MAE190: Design Of A Shaft Subjected To Alternating Loads

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1 Introduction

Designing a shaft, though simple at first glance, has a lot of nuances. From the loading the shaft will experience, the material, multiple desired factors, and environments, multiple parameters need to be considered in its design. Along with this, calculating the shaft's diameter itself with all the parameters required is a tedious and troublesome process that could lead to errors due to the iterative nature of the calculation.

In order to streamline this process in a smooth and efficient manner, an optimization code can be created that could receive user-inputted parameters and calculate the minimum shaft diameter within less of a second.

This, however, does provide some challenges with the multiple parameters required for shaft design, and the intricacies of some of these parameters, such as a different material specifications and multiple surface finishes.

The shaft optimization code presented in this report allows for a convenient, flexible, and robust experience for the user. Due to the UI nature of the code, the user is able to quickly change parameters and run the analysis without the need to input the same values over and over again.

Additionally, the code provides a variety of options for the user to where they will be able to experiment and analyze a plethora of situations to find what combination of parameters they would need for their shaft design.

To implement this code, the following equations are used:

Equations 1-2. *Alternating and Mean Moment and Torque Equations*

$$M_a = \frac{M_{max} - M_{min}}{2} \qquad M_m = \frac{M_{max} + M_{min}}{2} \tag{1}$$

where M_a and M_m are the alternating and mean moments.

$$T_a = \frac{T_{max} - T_{min}}{2} \qquad T_m = \frac{T_{max} + T_{min}}{2} \tag{2}$$

where M_a and M_m are the alternating and mean torques.

Equations 3-8. *Stress-Concentration Factor Equations*

$$K_t \cong A(\frac{r}{d})^b \tag{3}$$

where K_t is the geometric stress-concentration factor, and A and b are arbitrary variables dependent on $\frac{D}{d}$.

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \tag{4}$$

$$\sqrt{a} = 1.24 - 2.25(10^{-3})S_{ut} + 1.60(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \tag{5}$$

where $\sqrt{a}$ is the Neuber constant in English and Metric units, respectively, for bending or axial loading.

$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3 \tag{6}$$

$$\sqrt{a} = 0.958 - 1.83(10^{-3})S_{ut} + 1.43(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \tag{7}$$

where $\sqrt{a}$ is the Neuber constant in English and Metric units, respectively, for torsion loading.

$$K_f = 1 + \frac{K_t - 1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} \tag{8}$$

where K_f is the stress concentration factor.

Equations 9-15. *Marin Factor Equations*

$$k_a = aS_{ut}^b \tag{9}$$

where k_a is the surface factor, and a and b are arbitrary variables dependent on material surface finish.

$$k_b = 0.879d^{-0.107} \tag{10}$$

$$k_b = 0.91d^{-0.157} \tag{11}$$

where k_b is the size factor in English units, where $0.3 \leq$ d ≤ 2 inches and $2 <$ d ≤ 10 inches, respectively.

$$k_b = 1.24d^{-0.107} \tag{12}$$

$$k_b = 1.51d^{-0.107} \tag{13}$$

where k_b is the size factor in Metric units, where $7.62 \leq$ d ≤ 51 mm and $51 <$ d ≤ 254 mm, respectively.

$$k_d = 0.98 + 3.5(10^{-4})T_F - 6.3(10^{-7})T_F^2 \tag{14}$$

$$k_d = 0.98 + 5.9(10^{-4})T_C - 2.1(10^{-6})T_C^2 \tag{15}$$

where k_d is the temperature factor for English and Metric units, respectively.

Equations 16-17. *Endurance Limit Equations*

$$S'_e = 0.5S_{ut} \tag{16}$$

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{17}$$

where S'_e is the rotary-beam test specimen endurance limit and S_e is the endurance limit at the critical location

