# Al Mkni fi’l-jabr wa’l-muqābala "Exposition of Algebraic Operations" by Ibn Al-Ha’im

**A line by line translation by Ishraq Al-Awamleh**

1. By praising God I start my task,
   and I send my prayer along with a corresponding salutation
2. to his chosen Prophet [Muhammed], the best of mankind, and his family,
   and to his companions, then supplications continue
3. to [my teacher], renowned in his time, who belongs to Jelawa [tribe],
   Ali upon whom there are continuous rain clouds,
4. And then the science of Algebra is great,
   to which the best and most skillful people are attracted.
5. And I will contain its [algebra] core in a poem,
   through which the discerning person will be pleased and go forward.
6. And here I start with my undertaking; my endeavor,
   and I ask the help of the provider of reason.
7. Names of [unknown] kinds of numbers, their ranks and exponents
8. and by root, square and cube they are named,
   quantities to which it was not known, at the beginning, how to apply algebraic operations.
9. When multiplying a number by itself, the number is a root,
   and the result [of multiplication] is called a square.

10. And multiplying a thing by a square makes a cube,
    and from a square and a cube you produce various categories [secondary kinds];
11. so that you can say square square, then square cube;
    unknown kinds of numbers have defined exponents and ranks.
12. Root and thing are equivalent in one case,
    and they differ in other cases.
13. "Kaab" and "Mukaab" both meaning cube are used by the majority
    and their meaning differs in some other cases.
14. Their ranks are either primary, as their exponents [1, 2, 3],
    or secondary, where the next exponent is one more than the previous one.
15. The first [rank] is for the root, and its exponent is one,
    and the square follows with exponent two which exceeds the first exponent by one.
16. The third [rank] is for the cube and you know its exponent,
    it’s three, just like the positional base ten; and these are the primary [exponents of the unknown numbers].

17. Whatever exponent there is beyond [the three will be called secondary] name it;
double it or treble depending on what the seeker says.
18. [To find out the kind of a given exponent], You name square for each two and a cube
for each three and get the result.
19. Attach the square to the cube and put the square first [square cube]
[this] is the result for the learner.
20. For the opposite case [that is, to know the exponents from a given kind],combine the separated exponents [values]
by adding them, you obtain the exponent sought and this method is all inclusive.
21. Addition and Subtraction
22. Whatever kind is the same, and you want to add,
apply to it what you do with adding known numbers.
23. And you say, in subtracting unknown numbers, follow the same rule, but when the kinds differ,
addition is accomplished by adding “and”.
24. And subtraction [with different kinds] is accomplished by using [the method of] exception. If there is
an exception in one of them [subtrahend and minuend] or in both,
25. first, add the exception of each quantity to both
in order to balance [to achieve one of the six forms];
26. and in each case [i.e., subtraction and balancing], remove the exception,
and complete the work after removing the exception.
27. Multiplication and Division
28. Whenever you multiply a kind by a number,
the answer will be of the same kind a seeker asked about.
29. Whenever multiplying two kinds add the exponents of the two kinds
to obtain the exponent of the result, and then obtain the quantity of the result.
30. And you say, additive term multiplied by subtractive term gives subtractive term,
but when the terms are similar [either both are additive or subtractive],
[an] additive term is obtained.
31. When dividing two kinds of the same rank, the result is a number.
And if the ranks are different,
32. when the numerator has higher rank [than the denominator], the excess value of the exponents,
is the exponent of the resulting kind.
33. And [when] the reverse of this [the numerator has lower rank than the denominator], make the answer be the question,
and the same rule applies when dividing a number by a kind.

34. For the reverse [i.e., dividing any kind by a number], the result will have the same kind [as the question].
And for the cases [i.e., lower rank by higher rank, or number by kind] remove the division and equate the terms.
35. And with this method using mathematics,
you should improve your skills so that others will not surpass you.
36. The Six Equations [six canonical forms]
37. And take these six original forms,
ordered, and they are known as equations.
38. A number and a thing and a square, they [the six equations] encompass,
half of them are simple and the other half are the opposite [compound].
39. Roots and squares in the first one [first equation] are equated,
and squares in the intermediate one [the second equation] are equated with numbers.
40. And things [or roots] are equated with numbers in the last one [third equation] of the simple equations,
and then you follow what I say.
41. In the first two [simple equations], divide [both terms: roots and numbers, respectively ] by the number of squares,
and in the third [equation] divide the number by what it's equated to [the number of roots].
42. The result is a root except for the intermediate [second equation],
and your answer [in the second equation] will be square for the seeker.
43. And take AJM [A:Adad (number), J:Jadther(root) and M:Mal (Square)] to order the others [compound equations],
in the fourth [equation] the number is isolated.
44. And in the fifth [equation] root is isolated, and in the sixth [equation],
square is isolated and is equated [in the fifth, root is equated to square and numbers, and in the sixth, square is equated to roots and numbers]
45. [To solve the compound equations] In all of them, square [the] half of the roots
and add the result, except for the second [fifth compound equation],
46. to the number, and remember the [new] result,
then subtract the half of the roots from the [new] result and so the [final] result
47. is the root for the first [the fourth compound equation]. And in the sixth [the third compound equation] you have to add [the half of the root to the new result],
and the root of the square is the resulting outcome.
48. And in the fifth [compound equation] [ case 1] subtract the number from the square [of the half of the root],
and the root of the result indicates the target.

