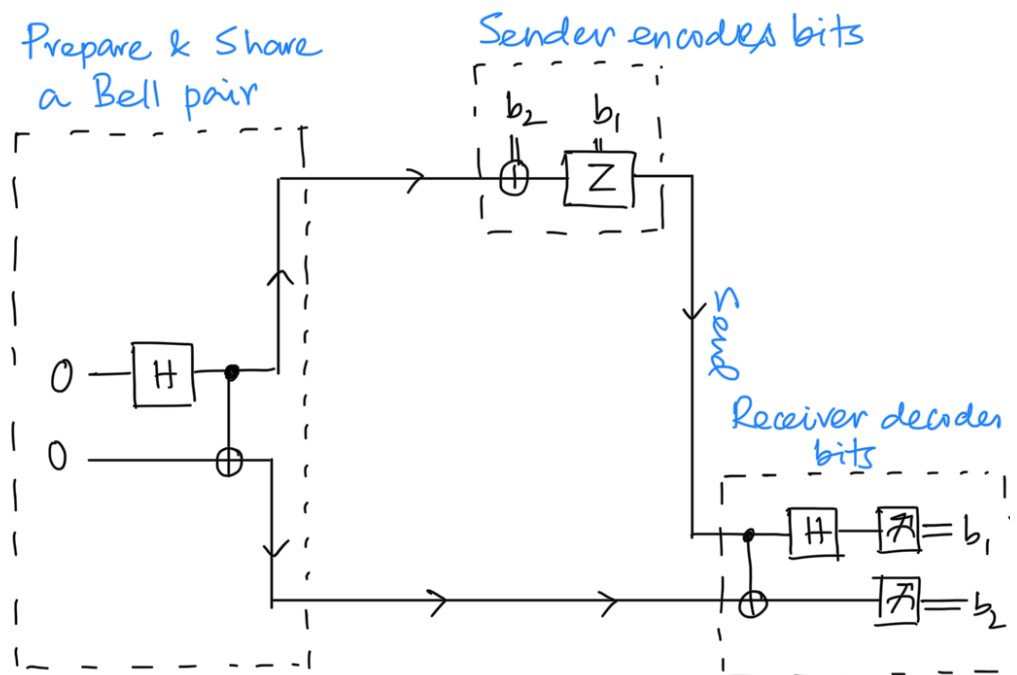


③ Make a presentation on dense coding.

Ans:



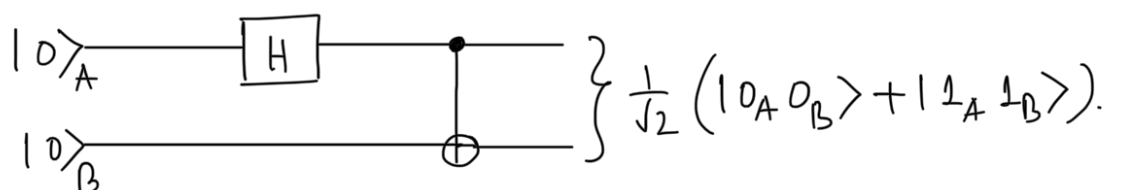
- Superdense coding, also referred to as dense coding is a quantum communication protocol to communicate a number of classical bits of information by only transmitting a smaller number of qubits, under the assumption of sender & receiver pre-sharing an entangled resource.
- In this protocol it involves two parties, Alice & Bob, which share a pair of maximally entangled qubits & allows Alice to transmit two bits (i.e. one of 00, 01, 10 or 11) to Bob by sending only one qubit.

Overview :

- Suppose Alice wants to send two classical bit of information (00, 01, 10 or 11) to Bob using qubits (instead of classical bits).
- To do this, an Bell state is prepared using Bell circuit & then sends one of this qubit to Alice & other to Bob.
- Once Alice obtains her qubit state she applies a certain quantum gate to her qubit depending on which two-bit message (00, 01, 10 or 11) she wants to send to Bob.
- Her entangled qubit is then sent to Bob who, after applying the appropriate quantum gate & making a measurement, can retrieve the classical two-bit message.

Protocol :

Preparation: The protocol starts with the preparation of an entangled state, which is later shared between Alice & Bob.


$$\left. \begin{array}{c} |0\rangle_A \text{---} \boxed{H} \text{---} \bullet \\ |0\rangle_B \text{---} \oplus \end{array} \right\} \frac{1}{\sqrt{2}} (|0_A 0_B\rangle + |1_A 1_B\rangle).$$

By the Bell circuit the Bell state

$$|+\rangle = \frac{1}{\sqrt{2}} (|0_A 0_B\rangle + |1_A 1_B\rangle) \text{ is prepared}$$

Sharing: After the preparation of $|+\rangle$, the qubit

denoted by subscript A is sent to Alice & the qubit denoted by subscript B is sent to Bob.

Encoding: By applying a quantum gate Alice can transform the entangled state $|+\rangle$ into any of the four Bell states (including $|+\rangle$).

