QUANTITATIVE METHODS

MODULE CODE: BIT 125

WELCOME

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COURSE CONTENTS

CHAPTER 05

MEASRUE OF CENTRAL TENDANCY

- ☐ Arithmetic mean
- ☐ Geometric mean
- ☐ Harmonic mean
- ☐ The median: quartiles; deciles and percentiles
- ☐ The mode
- ☐ Relation between mean, median and mode

5 Lectures Hours

MEASURE OF CENTRAL TENDENCY

The central tendency is measured by averages. These describe the point about which the various observed values cluster. In mathematics, an average or central tendency of data set refers to a measure of the middle value of the data set.

- ❑ Measure of central tendency gives an idea about the concentration of the values in the central part of the distribution.
- ❑ There are the five measure of central tendency that are in common use:
  - Arithmetic Mean
  - Geometric Mean
  - Harmonic Mean
  - Median
  - Mode

ARITHMETIC MEAN

Arithmetic Mean (AM)

The arithmetic mean is the sum of a set of observations, positive, negative or zero, divided by the number of observations.

If we have "n" real numbers $x_1, x_2, \dots \dots, x_n$ then their arithmetic mean, denoted by $\bar{x}$, can be expressed as:

$$\bar{x} = \frac{x_1 + x_2 + \dots \dots + x_n}{n}$$

For Example: Find the arithmetic mean of 9, 3 7, 3, 8, 10, 2

Solution:

$$\bar{x} = \frac{9+3+7+3+8+10+2}{7} = \frac{42}{7} = 6$$

So the arithmetic mean is 6.

ARITHMETIC MEAN

Arithmetic Mean (AM)

The arithmetic mean is the sum of a set of observations, positive, negative or zero, divided by the number of observations. Arithmetic mean can be either,

i) Simple arithmetic mean or, ii) Weighted arithmetic mean

Formula to Calculate arithmetic mean:

- Direct Method

$$\bar{x} = \frac{\Sigma fx}{n}$$

- Short-cut Method

$$\bar{x} = A + \frac{\Sigma fd}{n}$$

ARITHMETIC MEAN

Merits:

1. It is easy to understand and easy to calculate
2. It is based upon all the observations
3. It is familiar to common man and rigidly defined
4. It is capable of further mathematical treatment.
5. It is not affected by sampling fluctuations. Hence it is more stable.

ARITHMETIC MEAN

Demerits:

1. It cannot be determined by inspection
2. Arithmetic mean cannot be used if we are dealing with qualitative characteristics, which cannot be measured quantitatively like caste, religion, sex
3. Arithmetic mean cannot be obtained if a single observation is missing or lost
4. Arithmetic mean is very much affected by extreme values.

ARITHMETIC MEAN

Examples:

1. The following are the weight of 10 students of a class. Find the arithmetic mean

Roll No.	1	2	3	4	5	6	7	8	9	10
Weight (kg)	45	48	50	60	55	46	50	58	62	46

Solution:

Direct method:

Roll No.	Weight (kg) (X)
1	45
2	48
3	50
4	60
5	55
6	46
7	50
8	58
9	62
10	46
	$\Sigma X = 520$

$$\text{Arithmetic mean } (\bar{x}) = \frac{\Sigma X}{n}$$

$$= \frac{520}{10}$$

$$= 52\ kg$$

ARITHMETIC MEAN

Short-cut method:

Roll No.	Weight (kg) (X)	d = X – A = X - 50
1	45	45 – 50 = -5
2	48	48 – 50 = -2
3	50	0
4	60	10
5	55	5
6	46	-4
7	50	0
8	58	8
9	62	12
10	46	-4
	$\Sigma X = 520$	$\Sigma d = 20$

$$\text{Arithmetic mean } (\bar{x}) = A + \frac{\Sigma d}{n}$$

$$= 50 + \frac{20}{10}$$

$$= 50 + 2$$

$$= 52\ kg$$

ARITHMETIC MEAN

2. Find the arithmetic mean for the following data:

X	5	10	15	20	25	30	35	40	45	50
f	20	43	75	67	72	45	39	9	8	6

Solution:

