

# Immelmann turn

## A flight mechanics study

Irene Simó Muñoz  
irene.munoz@estudiantat.upc.edu

Prof: Àlex Ferrer Farré  
Fall 2021

This brief project aims to take a look at the flight mechanics involved in the Immelmann turn maneuver while analysing its performance in a realistic context.

For this, a first set of computations have been performed in order to obtain the models of the physic phenomena occurring, and a later study with aircraft values has been carried out to evaluate the previously obtained results. All the coding has been done with MATLAB<sup>1</sup> version R2021b (September 2021). The git repository of this project, including the L<sup>A</sup>T<sub>E</sub>X files, can be found in [this link](#).

This project has been developed for course 22027-Mecànica del vol in the Grau en Enginyeria en Tecnologies Aeroespacials degree, Universitat Politècnica de Catalunya, during fall 2021.

## Contents

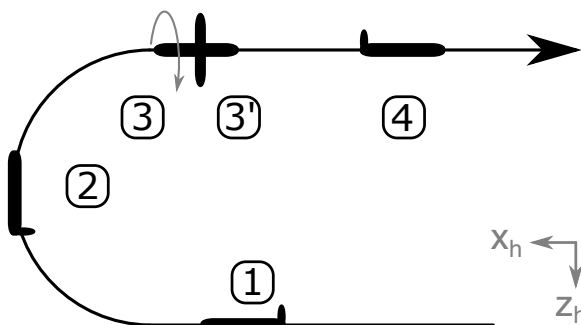
Introduction . . . . .	1
Maneuver Computations . . . . .	1
Maneuver Computations . . . . .	2
Maneuver Computations . . . . .	2
Maneuver Computations . . . . .	2
Maneuver Computations . . . . .	2
Maneuver study . . . . .	2
0.0.1 Evolution throughout time . . . . .	3
0.0.2 Trajectory analysis . . . . .	3
Introduction . . . . .	3

List of Figures . . . . .	4
---------------------------	---

List of Tables . . . . .	4
--------------------------	---

## Introduction

The Immelmann turn, named after German World War I flying ace Max Immelmann, is an aerobatic maneuver that results in level-flight of the aircraft in the opposite direction and higher altitude.



<sup>1</sup>MATrix LABoratory, language developed by The MathWorks Inc.

**Figure 1** Immelmann turn overview. Own elaboration.

The aircraft initiates this maneuver in straight levelled flight, and describes an upwards semicircle contained in its symmetry plane. This increases the flight altitude while changing the trajectory's to the original's opposite direction and leaves the plane turned upside down. It then rolls while still flying in a straight line, and returns to the initial condition of levelled flight.

This maneuver can be performed for various velocities and radius, but there are some dependencies and limitations to be taken into account, such as the maximum deflections or forces the control surfaces can provide or support, respectively.

In order to cover all these variables, a first approach through the dynamic and kinetic equations follows.

## Maneuver Computations

Let us assume symmetric flight (no lateral aerodynamic force  $Q = 0$  nor lateral thrust force as  $\nu = 0$ ), the thrust generated by the engines is parallel to the  $x$  wind axis (thus  $\epsilon = 0$ ), and that the maneuver takes place flawlessly, this is in the vertical plane too;  $\xi = \dot{\xi} = 0$ .

As the maneuver is a rather short, we will also assume that the fuel consumed is negligible and no mass fraction is lost;  $\dot{m} = 0$ .

The motion equations follow:

$$\begin{cases} T \cos(\epsilon) \cos(\nu) - D - mg \sin \gamma - m\dot{V} = 0 \\ T \cos(\epsilon) \sin(\nu) - Q + mg \cos \gamma \sin \mu + mV(\dot{\gamma} \sin \mu - \dot{\Xi} \cos \gamma \cos \mu) \\ -T \sin \epsilon - L + mg \cos \gamma \cos \mu + mV(\dot{\gamma} \cos \mu - \dot{\Xi} \cos \gamma \sin \mu) = 0 \\ \dot{x}_e = V \cos \gamma \cos \Xi \\ \dot{y}_e = V \cos \gamma \sin \Xi \\ \dot{x}_e = -V \sin \gamma \end{cases}$$

And for each of the phases that compose the maneuver, they can be simplified by substituting the flight conditions.

