NO CLONING

Let $|\psi_1\rangle, |\psi_2\rangle$ are two non orthogonal state then is no quantum copying machine $\exists$ it can make perfect copy.

$\Rightarrow$ Let U be the circuit by which we can copy, for two non-orthogonal state $|\psi_1\rangle$ and $|\psi_2\rangle$. and $|x\rangle$ is normalized.

Then, we get that,

$$U|\psi_1\rangle|x\rangle = e^{i\alpha}|\psi_1\rangle|\psi_1\rangle \quad \text{—— (1)}$$

$$U|\psi_2\rangle|x\rangle = e^{-\beta}|\psi_2\rangle|\psi_2\rangle. \quad \text{—— (2)}$$

Here, α, β does not effect the physical interpetation of the final state.

Now, (2) $\Rightarrow$

$$\langle\psi_2|\langle x|U^{\dagger} = e^{-i\beta}\langle\psi_2|\langle\psi_2| \quad \text{—— (3)}$$

(1) & (3) $\Rightarrow$

$$\langle\psi_2|\langle x|U^{\dagger}U|\psi_1\rangle|x\rangle = e^{i(\alpha-\beta)}\langle\psi_2|\langle\psi_2||\psi_1\rangle|\psi_1\rangle$$

$$\Rightarrow \langle\psi_2|\langle x||\psi_1\rangle|x\rangle = e^{-i(\alpha-\beta)}\langle\psi_2|\langle\psi_2||\psi_1\rangle|\psi_1\rangle.$$

$$\Rightarrow |\langle\psi_2|\psi_1\rangle\cdot\langle x|x\rangle| = |e^{-i(\alpha-\beta)}|\,|\langle\psi_2|\psi_1\rangle\cdot\langle\psi_2|\psi_1\rangle|$$

$$\Rightarrow |\langle\psi_2|\psi_1\rangle| = |\langle\psi_2|\psi_1\rangle|^2.$$

Since, x is normalized state, $|x^2| = |x|^2$.

$\therefore$ This gives that $\langle\psi_2|\psi_1\rangle = 0$ or 1.

(i) $\langle\psi_2|\psi_1\rangle = 0 \Rightarrow |\psi_1\rangle, |\psi_2\rangle$ are orthogonal

(ii) $\langle\psi_2|\psi_1\rangle = 1 \Rightarrow |\psi_1\rangle, |\psi_2\rangle$ are identical, only phase factor are different.