

Course Code: BIT 114

Course Name: Discrete Math

Program: BIT FALL 2019

Semester: 1 Year/ I semester



COLLEGE OF MANAGEMENT & INFORMATION TECHNOLOGY

BACHELOR IN INFORMATION TECHNOLOGY

ASSIGNMENT

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Year/ Semester: First, Fall 2019

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Date: 2020.06.27

1. Define contradiction and tautology. Show that the compound proposition $[p \wedge (p \rightarrow q)] \rightarrow q$ is tautology.

- ❖ A compound proposition is said to be contradiction if its truth value is always false. For eg:

P	$\sim P$	$P \wedge \sim P$
T	F	F
F	T	F

- ❖ A compound proposition is said to be tautology if its truth value is always true. For eg:

P	$\sim P$	$P \vee \sim P$
T	F	T
F	T	T

- ❖ Solution:

P	q	$p \rightarrow q$	$P \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Here, the above table shows that compound proposition $[p \wedge (p \rightarrow q)] \rightarrow q$ is tautology.

1. Without expanding show that

$$\begin{array}{|c|} \hline 1 \quad 1 \quad 1 \\ \hline a \quad b \quad c \quad = \\ (a-b)(b-c)(c-a) \quad bc \\ ca \quad ab \\ \hline \end{array}$$

Solution: Given,

Taking L.H.S

$$\begin{vmatrix} 1 & 1 & 1 & a & b \\ c & bc & ca & ab \end{vmatrix}$$

First step : $R_1 \rightarrow R_1 - R_2$

$$\begin{vmatrix} 1-1 & 1 & 1 & a-b & b \\ c & bc-ca & ca & ab \end{vmatrix}$$

taking (a-b) common in first row R_1 second step:

$R_2 \rightarrow R_2 - R_1$

$$= (a-b) \begin{vmatrix} 0 & 1-1 & 1 \\ 1 & b-c & c \\ -c & ca-ab & ab \end{vmatrix}$$

Taking (b-c) common in second row R_2

$$= \begin{vmatrix} 1 & 1 & c \\ 0 & 0 & 1 \\ -c & -a & ab \end{vmatrix} (a-b)(b-c)$$

Then,

$$\text{Here } = 1 \begin{vmatrix} 1 & 1 \\ -c & -a \end{vmatrix}$$

$$= 1 \times (-a) - (-c) \times 1$$

$$= (-a + c)$$

Hence, we have proved that $(a-b)(b-c)(c-a)$

L.H.S=R.H.S proved

2. Define homogeneous recurrence relation. Solve the recurrence relation.

$$a_n = 6a_{n-1} + 8a_{n-2} \text{ for } n \geq 2, a_0 = 4, a_1 = 10$$

❖ A homogeneous recurrence relation is said to be if the order of k of the form can be found as $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$

❖ Solution:

Given recurrence relation is $a_n = 6a_{n-1} + 8a_{n-2}$(i) This is the recurrence relation of second order. Its character equation is ; Or,

$$r^2 = 6r + 8$$

$$\text{Or, } r^2 - 6r - 8 = 0$$

$$\text{Or, } r(r-4) - 2(r-4) = 0$$

$$\text{Or, } (r-4)(r-2) = 0$$

Hence, $r_1 = 2, r_2 = 4$ The solution of

$$\text{eqn(i) is, } a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$$

$$a_n = \alpha_1 2^n + \alpha_2 4^n \dots\dots\dots\text{(ii)}$$

Put $n=0$ in eqn(ii), we get

$$a_0 = \alpha_1 2^0 + \alpha_2 4^0 \text{ Given } a_0 = 4, \text{ then}$$

$$\text{Or, } 4 = \alpha_1 + \alpha_2$$

Or, $\alpha_2 = 4 - \alpha_1$ again, put
 $n=1$

Or, $a_1 = \alpha_1 2 + \alpha_2 4$ Given $a_1 = 10$

Or, $10 = \alpha_1 2 + (4 - \alpha_1) 4$ Or,
 $5 = \alpha_1 + 8 - 2\alpha_1$

Or, $\alpha_1 = 3, \alpha_2 = 1$

Here, the equation is found to be $a_n = 3 \times 2^n + 1 \times 4^n$

3. Define the term tree, minimum spanning tree, simple, multiple and pseudo graph. Write prim's and kruskal's algorithm to construct minimum spanning tree. Explain the process of generating spanning tree using kruskal's algorithm? Assume example of your own.

