

1) Assignment + 3:

$$1) T(n) = \begin{cases} 1 & n=1 \\ 2T(\frac{n}{2}) + n & n>1 \end{cases}$$

$$T(n) = 2T(n/2) + n$$

$$T(n) = 2[2T(\frac{n}{2^2}) + \frac{n}{2}] + n$$

$$T(n/2) = 2[T(n/4) + \frac{n}{2}]$$

$$T(n) = 4T(\frac{n}{2^2}) + 2n - T(\frac{n}{2}) = 2T(\frac{n}{2^3}) + \frac{n}{2^2}$$

$$= 4[2T(\frac{n}{2^3}) + \frac{n}{2^2}] + 2n$$

$$= 8T(\frac{n}{2^3}) + 3n$$

$$= 2^3 T(\frac{n}{2^3}) + 3n$$

$$= 2^k T(\frac{n}{2^k}) + kn$$

$$n/2^k \leq 1 \Rightarrow k \leq \log_2 n$$

$$= 2^{\log_2 n} T(1) + n \log_2 n$$

$$= n^1 + 1 + n \log_2 n$$

$$= O(n \log n)$$



$$2) \quad T(n) = \begin{cases} 1 & n = 1 \\ 8T(n/2) + n^2 & n > 1 \end{cases}$$

$$T(n) = 8T(n/2) + n^2 = 8T(n/2) + (n/2)^2 + (n/2)^2$$

$$= 8 \left[ 8T(n/2) + (n/2)^2 \right] + n^2$$

$$= 8^2 T(n/2) + \frac{8n^2}{4} + n^2$$

$$= 8^2 T(n/2) + 3n^2$$

$$= 8^2 \left[ 8T(n/2) + (n/2)^2 \right] + 3n^2$$

$$= 8^3 T(n/2) + \frac{8^2 n^2}{16} + 3n^2$$

$$= 8^3 \left( T(n/2) \right) + 7n^2$$

$$= 8^k \left( T(n/2^k) \right) + (2^k - 1) n^2$$

$$\frac{n}{2^k} = 1 \rightarrow k = \log_2 n$$

$$= 8^{\log_2 n} (T(1) + (2^{\log_2 n} - 1) n^2)$$

$$= n^3 + (n-1)n^3$$

$$= n^3 + n^3 - n$$

$$= O(n^3)$$