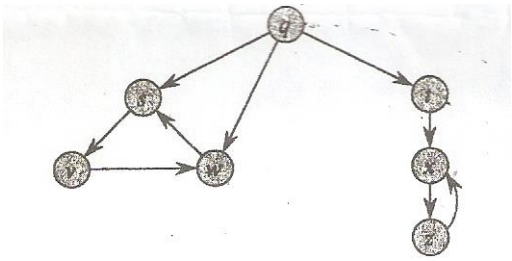
XC2454 – DESIGN AND ANALYSIS OF ALGORITHMS

1. Define correctness of an algorithm. Prove the correctness of Insertion sort algorithm using a loop invariant.
2. Write a recursive algorithm to find the maximum value of a set of n integers.give the running time of the algorithm.
3. Find the optimal lower bound for number of operations required to sort an array of n elements.
4. Discuss how the following criteria will be used to analyse algorithms:
    a. Amount of work done.
    b. Amount of memory used.
5. Describe the Tower of Hanoi problem. Give a recursive algorithm for this problem. Compute its running time. Demonstrate the algorithm for three discs.
6. Give a brief description about insertion sort with pseudocode. Demonstrate the algorithm for the array A = <8,12,4,7,2>. Compute its running time for the best, average and worst cases.
7. Discuss in detail, the Divide-and-Conquer approach of designing an algorithm with Merge sort as a suitable example. Write its pseudocode.
8. Show that the recurrence for its running time is:

$$T(n) = \begin{cases} a, & n = 1 \\ 2T(\lfloor n/2 \rfloor) + n, & n > 1 \end{cases}$$

where $a > 0$ is a constant. Show that $T(n) = \mathrm{O}(n \lg n)$.

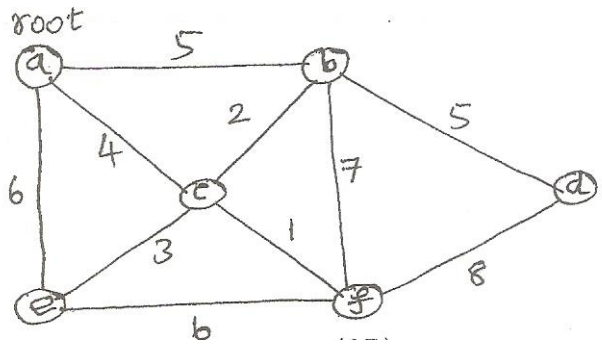
9. For any two non-negative functions f(n) and g(n), prove or disprove the following
(i) $f(n) = \mathrm{O}(g(n)) \Rightarrow g(n) = \Omega(f(n))$
(ii) $f(n) + g(n) = \Theta(min\{f(n), g(n)\})$.
10. Suppose that the splits at every level of Quicksort are in the proportion 1:5, find the minimum and maximum heights of leaf nodes in the recursion tree.
11. Why do we want the loop index of BUILD-HEAP to decrease from [length[A]/2] to 1 rather than increase from 1 to [length[A]/2]? Give examples, if necessary.
12. Construct a Heap of five vertices and write down its adjacency linked list representation.
13. Show that how DFS algorithm(Depth First Search) works on the following graph with pseudocode. Assume that the DFS procedure considers the vertices in alphabetical order and assume further that each adjacency list is ordered alphabetically. Write down the discovery and finishing times for each vertex as a parenthesis structure. Show the color and parent values at each step. Write also the DFS forest output.



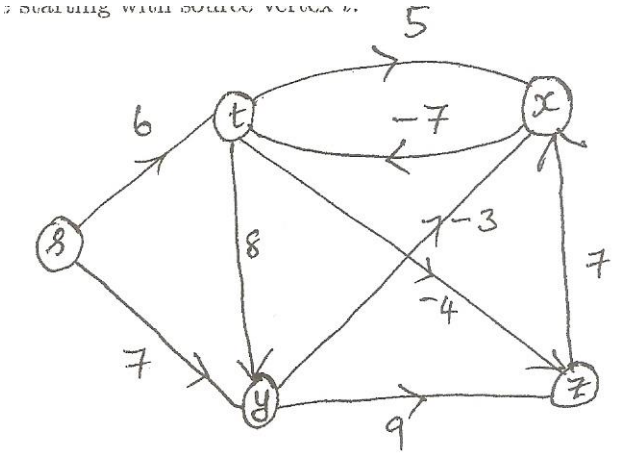
14. Write the pseudocode of PARTITION algorithm. Explain the simulation of the algorithm on the array A=<6,2,1,9,5>
15. Using iteration method, obtain the tight asymptotic bound for

