Red Black Tree

Properties and Visualization

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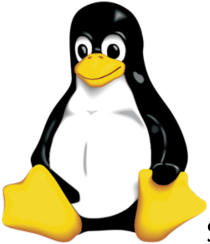
A Practical Problem



Suppose we have a some programmes to be run by our linux OS scheduler. The linux schdeduler will need a data structure to sort them by their lowest spent execution time. So, it might need a Binary Search tree to sort the programmes. But, a BST might grow linearly in worst case as new programmes are inserted to be managed by the scheduler which will increase the run-time from $\mathcal{O}(n \log n)$ to $\mathcal{O}(n^2)$. So, it will require a self height balancing BST which will always give the reduced searching time.

What is Red Black Tree?

- A red–black tree is a kind of self-balancing binary search tree in computer science.
- Each node of the binary tree has an extra bit, and that bit is often interpreted as the color (red or black) of the node.
- By constraining the node colors on any simple path from the root to a leaf, red-black trees ensure that no such path is more than twice as long as any other.
- The tree is thus approximately height balanced.

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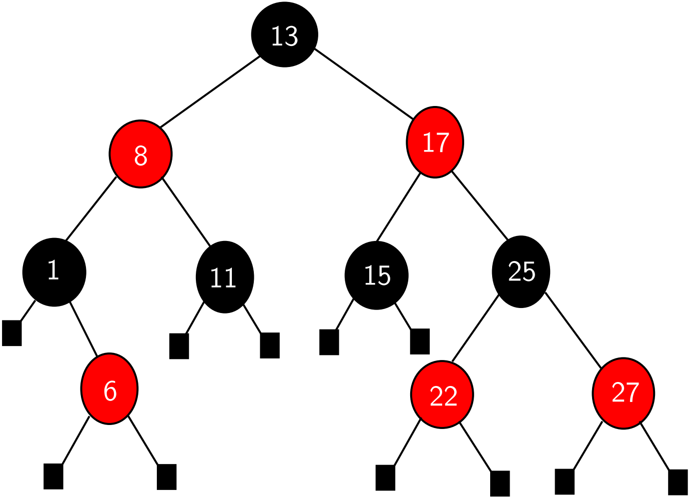
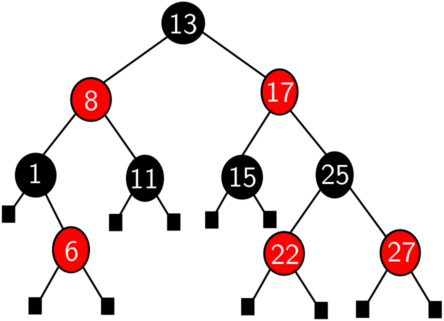
Properties of a Red Black Tree



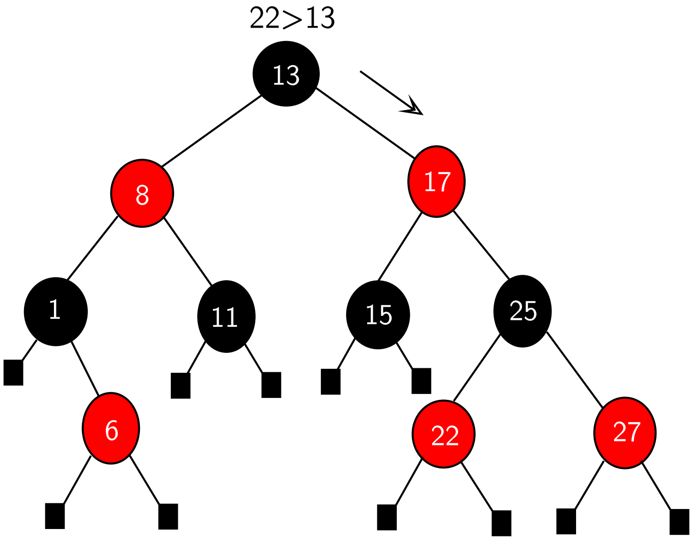
Properties of Red Black Tree

A Red Black Tree is a Binary Search Tree which have the following Red-Black properties:

1. Color Property: Each node is either red or black.
2. Root Property: The root is black
3. External Property: Every external node is black
4. Internal Property: Both children of a red node are black.
5. Depth Property: For each node, all simple paths from the node to descendant leaves contain the same number of black nodes.



