Project Report
Writing Robotic Manipulator

By

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Acknowledgement

I thank professor Chad Kessens for providing me an opportunity to work on this robot modelling project at University of Maryland College Park.

This Project was completed successfully with constant input and support from professor. This project helped me in building Modelling skills. This project gave me good intuition on how to simulate robots.

I am thankful to and fortunate enough to get constant support from the Ta’s and whose guidance lead to successful Project completion.

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Abstract

The Robotic Manipulator used was 6 Degrees of freedom arm. The Robot has a spherical wrist. The robot has six revolute joints and no prismatic joint was used in the robot. The main objective of this project was to perform a writing task with a robotic manipulator. The robot is designed using Peter Corke Toolbox. The end effector of the robot is a gripper which can hold pen (or any other writing element). The gripper is not incorporated in the robot design.

The letters in English are given as cartesian trajectories to the robot, the robot traces this trajectory by performing inverse kinematics on each of the point on the trajectory. The joint configuration of the robot is obtained for each point on the trajectory and this set of joint configurations is used to simulate the robot. The simulation of the manipulator is done in MATLAB environment. The robot was tested and validated for different joint configurations, and end effector orientations, both the forward and inverse kinematics of the robot are implemented.

Simulation results are provided with video of the simulation. The robot was validated by matching the ground truth values i.e the commanded robot pose and obtained pose.

Introduction

My main motivation for this project comes from my interest in making robots more autonomous and aiding humans not only in the repetitive and continuous tasks such as palletizing in warehouses, where human intervention is least required but also in performing complicated tasks such as surgery with ease. Today the manipulators are so robust and dextrous that they are used to perform surgery. Many surgeries such as brain surgery, cancer tumour treatment now a days are performed by Robotic manipulators. This motivated me to do this project that is making a 6 dof manipulator to write.

The report initially describes the assumptions that were made for the robot, Then a complete description of forward kinematics is provided with the transformation matrices that take us from one link to the other link, and also a transformation matrix is provide that represents end effector pose in base frame. This was also implemented in matlab.

Next part of report deals with the inverse kinematics problem of the manipulator which is very core to this project. Both approaches that is geometrical approach and numerical approach were described and used. For faster computation and smooth to get smooth tracing of curve numerical approach was preferred. Validation and testing is showed in this section.

The last section of this report describes the simulation of the robot, and simulation results are provided.

Assumptions

The links of the robot are rigid, The robot is firmly fixed to the ground.

The links are assumed to be fixed masses and mass of the links are not considered as I am not performing dynamics of the robot.

The robot links and joints are assumed to be frictionless.

The environment of the robot is not dynamic (i.e. the environment of the robot does not change with time.)

Spherical wrist is used in the robot, The robot joints have no limits on velocity.

The orientation of the end effector is considered to be fixed while computing inverse kinematics. There is only change in position of the end effector.

The robots writing task is only performed in yz plane (world frame) of when viewed from robots base frame.

Its assumed there is no obstacles in the path of the robot, no path planning is being implemented in this project.

The assumption of spherical wrist means that the three joint axis that form spherical wrist must intersect at the wrist centre.

