1. Given a 2-qubit state

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

it is not possible to find an entanglement of $|\psi\rangle$.

Suppose, $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

where $|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$

$|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$.

$|\alpha_1|^2 + |\beta_1|^2 = |\alpha_2|^2 + |\beta_2|^2 = 1$.

$\alpha_1, \alpha_2, \beta_1, \beta_2 \in \mathbb{C}$.

$|\psi_1\rangle \otimes |\psi_2\rangle$

$= \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$

$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

$\Rightarrow \quad \beta_1\alpha_2 = 0 \quad \& \quad \alpha_1\beta_2 = 0 \Rightarrow$

~~$\beta_1 = 0 \ \& \ \beta_2 = 0$~~

$\alpha_1\alpha_2 = \frac{1}{\sqrt{2}} \quad \& \quad \beta_1\beta_2 = \frac{1}{\sqrt{2}}$

$(\beta_1 = 0 \ \& \ \beta_2 = 0)$ or $(\alpha_2 = 0, \alpha_1 = 0)$

or $(\beta_1 = 0 \ \& \ \alpha_1 = 0)$ or $(\beta_2 = 0 \ \& \ \alpha_2 = 0)$.

all these Cases Contradict

$\alpha_1\alpha_2 = \frac{1}{\sqrt{2}} \ \& \ \beta_1\beta_2 = \frac{1}{\sqrt{2}}$.

Therefore $|\psi\rangle$ Can't be entangled.

2> Find the matrix representation of CCNOT gate & CSWAP gate.

CCNOT Gate:

If 1st & 2nd bits are 1 then only 3rd bit will be flipped.

Truth table.

$|000\rangle \xleftrightarrow{CCNOT} |000\rangle$

$|001\rangle \longrightarrow |001\rangle$

$|010\rangle \longrightarrow |010\rangle$

$|011\rangle \longrightarrow |011\rangle$

$|100\rangle \longrightarrow |100\rangle$

$|101\rangle \longrightarrow |101\rangle$

$|110\rangle \longrightarrow |111\rangle$

$|111\rangle \longrightarrow |110\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

C.SWAP :

1st state controls the swap of the last two bits.

if control bit = 1, then swap.

control bit = 0, then no swap.

$|00\rangle \mapsto |00\rangle$

$|01\rangle \mapsto |01\rangle$

$|10\rangle \mapsto |10\rangle$

$|11\rangle \mapsto |11\rangle$

$|100\rangle \mapsto |100\rangle$

$|101\rangle \mapsto |101\rangle$

$|110\rangle \mapsto |111\rangle$

$|111\rangle \mapsto |110\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$