Super dense coding

Suppose Alice wished to transmit two classical bit to Bob by using only a single qubit.

Now Alice is to do that using a quantum state to send this information. In quantum teleportation she had sent classical information through a classical channel and a quantum state was prepared.

Now the purpose is essentially to send a quantum bit to Bob and enable Bob to have classical information.

The difference is also is that she will be sending one bit of quantum information and bob should be able to extract two bits of classical information and hence the name dense coding.

In this process, Alice and Bob initially share a pair of qubits in the entangled state.

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right).$$

Alice is initially in possession of the 1st qubit, while Bob has possession of the 2nd qubit.

If Alice ~~whic~~ wishes to send the bit string '00' to Bob, then she do nothing at all to her qubit. If she whiches to send '01' then she applies ~~the phase flip Z to her qubit~~ the X to her qubit

If she wants to send '10' then she applies iY gate to her qubit.

If she ~~whi~~ wishes to send '11' then she applies the Z ~~iY~~ gate to her qubit.

- Alice and Bob start with Bell pair.

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Cbits	Alice' action
00	$I : |\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
01	$X \otimes I : |\beta_{01}\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$
10	$iY \otimes I : |\beta_{10}\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$
11	$Z \otimes I : |\beta_{11}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

Now Bob will apply CNOT Gate after Alice's action then, effect of Bob's CNOT is →

$$00 : \frac{|00\rangle + |11\rangle}{\sqrt{2}} \xrightarrow{\text{CNOT (Bob)}} \frac{|00\rangle + |10\rangle}{\sqrt{2}} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle$$

$$01 : \frac{|10\rangle + |01\rangle}{\sqrt{2}} \xrightarrow{\text{CNOT(Bob)}} \frac{|11\rangle + |01\rangle}{\sqrt{2}} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |1\rangle$$

$$10 : \frac{-|10\rangle + |01\rangle}{\sqrt{2}} \xrightarrow{\text{CNOT}} \frac{-|11\rangle + |01\rangle}{\sqrt{2}} = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \otimes |1\rangle$$

$$11 : \frac{|00\rangle - |11\rangle}{\sqrt{2}} \xrightarrow{\text{CNOT}} \frac{|00\rangle - |10\rangle}{\sqrt{2}} = \left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \otimes |0\rangle$$

Now we see that if the 2^{nd} Qubit is 0 Alice has sent 00 or 11.

If 2^{nd} Qubit is $|1\rangle$. Then Alice has sent 01 or 10

So this is the intimation that we get from measurement of 2nd qubit. But here 2nd qubit is collapsed, but the 1st qubit is still in a linear combination of states corresponding to the situation where the 2nd qubit is 0 in the first case or 2nd qubit it is 1 in the 2nd case.

Now Bob apply H-gate to 1st qubit, to make a measurement of the 1st qubit. If the 1st qubit is 0, Alice must have ~~sd~~ sent 00 or 01. $\left[H\left(\frac{|0\rangle+|1\rangle}{\sqrt{2}}\right) = |0\rangle\right]$

If 1st qubit 1, Alice must have sent 10 or 11. $\left[H\left(\frac{|0\rangle-|1\rangle}{\sqrt{2}}\right) = |1\rangle\right]$.

So by measuring 1st and 2nd qubit ~~we can~~ at a instance we can conform ~~what~~ what are the desired bit.

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