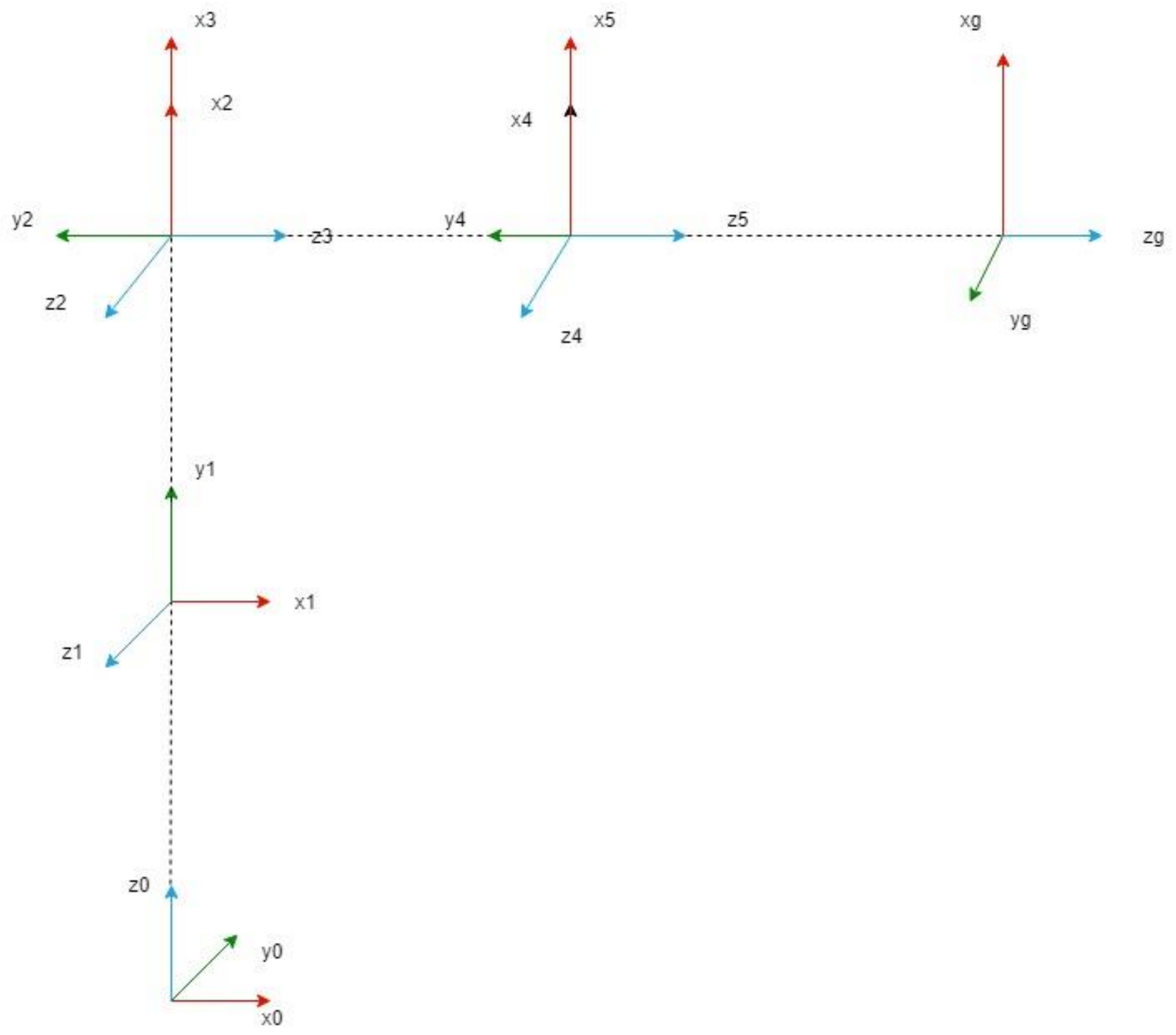
The gripper is assumed to be attached to end of the manipulator no physical appearance of the gripper is shown.

Joint limits are not specified for the robot only to prevent self-Collison joint 2 and joint 3 was constrained to move from 0 to 180 degrees, The length of the links are assumed to be fixed.

The trajectories given to robot are assumed to be smooth and continuous. No consideration of jerk (i.e it is assumed to be jerk free trajectories).

Forward kinematics

The frames of robot are positioned as shown in the image below.



(x0,y0,z0)------ represents robot base frame

(xg,yg,zg)------- represents robot end effector frame

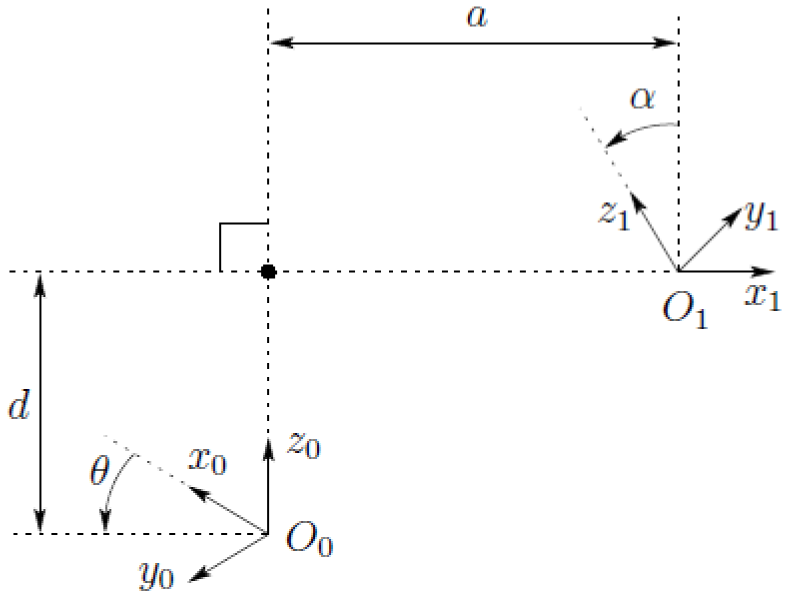
DH table:

DH convention is followed in writing the frames of robot.

Which says the following two conditions have to be met i.e

1) The x axis of the next joint x must be perpendicular to the previous z axis

2) The x axis must also intersect with the previous z axis.