Direct Method:

X	f	fx
5	20	100
10	43	430
15	75	1125
20	67	1340
25	72	1800
30	45	1350
35	39	1365
40	9	360
45	8	360
50	6	300
	$\Sigma f = n = 384$	$\Sigma fx = 8530$

Arithmetic mean $(\bar{x}) = \frac{\Sigma fx}{n}$

$$= \frac{8530}{384}$$

$$= 22.21$$

ARITHMETIC MEAN

Short-cut method:

X	f	d = X – A(25)	fd
5	20	-20	-400
10	43	-15	-645
15	75	-10	-750
20	67	-5	-335
25	72	0	0
30	45	5	225
35	39	10	390
40	9	15	135
45	8	20	160
50	6	25	150
	$\Sigma f = n = 384$		$\Sigma fd = -1070$

Arithmetic mean $(\bar{x}) = A + \dfrac{\Sigma fd}{n}$

$$= 25 + \frac{-1070}{384}$$

$$= 25 - 2.78$$

$$= 22.21$$

GEOMETRIC MEAN

The geometric mean is a type of average, usually used for growth rates, like population growth or interest rates. While the arithmetic mean **adds** items, the geometric mean **multiplies** items.

- **Individual Series**

$$\text{Geometric Mean (GM)} = (x_1, \times x_2 \times \cdots \times x_n)^{\frac{1}{n}}$$

Example:

1. Find the Geometric Mean of the following data:

   3, 2, 5, 6, 7

Solution:

$$\text{G.M} = (3 \times 2 \times 5 \times 6 \times 7)^{\frac{1}{5}} = 4.17$$

GEOMETRIC MEAN

- **Discrete/Continuous Series**

Geometric Mean (GM) $= ({x_1}^{f_1} \times {x_2}^{f_2} \times \cdots \times {x_n}^{f_n})^{\frac{1}{\Sigma f}}$

Example:

2. Find the Geometric Mean of the following data:

Solution:

x	f
2	1
3	2
4	5
5	4

$$\Sigma f = n = 12$$

Geometric Mean (GM) $= (2^1 \times 3^2 \times 4^5 \times 5^4)^{\frac{1}{12}} = 3.88$

GEOMETRIC MEAN

Example:

3. Find the Geometric Mean of the following data:

Class Interval	Frequency
0 – 10	3
10 – 20	2
20 – 30	7
30 – 40	5

HARMONIC MEAN

The **harmonic mean** is a type of numerical **average.** It is calculated by dividing the number of observations by the reciprocal of each number in the series. Thus, the **harmonic mean** is the reciprocal of the arithmetic **mean** of the reciprocals

- **Individual Series**

$$\text{Harmonic Mean (HM)} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$$

Example:

1. Find the Harmonic Mean of the following data:

3, 2, 4, 5, 6, 7

Solution:

$$\text{H.M} = \frac{6}{\frac{1}{3} + \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}} = 3.76$$

HARMONIC MEAN

- **Discrete/Continuous Series**

$$\text{Harmonic Mean (HM)} = \frac{\Sigma f}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \frac{f_3}{x_3} + \dots + \frac{fn}{xn}}$$

Example:

2. Find the Harmonic Mean of the following data:

X	3	4	5	7
f	6	2	3	5

Solution:

$$\text{H.M} = \frac{16}{\frac{6}{3} + \frac{2}{4} + \frac{3}{5} + \frac{5}{7}} = 4.19$$

HARMONIC MEAN

Example:

3. Find the Harmonic Mean of the following data:

Class Interval	Frequency
0 – 10	3
10 – 20	2
20 – 30	1
30 – 40	4

MEDIAN

Median (Md)

Median is the middle score in the distribution when the numbers have been arranged into numerical order, from either highest to lowest or lowest to highest. Like the mean, the median is a numerical representation of the center of the data set, and can be used with interval or ratio data. The median can also be used for ordinal data.

