Assignment 3

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Q1 Given a 2 cubit state $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$ — (1)

it is not possible to find $|\psi_1\rangle$ & $|\psi_2\rangle$ such that

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

Let us assume, for some Complex α_1, β_1 and α_2, β_2 that satisfy $|\alpha_1|^2 + |\beta_1|^2 = 1$ and $|\alpha_2|^2 + |\beta_2|^2 = 1$ the following holds:

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$
$$|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$
$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

$$= \left[\alpha_1|0\rangle + \beta_1|1\rangle\right] \otimes \left[\alpha_2|0\rangle + \beta_2|1\rangle\right]$$

$$= \alpha_1\alpha_2\left[|0\rangle \otimes |0\rangle\right] + \alpha_1\beta_2\left[|0\rangle \otimes |1\rangle\right] + \beta_1\alpha_2\left[|1\rangle \otimes |0\rangle\right] + \beta_1\beta_2\left[|1\rangle \otimes |1\rangle\right]$$

$$= \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle \quad \text{— (2)}$$

Comparing (1) & (2) we have

$$\alpha_1\alpha_2 = \frac{1}{\sqrt{2}} \quad \text{— (3)}$$

$$\alpha_1\beta_2 = 0 \quad \text{— (4)}$$

$$\beta_1\alpha_2 = 0 \quad \text{— (5)}$$

$$\beta_1\beta_2 = \frac{1}{\sqrt{2}} \quad \text{— (6)}$$

From (3) $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$

From (4) $\alpha_1 = 0$ or $\beta_2 = 0$ $\Rightarrow$ $\beta_2 = 0$ — (7)

From (5) $\beta_1 = 0$ or $\alpha_2 = 0$ $\Rightarrow$ $\beta_1 = 0$ — (8)

$\Rightarrow$ $\beta_1 \beta_2 = 0$

But in eqn (6) it is $1/\sqrt{2}$.

Hence there cannot be any $\alpha_1, \beta_1, \alpha_2, \beta_2$ and hence no ψ_1 & ψ_2 that satisfy eqn (1).

Q2 Find Matrix representation of CCNOT & CSWAP gate.

i) CCNOT

According to CCNOT, if the 1st & 2nd bits are 1 then third bit will be flipped.

State Transformations:

$$|000\rangle \longrightarrow |000\rangle$$
$$|001\rangle \longrightarrow |001\rangle$$
$$|010\rangle \longrightarrow |010\rangle$$
$$|011\rangle \longrightarrow |011\rangle$$
$$|100\rangle \longrightarrow |100\rangle$$
$$|101\rangle \longrightarrow |101\rangle$$
$$|110\rangle \longrightarrow |111\rangle$$
$$|111\rangle \longrightarrow |110\rangle$$

Hence Matrix Representation or permutation Matrix form is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

ii) CSWAP

According to CSWAP if first bit is 1 then next two bits are flipped.

State Transformations:

$$|000\rangle \rightarrow |000\rangle$$
$$|001\rangle \rightarrow |001\rangle$$
$$|010\rangle \rightarrow |010\rangle$$
$$|011\rangle \rightarrow |011\rangle$$
$$|100\rangle \rightarrow |100\rangle$$
$$|101\rangle \rightarrow |110\rangle$$
$$|110\rangle \rightarrow |101\rangle$$
$$|111\rangle \rightarrow |111\rangle$$

Hence the matrix representation or permutation matrix form is

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$