 (d)


periodogram of 64 samples
Power/frequency (dB/(rad/sample))
Normalized Frequency (×π rad/sample)
30
25
20
15
10
5
0
-5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


periodogram of 128 samples
Power/frequency (dB/(rad/sample))
Normalized Frequency (×π rad/sample)
40
30
20
10
0
-10
-20
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

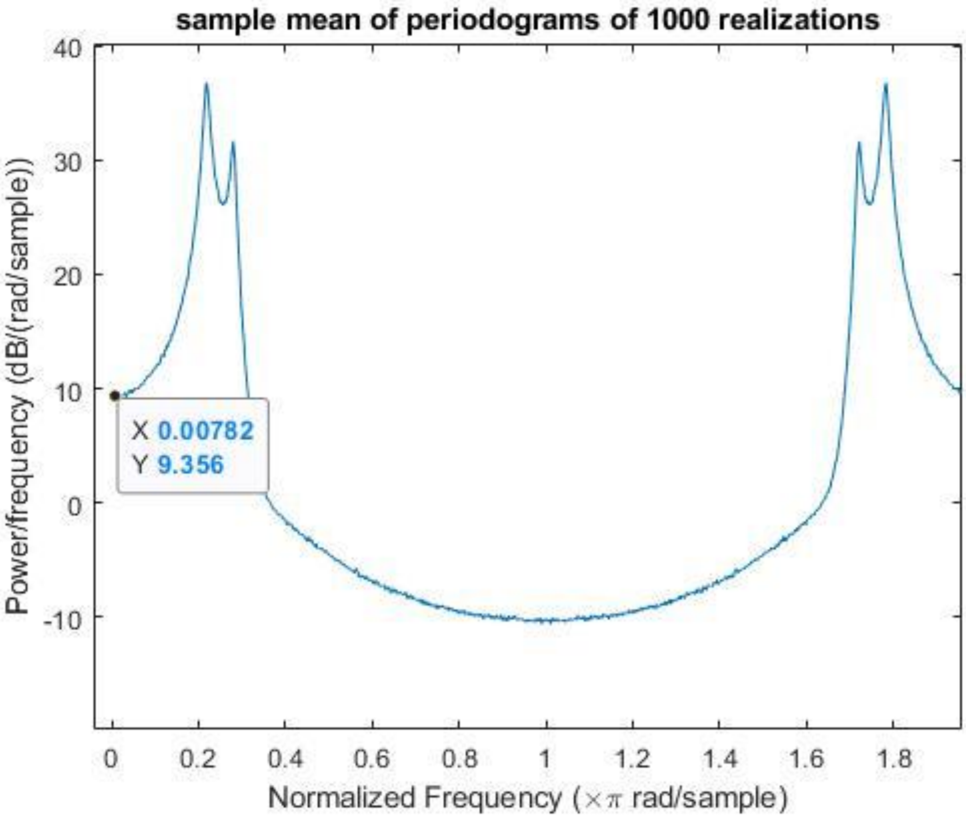

periodogram of 256 samples
Power/frequency (dB/(rad/sample))
Normalized Frequency (×π rad/sample)


periodogram of 512 samples
Power/frequency (dB/(rad/sample))
Normalized Frequency (×π rad/sample)

