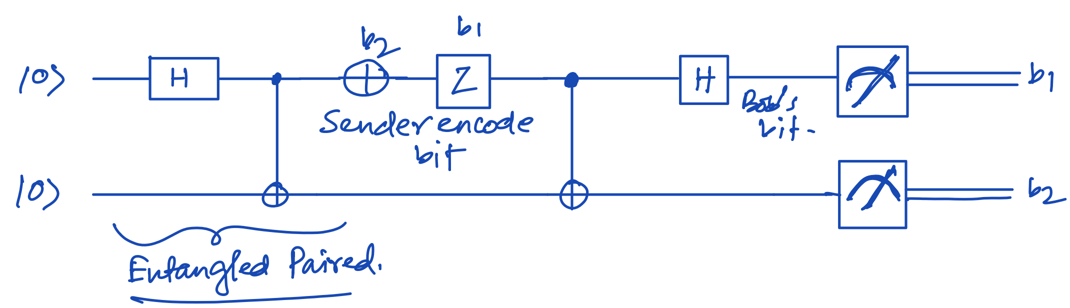
SUPER DENSE CODING.

Dense coding is a quantum communication protocol to communicate a fixed number of classical bit of information by only transmitting a smaller number of qubit, under the assumption of sender and recived preshering an entangled. state. In this procidure, Alice and Bob share an entangled state.

Now alice want to transfer two bit information to Bob by sending only one qubit.



when, the sender and reliver share a bell state and share a classical bit through one qubit. In this diagram line carring qubits and double lines are classical bit. Alice needs to perform on her entangled qubit depending on which classical two bit message, she wants to send Bob.

Here we get four possible cases corresponds to the four possible two bit string.

Case-I: If Alice wants to send classical two-bit string 00 to Bob, then she applies the I gates so that it remain unchanged. The resultant entangled state is, then, $|B_{00}\rangle = \frac{1}{\sqrt{2}} \; |00\rangle + |11\rangle$..

$|B_{00}\rangle$ is also need to remind us of the fact that Alice wants to send the two bit string $|00\rangle$.

Case-II: If Alice wants to send the classical two bit string 01 to bob., then she carries the quantum not gate $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ to here qubit that the resultant entangle state become.

$$|B_{01}\rangle = \frac{1}{\sqrt{2}} \left(|10\rangle + |01\rangle\right).$$

Case-III: If Alice wants to send the classical two bit string 01 to bob, she applies the quantum Z gate $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ to her qubit. So, that the resultant entangle state. become —

$$|B_{10}\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle - |11\rangle\right)$$

Case-IV: If Alice wants to send the classical two bit string 11 to Bob, then she appling the quantum gate $ZX = iY = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. So that the resultant entangle state become.

$$|B_{11}\rangle = \frac{1}{\sqrt{2}} \left(|01\rangle - |01\rangle\right)$$

The matrix X, Z and Y are Pauli Matrices. In order for bob to find out which classical bit Alice send he will perform the Cnot unitary operation, with A as carrol qubit and B. as target qubit. Then he will perform $H \otimes I$ unitary operation on the entangle qubit A.

If the resultant entangle state was B_{00} then after the application of above unitary operation the entangle state will become $|00\rangle$.

$$
\left.
\begin{array}{lll}
\text{i.e.} & B_{00} & \rightarrow |00\rangle \\
\text{Similarly,} & B_{01} & \rightarrow |01\rangle \\
 & B_{10} & \rightarrow |10\rangle \\
 & B_{11} & \rightarrow |11\rangle
\end{array}
\right\}
$$

These, operation performed by Bob can be seen as a measurement which project the entangle state one of four two qubit basis vector.

After the operations perform by Alice if the resultant entangled state was $B_{01} = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$. Then, a CNOT with A as carol bit and B as target bit will change B_{01} to $B_{01}' = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$. Therefore,

$$|B_{01}'\rangle \xrightarrow{H \otimes I} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |1\rangle + \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |$$

$$= \frac{1}{2}(|01\rangle - |11\rangle) + \frac{1}{2}(|01\rangle + |11\rangle)$$

$$= |01\rangle.$$