(1) Given a 2-qubit state $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$.

S.T. its not possible to find $|\psi_1\rangle$ & $|\psi_2\rangle$ s.t.

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle.$$

A: Let us assume, $|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$

& $|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$.

where, $\alpha_1, \beta_1 \in \mathbb{C}$ s.t. $|\alpha_1|^2 + |\beta_1|^2 = 1$

& $\alpha_2, \beta_2 \in \mathbb{C}$ s.t. $|\alpha_2|^2 + |\beta_2|^2 = 1$.

Now, $|\psi_1\rangle \otimes |\psi_2\rangle = \alpha_1\alpha_2|0\rangle \otimes |0\rangle + \alpha_1\beta_2|0\rangle \otimes |1\rangle$

$+ \beta_1\alpha_2|1\rangle \otimes |0\rangle + \beta_1\beta_2|1\rangle \otimes |1\rangle$

$$= \alpha_1\alpha_2 |00\rangle + \alpha_1\beta_2 |01\rangle + \alpha_2\beta_1 |10\rangle + \beta_1\beta_2 |11\rangle.$$

$$\therefore |\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = |\psi_1\rangle \otimes |\psi_2\rangle = \begin{pmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \alpha_2\beta_1 \\ \beta_1\beta_2 \end{pmatrix}.$$

$$\Rightarrow \left.\begin{aligned} \alpha_1\alpha_2 &= 1/\sqrt{2} \\ \alpha_1\beta_2 &= 0 \\ \alpha_2\beta_1 &= 0 \\ \beta_1\beta_2 &= 1/\sqrt{2} \end{aligned}\right\}$$

Now, $|\alpha_1 \alpha_2|^2 + |\alpha_1 \beta_2|^2 = 1/2$

$$\Rightarrow |\alpha_1|^2 \left(|\alpha_2|^2 + |\beta_2|^2\right) = 1/2$$

$$\Rightarrow |\alpha_1|^2 = 1/2 \quad \text{as } |\alpha_2|^2 + |\beta_2|^2 = 1.$$

again, $|\beta_1 \alpha_2|^2 + |\beta_1 \beta_2|^2 = 1/2$

$$\Rightarrow |\beta_1|^2 \left(|\alpha_2|^2 + |\beta_2|^2\right) = 1/2$$

$$\Rightarrow |\beta_1|^2 = 1/2 \quad \text{as. previous.}$$

Now, $\alpha_1 \beta_2 = 0$

$$\Rightarrow |\alpha_1 \beta_2|^2 = 0$$

$$\Rightarrow |\alpha_1|^2 |\beta_2|^2 = 0$$

$$\Rightarrow |\beta_2|^2 = 0 \text{ as } |\alpha_1|^2 = 1/2.$$

also, $\alpha_2 \beta_1 = 0$

$$\Rightarrow |\alpha_2 \beta_1|^2 = 0$$

$$\Rightarrow |\alpha_2|^2 |\beta_1|^2 = 0$$

$$\Rightarrow |\alpha_2|^2 = 0 \quad \left(\text{as } |\beta_1|^2 = 1/2\right)$$

$$\Rightarrow |\alpha_2|^2 + |\beta_2|^2 = 0$$

which is a contradiction to the assumption that $|\alpha_2|^2 + |\beta_2|^2 = 1$.

(2) Matrix representation of CCNOT GATE:

Input			Output		
a	b	c	a'	b'	c'
0	0	0	0	0	0
0	1	0	0	1	0
0	0	1	0	0	1
1	0	0	1	0	0
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

Matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Matrix representation of CSWAP GATE

Input			Output		
a	b	c	a'	b'	c'
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

Matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$