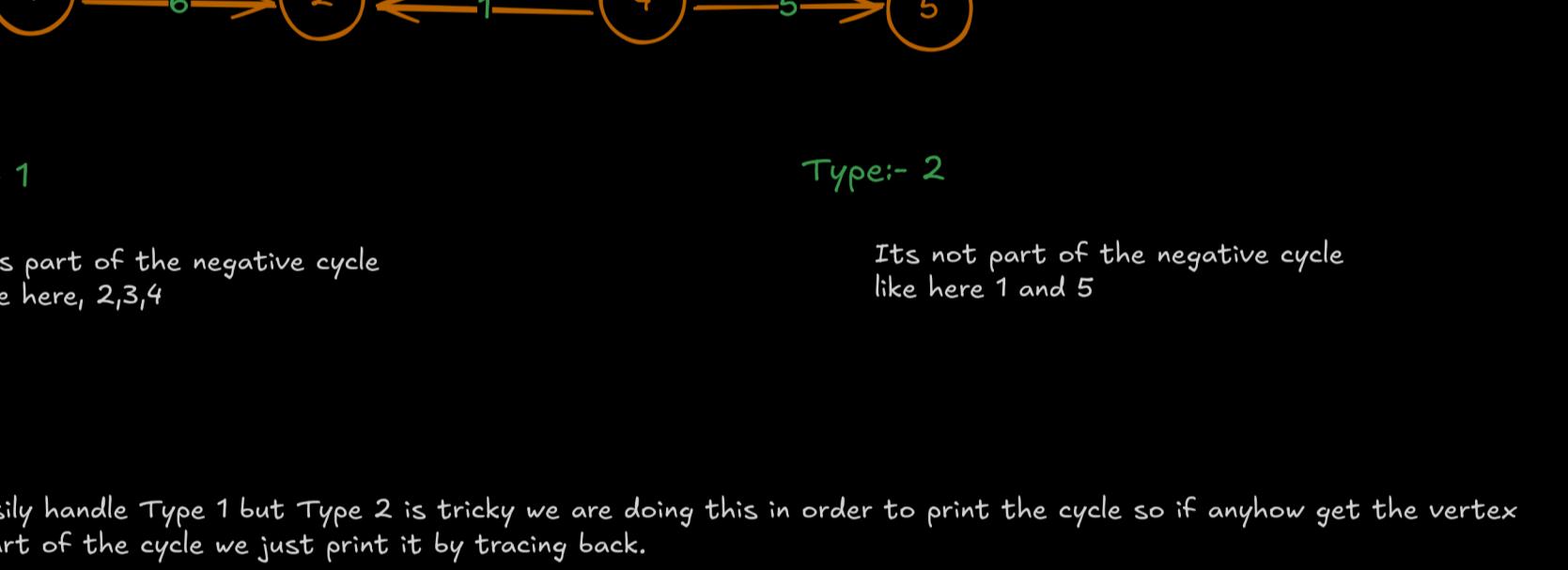
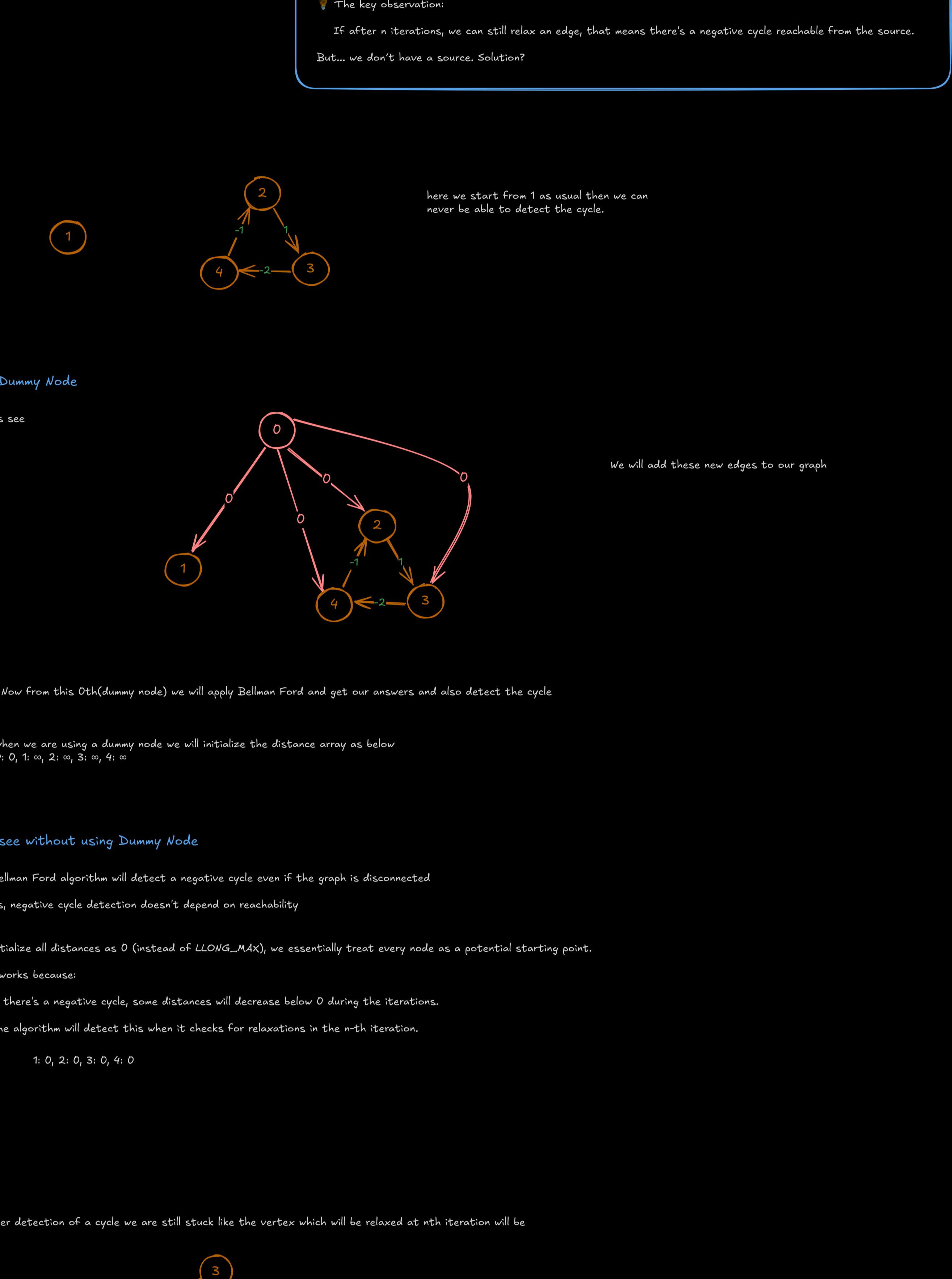