Equations 18-23. *Fatigue Failure Criteria Equations*

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2} \tag{18}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2} \tag{19}$$

$$d_{Goodman} = (\frac{16n}{\pi}(\frac{A}{S_e} + \frac{B}{S_{ut}}))^{\frac{1}{3}} \tag{20}$$

$$d_{Gerber} = (\frac{8nA}{\pi S_e}(1 + \sqrt{1 + (\frac{2BS_e}{AS_{ut}})^2}))^{\frac{1}{3}} \tag{21}$$

$$d_{ASME} = (\frac{16n}{\pi}\sqrt{4(\frac{K_f M_a}{S_e})^2 + 3(\frac{K_{fs} T_a}{S_e})^2 + 4(\frac{K_f M_m}{S_y})^2 + 3(\frac{K_{fs} T_m}{S_y})^2})^{\frac{1}{3}} \tag{22}$$

$$d_{SWT} = (\frac{16n}{\pi}\sqrt{(A^2 + AB)})^{\frac{1}{3}} \tag{23}$$

2 Results

The code was designed to emulate a UI, where the user is able to select different options to define or change the main parameters before running the optimization code. This allows for more flexibility for the user in case they make a mistake in inputting values without needing to reset the script. Though this design did provide a lot of challenges in terms of troubleshooting, the final product is effective and efficient while providing convenience to the user.

This section of the report will go through the specific subsections of the code and explain how each section works, along with some insight on their respective design choices.

2.1 Define Units and Main Menu

The **Define Units** section of the code asks the user which units they would like to do the optimization in—that being either English or Metric units.

Once selected, they will be brought to the **Main Menu**, where they will be able to select one of nine options, which will be elaborated upon later. Each of these sections have been defined as a specific function that the main menu script will call based on the user input.

After each section has been completed, the user will be brought to the **Main Menu**, where they will see the values they have inputted or selected, and can choose to either proceed to another section or redo a section.

The user is also able to exit the script at any time, where, if they inputted any parameters, will show a summary of all the parameters they have defined within the script.

2.2 Define Moment and Torque

The **Define Moments and Torques** section prompts the user to input two bending moments and two torques in any order they would like, as the code will automatically sort all the inputs as either the maximum or the minimum of each force types.

2.3 Define Material

The **Define Material** section prompts the user to select a material from a list of steels, such as carbon steels, titanium alloys, and aluminum alloys.

The user is able to select their desired material, which will then lead to another prompt where they will be asked which specification they would like, such as the chemistry range and limits for carbon steels (i.e. AISI 1010, AISI 1035), and such.

Once selected, the user will be prompted regarding which surface finish they would like. For majority of the carbon steels, they will be prompted whether or not their steel is either Hot-Rolled (HR) or Cold-Drawn (CD). As for the titanium alloys, they will be prompted whether or not their steel is either HR, CD, Machined, Polished, or As-Forged.

The aluminum alloys, however, are an optional material that would not provide much with the script due to the inherent lack of endurance limits found in aluminum. This, in turn, will allow the user to select the aluminum alloy but the optimization script will not work with this material. These materials were still left within the code just in case the user wants to know the strength properties of these aluminum alloys.

The strength properties of each material are saved within a matrix designated for each material and indexed through the material specification, surface finish, and units selected earlier in the **Define Units** section.

Finally, for flexibility, the user could also input a custom material, along with its yield and tensile strengths, and the material's surface finish.

2.4 Define Fatigue Factor of Safety

The **Define Fatigue Factor of Safety** section prompts the user to input their desired fatigue factor of safety. This value will be used later on to calculate the minimum shaft diameter, but will also be used later to check for yielding.

2.5 Define Reliability Factor

The **Define Reliability Factor** section prompts the user to input their desired reliability percentage. This value will be used to determine the reliability factor, which is stored in a matrix along with the percentages and will be called in the code via indexing.

2.6 Define Temperature Factor

The **Define Temperature Factor** section, one of the additional sections besides the requirements of the project, prompts the user to input their desired temperature, depending on which units they selected in the **Define Units** section. This value will be used to determine the temperature factor, which will be calculated later on in the **Shaft Optimization** section.

2.7 Define Shaft Geometry

The **Define Shaft Geometry** section prompts the user to input their desired diameter and fillet radius-diameter ratios (D/d and r/d, respectively). This value will be used to determine the stress concentration factors, which will be calculated later on in the **Shaft Optimization** section.

2.8 Define Preset Parameters

The **Define Preset Parameters** section, once selected, will overwrite all the other parameters of the code to the parameter values found in the **Proof of Calculation** section.

2.9 Shaft Optimization

The **Shaft Optimization** section is where the bulk of the calculations occur, such as the Marin Factors, the alternating and mean moments and torques, and others.

Firstly, if not all of the parameters have been defined, selecting this section will produce an error message asking the user to ensure that all parameters have been defined. Additionally, if your defined material is an aluminum alloy, selecting this section will produce an error message asking the user to select a different material.

Once all parameters have been defined, the user will be prompted with which fatigue failure design criteria they would like to use (Goodman, Gerber, ASME Elliptic, and SWT).