DH parameters for my robot

Link	Theta	d	a	alpha
1	θ_1 +0	17	0	$\pi/2$
2	$\theta_2+\pi/2$	0	21	0
3	θ_3+0	0	0	$\pi/2$
4	θ_4+0	25	0	$-\pi/2$
5	θ_5 +0	0	0	$\pi/2$
6	θ_6 +0	25	0	0

Performing forward kinematics:

General Transformation matrix for dh parameters is found by

$$T_1^0 = Rot_{z,\theta} * Trans_{z,d} * Trans_{x,a} * Rot_{x,\alpha}$$

This gives frame 1 represented in zero frame.

initial configuration is all thetas zero, the forward kinematics is as follows.

$$\text{A01} = \begin{pmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A12 = \begin{pmatrix} \cos\left(\theta_2+\frac{\pi}{2}\right) & -\sin\left(\theta_2+\frac{\pi}{2}\right) & 0 & 21\cos\left(\theta_2+\frac{\pi}{2}\right) \\ \sin\left(\theta_2+\frac{\pi}{2}\right) & \cos\left(\theta_2+\frac{\pi}{2}\right) & 0 & 21\sin\left(\theta_2+\frac{\pi}{2}\right) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A23 = \begin{pmatrix} \cos(\theta_3) & 0 & \sin(\theta_3) & 0 \\ \sin(\theta_3) & 0 & -\cos(\theta_3) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A34 = \begin{pmatrix} \cos(\theta_4) & 0 & -\sin(\theta_4) & 0 \\ \sin(\theta_4) & 0 & \cos(\theta_4) & 0 \\ 0 & -1 & 0 & 25 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A45 = \begin{pmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) & 0 \\ \sin(\theta_5) & 0 & -\cos(\theta_5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A56 = \begin{pmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ \sin(\theta_6) & \cos(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & 25 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where thetas are joint angles, The Transformation matrix from end effector to base frame is given as follows.

$$T_g^0 = A_1^0 * A_3^2 * A_4^3 * A_5^4 * A_6^5$$

It is given in image below.

$$
\begin{pmatrix}
\sin(\theta_6)\,\sigma_3 + \cos(\theta_6)\,\sigma_2 & \cos(\theta_6)\,\sigma_3 - \sin(\theta_6)\,\sigma_2 & \sin(\theta_5)\,\sigma_8 + \cos(\theta_5)\,\sigma_9 & 21\cos(\theta_1)\,\sigma_{14} + 25\sin(\theta_5)\,\sigma_8 + 25\cos(\theta_5)\,\sigma_9 + 25\cos(\theta_1)\cos(\theta_3)\,\sigma_{15} + 25\cos(\theta_1)\,\sigma_{14}\sin(\theta_3) \\
-\sin(\theta_6)\,\sigma_4 - \cos(\theta_6)\,\sigma_1 & \sin(\theta_6)\,\sigma_1 - \cos(\theta_6)\,\sigma_4 & \cos(\theta_5)\,\sigma_7 - \sin(\theta_5)\,\sigma_6 & 21\,\sigma_{14}\sin(\theta_1) - 25\sin(\theta_5)\,\sigma_6 + 25\cos(\theta_5)\,\sigma_7 + 25\cos(\theta_3)\sin(\theta_1)\,\sigma_{15} + 25\,\sigma_{14}\sin(\theta_1)\sin(\theta_3) \\
\cos(\theta_6)\,\sigma_5 - \sin(\theta_4)\sin(\theta_6)\,\sigma_{10} & -\sin(\theta_6)\,\sigma_5 - \cos(\theta_6)\sin(\theta_4)\,\sigma_{10} & \cos(\theta_4)\sin(\theta_5)\,\sigma_{10} - \cos(\theta_5)\,\sigma_{11} & 21\,\sigma_{15} - 25\cos(\theta_3)\,\sigma_{14} + 25\sin(\theta_3)\,\sigma_{15} - 25\cos(\theta_5)\,\sigma_{11} + 25\cos(\theta_4)\sin(\theta_5)\,\sigma_{10} + 17 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

where

$$\sigma_1 = \cos(\theta_5)\,\sigma_6 + \sin(\theta_5)\,\sigma_7$$

$$\sigma_2 = \cos(\theta_5)\,\sigma_8 - \sin(\theta_5)\,\sigma_9$$

$$\sigma_3 = \cos(\theta_4)\sin(\theta_1) + \sin(\theta_4)\,\sigma_{13}$$

$$\sigma_4 = \cos(\theta_1)\cos(\theta_4) + \sin(\theta_4)\,\sigma_{12}$$

$$\sigma_5 = \sin(\theta_5)\,\sigma_{11} + \cos(\theta_4)\cos(\theta_5)\,\sigma_{10}$$

$$\sigma_6 = \cos(\theta_1)\sin(\theta_4) - \cos(\theta_4)\,\sigma_{12}$$

$$\sigma_7 = \cos(\theta_3)\sin(\theta_1)\,\sigma_{15} + \sigma_{14}\sin(\theta_1)\sin(\theta_3)$$

$$\sigma_8 = \sin(\theta_1)\sin(\theta_4) - \cos(\theta_4)\,\sigma_{13}$$

$$\sigma_9 = \cos(\theta_1)\cos(\theta_3)\,\sigma_{15} + \cos(\theta_1)\,\sigma_{14}\sin(\theta_3)$$

$$\sigma_{10} = \cos(\theta_3)\,\sigma_{15} + \sigma_{14}\sin(\theta_3)$$

$$\sigma_{11} = \cos(\theta_3)\,\sigma_{14} - \sin(\theta_3)\,\sigma_{15}$$

$$\sigma_{12} = \cos(\theta_3)\,\sigma_{14}\sin(\theta_1) - \sin(\theta_1)\sin(\theta_3)\,\sigma_{15}$$

