QUANTITATIVE METHODS

MODULE CODE: BIT 125

WELCOME

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COURSE CONTENTS

CHAPTER 07

PROBABILITY

- ☐ Preliminaries
- ☐ Classical, empirical, axiomatic approaches of probability theory
- ☐ Conditional probability (Baye's Theorem)
- ☐ Inverse probability
- ☐ Probability distribution
- ☐ Mathematical expectation
- ☐ Variance of random variable

6 Lectures Hours

PROBABILITY

What is probability?

Most people use terms such as chance, likelihood, or probability to reflect the level of uncertainty about some issues or events. Examples in which these terms may be used are as follows:

- As you watch the news every day, you hear forecasters saying that there is a 70% chance of rain tomorrow.

- As you plan to enter a new business, an expert in the field tells you that the probability of making a first-year profit in this business is only 0.4, or there is a 40% chance that you will make a profit.

PROBABILITY

Experiment:

- A pre-planned process for the sake of producing data that can reveal the purpose of the process application or meet the objectives of the study in which the experiment is conducted.
- In the context of probability, when the term ‘experiment’ is used it typically indicates a process that can result in only one of several possible outcomes.

An outcome:

The result of a single trial of an experiment

An event:

A collection of one or more outcomes of an experiment

Random Experiment:

If an experiment or trial is repeated under the same conditions for any number of times, the result obtained is not unique, it is called as “*Random Experiment*”.

PROBABILITY

Sample space:

The set of all possible outcomes of a random experiment is known as "Sample Space" and denoted by set S (this is similar to Universal set in Set Theory). The outcomes of the random experiment are called sample points or outcomes. In tossing a coin, head of tail may turn up. Thus $S = \{H, T\}$ is the sample space.

Exhaustive Events:

The total number of all possible elementary outcomes in a random experiment is known as "*exhaustive events*". In other words, a set is said to be exhaustive, when no other possibilities exists. In tossing a coin, head or tail may turn up. Getting head and tail are two exhaustive cases of a coin tossing.

Favourable Events:

The elementary outcomes which entail or favour the happening of an event is known as "*favourable events*" i.e., the outcomes which help in the occurrence of that event. In throwing a die, the cases favourable to "get an even number" are 2, 4, 6.

Equally likely or Equi-probable Events:

Outcomes are said to be '*equally likely*' if there is no reason to expect one outcome to occur in preference to another.

i.e. among all exhaustive outcomes, each of them has equal chance of occurrence.

For example: Head (H) and tail (T) are two equally likely cases of a fair coin tossing

Complementary Events:

Let E denote occurrence of event. The complement of E denotes the non occurrence of event E. Complement of E is denoted by $\bar{E}$.

PROBABILITY

Mutually Exclusive Events:

Events are said to be "*mutually exclusive*" if the occurrence of an event totally prevents occurrence of all other events in a trial. In other words, two events A and B cannot occur simultaneously. In coin tossing experiment, head and tail are two mutually exclusive events.

Independent Events:

Two or more events are said to be 'independent', in a series of a trials if the outcome of one event is does not affect the outcome of the other event or vise versa.

For example: If a coin and a die are thrown then the occurrence of the head up in the coin will not effect the getting 4 on the die.

In other words, several events are said to be **"dependents"** if the occurrence of an event is affected by the occurrence of any number of remaining events, in a series of trials.

DIFFERENT APPROACHES OF PROBABILITY

1. CALSSICAL OR MATHEMATICAL APPROACH

In this approach we assume that an experiment or trial results in any one of many possible outcomes, each outcome being Equi-probable or equally-likely.

Definition: If a trial results in "n" exhaustive, mutually exclusive, equally likely and independent outcomes, and if "m" of them are favourable for the happening of the event E, then the probability "P" of occurrence of the event "E" is given by

$$P(E) = \frac{\text{Number of outcomes favourable to event E}}{\text{Exhaustive number of outcomes}} = \frac{m}{n}$$

Example: If a pair of fair dice is rolled,

- What is the probability of rolling a sum of two?
- What is the probability of rolling a sum of seven?
- What is the probability of rolling a sum of four?

