Fast, Safe, Pure-Rust Elliptic Curve Cryptography

Isis Lovecruft / Henry de Valence

RustConf 2017

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- It *isn't* aimed at cryptographers.
- Don't worry! We'll cover the basic terminology, and — with a tad of high-school level algebra — you should be able to follow along just fine.
- If not, still don't worry! All questions are welcome, and if you're shy please feel free to talk to us privately afterwards, either in person or online.

Overview

What is `curve25519-dalek`?

Implementing low-level arithmetic in Rust

Rust features we love, and features we want to improve

Implementing crypto with `-dalek`

What is `curve25519-dalek`?

Anatomy of an elliptic curve cryptography implementation

<table>
<tr><td colspan="2">Applications</td></tr>
<tr><td>Protocol</td><td>Protocol-specific library</td></tr>
<tr><td>Group</td><td rowspan="3"><code>curve25519-dalek</code></td></tr>
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Our implementation was originally based on Adam Langley's `ed25519` Go code, which was in turn based on the reference `ref10` implementation.

Historical Implementations

In order to talk about what `curve25519-dalek` is, and why we made it, it's important to revisit other elliptic curve libraries, their designs, and common problems.

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- Assumptions about how these lower-level pieces will be used aren't necessarily correct if someone wanted to reuse the code to implement a different protocol.
- Excessive copy-pasta with minor tweaks by other cryptographers (worsened by the fact that some cryptographers think that releasing unsigned tarballs of their implementations *inside* another tarball of a benchmarking suite is somehow an appropriate software distribution mechanism).

Historical Implementations: Part I (cont.)

This leads to large, monolithic codebases which are idiosyncratic, incompatible with one another, and highly specialised to perform only the single protocol they implement (usually, a signature scheme or Diffie-Hellman key exchange).

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- *I can keep going.*

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Implementing low-level arithmetic in Rust

Example: implementing multiplication in $\mathbb{F}_p$, $p = 2^{255} - 19$

Let's jump down to the lowest abstraction layer: using primitive types to implement field arithmetic.

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Specifically: how can we implement multiplication of two integers modulo $p = 2^{255} - 19$, using only the primitive operations provided by the CPU?

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Specifically: how can we implement multiplication of two integers modulo $p = 2^{255} - 19$, using only the primitive operations provided by the CPU?

Two questions:

- What are the primitive operations?
- What does multiplication in $\mathbb{F}_p$ look like?

Multiplication modes

Primitive types have a fixed size: `u8`, `i8`, …, `u64`, `i64`, etc., but numbers get bigger when you multiply them. What happens?

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2. Wrapping arithmetic (release): `8u8 * 40u8 == 64u8`

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4. Widening arithmetic: `8u8 * 40u8 == 320u16`

Rust has intrinsics for 1, 2, and 3, and we can get 4 by writing

`(x as T) * (y as T)`,

where `T` is the next-wider type.

Lowering widening multiplication to assembly on x86-64

```rust
#![feature(i128_type)]
// Test to see how rustc / LLVM lowers a widening mul on x64
pub fn widening_mul(x: u64, y: u64) -> u128 {
  (x as u128) * (y as u128)
}  // made with https://rust.godbolt.org
```

rustc nightly (Editor #1, Compiler #1)

rustc nightly | -O

.LX0: | .text | // | \s+ | Intel

```asm
example::widening_mul:
  push rbp
  mov rbp, rsp
  mov rax, rsi
  mul rdi
  pop rbp
  ret
```

rustc 1.21.0-nightly (a7e0d3a81 2017-08-11)- *3600ms*

rustc nightly (Editor #1, Compiler #2)

rustc nightly | C target_cpu=haswell

.LX0: | .text | // | \s+ | Intel

```asm
example::widening_mul:
  push rbp
  mov rbp, rsp
  mov rdx, rsi
  mulx rdx, rax, rdi
  pop rbp
  ret
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rustc 1.21.0-nightly (a7e0d3a81 2017-08-11)- *900ms*

Radix-2^{51} representation

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What does this mean? It means we write numbers x, y as

$$x = x_0 + x_1 2^{51} + x_2 2^{102} + x_3 2^{153} + x_4 2^{204} \qquad 0 \le x_i \le 2^{51}$$
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Since $2^{51} < 2^{64}$, we can write this as