1. If Alice wants to send two classical bit 00 to Bob, then she applies $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ gate to her qubit & she have

$$|B_{00}\rangle = \frac{1}{\sqrt{2}} (|0_A 0_B\rangle + |1_A 1_B\rangle)$$

2. To send classical two bit 01 she applies $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ gate to her qubit & she have

$$|B_{01}\rangle = \frac{1}{\sqrt{2}} (|1_A 0_B\rangle + |0_A 1_B\rangle).$$

3. To send the classical bit 10 she applies

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ quantum gate to her qubit}$$

& she have

$$|B_{10}\rangle = \frac{1}{\sqrt{2}} (|0_A 0_B\rangle - |1_A 1_B\rangle)$$

4. If Alice wants to send classical bit 11, she applies, $Z * X = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

quantum gate to her qubit & she have

$$|B_{11}\rangle = \frac{1}{\sqrt{2}} (|0_A 1_B\rangle - |1_A 0_B\rangle).$$

Sending: After having performed one of the operation in the Encoding stage, Alice now send her entangled qubit to Bob. i.e. Alice will send one of $\{B_{00}, B_{01}, B_{10}, B_{11}\}$ as she wants to send 00, 01, 10 or 11 respectively.

Decoding: After receiving the quantum entangled state B_{00}, B_{01}, B_{10} or B_{11} , Bob will first apply CNOT gate with A as control qubit & B as target qubit & then $H \otimes I$ unitary operation on the entangled qubit A.

1. If B_{00} is received after applying above unitary operation Bob will get $|00\rangle$.

$$|B_{00}\rangle = \frac{1}{\sqrt{2}} (|0_A 0_B\rangle + |1_A 1_B\rangle).$$

After CNOT : $\frac{1}{\sqrt{2}} (|0_A 0_B\rangle + |1_A 0_B\rangle).$

After $H \otimes I$:

$$\begin{aligned} & \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} (|0\rangle_A + |1\rangle_A) |0\rangle_B + \frac{1}{\sqrt{2}} (|0\rangle_A - |1\rangle_A) |0\rangle_B \right) \\ &= \frac{1}{2} \left[|0_A 0_B\rangle + |1_A 0_B\rangle + |0_A 0_B\rangle - |1_A 0_B\rangle \right] \\ &= |0_A 0_B\rangle. \end{aligned}$$

Then Bob knows Alice wants to send 00 classical bit

2. If B_{01} is received after applying above unitary operation Bob will get $|01\rangle$.

$$|B_{01}\rangle = \frac{1}{\sqrt{2}} (|0_A 1_B\rangle + |1_A 0_B\rangle).$$

After CNOT : $\frac{1}{\sqrt{2}} (|0_A 1_B\rangle + |1_A 1_B\rangle).$

After $H \otimes I$:

$$\begin{aligned} & \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} (|0\rangle_A + |1\rangle_A) |1\rangle_B + \frac{1}{\sqrt{2}} (|0\rangle_A - |1\rangle_A) |1\rangle_B \right) \\ &= \frac{1}{2} \left[|0_A 1_B\rangle + |1_A 1_B\rangle + |0_A 1_B\rangle - |1_A 1_B\rangle \right] \\ &= |0_A 1_B\rangle. \end{aligned}$$

Then Bob knows Alice wants to send 01 classical bit

3. If B_{10} is received after applying above unitary operation Bob will get $|10\rangle$.

$$|B_{10}\rangle = \frac{1}{\sqrt{2}} (|0_A 0_B\rangle - |1_A 1_B\rangle).$$

After CNOT : $\frac{1}{\sqrt{2}} (|0_A 0_B\rangle - |1_A 0_B\rangle).$

After $H \otimes I$:

$$\begin{aligned} & \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} (|0\rangle_A + |1\rangle_A) |0\rangle_B - \frac{1}{\sqrt{2}} (|0\rangle_A - |1\rangle_A) |0\rangle_B \right) \\ &= \frac{1}{2} \left[|0_A 0_B\rangle + |1_A 0_B\rangle - |0_A 0_B\rangle + |1_A 0_B\rangle \right] \\ &= |1_A 0_B\rangle. \end{aligned}$$

Then Bob knows Alice wants to send 10 classical bit

4. If B_{11} is received after applying above unitary operation Bob will get $|11\rangle$.

$$|B_{11}\rangle = \frac{1}{\sqrt{2}} (|0_A 1_B\rangle - |1_A 0_B\rangle).$$

After CNOT : $\frac{1}{\sqrt{2}} (|0_A 1_B\rangle + |1_A 1_B\rangle)$

After $H \otimes I$:

$$\begin{aligned} & \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} (|0\rangle_A + |1\rangle_A) |1\rangle_B + \frac{1}{\sqrt{2}} (|0\rangle_A - |1\rangle_A) |1\rangle_B \right) \\ &= \frac{1}{2} \left[|0_A 1_B\rangle + |1_A 1_B\rangle + |0_A 1_B\rangle - |1_A 1_B\rangle \right] \\ &= |0_A 1_B\rangle. \end{aligned}$$

Then Bob knows Alice wants to send 01 classical bit