❖ Tree: A connected undirected graph with no simple circuit is called tree .for a tree a graph must be simple graph.

❖ Minimum spanning tree: A spanning tree of n connected graph in which the sum of weight of it's edge in minimal is called minimum spanning tree.

❖ Simple graph:

❖ Multiple graph: ➔ Pseudo graph:

❖ Prim's algorithm:

Procedure: G: weighted connected undirected graph with n vertices.

$T :=$ a minimum weight edge.

For $P := 1$ to $n-2$. Begin $e :=$ an edge of minimum weight incident to a vertex in T and not forming a simple circuit in T if added to T .

$T := T$ with e added.

End{ T is a minimum spanning tree of G . }

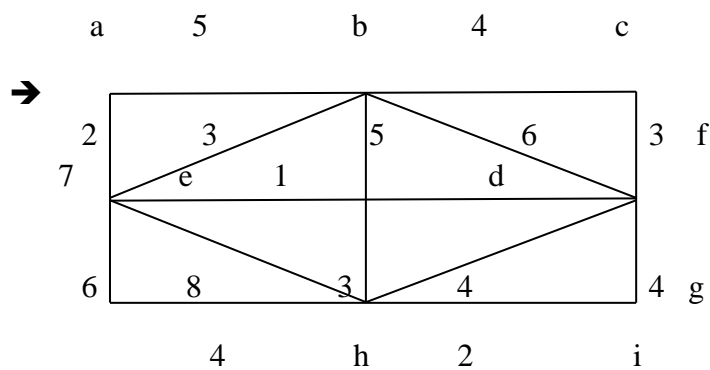
Kruskal's algorithm: Procedure:

Kruskal(G : a connected undirected weighted graph with n vertices.) T : = empty graph.

For : i : 1 to $n-1$.

Begin e := any edge in G with smallest weight and forming a simple circuit if added to T . T := tree with e added.

End{ T is minimum spanning tree of G .}



Solution:

First of all we list edges in ascending order.

(e,d)=1

(h,i)=2

(a,f)=2

(b,f)=3

(e,h)=3

(c,d)=3

(b,c)=4

(f,h)=8

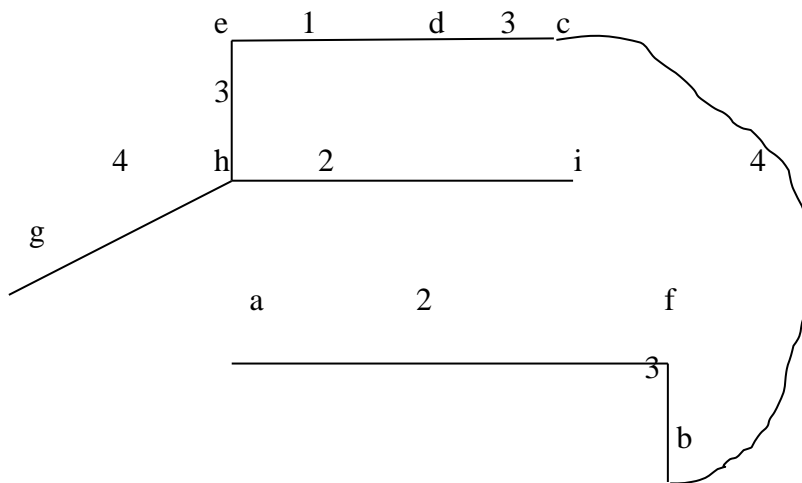
$$e \quad \quad \quad 1 \quad \quad \quad d$$
$$\begin{array}{ccccccc} e & & 1 & & d & & \\ \hline h & & 2 & & i & & a & & 2 & & f \\ \hline \end{array}$$

e 1 d 3 c b

h 2 i a 2 f

3 3

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here in step 4 the circuit is formed so they are excluded (d,h),(d,i) step 5

: adding (b,e),(a,b) having weight 5 on T. here (a,b),(b,e) is excluded

because it makes circuit

(f,g),(b,d),(e,f),(f,h) are also excluded

Therefore, minimum spanning tree is found to be 22 having weighted.