To find the median, one lines up the scores from highest to lowest (or lowest to highest) and simply finds the middle score. If there is an odd number of scores, the median is the middle number of the lined up data set. If there is an even number of scores, the median is found by averaging the two middle numbers (adding them up and dividing by two) and that average is the median for the data.

Unlike the mean, extreme scores in the data set, called outliers, have less of an effect on the median. When outliers are present, the mean is "pulled" in the direction of the outlier, meaning an extremely high score would result in a higher mean than if the outlier was not present. The median on the other hand, would be less affected by the outlier, often resulting in little or no change in the median.

MEDIAN

<u>Median of Individual Series</u>

In an ungrouped frequency distribution if the n values are arranged in ascending or descending order of magnitude, the median is the middle value if n is odd. When n is even, the median is the mean of the two middle values.

Example 1. Suppose we have the following series:

15, 19, 21, 7, 10, 33, 25, 18 and 5

Solution:

We have to first arrange it in either ascending or descending order. These figures are arranged in an ascending order as: 5,7,10,15,18,19,21,25,33

In this case, n is 9, as such

$$\frac{n+1}{2} = \frac{9+1}{2} = 5$$

That is, the size of the 5th item is the median. So median is 18

MEDIAN

Median of Discrete Series

Example 2.

From the following data calculate the median

Marks	45	55	25	35	5	15
No. of Students	40	30	30	50	10	20

Solution-

Step I- First we will find out the commutative frequency

Marks in ascending order (x)	No. of students (f)	Commutative frequency C.f
5	10	10
15	20	30
25	30	60
35	50	110
45	40	150
55	30	180
	N = 180	

MEDIAN

Step II - Size of $\frac{N+1}{2}$ item $=$ size of $\frac{181}{2}$ item $=$ **90.5^{th} item**

Step III- Commutative frequency which includes 90.5^{th} $= 110$

Median = size of item corresponding to 110 = 35

MEDIAN

Median of Continuous Series

Example 3.

From the following data calculate the median

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 - 60
No. of Students	10	20	30	50	40	30

Solution:

Step I – First we will find out the cumulative frequency

Marks	Frequency (f)	Cumulative frequency (c.f)
0 – 10	10	10
10 – 20	20	30
20 – 30	30	60
30 – 40	50	110
40 – 50	40	150
50 - 60	30	180
	N = 180	

MEDIAN

Step II – Size of $\frac{N}{2}$ item = Size of 180/2 item = 90th item

Step III - f = 50**,** c.f = 60, l = 30, N = 180, h = 10

Step IV - Median = $\mathrm{l} + \frac{\frac{N}{2} - c.f}{f} \times h$

$$= 30 + \frac{90-60}{50} \times 10$$

= 36

MEDIAN

Merits of Median

1. Even if the value of extreme item is much different from other values, it is not much affected by these values e.g. Median in case of 4, 7, 12, 18, 19 is 12 and if we add two values equal to 450 10000, new median is 18.
2. For open end intervals, it is also suitable one. As taking any value of the intervals, value of Median remains the same.
3. It can be easily calculated and is also easy to understand
4. Median is also used for other statistical devices such as Mean Deviation and skewness.
5. Extreme items may not be available to get Median. Only if number of terms is known, we can get median

MEDIAN

Demerits of Median

1. Even if the value of extreme items is too large, it does not affect too much, but due to this reason, sometimes median does not remain the representative of the series.
2. If the number of series is even, we can only make its estimate; as the A.M. of two middle terms is taken as Median.
3. It is affected much more by fluctuations of sampling than A.M.
4. Median cannot be used for further algebraic treatment. Unlike mean we can neither find total of terms as in case of A.M. nor median of some groups when combined.
5. In a continuous series it has to be interpolated. We can find its true-value only if the frequencies are uniformly spread over the whole class interval in which median lies.

QUARTILE

Quartile

The values of a variate that divide the series or the distribution into four equal parts are known as quartiles. Since three points are required to divide the data into four equal parts, we have three quartiles **Q1, Q2, Q3**.