$$\sigma_{13} = \cos(\theta_1)\sin(\theta_3)\,\sigma_{15} - \cos(\theta_1)\cos(\theta_3)\,\sigma_{14}$$

$$\sigma_{14} = \cos\left(\theta_2 + \frac{\pi}{2}\right)$$

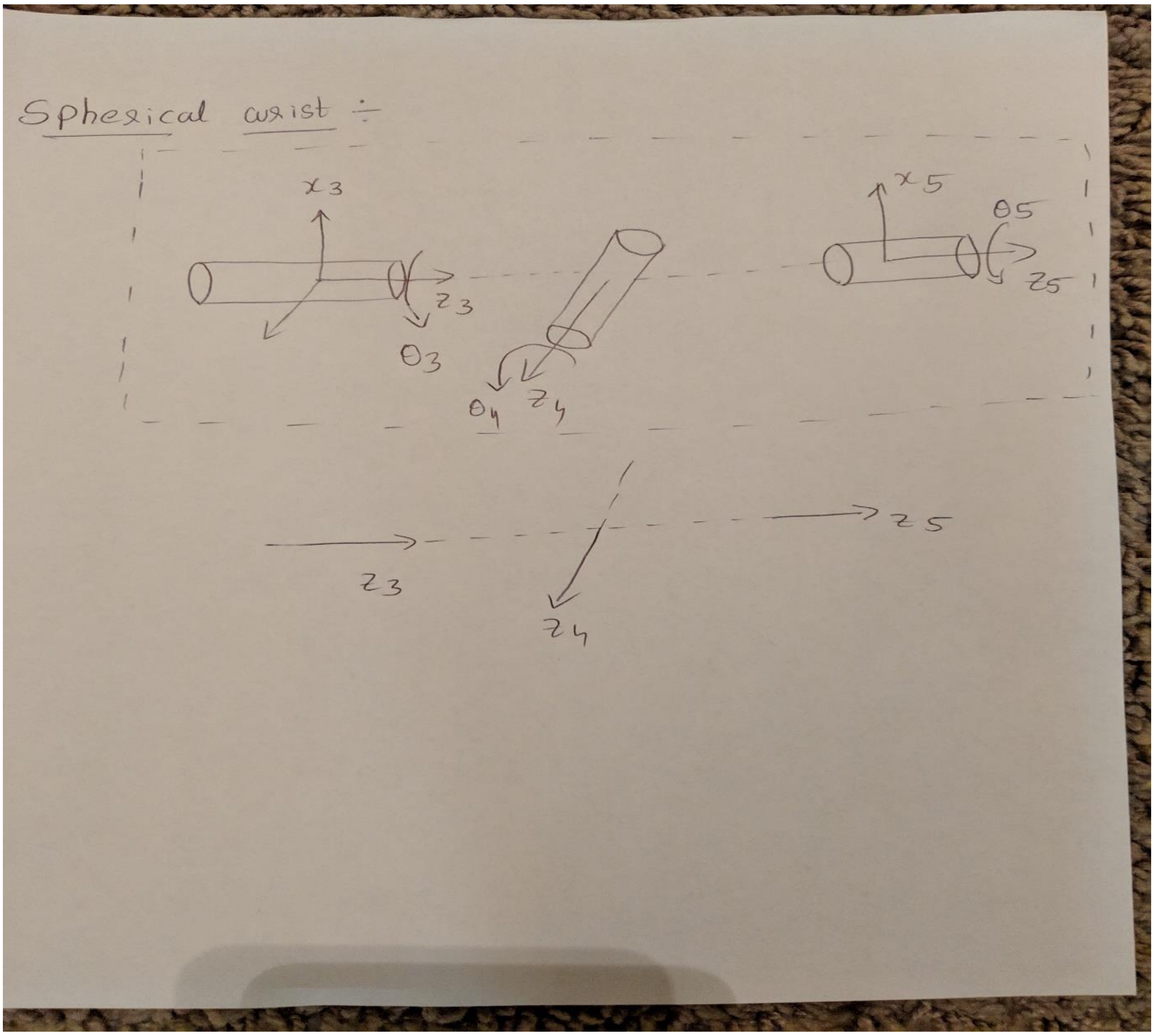
$$\sigma_{15} = \sin\left(\theta_2 + \frac{\pi}{2}\right)$$

Inverse Kinematics

Geometrical approach of Inverse kinematics is done only for one end effector pose, and this inverse kinematics is not used in simulation as I have multiple points, So I used the inbuilt ik solver which is numerical approach and was more efficient and gave me results in seconds. The inbuilt ik solver uses numerical approach called BFGS algorithm which is gradient descent-based algorithm.

Also the solved inverse kinematics gave correct result but it was computationally inefficient. Here given below is the geometrical approach for solving inverse kinematics of my robot.

The robot has spherical wrist with ZYZ configuration. As it can be observed from the robot.



