

1. Evaluate the following integral using Romberg Integration:

$$\int_1^2 \left(2x + \frac{3}{x}\right)^2 dx$$

Given the true value is 25.83333333333333, iterate until true percentage error < 1% or upto order of h^{16} .

2. Numerically integrate

$$\frac{dy}{dx} = x^3 - 5x^2 - x + 6$$

from $x = 0$ to $x = 4$ with a step size of 0.5. The initial condition at $x = 0$ is $y = 1$.

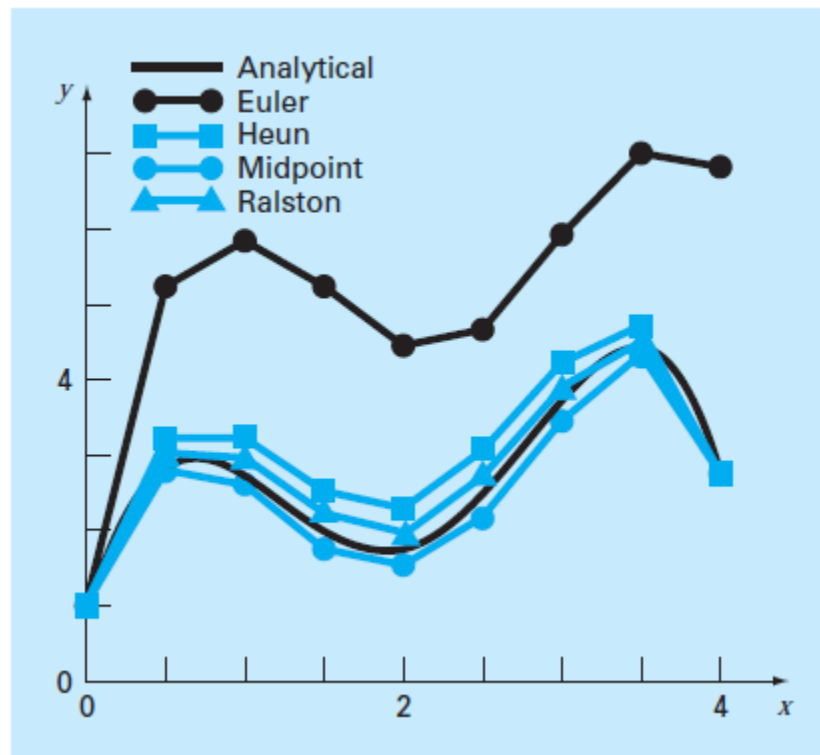
(a) using Euler's method

(b) Heun Method with a Single Corrector (second order RK method with $a_2 = 1/2$).

(c) The Midpoint Method (second order RK method with $a_2 = 1$)

(d) Ralston's Method (second order RK method with $a_2 = 2/3$)

Now, compare the solutions above with the analytical true solution and plot the graph like the following one:



(Figure 25.14 from Chapra's book)