49. Subtract it [ the target] from the half of the roots or add them together;
the root in both cases is the result.
50. And when the number exceeds the square [of the half of the roots] [case 2],
it's impossible [to find the root]. If they are equal [the number is equal to the square of the half of the root] [case 3],
51. then the half of the roots is the root sought,
and square is obtained from the root [by squaring the actual result].
52. Division [Last] Section
53. And whatever equations [compound equations] we mentioned earlier have only one square,
but if not, they will have a fraction of a square or more than one square.
54. For the square [term], restore the fraction,
and reduce the extra [squares - i.e., divide 1 by the number of squares and multiply the number of squares by this ratio].
55. And for the other two terms, do to them what you have done for the squares
and whatever you get, do to it what you did before the division [last section].
56. Or multiply the compound equation by the seen amount
of squares to find the means [corresponding equation to give rise to a new equation]
57. and find the new external number and adapt the construction[to find the root of your new equation],
and finally divide the root
58. by whatever you multiplied the number before. And after what we have seen,
whenever an equation is introduced to you, try to pick a suitable process [to transform it to one of the six equations].
59. And you have to master my method,
otherwise you won't desire to become an insider [to master algebra]
60. And what I have introduced is convincing,
and for God our praise continues.
61. Afterwards [our prayesrs] prayers are recited with peace to the
blessed Mohammed, the guide who has eminent distinction.
62. Of the best people, which spread over his early followers and then his family and his companions,
his wives with good manners [respectfulness].
63. And its verses are fifty nine composed,
in Al Aqsa in a good month [the third month in the Islamic calendar when Prophet Mohammed was born] so it's auspicious

64. Rabee [the third month in the Islamic calendar] of the year in which its count has been adjusted,

in "dal" [4] and "thad" [800] so absolute praise. [the Arabic letter "dal" has a corresponding number 4 and the letter "thad" has corresponding value 800 so 800 + 4 is 804 AH]