The first quartile (Q1) is also known as a lower quartile, is the value of a variate below which there are 25% of the observation and above which there are 75% of the observations.

The second quartile (Q2) is known as a middle quartile or median, is the value of a variate which divides the distribution into two equal parts. It means there are 50% of the observations above it and 50% of the observations below it.

The Third quartile (Q3) is known as an upper quartile, is the value of a variate below which there are 75% of the observations and above which there are 25% of the observations.

QUARTILE

Size of Quartile for Individual/Discrete Series

$$Q_1 = \text{size of } \frac{N+1}{4} \text{ th item}$$

$$Q_2 = \text{size of } \frac{2(N+1)}{4} \text{ th item}$$

$$Q_3 = \text{size of } \frac{3(N+1)}{4} \text{ th item}$$

QUARTILE

Example: From the following data calculate first, second and third quartile.

Marks	5	15	25	35	45	55
No. of students	10	20	30	50	40	30

Solution:

Step I – Calculation of Cumulative Frequencies

Marks	No. of Students (f)	Cumulative Frequency (c.f)
5	10	10
15	20	30
25	30	60
35	50	110
45	40	150
55	30	180
	N = 180	

QUARTILE

Step II –

$$Q_1 = \text{size of } \frac{N+1}{4} \text{ th item}$$

$$= \text{Size of } \frac{180+1}{4} \text{ th item}$$

$$= \text{size of } 45.25 \text{ th item}$$

$$= 25$$

$$Q_2 = size\ of\ \frac{2(N+1)}{4}\ th\ item$$

$$= \text{Size of } \frac{2(180+1)}{4} \text{ th item}$$

$$= \text{size of } 90.5 \text{ th item}$$

$$= 35$$

$$Q_2 = size\ of\ \frac{3(N+1)}{4}\ th\ item$$

$$= \text{Size of } \frac{3(180+1)}{4} \text{ th item}$$

$$= \text{size of } 135.7 \text{ th item}$$

$$= 45$$

QUARTILE

Size of Quartile for Continuous Series

$$Q_1 = \text{size of } \frac{N}{4} \text{ th item}$$

$$Q_2 = \text{size of } \frac{2N}{4} \text{ th item}$$

$$Q_3 = \text{size of } \frac{3N}{4} \text{ th item}$$

Computation of Quartile for Continuous Series

$$Q_1 = l + \frac{\frac{N}{4} - c.f}{f} \times h$$

$$Q_1 = l + \frac{\frac{N}{2} - c.f}{f} \times h$$

$$Q_1 = l + \frac{\frac{3N}{4} - c.f}{f} \times h$$

Example: Find first and third quartile from the following data.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 - 60
No. of Students	10	20	30	50	40	30

DECILES

The deciles of a variate that divide the series or the distribution into ten equal parts.

<u>Size of Deciles for Individual/Discrete Series</u>

$$D_i = \text{size of } \frac{i(N+1)}{10} \text{ th item}$$

$where,$

I = 1,2, 3, 4, 5, 6, 7, 8, 9

Example: From the following data calculate first and second Deciles

Marks	5	15	25	35	45	55
No. of students	10	20	30	50	40	30

DECILES

Solution:

Step I – Calculation of Cumulative Frequencies

Marks	No. of Students (f)	Cumulative Frequency (c.f)
5	10	10
15	20	30
25	30	60
35	50	110
45	40	150
55	30	180
	N = 180	

Step II –

D_1 = size of $\frac{N+1}{10}$ th item

= Size of $\frac{180+1}{10}$ th item

= size of 18.1 th item

= 15

D_2 = size of $\frac{2(N+1)}{10}$ th item

= Size of $\frac{2(180+1)}{10}$ th item

= size of 36.2 th item

= 25

DECILES

Size of Deciles for Continuous Series

$$D_i = \text{size of } \frac{iN}{10} \text{ th item}$$

$where,$

i = 1, 2, 3, 4, 5, 6, 7, 8, 9

Computation of Deciles for Continuous Series

$$D_1 = l + \frac{\frac{iN}{10} - c.f}{f} \times h$$

$where,$

i = 1, 2, 3, 4, 5, 6, 7, 8, 9

Example: Find D_5, D_9 and D_7 from the following data.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 - 60
No. of Students	10	20	30	50	40	30

PERCENTILES

The value of a variate which divides a given series or distribution into 100 equal parts are known as percentiles. Each percentile contains 1% of the total number of observations. Since ninety nine points are required to divide the data into 100 equal parts, we have 99 percentiles, $P_1\ to\ P_{100}$.

