⟨No - cloning theorem⟩

Here we try to make a copy of an unknown quantum state. It is not possible as. We will show it now.

Let we have a quantum system with two slots S and S'. Here slot S, the data slot, starts out in an unknown but pure quantum state, $|\psi\rangle$. We copy S to S', the target slot. We assume that the target slot starts out in some standard pure state $|s\rangle$.

So, initial state of copying system is

$$|\psi\rangle|s\rangle$$

Now some unitary evolution U now effects the copying procedure, ideally,..

$$|\psi\rangle \otimes |s\rangle \xrightarrow{U} U(|\psi\rangle \otimes |s\rangle) = |\psi\rangle|\psi\rangle$$

If this particular copying procedure works for two particular pure states $|\psi\rangle$ and $|\phi\rangle$, we have

$$U(|\psi\rangle|s\rangle) = |\psi\rangle|\psi\rangle$$
$$U(|\phi\rangle|s\rangle) = |\phi\rangle|\phi\rangle.$$

Now taking inner product we have

$$\langle s|\langle\psi|U^{+}U|\phi\rangle|s\rangle = \langle\psi|\langle\psi|\phi\rangle|\phi\rangle$$

$$\Rightarrow \langle\psi|\phi\rangle\langle s|s\rangle = \langle\psi|\phi\rangle\langle\psi|\phi\rangle$$

$$\Rightarrow \langle\psi|\phi\rangle = \langle\psi|\phi\rangle^2 \qquad [U^{+}U = I]$$

$$\Rightarrow \langle\psi|\phi\rangle = 0 \text{ or } \langle\psi|\phi\rangle = 1$$

$\Rightarrow$ $\boxed{|\psi\rangle = |\phi\rangle \text{ or } |\psi\rangle \text{ and } |\phi\rangle \text{ are orthogonal.}}$

Thus cloning device can only clone states which are orthogonal to one another, so general quantum cloning is not possible