

* General form of an state $|\psi\rangle$ after applying $H^{\otimes n}$

We know $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$\phi H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

combining these two, we can write —

$$H|a\rangle = \frac{1}{\sqrt{2}}((-1)^0|0\rangle + (-1)^a|1\rangle)$$

$$= \frac{1}{\sqrt{2}} \sum_{n \in \{0,1\}} (-1)^{a \cdot n} |n\rangle$$

Now let $|n\rangle = |n_{n-1}, n_{n-2}, \dots, n_0\rangle$ be an n -qubit.

So, therefore,

$$H^{\otimes n} |n\rangle = \frac{1}{2^{n/2}} \sum_{k_{n-1}=0}^1 \dots \sum_{k_0=0}^1 (-1)^{\sum x_i k_i} |k_{n-1} \dots k_0\rangle$$

And $n \cdot k = n_0 k_0 + n_1 k_1 + \dots + n_{n-1} k_{n-1}$, Sum of bitwise product.