HOMEWORK 4

1. Evaluate the following double integrals.

$$1) \iint_R y dx dy, \qquad R: \ 0 \le x \le \pi, \ 0 \le y \le \sin x.$$

$$2) \iint_R y dx dy, \qquad R: \ \text{bounded by } y = 0, \ x = 0 \text{ and } y = \ln x.$$

2. Find volume beneath a surface $z = f(x, y)$.

1) Find the volume of the solid that is bounded above by the cylinder $z = x^2$ and below the region enclosed by the parabola $y = 2 - x^2$ and the line $y = x$ in the xy-plane.

2) Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 4$, and the plane $z + y = 3$.

3. Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral.

$$1) \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1 + x^2 + y^2)^2} dy dx.$$

$$2) \int_{1}^{2} \int_{0}^{\sqrt{2-x^2}} \frac{1}{(x^2 + y^2)^2} dy dx.$$

4. Evaluate the integrals.

$$1) \iiint_R (x + y + z) dx dy dz, \qquad R: \ 0 \le x \le 1, \ 0 \le y \le 2, \ 0 \le z \le 3.$$

$$2) \int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz dy dx$$