$$T(n) = 2T(\lfloor n/2 \rfloor) + \Theta(n).$$

16. Discuss in detail, the procedure MAX-HEAPIFY(A,i) by writing its pseudocode. Explain the simulation of the algorithm MAX-HEAPIFY(A,2) on the array A=<16,4,10,14,7,9,3>
17. Discuss in detail with recurrence relation, the best, average and worst complexity cases of QUICK-SORT algorithm.
18. Define Minimum Spanning Tree T of a connected undirectional graphG(V,E).
19. Are all shortest-path weights well defined incase of a directed graph with negative weight edges? justify your answer with an example.
20. Construct a finite automation that accepts all strings of the form x= yz, where y = e or a string containing characters a,b and z= a^k, for even k.
21. Construct an automata which accepts strings that end with the pattern P = abba. Compute the final state function Ø for the text T = ababbabbababa. Explain how you verify that pattern P occurs in the text T with the help of final state function.
22. Explain Prim's algorithm with illustration on the following examples with indicated root vertex. What is its running time?



23. Write a brief description of Bellman-Ford algorithm and illustrate it with the following example starting with source vertex t.



24. Suppose we are comparing implementations of two algorithms A and B for a same computational problem on a same machine. For input of size n, algorithm A runs 8n^2 steps, while algorithm B runs in 64nlgn steps. For which values of n does algorithm A beat algorithm b?

25.

Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

26.

Suppose that the splits at every level of *Quick sort* are in the proportion $1/5$ and $4/5$. What are the minimum and maximum depths of leaf nodes in the recursion tree?

27.

Why do we want the loop index of *BUILD-HEAP* to decrease from $\lfloor length[A]/2 \rfloor$ to 1 rather than increase from 1 to $\lfloor length[A]/2 \rfloor$?

28.

Show that if an edge (u, v) is contained in some minimum spanning tree, then it is a light edge crossing some cut of the graph.

29.

Give an adjacency-list representation for a complete binary tree on 7 vertices. Assume that vertices are numbered from 1 to 7 as in a heap.

30.

Construct a finite automaton that accepts all strings of the form $x = yz$, where $y = \epsilon$ or a string containing characters a, b ending with b only and $z = a^k$, for even k.

31.

Find the prefix function $\pi[i]$ for the pattern $P = aabaabaabaab$.

32. Write down the language of binary representation of prime numbers.

33. Define a polynomial time verification algorithm.

34. Answer the following

a.

Show that the Hamiltonian-cycle problem is polynomial time reduced to the Travelling Salesman Problem (i.e., *HAM-CYCLE* $\leq_P$ **TSP**) by defining properly the transformation function with examples. Show that the running time of the transformation function is of polynomial time in input size. Further prove the correctness of the function.

b.

If $L_1, L_2 \subset \{0,1\}^*$ are any two concrete problems such that $L_1 \leq_P L_2$. Then Show that $L_2 \in \mathbb{P}$ implies $L_2 \in \mathbb{P}$.

35. Answer the following

a.

Discuss in detail, how the following criteria are used to analyze algorithms:

- Amount of work done
- Amount of memory space used.
- Optimality

b.

Using decision tree model prove that any comparison sort must necessarily make $\Omega(nlgn)$ comparisons in the worst case to sort a sequence of n elements.

c.

Discuss in detail, the Divide-and-Conquer approach of designing an algorithm for a given problem. Explain this concept with a pseudocode, by considering *Binary* ~~Sort~~ Search as a suitable example. Give the recurrence for the worst case running time

d.

