5) $128^{\log_2 7} \cdot n^2 = \Theta(n^9)$

i) big oh → $f(n) \leq c_1 g(n)$

simplify, $n^{\log_2 7} \cdot n^2 = \Theta(n^9)$

$n^7 \cdot n^2 = n^9 \Rightarrow f(n) = n^9$

$f(n) \leq c_1 g(n)$

$c_1 = 1, \quad n^9 \leq 1 \cdot n^9$

$\boxed{f(n) = O(n^9)}$

ii) omega → $f(n) \geq c_2 g(n)$

$c_2 = 1, \quad n^9 \geq 1 \cdot n^9$

$\boxed{f(n) = \Omega(n^9)}$

iii) Theta,

$f(n) = O(n^9)$ and $f(n) = \Omega(n^9)$

Hence $\boxed{f(n) = \Theta(n^9)}$

3) $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$

Simplify,

$n^{\log_2 64} \cdot n^{\log_2 32} = O(n^5)$

$n^{\log_2 2^6} \cdot n^{\log_2 2^5} = O(n^5)$

$n^6 \cdot n^5 \leq O(n^5) \Rightarrow n^{11} = O(n^5)$

$f(n) = n^{11}$, $g(n) = n^5$

$f(n) \leq c_1 g(n)$

$n^{11} \leq c_1 n^5$, it can only be equal if $c = n^{11}$ However c won't be constant

$n^{11} \leq n^c n^5$

Not valid

4) $\frac{4^n}{2^n} = O(2^n)$

$\frac{2^n 2^n}{2^n} = O(2^n)$

$2^n = O(2^n)$

$f(n) = 2^n$, $g(n) = 2^n$

$f(n) \leq g(n) \Rightarrow 2^n \leq c_1 2^n$

$c_1 = 1$, $2^n \leq 1 \cdot 2^n$

Valid

Assignment 2:

1) $f(n) = n-10$, $g(n) = n+10$, $f(n) = \Theta(g(n))$

Big O = $f(n) \leq c_1 g(n)$, omega → $f(n) \geq c_2 g(n)$

i) $n-10 \leq c_1(n+10)$ → $c_1 = 1$

$n-10 \leq n+10$ ✓

$f(n) \leq O(g(n))$

ii) $f(n) \geq c_2 g(n+10)$

$n-10 \geq c_2(n+10)$ — $n = 500$

$500-10 \geq c_2(500+10)$ → $490 \geq c_2(510)$

$c_2 = \frac{1}{2}$ → $490 \geq 510/2$ → $490 \geq 255$

$f(n) \geq \Omega(g(n))$

hence $f(n) \in \Theta(g(n))$

2) $f(n) = n$, $g(n) = n$

i) Big O $f(n) \leq c g(n)$ → $n \in 1 \cdot n$ $c = n$

$f(n) = O(gn)$

ii) omega $f(n) \geq c g(n)$

$n \geq 1 \cdot n$ where $c_2 = 1$

$f(n) \geq \Omega(gn)$

hence, $f(n) \in \Theta(gn)$