## Cycle Finding

```
You are given a directed graph,  
and your task is to find out if it contains a negative cycle, and also give an example of such a cycle.  
Input  
The first input line has two integers n and m: the number of nodes and edges.  
The nodes are numbered 1,2,...n.  
After this, the input has m lines describing the edges.  
Each line has three integers a, b, and c: there is an edge from node a to node b whose length is c.  
Output  
If the graph contains a negative cycle, print first "YES", and then the nodes in the cycle in their correct order.  
If there are several negative cycles, you can print any of them.  
If there are no negative cycles, print "NO".  
Constraints  
1 <= n <= 2500  
1 <= m <= 5000  
1 <= a,b <= n  
10^-9 <= c <= 10^9  
Example  
Input:  
4 5  
1 2 1  
2 4 1  
3 3 1  
4 1 -3  
4 3 -2  
Output:  
YES  
1 2 4 1
```



We will use a Dummy Node

Let's see

We will add these new edges to our graph

Now from this 0th(dummy node) we will apply Bellman Ford and get our answers and also detect the cycle

So when we are using a dummy node we will initialize the distance array as below  
0: 0, 1: 0, 2: 0, 3: 0, 4: 0

Now let's see without using Dummy Node

Bellman Ford algorithm will detect a negative cycle even if the graph is disconnected

As, negative cycle detection doesn't depend on reachability

We initialize all distances as 0 (instead of  $LLONG\_MAX$ ), we essentially treat every node as a potential starting point.

