Problem 134

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1 Definition

Consider the consecutive primes $p_1 = 19$ and $p_2 = 23$. It can be verified that 1219 is the smallest number such that the last digits are formed by p_1 whilst also being divisible by p_2.

In fact, with the exception of $p_1 = 3$ and $p_2 = 5$, for every pair of consecutive primes, $p_2 > p_1$, there exist values of n for which the last digits are formed by p_1 and n is divisible by p_2. Let S be the smallest of these values of n.

Find $\sum S$ for every pair of consecutive primes with $5 \leq p_1 \leq 1000000$.

2 Solution

The requested n will always be of the form $n = m10^d + p_1$, where d is the amount of digits in p_1. The requested property for n is

$$\begin{aligned}
n \bmod p_2 &= 0 && (1)\\
(m10^d + p_1) \bmod p_2 &= 0 && (2)\\
(m10^d \bmod p_2 + p_1 \bmod p_2) \bmod p_2 &= 0 && (3)\\
(m10^d \bmod p_2 + p_1) \bmod p_2 &= 0 && (4)\\
m10^d \bmod p_2 &= p_2 - p_1 && (5)
\end{aligned}$$

Following this reasoning, we proceed to check different m values for each p_1, p_2 pair, until Eq. 5 holds. Then we calculate $n = m10^d + p_1 = S$ and add it to the total.