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A3: Branched Cylinders: Dendritic Tree Approximations

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BM2102 Modelling and Analysis of Physiological Systems

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Contents

1 Question 01 and Question 02

General Solution

$$V_1(x) = A_1 e^{-x} + B_1 e^{x} \qquad 0 \le x \le L_1$$
$$V_{21}(x) = A_{21} e^{-x} + B_{21} e^{x} \qquad L_1 \le x \le L_{21}$$
$$V_{22}(x) = A_{22} e^{-x} + B_{22} e^{x} \qquad L_1 \le x \le L_{22}$$

Using Boundary conditions ; $x = 0$, I_{app} is applied

$$\therefore \left.\frac{dV_1}{dx}\right|_{x=0} = (-r_i \lambda_c)_1 I_{app}$$

$$(-r_i \lambda_c)_1 I_{app} = -A_1 e^{-x} + B_1 e^{x}$$
$$(-r_i \lambda_c)_1 I_{app} = -A_1 + B_1$$
$$(r_i \lambda_c)_1 I_{app} = A_1 - B_1 \quad — 1$$

Terminal ends of daughter branches are at rest,

$$V_{21}(L_{21}) = V_{22}(L_{22}) = 0.$$
$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0 \quad — 2$$
$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0 \quad — 3$$

Membrane potential is continuous at nodes,

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1)$$

$$A_1 e^{-L_1} + B_1 e^{L_1} = A_{21} e^{-L_1} + B_{21} e^{L_1} = A_{22} e^{-L_1} + B_{22} e^{L_1} \quad — 4$$

Considering current conservation at nodes,

$$\frac{-1}{(r_i\lambda_c)_1} \left.\frac{dV_1}{dx}\right|_{x=L_1} = \frac{-1}{(r_i\lambda_c)_{21}} \left.\frac{dV_{21}}{dx}\right|_{x=L_1} + \frac{-1}{(r_i\lambda_c)_{22}} \left.\frac{dV_{22}}{dx}\right|_{x=L_1}$$

$$\frac{-1}{(r_i\lambda_c)_1}\left(-A e^{-L_1} + B_1 e^{L_1}\right) = \frac{-1}{(r_i\lambda_c)_{21}}\left(-A_{21} e^{-L_1} + B_{21} e^{L_1}\right) + \frac{-1}{(r_i\lambda_c)_{22}}\left(-A_{22} e^{-L_1} + B_{22} e^{L_1}\right)$$

$$\frac{1}{(r_i\lambda_c)_1}\left(A_1 e^{-L_1} - B_1 e^{L_1}\right) = \frac{1}{(r_i\lambda_c)_{21}}\left(A_{21} e^{-L_1} - B_{21} e^{L_1}\right) + \frac{1}{(r_i\lambda_c)_{22}}\left(A_{22} e^{-L_1} - B_{22} e^{L_1}\right) \quad — 5$$

$$0 = -\frac{A_1 e^{-L_1}}{(r_i \lambda_c)_1} + \frac{B_1 e^{L_1}}{(r_i \lambda_c)_1} + \frac{A_{21} e^{-L_1}}{(r_i \lambda_c)_{21}} - \frac{B_{21} e^{L_1}}{(r_i \lambda_c)_{21}} + \frac{A_{22} e^{-L_1}}{(r_i \lambda_c)_{22}} - \frac{B_{22} e^{L_1}}{(r_i \lambda_c)_{22}}$$

$$\therefore -\frac{A_1 e^{-L_1}}{(r_i \lambda_c)_1} + \frac{B_1 e^{L_1}}{(r_i \lambda_c)_1} + \frac{A_{21} e^{-L_1}}{(r_i \lambda_c)_{21}} - \frac{B_{21} e^{L_1}}{(r_i \lambda_c)_{21}} + \frac{A_{22} e^{-L_1}}{(r_i \lambda_c)_{22}} - \frac{B_{22} e^{L_1}}{(r_i \lambda_c)_{22}} = 0 \quad \text{——} (5)$$

