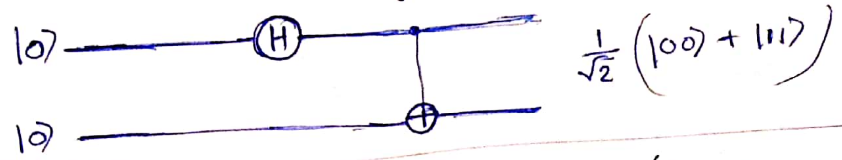
Assignment - 5

Q. 1) What is the matrix for this circuit?



Here $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Matrix representation for CNOT gate, is $\begin{pmatrix} I & 0 \\ 0 & X \end{pmatrix}$.

where, $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

So, matrix representation for the given circuit is

$$M = (CNOT)\,(H \otimes I)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} I & 0 \\ 0 & X \end{pmatrix} \begin{pmatrix} I & I \\ I & -I \end{pmatrix}$$

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

Verification

$$M|00\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\therefore \boxed{M|00\rangle = \frac{1}{\sqrt{2}}\,|00\rangle + |11\rangle}$$

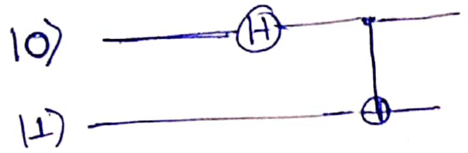
(verified)

Q. ② $\boxed{\text{Generate the qubit } \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)}$

Solution.

We start from $|01\rangle$ and we will apply some circuit as above we get the state $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Now.



$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$.

Here $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $\quad I|1\rangle = |1\rangle$.

Now $H|0\rangle \otimes I|1\rangle$

$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle$

$= \frac{1}{\sqrt{2}} |01\rangle + |11\rangle$

Now $|01\rangle \xrightarrow{CNOT} |01\rangle$

$|11\rangle \xrightarrow{CNOT} |10\rangle$

$\therefore \quad \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

So, $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ can be generalized when we apply the above circuit on the state $|01\rangle$.

So, the matrix will be the same as

$$\boxed{M = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}}$$

(3) Find M' such that $|00\rangle \xrightarrow{M'} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$,

where $M' \neq M$, M' is unitary.

$$M' = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Now,

$$M'|00\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

$$\boxed{M'|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)}$$

$$(M')^* = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$(M')\cdot(M')^* = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} = I_4.$$

Similarly, $(M')^* M = I_4$.

So, M' is a unitary matrix and $\boxed{M' \neq M}$.

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