Immelmann turn

A flight mechanics study

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This brief project aims to take a look at the flight mechanics involved in the Immenlman turn maneuver while analysing its performance in a realistic context.
For this, a first set of computations have been performed in order to obtain the models of the physic phenomena occurring, and a later sutdy with aircraft values has been carried out to evaluate the previously obtained results.
All the coding has been done with MATLAB[1] version R2021b (September 2021). The git repository of this project, including the LaTeXfiles, can be found in this link.
This project has been developed for course 22027-Mecànica del vol in the Grau en Enginyeria en Tecnologies Aeroespacials degree, Universitat Politècnica de Catalunya, during fall 2021.

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Introduction

The Immelmann turn, named after German World War I flying ace Max Immelmann, is an an aerobatic maneuver that results in level-flight of the aircraft in the opposite direction and higher altitude.

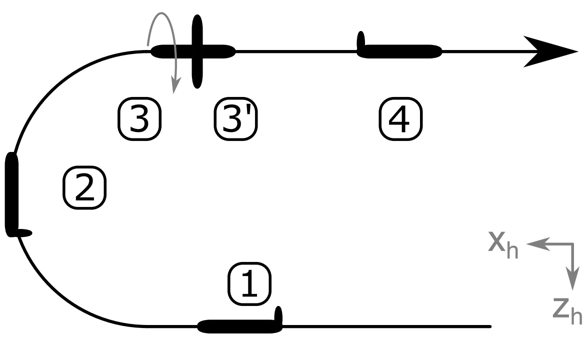

Figure 1 Immelmann turn overview. Own elaboration.

[1] MATrix LABoratory, language developed by The MathWorks Inc.

The aircraft initiates this maneuver in straight levelled flight, and describes an upwards semicircle contained in its symmetry plane. This increases the flight altitude while changing the trajectory's to the original's opposite direction and leaves the plane turned upside down. It then rolls while still flying in a straigth line, and returns to the initial condition of levelled flight.

This maneuver can be performed for various velocities and radius, but there are some dependencies and limitiations to be taken into account, such as the maximum deflections or forces the control surfaces can provide or support, respectively.
In order to cover all these variables, a first approach through the dynamic and kinetic equations follows.

Maneouver Computations

Let us assume symmetric flight (no lateral aerodynamic force $Q = 0$ nor lateral thrust force as $\nu = 0$), the thrust generated by the engines is parallel to the x wind axis (thus $\epsilon = 0$), and that the maneuver takes place flawlessly, this is in the vertical plane too; $\xi = \dot{\xi} = 0$.
As the maneuver is a rather short, we will also assume that the fuel consumed i negligible and no mass fraction is lost; $\dot{m} = 0$.
The motion equations follow:

$$
\begin{cases}
T\cos(\epsilon)\cos(\nu) - D - mg\sin\gamma - m\dot{V} = 0 \\
T\cos(\epsilon)\sin(\nu) - Q + mg\cos\gamma\sin\mu + mV(\dot{\gamma}\sin\mu - \dot{\Xi}\cos\gamma\cos\mu) \\
-T\sin\epsilon - L + mg\cos\gamma\cos\mu + mV(\dot{\gamma}\cos\mu - \dot{\Xi}\cos\gamma\sin\mu) = 0 \\
\dot{x}_e = V\cos\gamma\cos\Xi \\
\dot{y}_e = V\cos\gamma\sin\Xi \\
\dot{x}_e = -V\sin\gamma
\end{cases}
$$

And for each of the phases that compose the maneuver, they can be simplified by substituting the flight conditions.

	μ	$\gamma = \dot{\gamma}$	$\xi = \dot{\xi} = \nu = \epsilon = Q$
Cruise	0	0	0
Semicirle	0	f(t)	0
Inversion	f(t)	0	0

First phases - cruise

$$\begin{cases} T - D - m\dot{V} = 0 \\ 0 = 0 \\ -L + mg = 0 \\ \dot{x}_e = V \\ \dot{y}_e = 0 \\ \dot{x}_e = 0 \end{cases} \tag{1}$$

There are three equations and four variables; α, π and V Total degrees of freedom are:

Second phase - semicircle

$$\begin{cases} T - D - mg\sin\gamma - m\dot{V} = 0 \\ 0 = 0 \\ -L + mg\cos\gamma + mV\dot{\gamma} = 0 \\ \dot{x}_e = V\cos\gamma \\ \dot{y}_e = 0 \\ \dot{z}_e = -V\sin\gamma \end{cases} \tag{2}$$

There are four equations and four variables: α, π, V and γ. Timón de profundidad (elevator), palanca de gases (gas control lever or throttle). Total degrees of freedom are:

Third phase - inversion

$$\begin{cases} T - D - m\dot{V} = 0 \\ mg\sin\mu = 0 \\ -L - mg\cos\mu = 0 \\ \dot{x}_e = V \\ \dot{y}_e = 0 \\ \dot{z}_e = 0 \end{cases} \tag{3}$$

There are three equations and five variables; α, π, V and μ Ailerons para mu. Total degrees of freedom are:

Forth phase - cruise

$$\begin{cases} T - D - m\dot{V} = 0 \\ 0 = 0 \\ -L + mg = 0 \\ \dot{x}_e = V \\ \dot{y}_e = 0 \\ \dot{z}_e = 0 \end{cases} \tag{4}$$

There are three equations and four variables; α, π and V. Total degrees of freedom are:

Maneouver study

The higher the velocity is, the bigger the radius - this is, the greater the increase of altitude - given that the wing's flaps cannot deflect enough nor can support as much force to reduce the radius as to the same menauver for a lower velocity.
Note that the centripetal force follows:

$$F = \frac{mV^2}{R}$$

Where m, V and R are the aircraft's mass (in kg), velocity (in m/s) and radius (in m) respectively.

The only angles affected are gamma γ and mu μ if performed flawlessly. The former is the angle formed between the *x* axis of the horizontal set and the *x* axis of the wind set, while the latter is the one formed between the *y* axis of the same sets. Note that for the whole maneuver the body set remains consistent with the wind set.
Figure 2 shows how these change in the different phases of the Immelmann turn

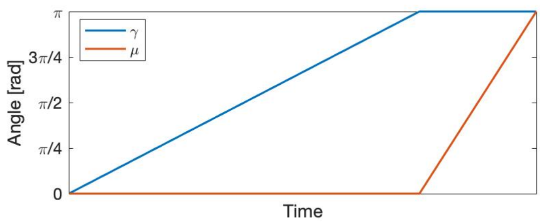

Figure 2 Angles evolution over time

Note that for γ the evolution is perfectly defined, as the increment of said angle needs to be positive in order to increase the flight altitude. Conversely, for μ the pilot could choose to rotate in the opposite direction thus changing μ from 0 to $-\pi$ instead - note that π and $-\pi$ are equivalent -, and the result would be virtually the same.
Furthermore, in order to describe a perfect semicircle the angular speed $\dot{\gamma}$ needs to remain constant. This results in the necessary condition of gamma's evolution to be a straight slope. For μ, however, the pilot could also perform the roll at non-constant rotating speed without this having an impact on the maneuver performance. This would be reflected as a curve on mu's temporal evolution, instead of a straight line.
For the following computations, some values will be needed in order to numerically solve the Ordinal Differential Equation systems. The ENAERT T-35 Pillán, a Chilean military small aircraft and its data [2] have been taken as references.

Figure 3 ENAERT T-35 Pillán. Extracted from [1]

The used information is:

	Value		Value
Mass	1.300 kg	Wingspan	8.84 m
Airfoil	63_3-414	Wing area	13.69 m^2
Stall s.		Cruise s.	
Thrust	224 kW	Air den.	1.225 kg/m^3
$C_L = 12\alpha + 0.33$			

Evolution throughout time

For the totality of the maneuver, the fixated parameters are the gas control lever (π) and the elevator (α), which are set as constants at 1.5 kN and 0.3 rad respectively. From the remaining variables, the position coordinates evolution ($\dot{x}$ and $\dot{z}$) are always left as dependant in order to be solved by the ODE solver. With the obtained results, the trajectory is plot - see the following section Trajectory analysis, where both of them are studied in moer depth -.
For the first and last phases, both cruise flight, the remaining variable (not counting $\dot{x}$ and $\dot{z}$) is velocity (V).
Evaluate L, D, V, γ, z, x

Cruise flight

Half loop

Taking the first equation of each of the phases' systems we can operate to integrate the velocity through time. For the most generic case:

$$\begin{cases} \frac{\partial V}{\partial t} &= \frac{1}{m}\left(T - D - mg\sin\gamma\right) \\ \frac{\partial \gamma}{\partial t} &= \frac{1}{mV}\left(L - mg\cos\gamma\right) \end{cases}$$

$$dt = \frac{m}{T - D - mg\sin\gamma} dV$$

$$\int dt = \int \frac{m}{T - D - mg\sin\gamma} dV$$

Subsituting drag's definition,

$$= \int \frac{m}{T - \frac{1}{2}\rho S V^2 (C_{D_0} + kC_L^2) - mg\sin\gamma} dV$$

The three force configurations during the maneuver correspond to the free body diagrams at stages 1, 2 and 3 (see Figure 1).