Let's define
$$x = \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix}$$

Let's simplify $Ax = b$

H.S
$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ -e^{-L_1}/(r_i\lambda_c)_1 & e^{L_1}/(r_i\lambda_c)_1 & e^{-L_1}/(r_i\lambda_c)_{21} & -e^{L_1}/(r_i\lambda_c)_{21} & e^{-L_1}/(r_i\lambda_c)_{22} & -e^{L_1}/(r_i\lambda_c)_{22} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{bmatrix}$$

$$\begin{bmatrix} A_1 - B_1 \\ e^{-L_{21}} A_{21} + e^{L_{11}} B_{21} \\ e^{-L_{22}} A_{22} + e^{L_{22}} B_{22} \\ A_1 e^{-L_1} + B_1 e^{L_1} - A_{21} e^{-L_1} - B_{21} e^{-L_1} \\ A_{21} e^{-L_1} + B_{21} e^{L_1} - A_{22} e^{-L_1} - B_{22} e^{L_1} \\ -\frac{A_1}{(r_i\lambda_c)_1} e^{-L_1} + \frac{B_1}{(r_i\lambda_c)} e^{L_1} + \frac{A_{21}}{(r_i\lambda_c)_{21}} e^{-L_1} - \frac{B_{21}}{(r_i\lambda_c)_{21}} e^{-L_1} + \frac{A_{22}}{(r_i\lambda_c)_{22}} e^{-L_1} - \frac{B_{22}}{(r_i\lambda_c)_{22}} e^{-L_1} \end{bmatrix}$$

Substituting from (1), (2), (3), (4) and (5)

$$\begin{bmatrix} (r_i \lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = b \implies \therefore \underline{\underline{Ax = b}}.$$

2 Question 3

```
% electrical constants and derived quantities for typical
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;        % cm
% d21 = 30e-4;       % cm
% d22 = 15e-4;       % cm
d21 = 47.2470e-4;        % E9 cm
d22 = d21;                % E9 cm

l1 = 1.5;      % dimensionless
l21 = 3.0;       % dimensionless
l22 = 3.0;       % dimensionless

% Electrical properties of compartments

Rm = 6e3;      % Ohms cm^2
Rc = 90;       % Ohms cm
Rs = 1e6;      % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);   % Ohms
rl21 = c1*d21^(-3/2);   % Ohms
rl22 = c1*d22^(-3/2);   % Ohms


% Applied current

iapp = 1e-9;  % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-
        l1)/rl22 -rl1*exp(-l1)/rl22];

b = [rl1*iapp 0 0 0 0 0]';

X= A\b;
display(X)
```

```
X =

   1.0e-03 *

    0.7189
   -0.0014
    0.7275
   -0.0018
    0.7275
   -0.0018
```

Figure 1: Values of the coefficients obtained by the code

3 Question 04

```
y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);
v1=X(1)*exp(-y1)+X(2)*exp(y1);
v21=X(3)*exp(-y21)+X(4)*exp(y21);
v22=X(5)*exp(-y22)+X(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady state voltage - E5');
```

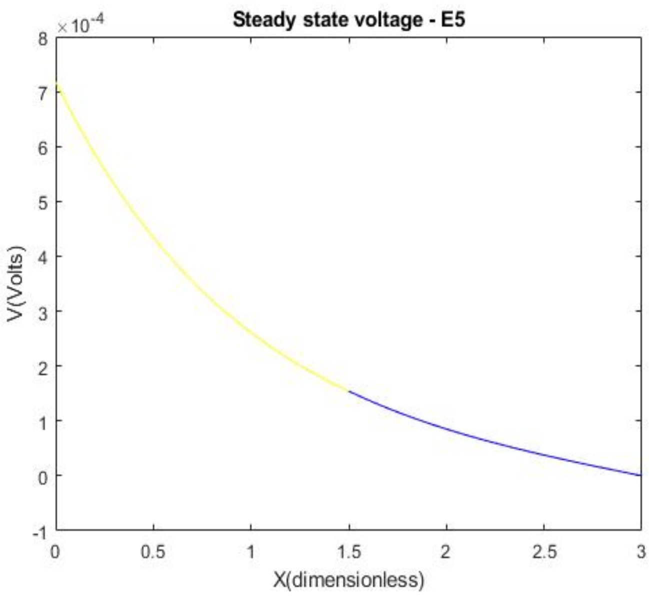

Figure 2: Steady State profile in each branch

The parent branch's steady-state profile is shown by the yellow line in the graph, and the steady profiles of the two daughter branches are represented by the blue and red lines. Notably, the graph does not show the red color. This could mean that the blue and red lines are drawn so close to one another or on top of each other that the red line is hidden.

As a result, we can assume that the daughter branches' steady-state voltage is probably equal. This conclusion is based on the observation that the red line would be clearly visible on the graph if it differed from the blue line by a substantial amount. Because of this, the red and blue lines' convergence indicates that their voltage profiles are comparable, suggesting the equllibrium of the steady state voltages in daughter branches.

