1. Given a 2-qubit state $|\phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Show that there does not exists $|\psi_1\rangle, |\psi_2\rangle$ such that $|\phi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$.

<u>Soln:</u> Let, $|\phi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$,

where, $|\psi_1\rangle = \alpha|0\rangle + \beta|1\rangle$, $|\psi_2\rangle = \gamma|0\rangle + \delta|1\rangle$

So, $|\phi\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes (\gamma|0\rangle + \delta|1\rangle)$

$= \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$

Now,

$\alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$\Rightarrow \alpha\gamma = \beta\delta = \frac{1}{\sqrt{2}}$ and $\alpha\delta = \beta\gamma = 0$

$\alpha\delta = 0 \Rightarrow$ either $\alpha = 0$ or $\delta = 0$

$\beta\gamma = 0 \Rightarrow$ either $\beta = 0$ or $\gamma = 0$

So, each value of $\alpha, \beta, \gamma, \delta$ we arrived at a contradiction that $\alpha\gamma = 0$ or $\beta\delta = 0$.

So, $|\phi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle$. [Proved]

2. Find the matrix representation of CCNOT gate and CCSWAP gate.

<u>Soln:</u>

- <u>CCNOT gate:</u> It is a 3 qubit gate. If first two bit is 1 then the third bit is flips.

<u>Truth Table:</u>

Input			Output		
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

$$M_{CCNOT} = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&1&0 \end{pmatrix}$$

- <u>CSWAP gate:-</u> It is also a 3-qubit gate. Here if the control bit is 1, then 2nd and 3rd bit are swap, otherwise remain same.

<u>Truth Table:-</u>

Input	Output
0 0 0	0 0 0
0 0 1	0 0 1
0 1 0	0 1 0
0 1 1	0 1 1
1 0 0	1 0 0
1 0 1	1 1 0
1 1 0	1 0 1
1 1 1	1 1 1

$$M_{CSWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$