<u>Size of Percentiles for Individual/Discrete Series</u>

P_i = size of $\frac{i(N+1)}{100}$ th item

$where,$
i = 1,2, 3, … , 99

Example: From the following data calculate $P_8\ and\ P_{70}$

Marks	5	15	25	35	45	55
No. of students	10	20	30	50	40	30

PERCENTILES

Size of Percentiles for Continuous Series

$$P_i = \text{size of } \frac{iN}{100} \text{ th item}$$

where,

i = 1, 2, 3, … , 100

Computation of Percentiles for Continuous Series

$$P_1 = l + \frac{\frac{iN}{100} - c.f}{f} \times h$$

where,

i = 1, 2, 3, … , 100

Example: Find P_5, P_{29} and P_{50} from the following data.

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 - 60
No. of Students	10	20	30	50	40	30

MODE

Mode (Mo)

Mode is often said to be that value in a series which occurs most frequently or which has the greatest frequency. But it is not exactly true for every frequency distribution. Rather it is that value around which the items tend to concentrate most heavily.

<u>Mode of Individual Series</u>

The number of times each value occurs is counted and the value which is repeated maximum number of times is the modal value.

Example: Find the mode of the following series: 8, 9, 11, 15, 16, 12, 15, 3, 7, 15

Solution: There are ten observations in the series wherein the figure 15 occurs maximum number of times three. The mode is therefore 15.

MODE

Mode of Discrete Series

Example: From the following table calculate mode.

Marks	5	10	11	12	13	14	15	16	18	20
No. of Students	4	6	5	10	20	22	24	6	2	1

Solution:

Marks	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6
5	4		x		x	x
10	6	10		15		x
11	5		11		21	
12	10	15				35
13	20		30	**52**		
14	22	**42**			**66**	
15	**24**		**46**			**52**
16	6	30		32		
18	2		8		9	x
20	1	3	x	x		x

MODE

Analysis Table

Column No.	Marks									
	5	10	11	12	13	14	15	16	18	20
1							1			
2					1	1				
3						1	1			
4				1	1	1				
5					1	1	1			
6						1	1	1		
Total				1	3	**5**	4	1		

The highest total in the analysis table is five. The item corresponding to it is 14. Therefore, the mode is 14

Mode of Continuous Series

Mode of continuous series is determined by the following formula

$$Mo = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Example 3: From the following table calculate mode.

Marks	58 – 60	60 – 62	62 – 64	64 – 66	66 – 68	68 – 70	70 – 72
No. of Students	12	18	25	30	10	3	2

Solution:

Marks	No. of Students
58 – 60	12
60 – 32	18
62 – 64	25
64 – 66	30
66 – 68	10
68 – 70	3
70 – 72	2

MODE

Since the maximum frequency is 30 is in the class 64-66, therefore 64-66 is the modal class.

And,

$$f_0 = 25, \quad f_1 = 30, \quad f_2 = 10$$

$$l = 64, \ h = 2$$

Now,

$$Mo = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 64 + \frac{30 - 25}{60 - 25 - 10} \times 2$$

$$= 64.4$$

MODE

Merits of Mode

1. In many cases it can be found by inspection.
2. It is not affected by extreme values.
3. It can be calculated for distributions with open end classes.
4. It can be located graphically.
5. It can be used for qualitative data

Demerits of Mode

1. It is not based on all values.
2. It is not capable of further mathematical treatment.
3. It is much affected by sampling fluctuations.


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