4 Section 04

4.1 Steady State Voltage Profile for 2(a)

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0]

X = A\b;
y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);
v1=X(1)*exp(-y1)+X(2)*exp(y1);
v21=X(3)*exp(-y21)+X(4)*exp(y21);
v22=X(5)*exp(-y22)+X(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady state voltage - E5')
```

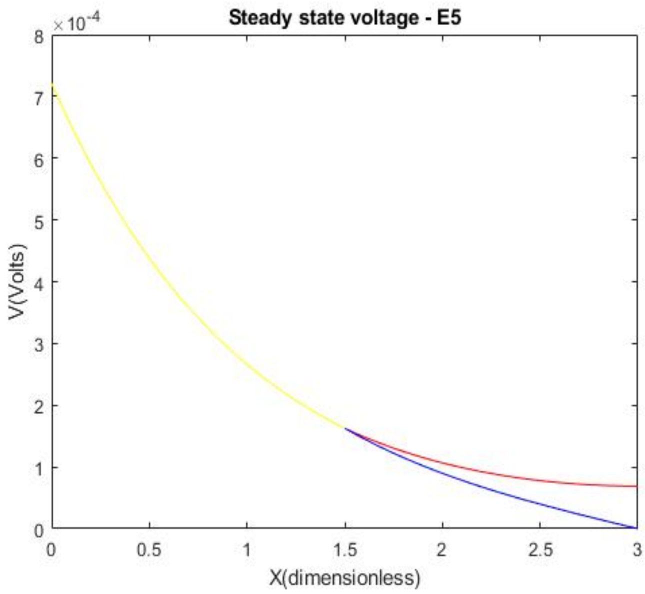

Figure 3: Steady State profile in each branch - 2(a)

4.2 Steady State Voltage Profile for 2(b)

```
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)]
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0]

X = A\b;
y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);
v1=X(1)*exp(-y1)+X(2)*exp(y1);
v21=X(3)*exp(-y21)+X(4)*exp(y21);
v22=X(5)*exp(-y22)+X(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady␣state␣voltage␣-␣E5')
```

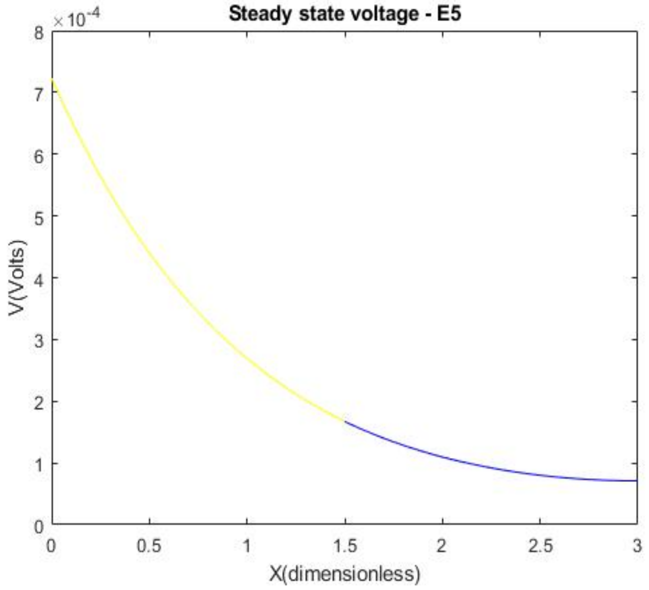

Figure 4: Steady State profile in each branch - 2(b)

4.3 Steady State Voltage Profile for 2(c)

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0]
b(1) = 0; b(2) = rl21*iapp

X = A\b;
y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);
v1=X(1)*exp(-y1)+X(2)*exp(y1);
v21=X(3)*exp(-y21)+X(4)*exp(y21);
v22=X(5)*exp(-y22)+X(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady state voltage - E5');
```

