# No cloning ~~no~~ theorem :-

There is ~~not~~ no quantum copying machine that can make two perfect copies of two nonorthogonal state.

$\Rightarrow$ ~~suppose~~ If possible let we can make a quantum circuit $U$, which can ~~copy~~ make two perfect copies of two nonorthogonal state.

Let $|\psi\rangle$ & $|\phi\rangle$ are two ~~normalit~~ non orthogonal state, and let $|s\rangle$ is some standard pure state, Then we have —

$$U|\psi\rangle|s\rangle = |\psi\rangle|\psi\rangle$$

$$U|\phi\rangle|s\rangle = |\phi\rangle|\phi\rangle.$$

Now taking inner product we have —

$$\langle(U|\psi\rangle|s\rangle) | (U|\phi\rangle|s\rangle)\rangle = \langle\psi|\langle\psi|\phi\rangle|\phi\rangle$$

$$\Rightarrow (U|\phi\rangle|s\rangle)^{\dagger}\, U|\psi\rangle|s\rangle = \langle\psi|\phi\rangle\langle\psi|\phi\rangle$$

$$\Rightarrow \langle s|\langle\phi|\, U^{\dagger}U\, |\psi\rangle|s\rangle = (\langle\psi|\phi\rangle)^2$$

as $U$ is an unitary matrix so, —

$$U^{\dagger}U = I_n = UU^{\dagger}$$

$$\Rightarrow \langle s|\langle\phi|\psi\rangle|s\rangle = (\langle\psi|\phi\rangle)^2$$

$$\Rightarrow \langle s|s\rangle\langle\phi|\psi\rangle = (\langle\psi|\phi\rangle)^2$$

here we choose $|s\rangle$ as unite vector —

So, $\langle S|S\rangle = 1$.

$\Rightarrow |\langle\phi|\psi\rangle| = |\langle\psi|\phi\rangle|^2$

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$\Rightarrow |\langle\psi|\phi\rangle| = 0$ or $1$.

$\Rightarrow |\langle\psi|\phi\rangle| = 0$ or $|\langle\psi|\phi\rangle| = 1$

$\Rightarrow$ $|\psi\rangle$ & $|\phi\rangle$ are orthogonal or $|\psi\rangle = |\phi\rangle$.

Thus cloning device can only clone states which are orthogonal to one another, So, ~~quantum mechanics~~ is in general quantum cloning is not possible.