

Course Code: BIT 116

Course Name: Mathematics I

Program: BIT FALL 2019

Semester: 1 Year/ I semester



COLLEGE OF MANAGEMENT & INFORMATION TECHNOLOGY

BACHELOR IN INFORMATION TECHNOLOGY

ASSIGNMENT

Submitted by:

Submitted to:

Name: Bishal Bhattarai

Lincoln University

Year/ Semester: First, Fall 2019

LCID: LC00017000753

Date: 2020.06.27

1. Find the Domain and Range of $f(x) = \sqrt{2 - x - x^2}$

Solution: Here,

$$2 - x - x^2 = 0$$

$$\text{Or, } (2+x)(1-x) = 0$$

Either, $x = 1$ or, -2

The interval be $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

Now, the sign conversion,

| | $2+x$ | $1-x$ | $(2+x)(1-x)$ |
|-----------------|-------|-------|--------------|
| $(-\infty, -2)$ | - | + | - |
| $(-2, 1)$ | + | + | + |
| $(1, \infty)$ | + | - | - |

Here, only the positive sign is taken. So, the Domain is found to be $(-2, 1)$.

For range,

$$y = \sqrt{2 - x - x^2}$$

$$= \sqrt{(2+x)(1-x)}$$

For, minimum value we take $x=0$, then the output will be maximum value. So the Range is $(0, \sqrt{2})$.

2. Find the solution of: $\frac{dx}{x^2+1} + \frac{dy}{y+1} = 0$

➤ Solution:

Here,

Integrating both side;

$$\int \frac{dx}{x^2+1} + \int \frac{dy}{y+1} = 0$$

$$\text{Or, } \tan^{-1}x + \log(y+1) = 0$$

Differentiate: $y = 4\sec t + \tan t$

➤ Solution: Given, $y = 4\sec t + \tan t$

Differentiate with respect to t we get,

$$\text{Or, } \frac{dy}{dt} = \frac{d(4\sec t + \tan t)}{dt}$$

$$\therefore \frac{dy}{dt} = 4\sec t \cdot \tan t + \sec^2 t$$

2. If $f''(x) = 20x^3 - 12x^2 + 6x$, then find $f(x)$.

➤ Solution: Given, $f''(x) = 20x^3 - 12x^2 + 6x$

Integrating both sides we get,

$$\text{Or, } \int f''(x) = \int 20x^3 - 12x^2 + 6x$$

$$\text{Or, } f'(x) = \int 20x^3 - 12x^2 + 6x$$

$$\text{Or, } f'(x) = \frac{20x^4}{4} - \frac{12x^3}{3} + \frac{6x^2}{2}$$

$$\text{Or, } f'(x) = 5x^4 - 4x^3 + 3x^2$$

Integrating both sides we get,

$$\text{Or, } \int f'(x) = \int 5x^4 - 4x^3 + 3x^2$$

$$\text{Or, } f(x) = \frac{5x^5}{5} - \frac{4x^4}{4} + \frac{3x^3}{3}$$

$$\text{Or, } f(x) = x^5 - x^4 + x^3$$

Hence, $f(x)$ is found to be $x^5 - x^4 + x^3$.

3. Find the area enclosed between x axis, the curve $y = x^3 - 2x + 5$ and the ordinates $x = 1$ and $x = 2$.

➤ Solution:

