Assignment - 3

(1) Given a 2-qubit state $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

Show that it is not possible to find $|\psi_1\rangle$ & $|\psi_2\rangle$ such that $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$.

$\Rightarrow$ let $|\psi_1\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle$.

& $|\psi_2\rangle = \alpha_2 |0\rangle + \beta_2 |1\rangle$.

if $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

$$= (\alpha_1 |0\rangle + \beta_1 |1\rangle) \otimes (\alpha_2 |0\rangle + \beta_2 |1\rangle)$$

$$= \alpha_1 \alpha_2 |00\rangle + \alpha_1 \beta_2 |01\rangle + \beta_1 \alpha_2 |10\rangle + \beta_1 \beta_2 |11\rangle$$

$$= \begin{pmatrix} \alpha_1 \alpha_2 \\ \alpha_1 \beta_2 \\ \beta_1 \alpha_2 \\ \beta_1 \beta_2 \end{pmatrix}$$

as $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$.

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

So, by comparing both we get —

$$\alpha_1\alpha_2 = \beta_1\beta_2 = \frac{1}{\sqrt{2}}$$

$$\& \quad \alpha_1\beta_2 = \beta_1\alpha_2 = 0.$$

~~Also,~~ $\cancel{|\alpha_1\alpha_2|^2 + |\alpha_1\beta_2|^2 + |\beta_1\alpha_2|^2 + |\beta_1\beta_2|^2}$

$$\cancel{= \sqrt{\left(\tfrac{1}{2}\right)^2 + \tfrac{1}{2}} = 1}.$$

~~or,~~ $\cancel{|\alpha_1|^2(|\alpha_2|^2 + |\beta_2|^2) +}$

Now, $\beta_1\beta_2 = \frac{1}{\sqrt{2}}$ & $\alpha_1\beta_2 = 0$

$$\Rightarrow |\beta_1\beta_2|^2 + |\alpha_1\beta_2|^2 = \frac{1}{2} + 0 = \frac{1}{2}$$

$$\Rightarrow |\beta_2|^2(|\alpha_1|^2 + |\beta_1|^2) = \frac{1}{2}$$

$$\Rightarrow |\beta_2|^2 = \frac{1}{2} \Rightarrow |\beta_2| = \frac{1}{\sqrt{2}}.$$

Similarly, $|\alpha_2| = \frac{1}{\sqrt{2}}$.

$$\Rightarrow \beta_2 \neq 0 \;\&\; \alpha_2 \neq 0$$

as $\alpha_1\alpha_2 = \frac{1}{\sqrt{2}} \Rightarrow \alpha_1 \neq 0$

also, $\beta_1\beta_2 = \frac{1}{\sqrt{2}} \Rightarrow \beta_1 \neq 0$

$$\Rightarrow \alpha_1 \neq 0,\ \alpha_2 \neq 0,\ \beta_1 \neq 0,\ \beta_2 \neq 0$$

then $\alpha_1\beta_2 = \beta_1\alpha_2 = 0$ can't be possible as $\mathbb{C}$ is an integral domain

So, $\alpha_1\beta_2 = 0 \Rightarrow \alpha_1 = 0$ or $\beta_2 = 0$.

a contradiction.

CSWAP gate.

⇒ ~~CCNOT is a 3-qubit gate. It negates the second bit if the first bit is 1, and does nothing if the first bit is 0~~

CCNOT is a 3-qubit gate. If first & 2nd bit are 1 then 3rd bit will flipped & fixed on other cases.

So, if A be the matrix rep of CCNOT gate

1) $A|000\rangle = |000\rangle$

$A|001\rangle = |001\rangle$

$A|010\rangle = |010\rangle$

$A|011\rangle = |011\rangle$

$A|100\rangle = |100\rangle$

$A|101\rangle = |101\rangle$

$A|110\rangle = |111\rangle$

$A|111\rangle = |110\rangle$

2) $A = \begin{pmatrix} I_{6\times6} & O_{6\times2} \\ O_{2\times6} & X_{2\times2} \end{pmatrix}$ where $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

I is identity matrix.

$= \begin{pmatrix} I & O \\ O & X \end{pmatrix}$ ~~as matrix~~

is the matrix representation of CCNOT gate.

& for CSWAP gate if A be the matrix rep then ———

$$A|000\rangle = |000\rangle$$
$$A|001\rangle = |001\rangle$$
$$A|010\rangle = |010\rangle$$
$$A|011\rangle = |011\rangle$$
$$A|100\rangle = |100\rangle$$
$$A|101\rangle = |110\rangle$$
$$A|110\rangle = |101\rangle$$
$$A|111\rangle = |111\rangle$$

where first bit is 0 then 2nd & 3rd bit fixed & if first bit is 1 then 2nd & 3rd bits swap.

$$\Rightarrow A = \begin{pmatrix} I_{5\times5} & O_{5\times2} & O_{5\times1} \\ O_{2\times5} & X_{2\times2} & O \\ O_{1\times5} & O_{1\times2} & I_{1\times1} \end{pmatrix} \equiv \begin{pmatrix} \begin{pmatrix} I_5 & 0 \\ 0 & X \end{pmatrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & I_1 \end{pmatrix}$$

where $I_1 = (1)$.