**The Poem in Arabic.**

قصيدة ابن الهائم المقنع في الجبر والمقابلة

| | | |
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| .1 | بحمد الله ابتدي ما أحاول | وأهدي صلاة مع سلام يشاكل |
| .2 | على المصطفى خير الأنام وآله | واصحابه ثم الدعا يتواصل |
| .3 | لفخر الزمان المنتمي لجلاوة | علي عليه سحب جود هواطل |
| .4 | وبعد فعلم الجبرعلم معظم | يميل اليه المتقنون الافاضل |
| .5 | واني لحاو لبه في قصيدة | بها يكتفي ذو فطنة ويطاول |
| .6 | وها أنا ساع في الذي قد قصدته | وعونا من المولى الحجي انا سائل |
| .7 | أسماء الأنواع ومراتبها واسوسها | |
| .8 | وبالجذر ثم المال فالكعب لقبوا | مقادير لم تدر ابتداء تحاول |
| .9 | فما ضربه في مثله هو جذرهم | وبالمال سموا ما بذا هو حاصل |
| .10 | وذا ضربه في ذاك يبدو مكعبا | ومن ذين اسماً البواقي تناول |
| .11 | فقل مال مال ثم مال مكعب | اسوس لها معلومة ومنازل |
| .12 | وجذر وشيء في محل تصادق | وبينهما في آخرين تفاضل |
| .13 | وبالكعب سوى الاكثرون مكعبا | وبين كلا العرفين قطعا تفاضل |
| .14 | منازلها اصلية كاسوسها | وفرعية بواحد تتفاضل |
| .15 | فالأولى لجذر اسها واحد وما | تلتها المال اسها اثنان فاضل |
| .16 | وثالثة للكعب فادرى واسها | ثلاث كما في العدّ فهي الأصائل |
| .17 | وما زاد فرع أس كل سمّيه | فثنه وثلث حسب ما هو قائل |
| .18 | ومالا بكل اثنين خذ ومكعبا | لكل ثلاث ثم ما هو حاصل |
| .19 | أضف بعض للبعض والمال قدمن | يكن ما بدا جواب من هو سائل |
| .20 | وفي عكسه ركب اسوسا تفصلت | بجمع تقربا بالقصد فالضبط شامل |
| .21 | الجمع والطرح | |
| .22 | وما يتفق نوعا وقد رمت جمعه | ففيه أعملن ما انت بالعد عامل |
| .23 | وقل هكذا طرح وعند تخالف | فجمع بواو العطف قل يتناول |
| .24 | وفي الطرح الاستثنا اعتمد ثم ان يكن | على واحد او فيهما هو داخل |
| .25 | ففي البدء مستثناهما زد عليهما | كذا ذوا اختصاص مثل ما يتعادل |
| .26 | ففي كل باب منهما لفظه أزل | والأعمال تمم بعدما هو زائل |
| .27 | الضرب والقسمة | |
| .28 | ومهما ضربت النوع في عدد يك ال | جواب من النوع الذي قال سائل |
| .29 | واسّي كلا النوعين فاجمع فما بدا | فاس جواب ثم كمّ يحاول |
| .30 | وقل زائد في ناقص هو ناقص | وعند اتفاق زائد هو شامل |
| .31 | ويخرج عد ان قسمت موافقا | وان كان بين الرتبتين تفاضل |
| .32 | ومقسومك الأعلى فزائد أسّه | هو الاس للنوع الذي هو حاصل |
| .33 | وفي عكسه اجعل كالسؤال جوابه | وعدّ على نوع لهذا يماثل |
| .34 | وفي العكس يبدو نوع ما قد قسمته | وقسما بمثلويّه نحيّي المعادل |
| .35 | ومناهجه يدرى بنوع تحيل | فحصل قواه لاعدتك الفضائل |
| .36 | المسائل السّت | |
| .37 | وهاك ضروبا ستة قد تاصّلت | مرتبة في العرف فهي مسائل |
| .38 | على عدد والشيء والمال دورها | فنصف بسيط ثم نصف مقابل |
| .39 | جذور وأموال في الأولى تعادلا | والأموال في الوسطى لعد تعادل |
| .40 | والأشيا عدا عادلت في اخيرة ال | بسيطات فاعمل بعد ما انا قائل |
| .41 | ففي الأولين أقسم على المال عدله | وفي ثالث عدا على ما يعادل |
| .42 | فما كان فهو الجذر من غير اوسط | وفيه اجب بالمال من هو سائل |
| .43 | وخذ عجما ضبطا لترتيب مقرن | ففي رابع أفراد عد يقابل |

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| .44 | وفي خامس افراد جذر وسادس | تفرد مال واقتران يعادل |
| .45 | وفي كلها نصف الجذور فربعا | وزد في سوى الثاني الذي هو حاصل |
| .46 | على العدد واحفظ جذر ما هو كائن | فنصف الجذور أطرحه منه ففاضل |
| .47 | هو الجذر في الأولى و زده بسادس | عليه فجذر المال ما هو عائل |
| .48 | وفي الخامس اطرح عده من مربع | وجذر الذي يبقى على القصد دالل |
| .49 | فالقه من التصنيف أو فاجمعنهما | يك الجذر في الحالين ما هو حاصل |
| .50 | وحيث يفوق العد فيه مربّعا | فذاك محال أو تراه يماثل |
| .51 | فنصف الجذور الجذر وهو كجذرهم | فعلم بقدر المال ما عنه حاصل |
| .52 | فصل | |
| .53 | وما مر حيث المال في الضرب واحد | فان لم يكن بل كسر مال وعائل |
| .54 | فللمال كما كسر مال بجبره | ورد بحط زائدا والمعادل |
| .55 | وما قارن اصنع فيه ما قد صنعته | فما كان فاعمل فيه ما انت عامل |
| .56 | او اضرب لدى التركيب قدر الذي يرى | من المال في عد لتدرى الوسائل |
| .57 | وقدر كعد خارجا والبنا اعتمد | وفي الاخر أقسم ما لجذر يقابل |
| .58 | على ما ضربت العد فيه وبعده | تناول تحيل حين تاتي المسائل |
| .59 | ولا بد من إتقان نحو وسيلتي | والا فلا تطمع بأنك داخل |
| .60 | وهذا الذي أوردته فيه مقنع | ولله حمد دائم يتواصل |
| .61 | وتتلو صلاة مع سلام على الرضى | محمد الهادي الكريم الشمائل |
| .62 | نعم الاؤلى ثم آله ثم صحبه | وازواجه الغر الكرام الافاضل |
| .63 | وابياتها تسع وخمسون انشأت | بالاقصى وشهر اليمن فهي تطاول |
| .64 | ربيع من العام الذي ضبط عدّه | بدال وضاد فالثنا يتكامل |