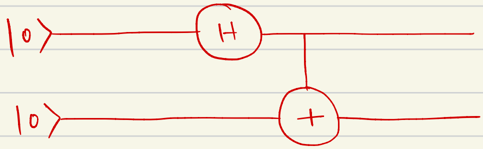
Assignment 5

Soham Sanjay Zemse
CRS2012

Q1 What is the matrix for this circuit?



$\frac{1}{\sqrt{2}}\left[\,|00\rangle + |11\rangle\,\right]$

First of all, the $H \otimes I$ is operated on $|00\rangle$ state and after that CNOT gate is operated on the previous resulting state.

So, the matrix for the above circuit in the matrix multiplication of CNOT and $H \otimes I$

Now, $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$H \otimes I = \begin{bmatrix} \frac{1}{\sqrt{2}} \cdot 1 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \frac{1}{\sqrt{2}} \cdot 1 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \frac{1}{\sqrt{2}} \cdot 1 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \frac{1}{\sqrt{2}} (-1) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

We know

$$M_{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

So, the matrix for the above circuit

$$M = M_{CNOT} \cdot (H \otimes I)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

Verification:

$$M|00\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$= \frac{1}{\sqrt{2}} \left[|00\rangle + |11\rangle \right]$$

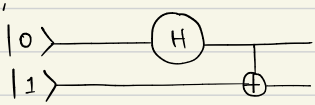
So, $M|00\rangle = \frac{1}{\sqrt{2}} \left[|00\rangle + |11\rangle \right]$

Hence M is the required Matrix

Q2 Generate the qubit $1/\sqrt{2} \left[|01\rangle + |10\rangle \right]$

We can start from state $|01\rangle$ and apply same circuit as above to get the state $1/\sqrt{2} \left[|01\rangle + |10\rangle \right]$

Now,



$1/\sqrt{2} \left[|01\rangle + |10\rangle \right]$

Firstly, $H|0\rangle = 1/\sqrt{2} \left[|0\rangle + |1\rangle \right]$, $I|1\rangle = |1\rangle$

Now $\left[H|0\rangle \otimes I|1\rangle \right]$

$$= \frac{1}{\sqrt{2}} \left[|0\rangle + |1\rangle \right] \otimes |1\rangle$$

$$= \frac{1}{\sqrt{2}} \left[|01\rangle + |11\rangle \right]$$

Now, $|01\rangle \xrightarrow{CNOT} |01\rangle$

$|11\rangle \xrightarrow{CNOT} |10\rangle$

So, when apply CNOT on $\frac{1}{\sqrt{2}}\left[|01\rangle + |11\rangle\right]$

We will get $\frac{1}{\sqrt{2}}\left[|01\rangle + |10\rangle\right]$

So, $\frac{1}{\sqrt{2}}\left[|01\rangle + |10\rangle\right]$ can be generated when we apply the above circuit on the state $|01\rangle$.

The matrix for the transformation will be same as question ① i.e.

$$M = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

Q3 Find M' such that $|00\rangle \xrightarrow{\quad M' \quad} \frac{1}{\sqrt{2}}\left[|00\rangle + |11\rangle\right]$
where $M' \neq M$, M' is unitary.

$$M' = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$M'|00\rangle = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{bmatrix} = 1/\sqrt{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 1/\sqrt{2} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$= 1/\sqrt{2} \left[|00\rangle + |11\rangle \right]$$

Now, $$(M')^* = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$M' \cdot (M')^* = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= I_4$$

$$(M')^* \cdot M' = \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= I_4$$

So, M' is a unitary matrix & also $M' \neq M$.