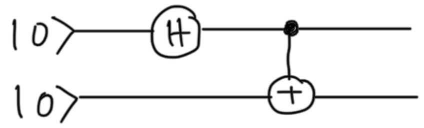
① 

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

What is the matrix for this circuit?

<u>Ans</u>: First of all $H \otimes I$ is operated on $|00\rangle$ state and after that CNOT gate is operated on the previous resulting state.

So, the matrix for the above circuit is the matrix multiplication of CNOT and $H \otimes I$.

Now, $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

$$H \otimes I = \begin{bmatrix} \frac{1}{\sqrt{2}}\cdot 1 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \frac{1}{\sqrt{2}}\cdot 1 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \frac{1}{\sqrt{2}}\cdot 1 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \frac{1}{\sqrt{2}}\cdot (-1) \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}.$$

We know matrix of CNOT,

$$M_{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

So, the matrix for the above circuit

$$M = M_{CNOT} \cdot (H \otimes I)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}.$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

<u>Verification</u>:

$$M|00\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

$$= \frac{1}{\sqrt{2}} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}. = \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right).$$

So, $M|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

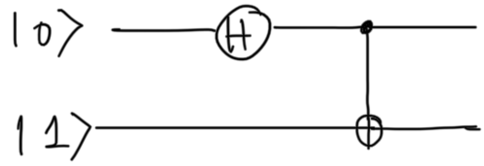
Hence M is the required matrix.

② Generate the qubit $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Ans:

Here we will start from the state $|01\rangle$ & we will apply same circuit as above to get the state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Now,



$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Firstly $\quad H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$., $I|1\rangle = |1\rangle$.

Now $\quad (H|0\rangle \otimes I|1\rangle)$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle.$$

$$= \frac{1}{\sqrt{2}}\left[|01\rangle + |11\rangle\right].$$

Now,

$$|01\rangle \xrightarrow{CNOT} |01\rangle.$$

$$|11\rangle \xrightarrow{CNOT} |10\rangle.$$

So, when apply CNOT on $\frac{1}{\sqrt{2}}[|01\rangle + |11\rangle]$

we will get $\frac{1}{\sqrt{2}}[|01\rangle + |10\rangle]$.

So, $\frac{1}{\sqrt{2}}[|01\rangle + |10\rangle]$ can be generated when we apply the above circuit on the state $|01\rangle$.

The matrix for the transformation will be same as question ① i.e.

$$M = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

③ Find M' such that $|00\rangle \text{ --- } \boxed{M'} \text{ --- } \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ where $M' \neq M$, M' is unitary.

$$M' = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

Now, $M'|00\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

$$= \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle \right)$$

Now, $(M')^* = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

$$M' \cdot (M')^* = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} . \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_4.$$

$$(M')^* \cdot M' = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_4.$$

So, M' is a unitary matrix. & also $M' \neq M$.