Once selected, the user will be able to input their desired convergence limit in %. Though this value might not be familiar with the user, the convergence factor should be below 1%.

Once all of these values have been selected, the script will calculate the Alternate and Mean Moments and Torques, the Stress Concentration Factors, the Marin Factors, the Endurance Limits, and then finally the shaft diameter.

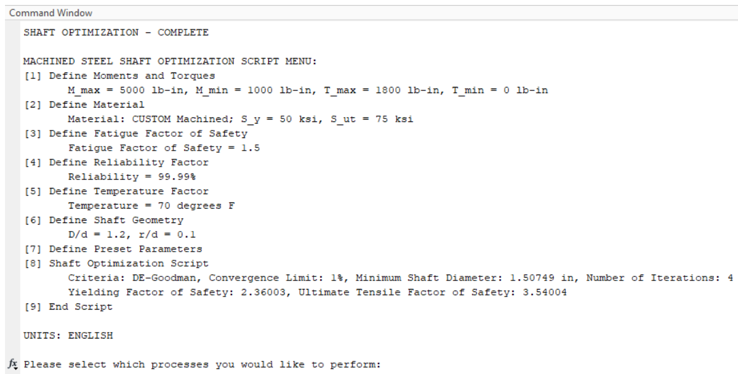
The code, then, undergoes an iterative process until the % error between each iteration's diameters is less than the desired convergence limit the user has defined earlier on in the section. This iterative process uses the same equations for the endurance limits, the size and stress concentration factors, and shaft diameters.

Along with this, the iterative process contains a safety check for the shaft through iterative calculations of the yielding and ultimate factors of safety. If either of these factors of safety, more commonly the yielding factor of safety, is less than the previously defined fatigue factor of safety, then the main menu will specify that the design will yield and is not viable.

Once the shaft optimization has been completed, the user will be brought back to the **Main Menu**, where the user will be able to see the minimum diameter, their chosen failure criteria and convergence limit, number of iterations, and the yielding and ultimate factors of safety (this will be shown in the discussion section of the report).

3 Discussion

The script was first tested with the parameters found in the Proof of Concept calculations.

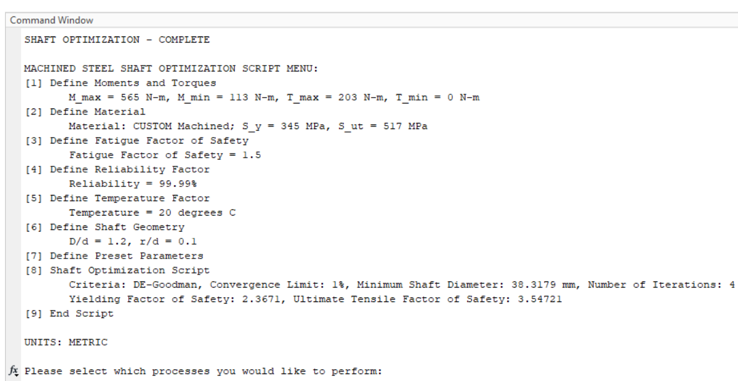


```
Command Window
SHAFT OPTIMIZATION - COMPLETE

MACHINED STEEL SHAFT OPTIMIZATION SCRIPT MENU:
[1] Define Moments and Torques
      M_max = 5000 lb-in, M_min = 1000 lb-in, T_max = 1800 lb-in, T_min = 0 lb-in
[2] Define Material
      Material: CUSTOM Machined; S_y = 50 ksi, S_ut = 75 ksi
[3] Define Fatigue Factor of Safety
      Fatigue Factor of Safety = 1.5
[4] Define Reliability Factor
      Reliability = 99.99%
[5] Define Temperature Factor
      Temperature = 70 degrees F
[6] Define Shaft Geometry
      D/d = 1.2, r/d = 0.1
[7] Define Preset Parameters
[8] Shaft Optimization Script
      Criteria: DE-Goodman, Convergence Limit: 1%, Minimum Shaft Diameter: 1.50749 in, Number of Iterations: 4
      Yielding Factor of Safety: 2.36003, Ultimate Tensile Factor of Safety: 3.54004
[9] End Script

UNITS: ENGLISH

fx Please select which processes you would like to perform:
```

Figure 1: Main Menu Command Window - Proof of Concept (English) Verification.