Solution:

$$P(B) = P(roll\ a\ seven) = \frac{6}{36} = \frac{1}{6}$$

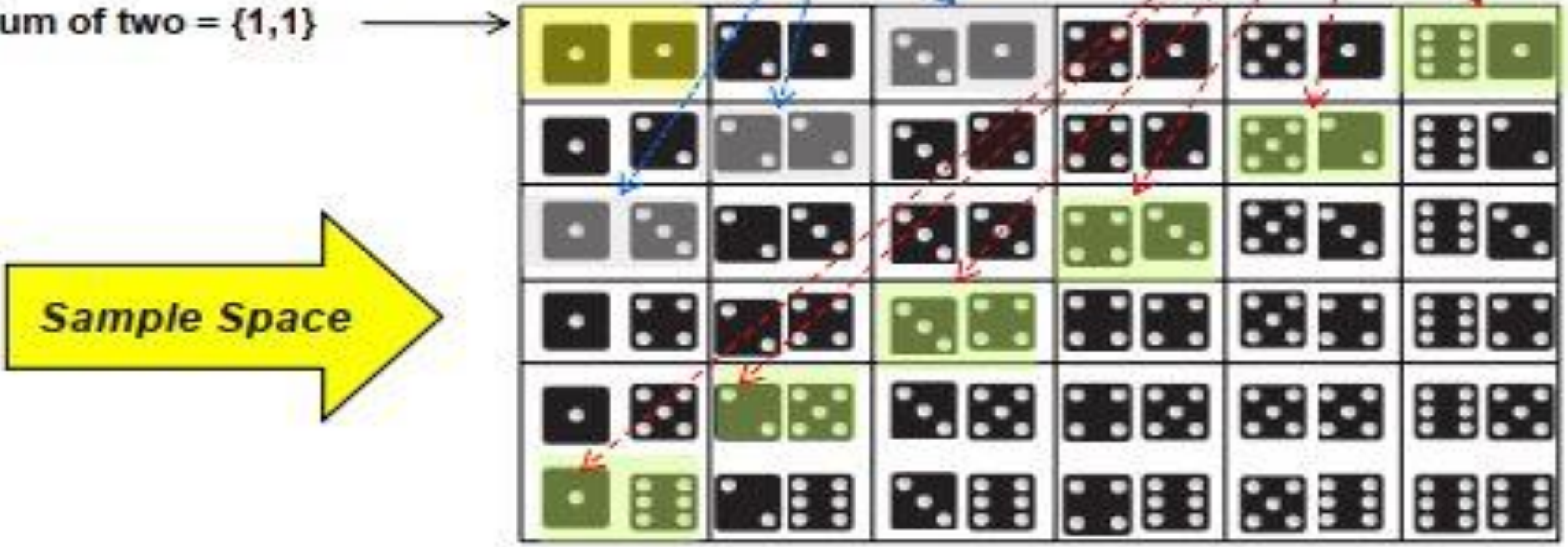
Sum of seven = {(6,1), (1,6), (5,2), (2,5), (4,3), (3,4)}

$$P(C) = P(roll\ a\ four) = \frac{3}{36} = \frac{1}{12}$$

Sum of four = {(1,3), (3,1), (2,2)}

$$P(A) = P(roll\ a\ two) = \frac{1}{36}$$

Sum of two = {1,1}



2. EMPIRICAL OR STATISTICAL APPROACH

This approach is also called the “frequency” approach to probability. Here the probability is obtained by actually performing the experiment large number of times. As the number of trials n increases, we get more accurate result.

Definition: Consider a random experiment which is repeated large number of times under essentially homogeneous and identical conditions. If “n” denotes the number of trials and “m” denotes the number of times an event A has occurred, then, probability of event A is the

limiting value of the relative frequency $\frac{m}{n}$.

