Assignment – 6.

① Given $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ & $|\psi^*\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$
prove that they are orthogonal & find a unitary matrix B such that it transform $|\psi\rangle$ to $|0\rangle$ & $|\psi^*\rangle$ to $|1\rangle$.

$\Rightarrow$ $|\psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix}$ & $|\psi^*\rangle = \begin{pmatrix}\beta^*\\ -\alpha^*\end{pmatrix}$

Then, $\langle\psi|\psi^*\rangle = (\alpha^* \ \beta^*)\begin{pmatrix}\beta^*\\ -\alpha^*\end{pmatrix}$

$= \alpha^*\beta^* - \beta^*\alpha^*$

$= 0$.

$\Rightarrow$ ~~$|\psi\rangle$ & $|\psi^*\rangle$ are orthogonal.~~

$|\psi\rangle$ & $|\psi^\perp\rangle$ are orthogonal.

first of all we ~~find~~ want to find a matrix A, which satisfies,

$A(|0\rangle) = |\psi\rangle$ & $A(|1\rangle) = |\psi^\perp\rangle$

so, basically, $A = \begin{pmatrix}\alpha & +\beta^*\\ \beta & -\alpha^*\end{pmatrix}$

& α, ~~$|\beta|$~~ ~~[illegible]~~ $|\alpha|^2 + |\beta|^2 = 1$ So, A is an unitary matrix & so, A^+ is also

& $A^{-1} = A^+ = (A^*)^T = \begin{pmatrix} \alpha^* & \beta \\ \beta^* & -\alpha \end{pmatrix}^T$

$= \begin{pmatrix} \alpha^* & \beta^* \\ \beta & -\alpha \end{pmatrix}$.

& $A^{-1}(|\psi\rangle) = |0\rangle$ & $A^{-1}(|\psi^\perp\rangle) = |1\rangle$ ..