

1. Evaluate the following integral using Romberg Integration:

$$\int_{-3}^3 \frac{1}{1+x^2} dx$$

Given the true value is 2.498091544796509, iterate until true percentage error < 1% or upto order of  $h^{16}$ .

2. Numerically integrate

$$\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$$

from  $x = 0$  to  $x = 4$  with a step size of 0.5. The initial condition at  $x = 0$  is  $y = 1$ .

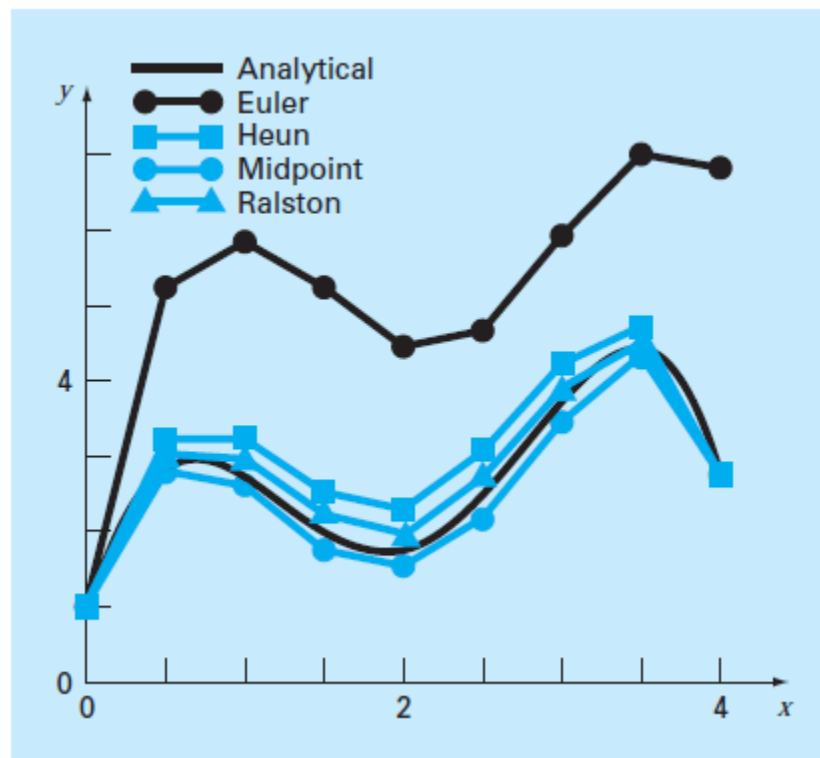
(a) using Euler's method

(b) Heun Method with a Single Corrector (second order RK method with  $a_2 = 1/2$ ).

(c) The Midpoint Method (second order RK method with  $a_2 = 1$ )

(d) Ralston's Method (second order RK method with  $a_2 = 2/3$ )

Now, compare the solutions above with the analytical true solution and plot the graph like the following one:



(Figure 25.14 from Chapra's book)