Using iteration method, obtain the tight asymptotic bound for

$$T(n) = \begin{cases} 1, & n = 1 \\ 2T(\lfloor n/2 \rfloor) + n, & n > 1 \end{cases}$$

36. Answer the following

a.

Starting with the procedure, discuss in detail *Insertion Sort* by writing its pesudocode. Compute its running time for the best, average and worst cases. Illustrate the simulation of insertion sort on the array $A =< 10, 4, 3, 2, 1, 9, 5 >$

b.

Explain, how do you construct a heap from given array of integers. Illustrate the operation of *BUILD-HEAP* on the array $A =< 4, 14, 7, 9, 16, 2, 8 >$

c.

Describe with pseudocode, the procedure *PARTITION* involved in **QUICKSORT** algorithm. Analyze the best, worst and balanced cases of this *PARTITION* procedure. Illustrate its operation on the array $A =< 2, 8, 7, 1, 3, 5. 6, 4 >$

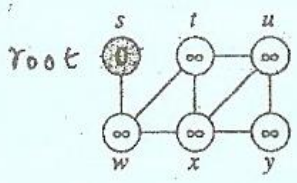
d.

Illustrate the *Merge Sort* algorithm for the array $A =< 4, 1, 3, 2, 16, 9, 10, 14 >$.

37. Answer the following

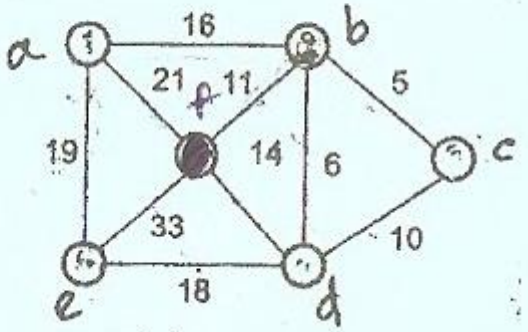
a.

Give a brief description of *Breadth-First Search* algorithm with the pseudocode. Show the d and π values that result from running *breadth-first search* on the following graph, using indicated source vertex.



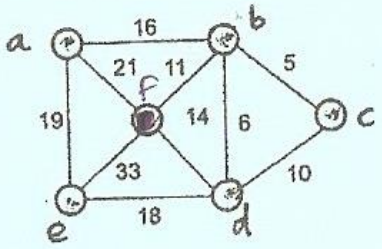
b.

Demonstrate *Kruskal's* algorithm for the following graph.



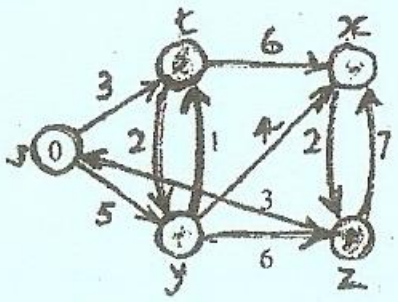
c.

Find the minimum spanning tree using *Prim's* algorithm for the following graph with a as the root vertex. Explain the algorithm with its pseudocode. Write down its running time



d.

Illustrate *Dijkstra's* algorithm for the following example starting with source vertex t.



38. Answer the following

a.

Construct an automata which accepts strings that end with the pattern $P = abab$. Compute the final state function ϕ for the text $T = abaababbbababbbaba$. Explain that how you verify that pattern P occurs in the text T with the help of final state function.

b.

Explain with pseudocode the algorithm *COMPUTE-TRANSITION-FUNCTION* which computes the transition function for string-matching automaton corresponds to a given pattern. Give the running time for this algorithm. Illustrate this algorithm for the pattern $P = abababa$.

c.

Write the pseudocode of *Knuth-Morris-Pratt* algorithm for string matching. Illustrate the algorithm with the example T=abababaabcababaca and P=ababaca.

d.

Write algorithm *COMPUTE-PREFIX-FUNCTION*(P) for the computation of prefix function of a given pattern. Illustrate the algorithm with the example for pattern $P = abacab$ for pattern. Give its running time.