As we can see from the image above which shows the spherical wrist of my robot. The three joint axis of the wrist coincide at point. This point is called wrist center. The wrist center is at a distance from the gripper positon. Since this is robot with six degrees of freedom and spherical wrist. We can do kinematic decoupling that simplifies our problem in doing inverse position kinematics, and inverse orientation kinematics.

$$\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} o_x - d_6 r_{13} \\ o_y - d_6 r_{23} \\ o_z - d_6 r_{33} \end{bmatrix}$$

Where d6 is the distance between the wrist centre and gripper frame. In our case d6=25.

r13,r23,r33 are obtained from the end effector pose which is given.

Now that wrist centre position is fixed that is position of the end effector fixed. The orientation of the end effector is determined by the first three joint variables that is theta1,theta2and theta3 in our case.

Below is the calculation for finding theta1 theta2 theta3(please see next page).

Here L0=17; L1=21; L2=25 ; xc, yc, zc are the coordinates of the wrist centre obtained from the method shown previous.Now we know the values of theta1 theta2 theta3 we can calculate the other theta values by using this formula.

$$R = R_3^0 R_6^3$$

Here R is known orientation of end effector in base frame, we know R03 from theta1 to theta3 values,Now we compare R36 with the euler angle matrix for spherical wrist given as follows.

Inverse Orientation:

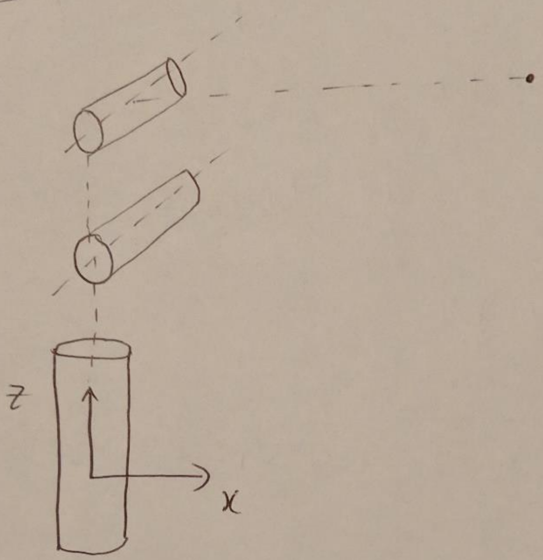
$$R_6^3 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}$$

$$\theta_5 = \text{atan2}\left(s_1r_{13} - c_1r_{23}, \pm\sqrt{1 - (s_1r_{13} - c_1r_{23})^2}\right)$$

$$\begin{aligned} \theta_4 &= \text{atan2}(c_1c_{23}r_{13} + s_1c_{23}r_{23} + s_{23}r_{33}, \\ &\qquad -c_1s_{23}r_{13} - s_1s_{23}r_{23} + c_{23}r_{33}) \\ \theta_6 &= \text{atan2}(-s_1r_{11} + c_1r_{21}, s_1r_{12} - c_1r_{22}) \end{aligned}$$

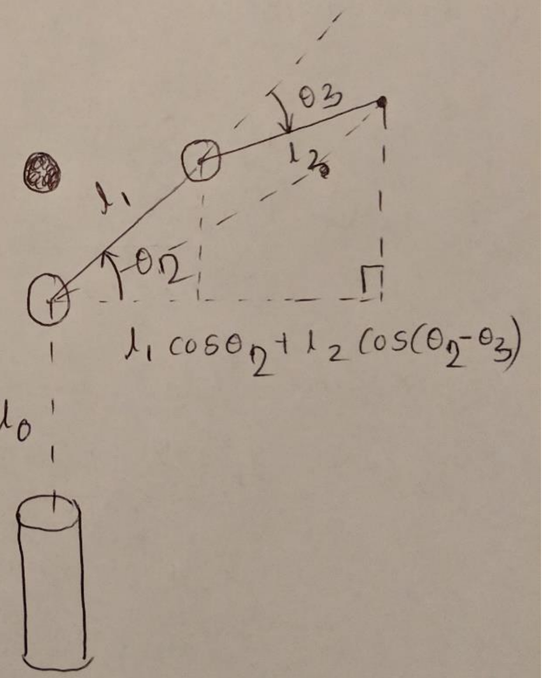
Inverse Position:

Inverse Kinematics Geometric approach :



this is robot configuration

Now Viewing side view :

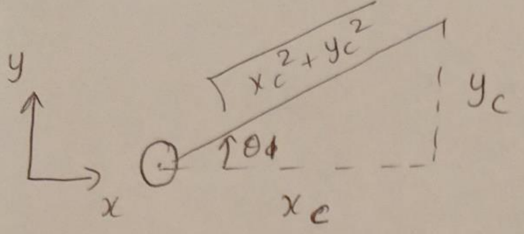


$$z_c = l_0 + l_1 \sin\theta_2 + l_2 \sin(\theta_2 - \theta_3)$$

we know z_c, l_0, l_1, l_2

Now consider the top view of the robotic manipulator.

top view



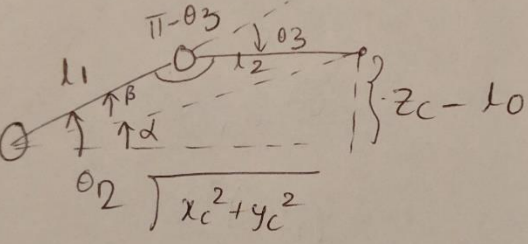
$\theta_\phi = \tan^{-1}\left(\frac{y_c}{x_c}\right)$

we know x_c, y_c so

we found one Joint variable 'θ_1'.