DIFFERENT APPROACHES OF PROBABILITY

Example 1: A college basketball coach is trying to form a team from 190 players each is equally qualified to play. 70 players are freshmen, 60 are sophomore, and 60 are senior. If he picks a player randomly:

- What is the chance that the player will be freshman?
- What is the chance that the player will be sophomore?
- What is the chance that the player will be senior?

Solution:

Student Level	Number of Students (f_i)
Freshman Sophomore Senior	70 60 60
Total	$\sum f_i = 190$

$$P(\text{freshman}) = \frac{70}{190}$$

$$P(\text{sophomore}) = \frac{60}{190}$$

$$P(\text{senior}) = \frac{60}{190}$$

DIFFERENT APPROACHES OF PROBABILITY

Example 2: In a recent CNN poll, 2000 people were selected to ask whether second-hand smoke is harmful. 1450 said second-hand smoke is harmful, and 300 said it is not harmful, and the remainder had no opinion. Based on the results of this survey.

- What is the probability that a randomly selected adult American believes that secondhand smoke is harmful?
- What is the probability that a randomly selected adult American believes that secondhand smoke is not harmful?
- What is the probability that a randomly selected adult American will have no opinion about secondhand smoke?

Solution:

Adult opinion	Number of students (f_i)	P(event i)
Secondhand smoke is harmful	1450	**1450/2000= 0.725**
Secondhand smoke is not harmful	300	**300/2000= 0.15**
No opinion	250	**250/2000= 0.125**
Total	$\Sigma f_i = 2000$	$\Sigma p_i = 1$

DIFFERENT APPROACHES OF PROBABILITY

AXIOMETIC APPROACH

This approach was proposed by Russian Mathematician A. N. Kolmogorov in1933. “Axioms” are statements which are reasonably true and are accepted as such, without seeking any proof.

Definition: Let S be the sample space associated with a random experiment. Let E be any event in S. then P(E) is the probability of occurrence of E if the following axioms are satisfied.

1. P(E) > 0, where E is any event.

2. The sum of the probability of all the events in the sample space is 1.

Let S = $\{E_1, E_2, \ldots, E_n\}$

then P(S) = $\sum_{i=1}^{n} P(E_i)$ =1.

3. Let $E_1, E_2, \ldots, E_n$ are n disjoint (mutually exclusive) events then,

P(E_1,∪ E_2 ∪ … ∪ E_n) = $\sum_{i=1}^{n} P(E_i)$

SOME FACTS OF PROBABILITY

i. The probability of an event is between 0 and 1. A probability of 1 is equivalent to 100% certainty. Probabilities can be expressed at fractions, decimals, or percent.

$$0 \leq P(A) \leq 1$$

ii. The sum of the probability of an event occurring and it not occurring is 1.

$$P(A) + P(not\ A) = 1 \text{ or } P(not\ A) = 1 - P(A)$$

Example: A bag contains 5 red marbles, 3 blue marbles, and 2 green marbles.

$P(red) + P(not\ red) = 1$

i.e. $\frac{3}{10} + \frac{7}{10} = 1$

iii. If event *B* is a subset of event *A*, then the probability of B is less than or equal to the probability of A. $pr(B) \leq pr(A)$.

Example: There are 20 people in the room: 12 girls (5 with blond hair and 7 with brown hair) and 8 boys (4 with blond hair and 4 with brown hair).

$P(girl\ with\ brown\ hair) \leq pr(girl) \Longrightarrow \frac{7}{20} \leq \frac{12}{20}$

SOME FACTS OF PROBABILITY

iv. ADDITIVE RULE:

If two events *A* and *B* are mutually exclusive (meaning *A* cannot occur at the same time as *B* occurs), then the probability of either *A* or *B* occurring is the sum of their individual probabilities.