Searching 22



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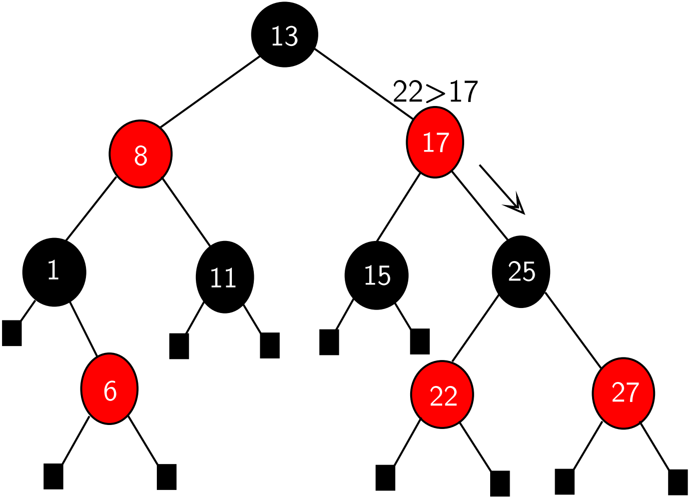

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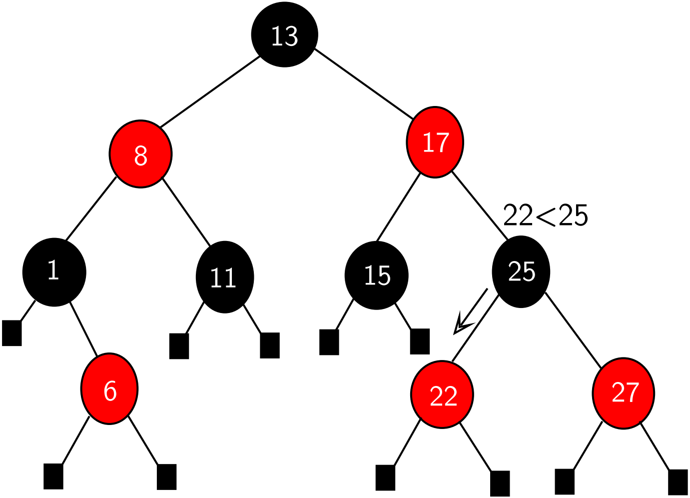

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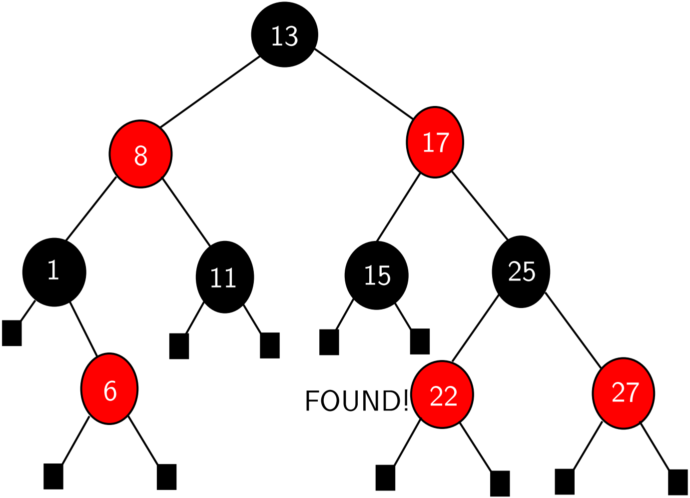
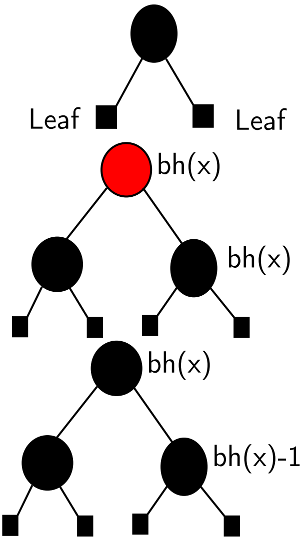


Why Red Black Tree is Height Balanced

Red Black Tree for storing n items will have a height of $\mathcal{O}(n)$. So, it will remain height balanced.

1. the subtree rooted at any node x contains at least $2^{bh(x)} - 1$ internal nodes.
2. If $h(x) = 0$ then x is a leaf, so the subtree rooted at x contains $2^0 - 1 = 0$ internal nodes.
3. For a node x with positive height and two children, the black height of the children will be $bh(x)$ (for red internal node) or $bh(x) - 1$ (for black internal node).



Why Red Black Tree is Height Balanced

- So, using induction, we can prove subtree rooted at x contains at least $(2^{bh(x)-1} - 1) + (2^{bh(x)-1} - 1) + 1 = (2^{bh(x)} - 1)$ internal nodes.
- According to the internal property the black-height of the root must be at least $\frac{h}{2}$; thus
- $n \geqslant 2^{\frac{h}{2}} - 1$
- $\log(n+1) \geqslant \frac{h}{2}$
- $h \geqslant 2\log(n+1)$
- Thus $h = \mathcal{O}(\log(n))$

Red Black Tree Insertion

- We can insert each node in Red Black Tree in $\mathcal{O}\log(n)$ time
- We use left rotation, right rotation and recoloring of nodes to fix the property violation which is caused by inserting a new node.
- There are roughly three cases in Red Black Tree Insertion.
- We are simulating the insertion of 5 keys: $2, 3, 5, 7, 4$ consecutively in an empty RBT.