$\theta_1 = a\tan 2(x_c, y_c)$

we know hypotenuse $\sqrt{x_c^2+y_c^2} = l_1\cos\theta_2 + l_2\cos(\theta_2-\theta_3)$



$\alpha = \tan^{-1}\left(\frac{z_c - l_0}{\sqrt{x_c^2+y_c^2}}\right)$ //

using cosine law we get

$$\cos\beta = \frac{l_1^2 + x_c^2 + y_c^2 + (z_c - l_0)^2 - l_2^2}{2 l_1 \sqrt{x_c^2+y_c^2+(z_c-l_0)^2}} //$$

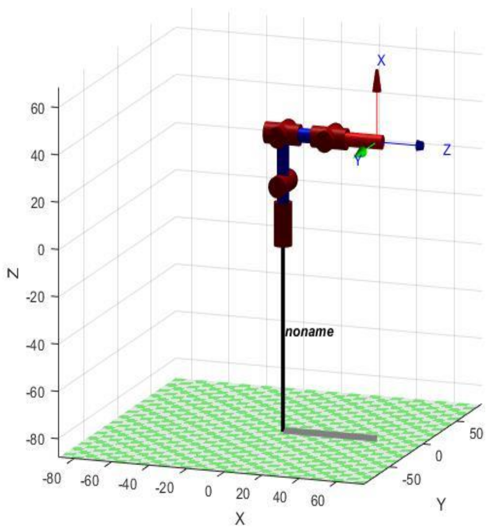
$\theta_2 = \alpha + \beta$ //

$$\cos(\pi - \theta_3) = \frac{l_1^2 + l_2^2 - \left[(x_c^2+y_c^2) + (z_c - l_0)^2\right]}{2 l_1 l_2} //$$

from this we get θ_3 //

Simulation

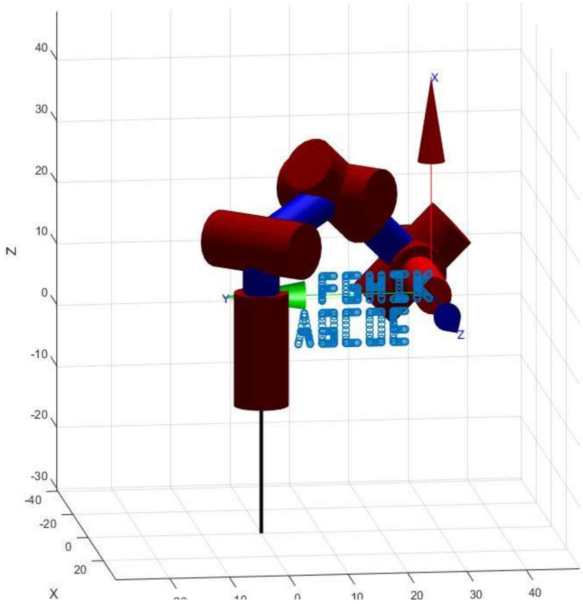
The simulation was performed in matlab using peter corke toolbox. Robot was made by using serial link command which uses the DH parameters of the robot to form robot's structure. To run the simulation I have to run peter corke toolbox so that I would be able to use the functions from the toolbox. The robot obtained is shown in the figure below



The robot is the given the trajectory as series of points each point is in the form of pose transformation matrix which gives the location of each point and its orientation in base frame. As its assumed the orientation of end effector remains fixed for all points only the translational part of the matrix is varying. The trajectory is obtained by using cartesian trajectory generator "ctraj" which uses the pose of initial point and final point to obtain trajectory between those two points. The alphabets that robot has to write is given as series of trajectories.

Next step since we know the position of each of these points in the robots base frame, now we need to get joint angles of the robot to make robot trace the trajectory. The joint angles are obtained by doing inverse kinematics on each of these points.

Now the robot knows the joint angles it moves end effector accordingly and traces the letters. This was accomplished successfully, and the simulation result is shown below.



As you can see in the above result the robot has written the alphabets(A,B,C,D,E,F,G,H,I,K).The robot traces these letters in ZY plane of the robots base frame.

The robot was able to successfully trace these alphabets without any collision with its own body. Here is the video link attached below that shows the simulation of robot.