	$\mu$	$\gamma = \dot{\gamma}$	$\xi = \dot{\xi} = \nu = \epsilon = Q$
Cruise	0	0	0
Semicircle	0	f(t)	0
Inversion	f(t)	0	0

### First phases - cruise

$$\begin{cases} T - D - m\dot{V} = 0 \\ 0 = 0 \\ -L + mg = 0 \\ \dot{x}_e = V \\ \dot{y}_e = 0 \\ \dot{z}_e = 0 \end{cases} \quad (1)$$

There are three equations and four variables;  $\alpha$ ,  $\pi$  and  $V$ . Total degrees of freedom are:

### Second phase - semicircle

$$\begin{cases} T - D - mg \sin \gamma - m\dot{V} = 0 \\ 0 = 0 \\ -L + mg \cos \gamma + mV\dot{\gamma} = 0 \\ \dot{x}_e = V \cos \gamma \\ \dot{y}_e = 0 \\ \dot{z}_e = -V \sin \gamma \end{cases} \quad (2)$$

There are four equations and four variables:  $\alpha$ ,  $\pi$ ,  $V$  and  $\gamma$ . Timón de profundidad (elevator), palanca de gases (gas control lever or throttle). Total degrees of freedom are:

### Third phase - inversion

$$\begin{cases} T - D - m\dot{V} = 0 \\ mg \sin \mu = 0 \\ -L - mg \cos \mu = 0 \\ \dot{x}_e = V \\ \dot{y}_e = 0 \\ \dot{z}_e = 0 \end{cases} \quad (3)$$

There are three equations and five variables;  $\alpha$ ,  $\pi$ ,  $V$  and  $\mu$ . Ailerons para mu. Total degrees of freedom are:

### Forth phase - cruise

$$\begin{cases} T - D - m\dot{V} = 0 \\ 0 = 0 \\ -L + mg = 0 \\ \dot{x}_e = V \\ \dot{y}_e = 0 \\ \dot{z}_e = 0 \end{cases} \quad (4)$$

There are three equations and four variables;  $\alpha$ ,  $\pi$  and  $V$ . Total degrees of freedom are:

## Maneuver study

The higher the velocity is, the bigger the radius - this is, the greater the increase of altitude - given that the wing's flaps cannot deflect enough nor can support as much force to reduce the radius as to the same maneuver for a lower velocity.

Note that the centripetal force follows:

$$F = \frac{mV^2}{R}$$

Where  $m$ ,  $V$  and  $R$  are the aircraft's mass (in kg), velocity (in m/s) and radius (in m) respectively.

The only angles affected are gamma  $\gamma$  and mu  $\mu$  if performed flawlessly. The former is the angle formed between the  $x$  axis of the horizontal set and the  $x$  axis of the wind set, while the latter is the one formed between the  $y$  axis of the same sets. Note that for the whole maneuver the body set remains consistent with the wind set.

Figure 2 shows how these change in the different phases of the Immelmann turn

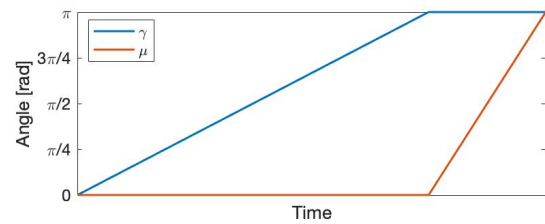


Figure 2 Angles evolution over time

Note that for  $\gamma$  the evolution is perfectly defined, as the increment of said angle needs to be positive in order to increase the flight altitude. Conversely, for  $\mu$  the pilot could choose to rotate in the opposite direction thus changing  $\mu$  from 0 to  $-\pi$  instead - note that  $\pi$  and  $-\pi$  are equivalent -, and the result would be virtually the same.