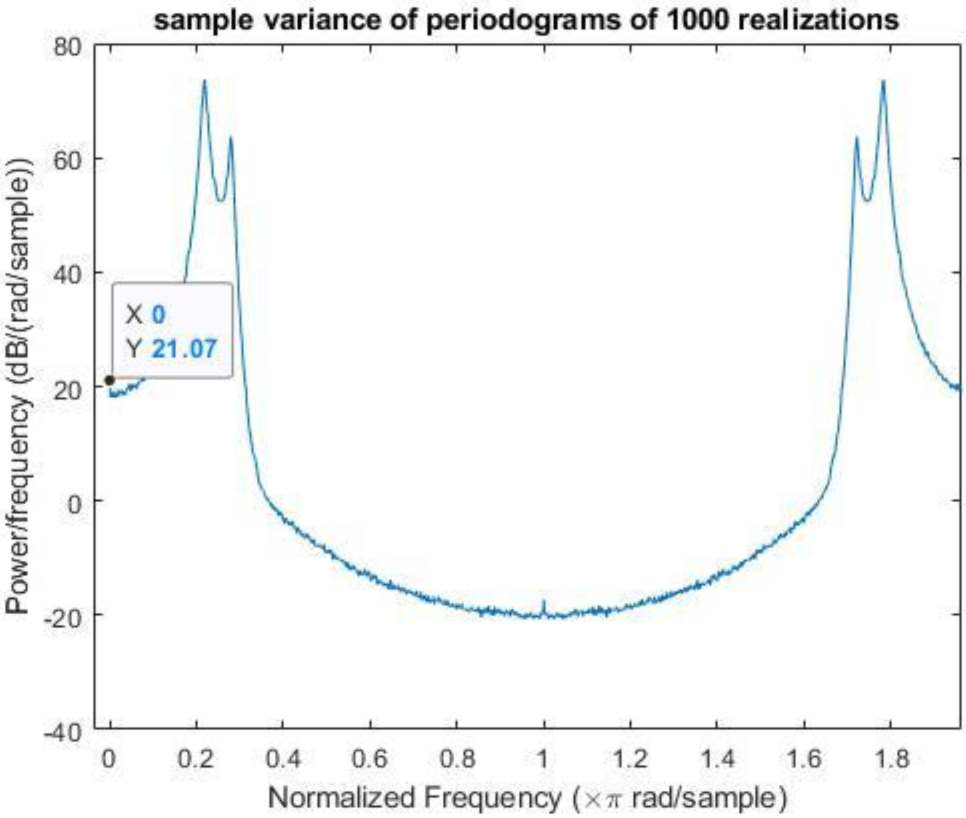


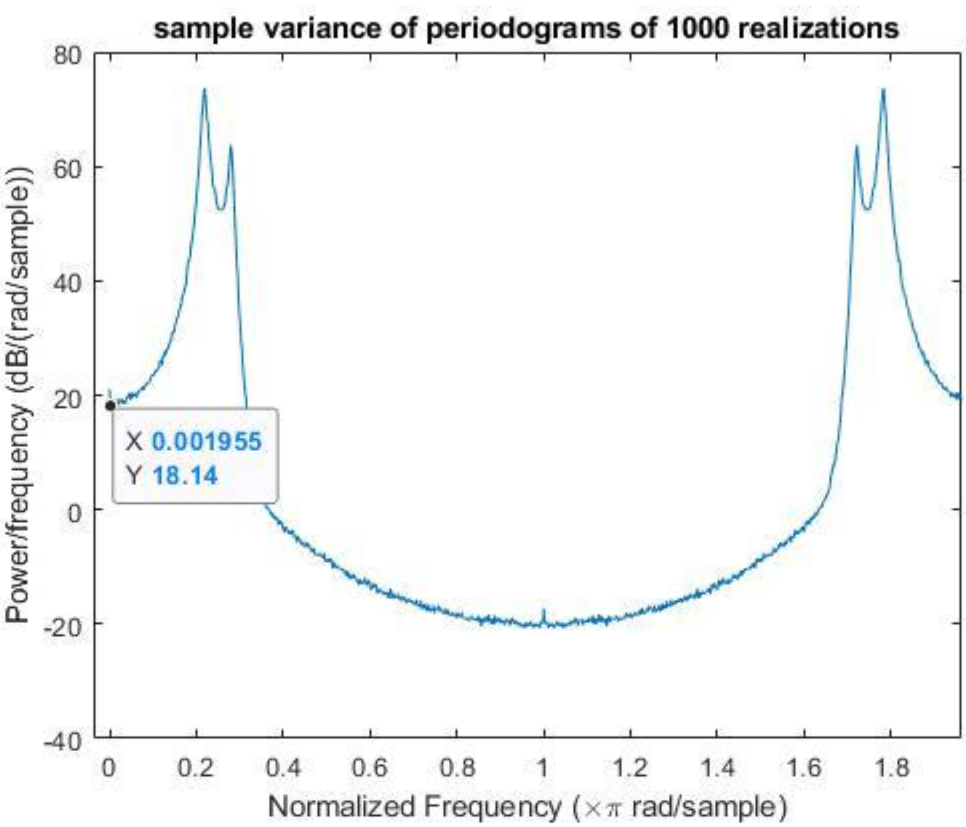
As we can see, as more and more samples are used for estimation, the periodogram tends to be better estimate of the true PSD.

P2 (e)



As we can see, the sample mean of the periodograms of the 1000 realizations is almost identical to the true PSD.


sample variance of periodograms of 1000 realizations
Power/frequency (dB/(rad/sample))
Normalized Frequency (×π rad/sample)
X 0
Y 21.07
80
60
40
20
0
-20
-40
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


sample variance of periodograms of 1000 realizations
Power/frequency (dB/(rad/sample))
Normalized Frequency (×π rad/sample)
X 0.001955
Y 18.14
80
60
40
20
0
-20
-40
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

According to the lecture notes, the sample variance of the periodograms of the 1000 realizations should approach the square of true PSD at nonzero frequencies, and twice the square of the true PSD at DC (zero frequency). So in dB, the samples variance should be double of the true PSD at nonzero frequencies, and 3dB larger than the double at DC (zero frequency). This is reflected in the plots above.