No Cloning

There is no quantum copying machine that can make two perfect copies of two nonorthogonal state.

Ans:- Suppose the copying circuit $|U\rangle$ can be made to work for two nonorthogonal normalized states $|\psi\rangle$ and $|\phi\rangle$ and assume that $|x\rangle$ is normalized. Then we have.

$$U|\psi\rangle|x\rangle = e^{i\alpha}|\psi\rangle|\psi\rangle \quad \text{——} ①$$

$$\text{and } U|\phi\rangle|x\rangle = e^{i\beta}|\phi\rangle|\phi\rangle \quad \text{——} ②$$

Where we have allowed for phases α and β, which do not affect the physical interpretation of the final state.

From ② we have:

$$\langle\phi|\langle x|U^{+} = e^{-i\beta}\langle\phi|\langle\phi| \quad \text{——} ③$$

From ① and ③ we have.

$$\langle\phi|\langle x|U^{+}U|\psi\rangle|x\rangle = e^{i(\alpha-\beta)}\langle\phi|\langle\phi||\psi\rangle|\psi\rangle$$

$$\Rightarrow \langle\phi|\langle x||\psi\rangle|x| = e^{i(\alpha-\beta)}\langle\phi|\langle\phi||\psi\rangle|\psi\rangle \quad \text{Since } U^{+}U = \mathbb{I}$$

$$\Rightarrow |\langle\phi|\psi\rangle\cdot\langle x|x\rangle| = |e^{i(\alpha-\beta)}\langle\phi|\psi\rangle\cdot\langle\phi|\psi\rangle|$$

$$\Rightarrow |\langle\phi|\psi\rangle| = |\langle\phi|\psi\rangle|^{2}$$

since x is normalized state and $|z^{2}| = |z|^{2}$

This equation has two solution either $|\langle\phi|\psi\rangle| = 0$ or 1. In the first case the two state are orthogonal and in the second since we have assumed that both of them are normalized, they are identical apart form phase factor.