Q.) What the general form of an state $|x\rangle$ underer $H^{\otimes n}$.

We know $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Combining these two, we can write.

$$H|a\rangle = \frac{1}{\sqrt{2}}\left((-1)^0|0\rangle + (-1)^a|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}} \sum_{x \in \{0,1\}} (-1)^{a \cdot x} |x\rangle.$$

Now let

$|x\rangle = |x_{n-1}, x_{n-2}, \ldots, x_0\rangle$ be an n-qubit

So therefore

$$|x\rangle \xrightarrow{H^{\otimes n}} \frac{1}{2^{n/2}} \sum_{k_{n-1}=0}^{1} \cdots \sum_{k_0=0}^{1} (-1)^{\sum x_j k_j} |k_{n-1} \ldots k_0\rangle$$

So, we can write in general

$$\boxed{H^{\otimes n}|x\rangle = \frac{1}{2^{n/2}} \sum_{k} (-1)^{x \cdot k} |k\rangle}$$

$|k\rangle = |k_{n-1} \ldots k_0\rangle$

$x \cdot k = x_0 k_0 + x_1 k_1 + \cdots + x_{n-1} k_{n-1}$

sum of bitwise product.