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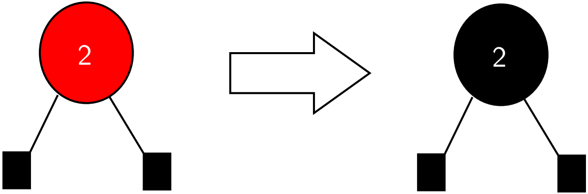
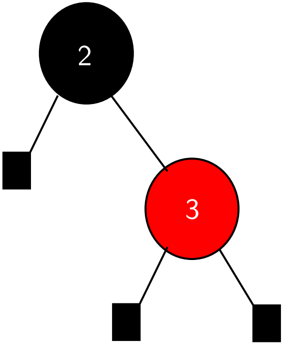
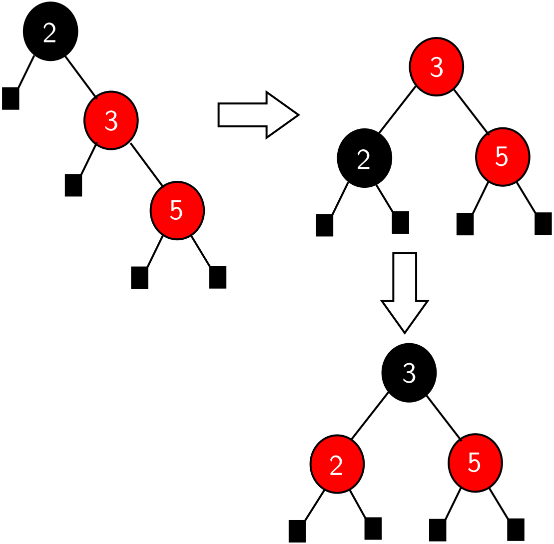
Insert 2



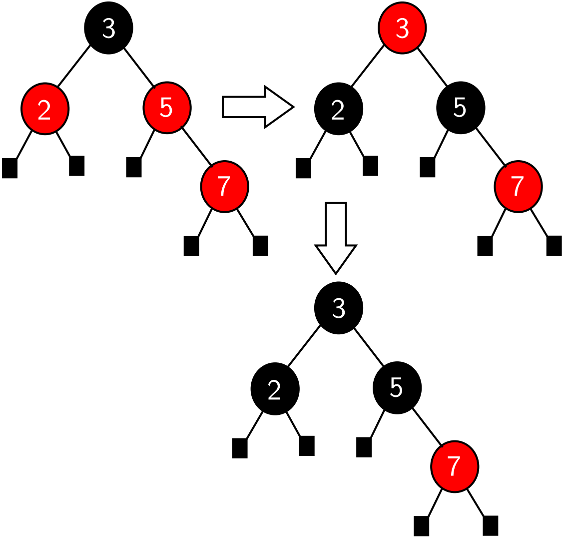
Insert 3



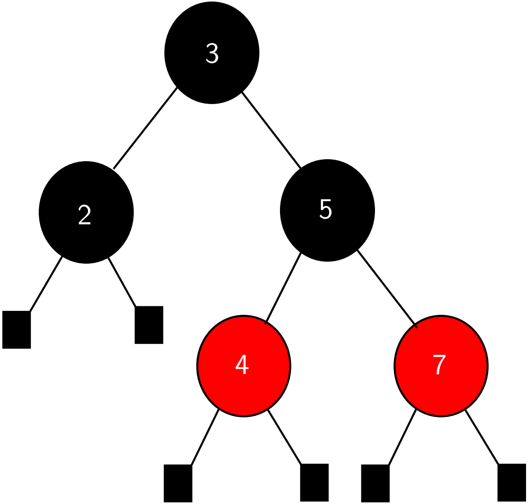
Insert 5



Insert 7



Insert 4



Red Black Tree Insert

```
RB Insert(T,z)
y ← T.nil
x ← T.root
while x ≠ T.nil do
    y ← x;
    if z.key < x.key then
        x ← x.left
    else
        x ← x.right
    end
end
z.p ← y
if y = T.nil then
    T.root ← z
else if z.key < y.key then
    y.left ← z
else
    y.right ← z
z.left ← T.nil
z.right ← T.nil
y.color ← red
RB-Insert-Fixup(T, z)
```

Red Black Tree Deletion

- Similar to Red Black Tree Insertion, Deletion also runs in $\mathcal{O}\log(n)$ time
- Deletion is a bit more complicated than Insertion
- There are mainly three cases in RBT Deletion; in one of the cases, a double black node is created which can be handled through six different cases.

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THANK YOU
ANY QUESTIONS ?