```
struct FieldElement64([u64;5])
```

and use the widening multiplication

```
(x[i] as u128) * (y[j] as u128)
```

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\begin{aligned}
z_0 &= x_0 y_0 && 1 \\
z_1 &= x_0 y_1 + x_1 y_0 && 2^{51} \\
z_2 &= x_0 y_2 + x_1 y_1 + x_2 y_0 && 2^{102}
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z_5 &= x_1 y_4 + x_2 y_3 + x_3 y_2 + x_4 y_1 && 2^{255} \\
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z_7 &= x_3y_4 + x_4y_3 && 2^{357}\\
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We can combine this with the formulas on the previous slide:

$$z_0 = x_0 y_0 + 19(x_1 y_4 + x_2 y_3 + x_3 y_2 + x_4 y_1) \qquad 1$$

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$$\equiv (z_0 + 19z_5) + (z_1 + 19z_6)2^{51} + (z_2 + 19z_7)2^{102} + (z_3 + 19z_8)2^{153} + z_4 2^{204} \pmod{p}$$

We can combine this with the formulas on the previous slide:

$$z_0 = x_0y_0 + 19(x_1y_4 + x_2y_3 + x_3y_2 + x_4y_1) \qquad 1$$
$$z_1 = x_0y_1 + x_1y_0 + 19(x_2y_4 + x_3y_3 + x_4y_2) \qquad 2^{51}$$

Multiplication, part II

Since $p = 2^{255} - 19$, we have $2^{255} \equiv 19 \pmod{p}$.

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Rust implementation, part I

Let's write this in Rust:

```
impl<'a, 'b> Mul<&'b FieldElement64> for &'a FieldElement64 {
    type Output = FieldElement64;
    fn mul(self, _rhs: &'b FieldElement64) -> FieldElement64 {
        #[inline(always)]
        fn m(x: u64, y: u64) -> u128 { (x as u128) * (y as u128) }

        // Alias self, _rhs for more readable formulas
        let a: &[u64; 5] = &self.0; let b: &[u64; 5] = &_rhs.0;
        // 64-bit precomputations to avoid 128-bit multiplications
        let b1_19 = b[1]*19; let b2_19 = b[2]*19; let b3_19 = b[3]*19; let b4_19 = b[4]*19;

        // Multiply to get 128-bit coefficients of output
        let c0 = m(a[0],b[0]) + m(a[4],b1_19) + m(a[3],b2_19) + m(a[2],b3_19) + m(a[1],b4_19);
        let c1 = m(a[1],b[0]) + m(a[0],b[1])  + m(a[4],b2_19) + m(a[3],b3_19) + m(a[2],b4_19);
        let c2 = m(a[2],b[0]) + m(a[1],b[1])  + m(a[0],b[2])  + m(a[4],b3_19) + m(a[3],b4_19);
        let c3 = m(a[3],b[0]) + m(a[2],b[1])  + m(a[1],b[2])  + m(a[0],b[3])  + m(a[4],b4_19);
        let c4 = m(a[4],b[0]) + m(a[3],b[1])  + m(a[2],b[2])  + m(a[1],b[3])  + m(a[0],b[4]);
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```

However, the c_i are too big: we want `u64`s, not `u128`s.

Rust implementation, part II

To finish, we reduce the size of the coefficients by carrying their values upwards into higher coefficients: $(c_{i+1}, c_i) \leftarrow (c_{i+1} + \lfloor c_i/2^{51} \rfloor, c_i \mod 2^{51})$

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```
        let low_51_bit_mask = (1u64 << 51) - 1;
        c1 +=  c0 >> 51;
        let mut c0: u64 = (c0 as u64) & low_51_bit_mask;
        c2 +=  c1 >> 51;
        let c1: u64 = (c1 as u64) & low_51_bit_mask;
        c3 +=  c2 >> 51;
        let c2: u64 = (c2 as u64) & low_51_bit_mask;
        c4 +=  c3 >> 51;
        let c3: u64 = (c3 as u64) & low_51_bit_mask;
        c0 += ((c4 >> 51) as u64) * 19;
        let c4: u64 = (c4 as u64) & low_51_bit_mask;