This works because:

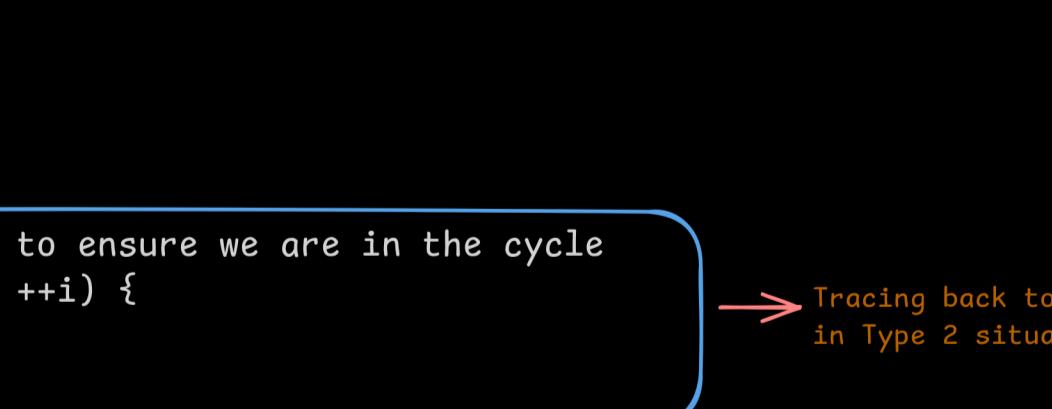
If there's a negative cycle, some distances will decrease below 0 during the iterations.

The algorithm will detect this when it checks for relaxations in the  $n$ -th iteration.

0: 0, 1: 0, 2: 0, 3: 0, 4: 0

## Cycle Detection

Next after detection of a cycle we are still stuck like the vertex which will be relaxed at  $n$ -th iteration will be



Type:- 1

Its part of the negative cycle  
like here, 2,3,4

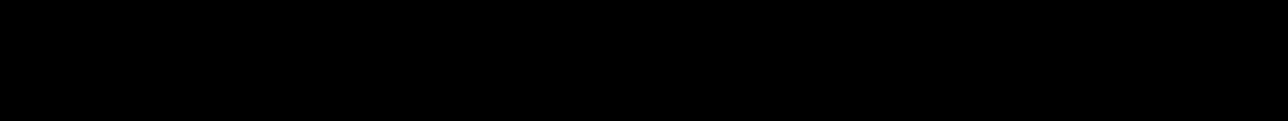
Type:- 2

Its not part of the negative cycle  
like here 1 and 5

We can easily handle Type 1 but Type 2 is tricky we are doing this in order to print the cycle so if anyhow get the vertex which is part of the cycle we just print it by tracing back.

So in Type 2 we will first want to reach the vertex which is part of negative cycle so for that we will maintain a relaxant array which will tell who relaxed whom necessary for us to trace back.

Now to reach the cycle, in worst case we have to trace back  $n$  times like in this case



If this is detected

So just do  
 $x = \text{relaxant}[x]$

And for relaxing



inside the Bellman Ford condition

```
path[v] = path[u] + wt;  
relaxant[v] = u;  
x = v;
```

```
#include <bits/stdc++.h>  
using namespace std;  
  
typedef long long ll;  
typedef tuple<int, int, int> Edge;  
  
int main() {  
    ios::sync_with_stdio(false);  
    cin.tie(nullptr);  
  
    int n, m;  
    cin >> n >> m;  
  
    vector<Edge> edges;  
    for (int i = 0; i < m; ++i) {  
        int u, v, wt;  
        cin >> u >> v >> wt;  
        edges.push_back({u, v, wt});  
    }  
  
    vector<ll> dist(n + 1, 0);  
    vector<int> relaxant(n + 1, -1);  
  
    int x = -1;  
    for (int i = 0; i < n; ++i) {  
        x = -1;  
        for (auto [u, v, w] : edges) {  
            if (dist[u] + w < dist[v]) {  
                dist[v] = dist[u] + w;  
                relaxant[v] = u;  
                x = v;  
            }  
        }  
    }  
  
    if (x == -1) {  
        cout << "NO\n";  
        return 0;  
    }  
  
    // Move x back n times to ensure we are in the cycle  
    for (int i = 0; i < n; ++i) {  
        x = relaxant[x];  
    }  
  
    vector<int> cycle;  
    for (int curr = x; curr == relaxant[curr]);  
        cycle.push_back(curr);  
        if (curr == x && cycle.size() > 1) {  
            break;  
        }  
    }  
  
    reverse(cycle.begin(), cycle.end());  
  
    cout << "YES\n";  
    for (int node : cycle) {  
        cout << node << " ";  
    }  
    cout << "\n";  
}  
}
```

Time Complexity

$O(n \cdot m)$

Space Complexity

$O(n + m)$

## Common Questions

1. Why do we need the dummy node 0?

2. What happens if we don't reverse the cycle?

3. How does relaxant[] help us trace the cycle?

4. Can we use DFS/BFS for this? Why not?

5. What if the graph had no cycles -- how do we know?