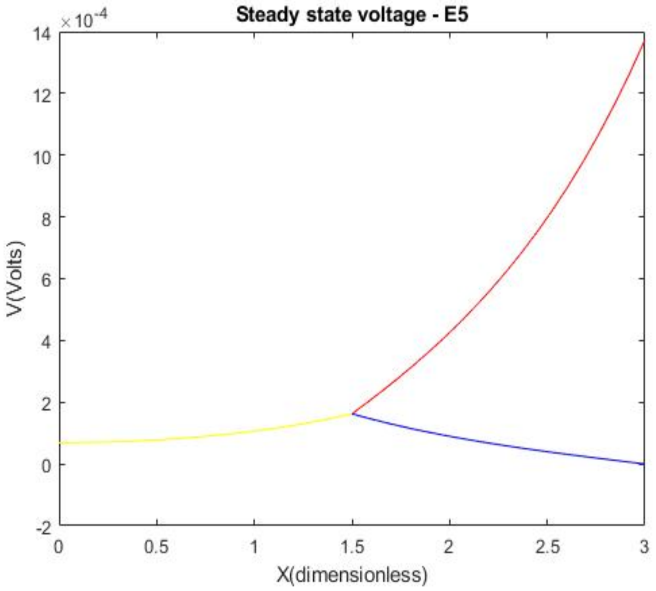

Figure 5: Steady State profile in each branch -2(c)

4.4 Steady State Voltage Profile for 2(d)

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0]
b(1) = 0; b(2) = rl21*iapp
b(3) = rl22*iapp

X = A\b;
y1=linspace(0,l1,20);
y21=linspace(l1,l21,20);
y22=linspace(l1,l22,20);
v1=X(1)*exp(-y1)+X(2)*exp(y1);
v21=X(3)*exp(-y21)+X(4)*exp(y21);
v22=X(5)*exp(-y22)+X(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady state voltage - E5');
```

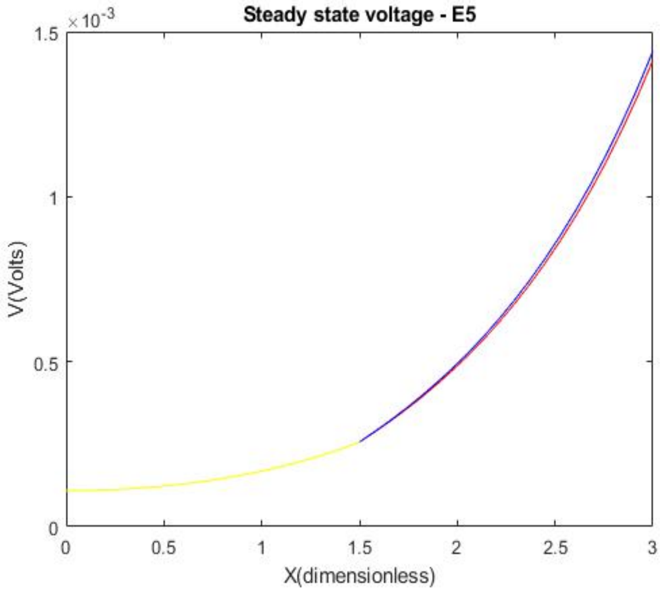

Figure 6: Steady State profile in each branch - 2(d)

5 Question 05

How quickly the membrane voltage varies over a specific distance is represented by the membrane voltage gradient. Because the membrane voltage rises with distance from the node, the membrane voltage gradient for the daughter branches is positive at the rightmost nodes. This happens because a signal is being sent to another neuron or branch by the daughter branches' rightmost nodes.

The electrical signal travels down the neuron's axon as a wave of depolarization. The rightmost node of a daughter branch depolarizes when this depolarization wave reaches it. As you go away from that original node, the depolarization then extends down the remainder of the daughter branch, increasing the membrane voltage.

A positive voltage gradient is produced at the rightmost daughter branch nodes as a result of the rise in membrane voltage there. The electrical signal can be efficiently transmitted from the parent branch to the daughter branches thanks to the positive voltage gradient.

6 Question 06

```
x =

   1.0e-03 *

    0.7216
    0.0013
    0.7132
    0.0018
    0.7132
    0.0018
```

Figure 7: Values of the coefficients obtained by the code - 2(b)

```
x =

   1.0e-03 *

    0.0545
    0.0545
   -0.2768
    0.0710
   -0.3058
    0.0725
```

Figure 8: Values of the coefficients obtained by the code - 2(d)

The equal transmission of electrical impulses to both daughter branches is indicated by the convergence of the blue and red graphs in the steady state plot shown in Figure 2(b). This alignment points to a balanced transmission along each branch and a symmetrical distribution of electrical activity.

On the other hand, the steady state plot seen in Figure 2(d) shows clearly separated blue and red graphs, which suggest that the electrical impulses from the parent branch are not transmitted equally. This discrepancy points to an uneven dispersion of electrical activity, where a higher percentage of impulses reach one daughter branch than the other.

Such changes in transmission may result from variations in electrical qualities, branching features, or outside factors that impact the conduction channel.