SUPERDENSE COADING

In quantum information theory, Superdense Coding(also referred to as *dense coding*) is a quantum communication protocol to communicate a number of classical bits of information by only transmitting a smaller number of qubits, under the assumption of sender and received per-sharing an entangled state.

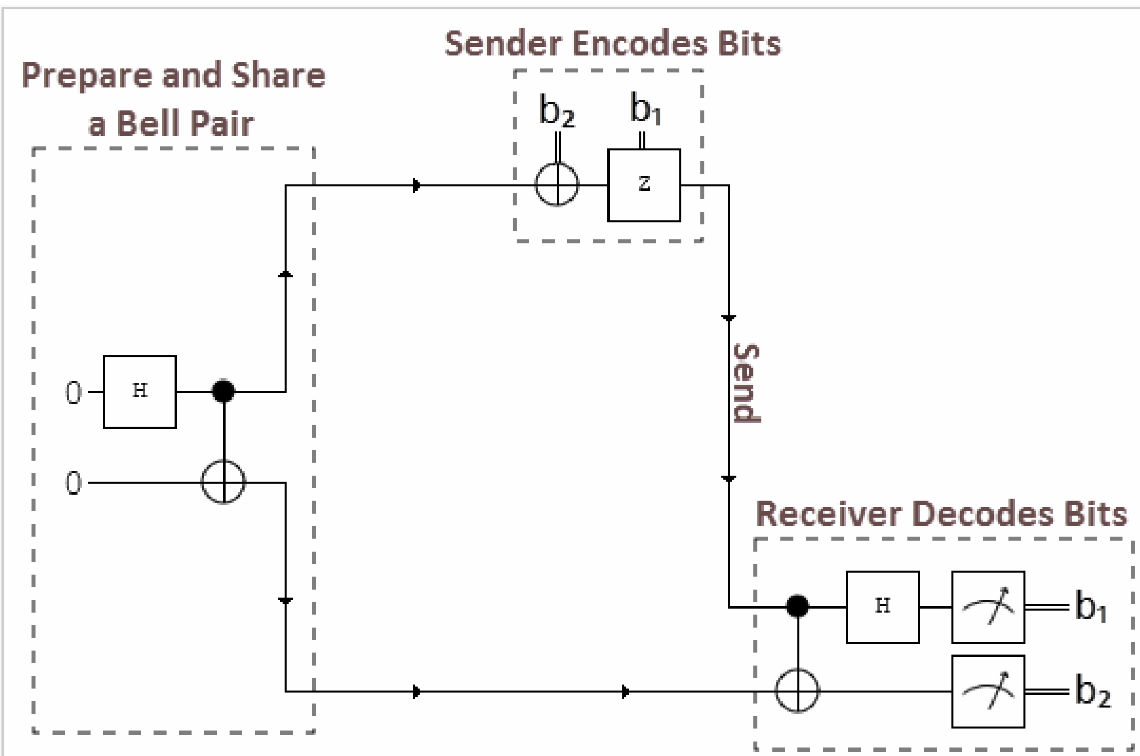

Figure: When the sender and receiver share a Bell state, two classical bits can be packed into one qubit. In the diagram, lines carry qubits, while the doubled lines carry classical bits. The variables $b_1$and b_2 are classic boolean, and the zeroes at the left hand side represent the pure quantum state. See the section named "The protocol" below for more details regarding this picture.

Suppose Alice wants to send two classical bits of information (00, 01, 10, or 11) to Bob using qubits (instead of classical bits). To do this, an entangled state (e.g. a Bell state) is prepared using a Bell circuit or gate by Charlie, a third person. Charlie then sends one of these qubits (in the Bell state) to Alice and the other to Bob. Once Alice obtains her qubit in the entangled state, she applies a certain quantum gate to her qubit depending on which two-bit message (00, 01, 10 or 11) she wants to send to Bob. Her entangled qubit is then sent to Bob who, after applying the appropriate quantum gate and making a measurement, can retrieve the classical two-bit message.

Encoding Strtegy:

Alice needs to perform on her entangled qubit, depending on which classical two-bit message she wants to send to Bob.There are four cases, which correspond to the four possible two-bit strings that Alice may want to send.

1. If Alice wants to send the classical two-bit string 00 to Bob, then she applies the identity quantum gate, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ to her qubit, so that it remains unchanged. The resultant entangled state is then

$$|B_{00}\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

$|B_{00}\rangle$ is also used to remind us of the fact that Alice wants to send the two-bit string 00.

2. If Alice wants to send the classical two-bit string 01 to Bob, then she applies the quantum NOT gate X= $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to her qubit, so that the resultant entangle entangle state become

$$|B_{01}\rangle = \frac{1}{\sqrt{2}} (|10\rangle + |01\rangle)$$

3. If Alice wants to send the classical two-bit string 10 to Bob, then she applies the quantum Z gate to her qubit, so that the resultant entangle entangle state become

$$|B_{10}\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle)$$

4. f Alice wants to send the classical two-bit string 01 to Bob, then she applies the quantum gate Z*X=iY = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ so that the resultant entangle entangle state become

$$|B_{11}\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle)$$

The matrices X,Z and Y are Pouli Matrices.

Decoding Strtegy:

In order for Bob to find out which classical bits Alice sent he will perform the CNOT unitary operation, with A as control qubit and B as target qubit(A for Alice's bit and B for bob's bit). Then, he will perform $H \otimes I$ unitary operation on the entangled qubit A.

1.If the resultant entangled state was B_{00} then after the application of the above unitary operations the entangled state will become $|00\rangle$

2.If the resultant entangled state was B_{01} then after the application of the above unitary operations the entangled state will become $|01\rangle$

3.If the resultant entangled state was B_{10} then after the application of the above unitary operations the entangled state will become $|10\rangle$

3.If the resultant entangled state was B_{11} then after the application of the above unitary operations the entangled state will become $|11\rangle$

These operation performed by Bob can be seen as a measurement which project the entangled state onto one of the four two-qubit basis vector $|00\rangle$, $|01\rangle$, $|10\rangle$ or $|11\rangle$

Calculation:

After the operations performed by Alice if the the resultant entangled state was $B_{01} = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right)$ Then a CNOT with A as control bit and B as target bit will change B_{01} to $B'_{01} = \frac{1}{\sqrt{2}}\left(|11\rangle + |01\rangle\right)$ Now the Hadamard gate is applied only to A to obtain

$$B''_{01} = \frac{1}{\sqrt{2}}\left(\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right) \otimes |1\rangle + \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) \otimes |1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left(|01\rangle - |11\rangle\right) + \frac{1}{\sqrt{2}}\left(|01\rangle + |11\rangle\right)\right]$$

$$= \frac{1}{2}\left(|01\rangle + |01\rangle\right)$$

$$= |01\rangle$$