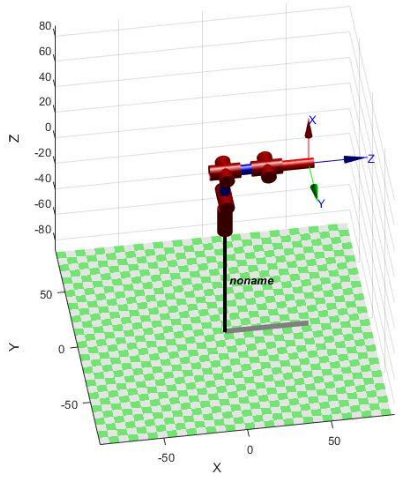
https://youtu.be/Mmkt_qAHH18

Validation and Testing:

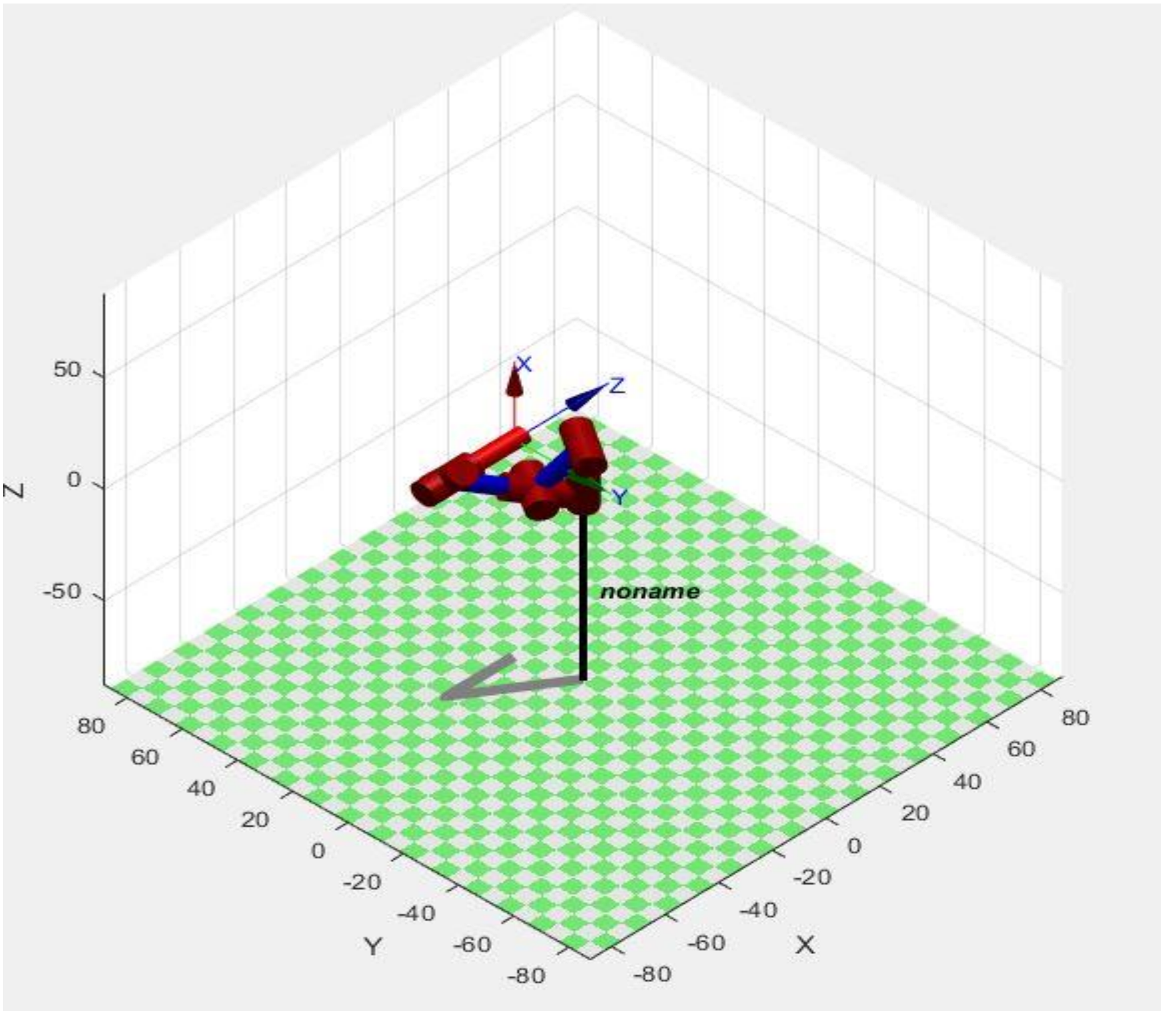
The robot was able to achieve commanded position and commanded orientation. The results are given below.

When commanded position without change in orientation of end effector.

Initial configuration of robot:



Final configuration of robot:



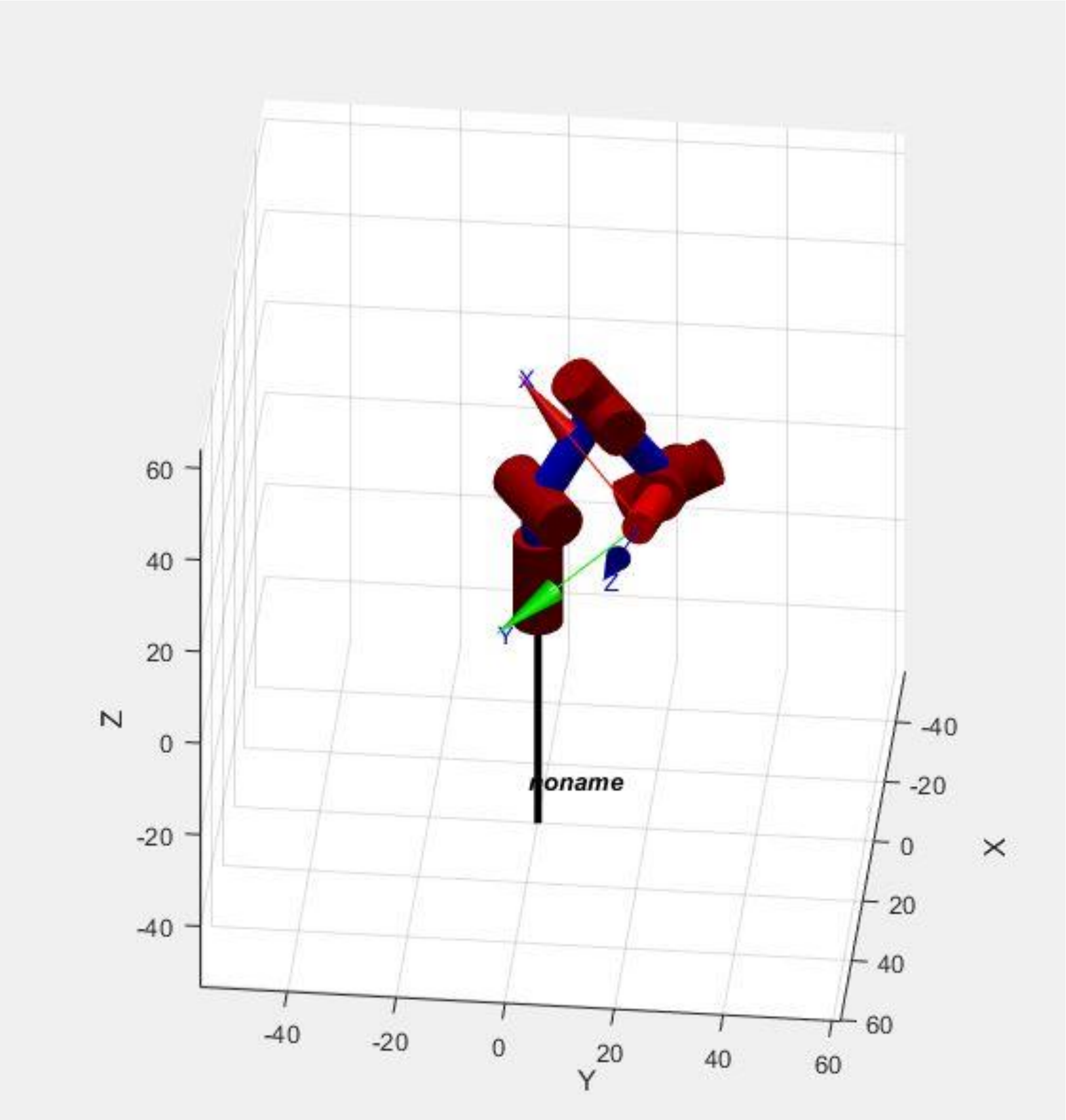
As you can see above the robot's end effector frame is same as initial configuration and the robot reached the commanded position of

(-5,20,10) in x,y,z respectively.

The joint angles that were obtained are as follows (theta1,theta2,theta3,theta4,theta5,theta6):

(2.5536 -2.4774 2.8687 -2.0905 -2.4483 0.9313).

Now changing both orientation and position with same initial configuration,



The result is as above the robot was able to move to commanded orientation of orientation and commanded position of (10,20,20).

The joint angles that were obtained are as follows in the order (theta1,theta2,theta3,theta4,theta5,theta6):

(-1.0162 0.6284 -2.8034 1.9885 -2.1572 -2.9521)

The validation of inverse kinematics was also done with the geometrical approach and joint angles obtained was found to be matching with the numerical ik solver.

The forward kinematics is also validated by showing that robot was able to move to commanded pose.

The Ground truth values of the points in alphabets also match with position attained by the robot end effector.

Conclusion:

To conclude the with I would like to mention the robot was able to trace all the alphabets that were given within the workspace of the robot. Since it's a 6 dof robot it was able to reach any pose given to robot within its reachable workspace. Robot was provided with the alphabets as series of trajectories. Robot was able to compute inverse kinematics of each point on the trajectory and the joint angles that were obtained are used to move robots end effector to the desired position and orientation. Robot was validated and tested with both geometrical approach and in built ik solver. The results are also provided that showed that robot was able to reach the desired pose A simulation was made that proves the robot chosen can perform the writing task.

References:

Robot Modeling and Control- First Edition: Mark W. Spong, Seth Hutchinson, and M. Vidyasagar .

http://petercorke.com/wordpress/toolboxes/robotics-toolbox

http://petercorke.com/wordpress/toolboxes/robotics-toolbox#Documentation

https://www.mathworks.com/help/robotics/ug/inverse-kinematics-algorithms.html