<u>Question-1</u>: Let, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Then, find an unitary matrix M such that, $M|\phi\rangle = |0\rangle$ and $M|\phi^{\perp}\rangle = |1\rangle$., where $|\phi^{\perp}\rangle = \beta^*|0\rangle - \alpha^*|1\rangle$.

<u>Soln</u>:-

Let, $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be an unitary matrix which

Satisfy all the condition.

Now, $M|\phi\rangle = |0\rangle$

$$\Rightarrow |\phi\rangle = M^{\dagger}|0\rangle = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} a^* \\ b^* \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \alpha^* \\ \beta^* \end{pmatrix}$$

Also, $M|\phi^{\perp}\rangle = |1\rangle$

$$\Rightarrow |\phi^{\perp}\rangle = M^* \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \beta^* \\ -\alpha^* \end{pmatrix} = \begin{pmatrix} c^* \\ d^* \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} \beta \\ -\alpha \end{pmatrix}$$

$$\therefore \quad M = \begin{pmatrix} \alpha^* & \beta^* \\ \beta & -\alpha \end{pmatrix}$$