Problem 134

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1 Definition

Consider the consecutive primes $p_1 = 19$ and $p_2 = 23$. It can be verified that 1219 is the smallest number such that the last digits are formed by p_1 whilst also being divisible by p_2.

In fact, with the exception of $p_1 = 3$ and $p_2 = 5$, for every pair of consecutive primes, $p_2 > p_1$, there exist values of n for which the last digits are formed by p_1 and n is divisible by p_2. Let S be the smallest of these values of n.

Find $\sum S$ for every pair of consecutive primes with $5 \leq p_1 \leq 1000000$.

2 Solution

The requested n will always be of the form $n = n \cdot 10^d + p_1$, where d is the amount of digits in p_1. The requested property for n is

$$n \bmod p_2 = 0 \tag{1}$$
$$(m10^j + p_1) \bmod p_2 = 0 \tag{2}$$
$$(m10^j \bmod p_2 + p_1 \bmod p_2) \bmod p_2 = 0 \tag{3}$$
$$(m10^j \bmod p_2 + p_1) \bmod p_2 = 0 \tag{4}$$
$$m10^j \bmod p_2 = p_2 - p_1 \tag{5}$$