The minimum shaft diameter the optimization script calculated is fairly similar to the diameter calculated from the proof of concept calculations. This difference is caused by the difference in the stress concentration factors. The proof of concept calculation simply assumes a K_t and K_{ts} for a $\frac{D}{d}$ of 1.2 and $\frac{r}{d}$ of 0.1. However, the optimization code uses Equation 3 found in Section 1, with variable $\frac{D}{d}$ and $\frac{r}{d}$.

The script was also tested with these same parameters, but in metric units in order to validate that the conversions are correct.



```
Command Window
SHAFT OPTIMIZATION - COMPLETE

MACHINED STEEL SHAFT OPTIMIZATION SCRIPT MENU:
[1] Define Moments and Torques
      M_max = 565 N-m, M_min = 113 N-m, T_max = 203 N-m, T_min = 0 N-m
[2] Define Material
      Material: CUSTOM Machined; S_y = 345 MPa, S_ut = 517 MPa
[3] Define Fatigue Factor of Safety
      Fatigue Factor of Safety = 1.5
[4] Define Reliability Factor
      Reliability = 99.99%
[5] Define Temperature Factor
      Temperature = 20 degrees C
[6] Define Shaft Geometry
      D/d = 1.2, r/d = 0.1
[7] Define Preset Parameters
[8] Shaft Optimization Script
      Criteria: DE-Goodman, Convergence Limit: 1%, Minimum Shaft Diameter: 38.3179 mm, Number of Iterations: 4
      Yielding Factor of Safety: 2.3671, Ultimate Tensile Factor of Safety: 3.54721
[9] End Script

UNITS: METRIC

fx Please select which processes you would like to perform:
```

Figure 2: Main Menu Command Window - Proof of Concept (Metric) Verification.

Then, the script was tested with the same parameters EXCEPT with a different material: AISI 1018 CD.

```
Command Window
SHAFT OPTIMIZATION - COMPLETE

MACHINED STEEL SHAFT OPTIMIZATION SCRIPT MENU:
[1] Define Moments and Torques
      M_max = 5000 lb-in, M_min = 1000 lb-in, T_max = 1800 lb-in, T_min = 0 lb-in
[2] Define Material
      Material: AISI 1018 CD; S_y = 54 ksi, S_ut = 64 ksi
[3] Define Fatigue Factor of Safety
      Fatigue Factor of Safety = 1.5
[4] Define Reliability Factor
      Reliability = 99.99%
[5] Define Temperature Factor
      Temperature = 70 degrees F
[6] Define Shaft Geometry
      D/d = 1.2, r/d = 0.1
[7] Define Preset Parameters
[8] Shaft Optimization Script
      Criteria: DE-Goodman, Convergence Limit: 1%, Minimum Shaft Diameter: 1.57893 in, Number of Iterations: 4
      Yielding Factor of Safety: 2.92156, Ultimate Tensile Factor of Safety: 3.46259
[9] End Script

UNITS: ENGLISH

fx Please select which processes you would like to perform:
```

Figure 3: Main Menu Command Window - Proof of Concept (AISI 1018 CD) Verification.

Due to the smaller ultimate tensile strength, using AISI 1018-CD requires a larger diameter to withstand the same bending and torsion loading as the custom material aforementioned. This also results in a smaller tensile safety factor, while the bigger yield strength results in a bigger yield safety factor.

4 Conclusion

The shaft optimization code provides a lot of flexibility for the user, as the UI design of the code makes it easy for the user to compare the diameters for different parameters without having to re-input all of the parameters again. The code is also robust enough to where the user can change a lot of parameters, such as the shaft geometry and temperature factor (which are not included in the requirements for the project), and are able to create multiple sets of parameters to analyze.

The code, however, still has some improvements that could be made. Currently, the code only takes bending and torsion loading and not axial loading. Implementing this could allow for more combinations of forces, along with forcing other parameters of the code to be a lot more robust.

An example of this would be the load factor that is present with the endurance limit equation. Currently, the load factor is set to be equal to 1, assuming that both bending and torsion loading are applied to the shaft. Once you introduce axial loading, the value for the load factor might change. Even without the presence of axial loading, the load factor should change when only one of either bending or torsion moment is applied on the analyzed shaft.

Another consideration for improvements would be to implement a larger library of materials to be used in the code itself. This would inherently provide more options for the user on which material they would like to use for the shaft.

In conclusion, the optimization code as it is already provides a lot of flexibility and robustness for the user while remaining convenient, but still has a lot of room for improvements and modifications.