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Vellore Institute of Technology

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Vellore – 632014, Tamil Nadu, India.

SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS
FALL SEMESTER – 2024∼2025
PMAT501L – Probability and Statistics
E2+TE2 Slot

DIGITAL ASSIGNMENT – I

Instructions:

(*i*). Last Date for Submission of Digital Assignment – I (DA–I) : **12th October 2024.**

(*ii*). Mention the Register Number, Name, Slot Details, Course Code and Course Title in the First Page of the Assignment. Also mention the Register Number and Name in every page of the document and write the problem statement for each question.

(*iii*). Submit the soft copy of answers for all the following questions as a single PDF document into V-Top Login properly on or before the mentioned last date.

Answer ALL the Questions

1. A box contains 5 black, 7 red and 6 green balls. Three balls are drawn from this box one after the other without replacement. What is the probability that the three balls are (*i*). all black colors, (*ii*). of different colors and (*iii*). two black and one green colors.

2. A card is drawn from a standard deck of 52 cards, and without replacing it, a second card is drawn. If the first card is a heart, what is the probability of second card being a heart?

3. An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn, and then 1 ball is taken at random from the later. What is the probability that it is a white ball?

4. Find the value of a, $P(X < 3)$, $P(1.3 \leq X \leq 6.7)$, $P(\frac{1}{2} < X < \frac{15}{2}/X > 3)$, the cumulative distribution function, the standard deviation, $E(2X \pm 3)$ and $Var(2X \pm 3)$ of the discrete random variable (X) with the following probability distribution.

$X = x$:	0	1	2	3	4	5	6	7	8
$p = P(X = x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

5. Suppose X is a continuous random variable with the probability density function given by

$$f(x) = \begin{cases} kxe^{-\lambda x}; & x \geq 0, \lambda > 0, \\ 0; & \text{Otherwise.} \end{cases}$$

Find the value of k, $P(X < 5)$, $P(1 \leq X \leq 100/X \leq 5)$, the cumulative distribution function and the standard deviation of X.

6. Two dimensional random variable (X, Y) has the joint probability function given by $P(X = x, Y = y) = k(3x + 5y)$, for $x = 0, 1, 2, 3$ and $y = 0, 1$.

   (*i*). Find the value of k.

   (*ii*). Find all the marginal and conditional distributions of X and Y.

   (*iii*). Find the probability distribution of Z, mean and variance of Z, where $Z = X + Y$.

7. Two random variables X and Y have the following joint probability density function:

$$f_{XY}(x, y) = \begin{cases} 2 - x - y, & 0 \leq x \leq 1, 0 \leq y \leq 1; \\ 0, & otherwise, \end{cases}$$

   Find

   (*i*). $P(X > \frac{1}{2})$, $P(Y < \frac{1}{2})$ and $P(X > \frac{1}{2}/Y < \frac{1}{2})$,

   (*ii*). the marginal probability density functions of X and Y,

   (*iii*). the conditional density functions $f_{Y/X}(y/x)$ and $f_{X/Y}(x/y)$,

   (*iv*). the variances $Var(X)$ and $Var(Y)$,

   (*v*). the co-variance between X and Y.

8. If X represents the outcome, when a fair die is tossed, then find the moment generating function of X and hence find $E(X)$ and $Var(X)$.

9. Compute the coefficient of correlation between the following two set of measures X and Y and hence obtain the equation of regression lines.

X :	65	67	66	71	67	70	68	69
Y :	67	68	68	70	64	67	72	70

10. Write and execute a program in MATLAB / *R* Softwares for any problem in Statistical or Probabilistic Measures.
    (Include the soft copy of the problem statement and the programming code with output for the problem).

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