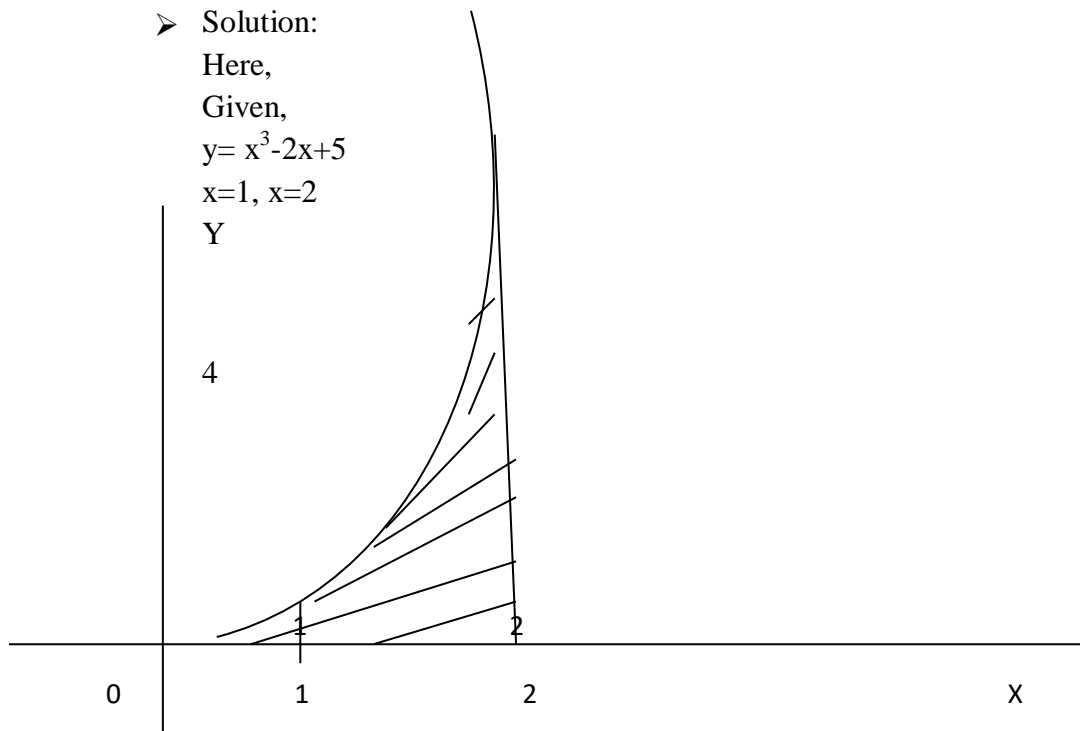
Here,

Given,

$$y = x^3 - 2x + 5$$

$$x=1, x=2$$

Y



From the formula;

$$\text{Area} = \int_a^b (\text{upper function} - \text{lower function}) dx$$

Now $a=1$ and $b=2$ then the function becomes

$$A = \int_1^2 x^3 - 2x + 5$$

$$A = \left[\frac{x^4}{4} - \frac{2x^2}{2} + 5x \right]_1^2$$

$$A = [4 - 4 + 10] - \left[\frac{1}{4} + 4 \right]$$

$$A = 10 - \frac{17}{4}$$

$$\therefore A = 5.75 \text{ sq. unit}$$

Hence, the area enclosed between x-axis is found to be 5.75 sq unit.

4. Find $\int \frac{dx}{e^x + 1}$ (Antiderivatives)

➤ Solution: Given,

$$\text{Or, } \int \frac{dx}{e^x + 1}$$

$$\text{Or, } \int \frac{e^x + 1 - e^x}{e^x + 1} dx$$

$$\text{Or, } \int \frac{e^x + 1}{e^x + 1} dx - \int \frac{e^x}{e^x + 1} dx$$

$$\text{Or, } \int dx - \ln(e^x + 1)$$

$$\text{Or, } x - \ln(e^x + 1) + C$$

\therefore Antiderivatives of the function $\int \frac{dx}{e^x + 1}$ is found to be $x \ln(e^x + 1) + C$.

State and Verify mean value theorem for $f(x) = x^3 - x$ in $[0, 2]$ ➤ Solution:

Here,

Given function is $f(x) = x^3 - x$ which is continuous on close interval $[a, b]$

Then, there exist some number in c in $[a, b]$ such that $f'(c)$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$[a, b] = [0, 2]$$

Then, $a = 0$ and $b = 2$.

Now, $f(a) = x^3 - x$ Or,

$$f(0) = 0 \quad f(b) = x^3 - x \quad f(2) = 6$$

$$\text{Or, } f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\text{Or, } f'(c) = \frac{6 - 0}{2 - 0}$$

$$\therefore f'(c) = 3$$

$$\text{Again, } f'(c) = 3x^2$$

From above we found $f'(c) = 3$

$$\text{Or, } 3 = 3x^2$$

$$\text{Or, } x = 1$$

Therefore, 1 lies between the interval 0 and 2. Hence, it proves mean value theorem.