        // Now all c_i fit in u64; reduce again to enforce c_i < 2^51
        FieldElement64::reduce([c0,c1,c2,c3,c4])
    }
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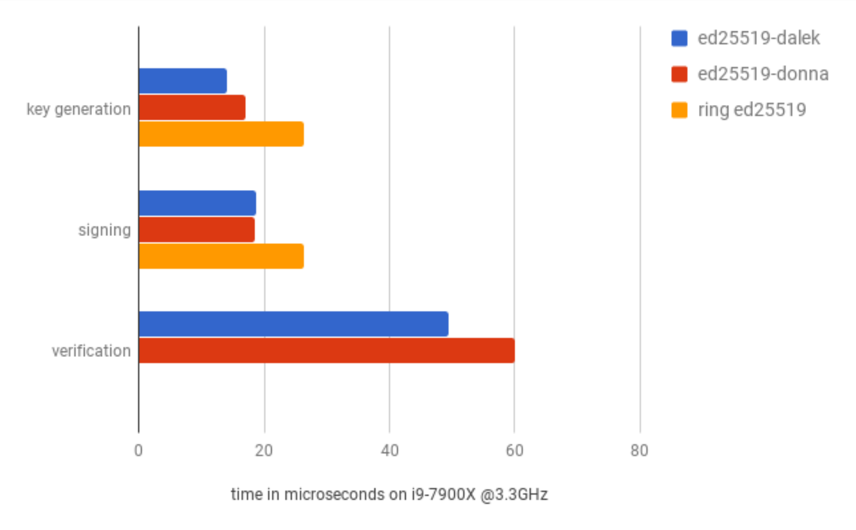
```

And... except for some comments and debug assertions, that's essentially the implementation we use!

How fast is it?



Rust features we love, and features we want to improve

Constant-time code and LLVM

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In the future, we'd like to do CI testing of the generated binaries: Rust, but verify.

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  Basic idea: statically track the sizes of intermediate values, and use specialization to insert reductions only when necessary.

Implementing crypto with `-dalek`

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This creates a `dleq` module with all the code for creating and verifying these proof statements, using Serde to convert to/from wire format.

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Basic idea: to prove $x \in [0, b^n]$, write x in base b as $x = \sum_{i=0}^{n-1} x_i b^i$, and prove that each digit is in range: $x_i \in [0, b]$.

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We implemented the Back-Maxwell rangeproof, which uses $b = 3$ and shares data between digits to save space.

Implementing rangeproofs with -dalek: (partial) code

```
// mi_H[i] = m^i * H = 3^i * H in the loop below, construct these serially here:
let mut mi_H = vec![*H; n];
let mut mi2_H = vec![*H; n];
for i in 1..n {
    mi2_H[i-1] = &mi_H[i-1] + &mi_H[i-1];
    mi_H[i] = &mi_H[i-1] + &mi2_H[i-1];
}
mi2_H[n-1] = &mi_H[n-1] + &mi_H[n-1];

// Need to collect into a Vec to get par_iter()
let indices: Vec<_> = (0..n).collect();
let compressed_Ris: Vec<_> = indices.par_iter().map(|j| {
    let i = *j;

    let Ci_minus_miH = &self.C[i] - &mi_H[i];
    let P = vartime::multiscalar_mult(&[self.s_1[i], -&self.e_0], &[G, Ci_minus_miH]);
    let ei_1 = Scalar::hash_from_bytes::<Sha512>(P.compress().as_bytes());

    let Ci_minus_2miH = &self.C[i] - &mi2_H[i];
    let P = vartime::multiscalar_mult(&[self.s_2[i], -&ei_1], &[G, Ci_minus_2miH]);
    let ei_2 = Scalar::hash_from_bytes::<Sha512>(P.compress().as_bytes());

    let Ri = &self.C[i] * &ei_2;

    Ri.compress()
}).collect();
```

Thank you!

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