Furthermore, in order to describe a perfect semicircle the angular speed  $\dot{\gamma}$  needs to remain constant. This results in the necessary condition of gamma's evolution to be a straight slope. For  $\mu$ , however, the pilot could also perform the roll at non-constant rotating speed without this having an impact on the maneuver performance. This would be reflected as a curve on mu's temporal evolution, instead of a straight line.

### 0.0.1 Evolution throughout time

Evaluate  $L$ ,  $D$ ,  $V$ ,  $\gamma$ ,  $z$ ,  $x$

Taking the first equation of each of the phases' systems we can operate to integrate the velocity through time. For the most generic case:

$$\begin{cases} \frac{\partial V}{\partial t} = \frac{1}{m} (T - D - mg \sin \gamma) \\ \frac{\partial \gamma}{\partial t} = \frac{1}{mV} (L - mg \cos \gamma) \end{cases}$$

$$dt = \frac{m}{T - D - mg \sin \gamma} dV$$

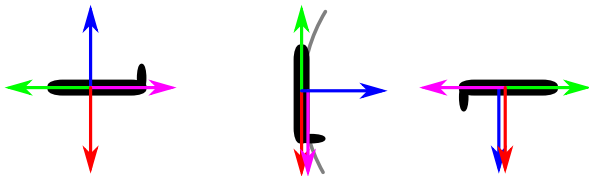
$$\int dt = \int \frac{m}{T - D - mg \sin \gamma} dV$$

Substituting drag's definition,

$$= \int \frac{m}{T - \frac{1}{2} \rho S V^2 (C_{D_0} + k C_L^2) - mg \sin \gamma} dV$$

### 0.0.2 Trajectory analysis

The three force configurations during the maneuver correspond to the free body diagrams at stages 1, 2 and 3 (see Figure 1).



**Figure 3** Free body diagrams during the semiloop. Own elaboration.

Note that at all times the lift behaves as the centripetal force, thus indicating that both the velocity and radius limitations to perform the Immelmann turn are inherent to the wing design and its maximum and minimum lift generation.

AS the motion will be circular, the normal and tangential accelerations will follow:

$$a_n = \frac{V^2}{R} = V\dot{\gamma} = \dot{\gamma}^2 R \quad a_t = V^2 R$$

As a result, the radius can be written as a function of the angle of attack. By operating with the third equation of system number 2, which is in wind system of reference, very convenient as is also polar system:

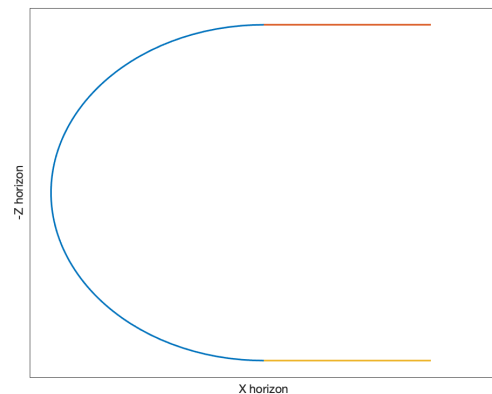
$$\begin{aligned} L &= m \left( \frac{V^2}{R} + g \cos(\gamma) \right) \\ \frac{1}{2} \rho S V^2 C_L(\alpha) &= m \left( \frac{V^2}{R} + g \cos(\gamma) \right) \\ R &= \frac{2mV^2}{2W \cos(\gamma) + \rho S V^2 C_L(\alpha)} \end{aligned}$$

As the plane must describe a semicircle, the radius must be constant.

Plot  $x$ ,  $z$  with time  $y$  despues en plano (ver trayectoria). We can consider the maneuver coordinates:

$$\begin{aligned} \dot{x}_e &= V \cos \gamma & x_e &= Vt \cos \gamma \\ \dot{y}_e &= 0 & y_e &= 0 \\ \dot{z}_e &= -V \sin \gamma & z_e &= -Vt \sin \gamma \end{aligned}$$

If we take both  $V$  (controlled and  $\gamma$  by the gas control lever)



### Other data

# List of Figures

1	Immelmann turn overview. Own elaboration. . . . .	1
2	Angles evolution over time . . . . .	2
3	Free body diagrams during the semiloop. Own elaboration. . . . .	3

# List of Tables