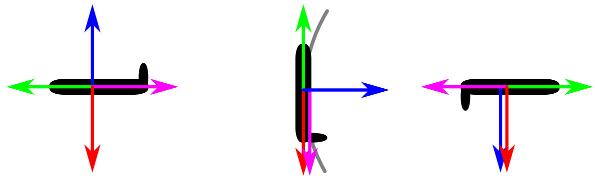

Figure 4 Free body diagrams during the semiloop. Own elaboration.

Note that at all times the lift behaves as the centripetal force, thus indicating that both the velocity and radius limitations to perform the Immelmann turn are inherent to the wing design and its maximum and minimum lift generation.
AS the motion will be circular, the normal and tangential accelerations will follow:

$$a_n = \frac{V^2}{R} = V\dot{\gamma} = \dot{\gamma}^2 R \qquad a_t = V^2 R$$

As a result, the radius can be writen as a function of the angle of attack. By operating with the third equation of system number 2, which is in wind system of reference, very convenient as is also polar set of axis:

$$L = m\left(\frac{V^2}{R} + g\cos(\gamma)\right)$$

$$\frac{1}{2}\rho S V^2 C_L(\alpha) = m\left(\frac{V^2}{R} + g\cos(\gamma)\right)$$

$$R = \frac{2mV^2}{2W\cos(\gamma) + \rho S V^2 C_L(\alpha)}$$

As the plane must describe a semicircle, the radius must be constant.

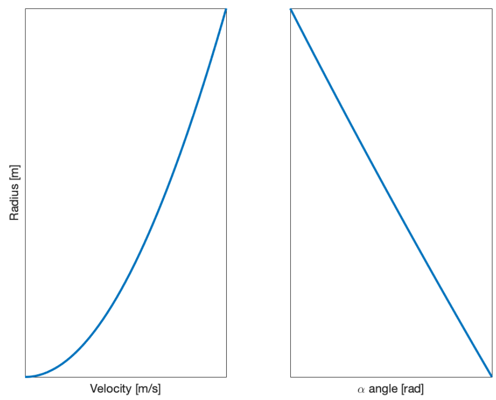

Figure 5 Radius dependance on V and αt. Own elaboration.

If we take both V (controlled by the gas control lever) and γ

Roll

Trajectory analysis

The expected result of the trajectory is portrayed in the following image:

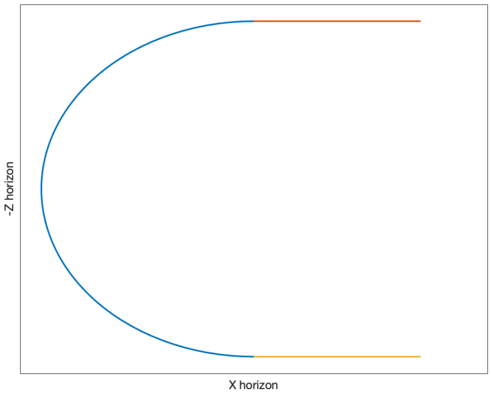

Figure 6 Expected trajectory output. Own elaboration.

We can consider the maneuver coordinates:

$$\begin{aligned} \dot{x}_e &= V\cos\gamma \\ \dot{y}_e &= 0 \\ \dot{x}_e &= -V\sin\gamma \end{aligned} \quad \rightarrow \quad \begin{aligned} x_e &= Vt\cos\gamma \\ y_e &= 0 \\ z_e &= -Vt\sin\gamma \end{aligned}$$

Other data

this is a biblatex test [3]

Bibliography

[1] Cristian Marambio Defensa.com. Proyecto t-35 pillán ii: Nueva vida para el pillán, Apr 2018.

[2] Frederick Thomas Jane. *Jane's All the World's Aircraft 1913*. David & Charles Reprtints, 1969.

[3] Miguel Ángel Gómez Tierno, Manuel Pérez Cortés, and César Puentes Márquez. *Mecánica del vuelo*. Ibergaceta, 2012.

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Appendix: Curve fitting

In order to plot the evolution of both Lift and Drag for each phase, the computated velocity - solved from each respective ODE system - has been fitted from the resultant set of data points into a curve.
For this, the native `polyfit` and `poly2sym` MATLAB functions have ben employed.

To correctly fit a set of points into a curve and avoid overfitting the polynomial (thus dealing with excessive oscillations and Runge's phenomemenon among others), the lienar Least Squares Method (LLS) has been used. The following plots show the square root sum and the norm of the squared regression value for each phase's velocity fit.