$P(A \text{ or } B) = P(A) + P(B)$

Example: A bag contains 5 red marbles, 3 blue marbles, and 2 green marbles.

$$\mathrm{P}(red \text{ or } green) = P(red) + P(green) = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

SOME FACTS OF PROBABILITY

v. **ADDITIVE RULE:**

If two events *A* and *B* are not mutually exclusive (meaning it is possible that *A* and *B* occur at the same time), then the probability of either *A* or *B* occurring is the sum of their individual probabilities minus the probability of both *A* and *B* occurring.

P(A or B) = p(A) + P(B) – P(A and B)

Example: There are 20 people in the room: 12 girls (5 with blond hair and 7 with brown hair) and 8 boys (4 with blond hair and 4 with brown hair). There are a total of 9 blonds and 11 with brown hair. One person from the group is chosen randomly.

P(girl or blond) = P(girl) + P(blond) – P(girl and blond)

$$\frac{12}{20}+\frac{9}{20}-\frac{5}{20}=\frac{16}{20}$$

SOME FACTS OF PROBABILITY

vi. **MULTIPLICATICE RULE:**

If two events A and B are independent (this means that the occurrence of A has no impact at all on whether B occurs and vice versa), then the probability of A and B occurring is the product of their individual probabilities.

$$\boldsymbol{P(A \text{ and } B) = P(A) \cdot P(B)}$$

Example: Roll a die and flip a coin.

$$P(\text{heads and roll a } 3) = P(H) \text{ and } P(3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

PRACTICE PROBLEMS

1. A bag contains 5 white and 6 black balls. One ball is selected at random. What is the probability that it is white?
2. Two fair coins are drawn at random. What is the probability of getting
   - 2 heads
   - no heads
   - one head and one tail
   - at least one head
   - at most one tail
3. A card is drawn from a pack of 52 cards at random what is the probability that it is

   i. Black ii. Heart iii. Face card iv. An ace v. red queed vi. Knave of heart vii. King or queen viii. Spade or heart ix. A red 2 or black 7 x. club or ace xi. Knave or face card xii. Non-face card.

CONDITIONAL PORBABILITY

CONDITIONAL PROBABILITY

The probability of an event from a *conditional* distribution, we write *P*(**B**|**A**) and pronounce it "the probability of **B** *given* **A**."

- A probability that takes into account a given condition is called a conditional probability.
- Let A and B be two dependent events. Then the probability of occurrence of event A when it is given that the event B has already occurred is known as the conditional probability. It is denoted by P(A|B).
- To find the probability of the event **B** *given* the event **A**, we restrict our attention to the outcomes in **A**. We then find in what fraction of *those* outcomes **B** also occurred.

$$P(B|A) = \frac{P(A \cap B)}{P(A)},$$ *P*(**A**) cannot equal 0, since we know that **A** has occurred.

CONDITIONAL PORBABILITY

EXAMPLE:

In an examination, 60% of the students have passed in Quantitative Method, 40% of the students have passed in Mathematics and 20% have passed in both subjects. A student is selected at random. What is the probability that the student has passed in QM if it is known that he has passed in Mathematics.

Solution:

Let A and B be the events that a student selected at random pass in QM and Mathematics respectively.

Then, P(A) = 60/100 = 0.6

P(B) = 40/100 = 0.4

P(A and B) = 20/100 = 0.2

The required probability the student selected has passed in QM given that he has passed in Mathematics is

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = 0.5$$

CONDITIONAL PORBABILITY

BAYES'S THEOREM

If $E_1, E_2, \dots, E_n$ be the set of n mutually exclusive and exhaustive events whose union is the random sample space S of an experiment and A be any arbitrary event of the sample space of the above experiment with P(A) ≠ 0, then the probability of an event E_i when the even A has already occurred, denoted by P($E_i|A$) and defined by

$$P(E_i|A) = \frac{P(E_i).P(A|E_i)}{P(E_1).P(A|E_1)+P(E_2).P(A|E_2)+\cdots+P(E_n).P(A|E_n)}$$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