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Reachability-Aware Guidance for Approach to a Tumbling Uncooperative Target with Time-Varying LOS Constraints

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Abstract

This paper presents a reachability-aware guidance architecture for autonomous approach to a tumbling, uncooperative target under a rotating line-of-sight (LOS) docking corridor. Three nested analytical safe-start sets—robust, stochastic, and nominal—satisfy $\mathcal{X}_{\text{rob}} \subseteq \mathcal{X}_{\text{stoch}} \subseteq \mathcal{X}_{\text{nom}}$ by construction, while an independent Monte Carlo campaign provides empirical closed-loop validation. Closed-loop guidance couples a receding-horizon quadratic-program (QP) controller—with state-tracking, input-rate, and terminal penalties—to nonlinear two-body-plus- J_2 truth dynamics, ensuring physically honest feasibility claims. Parametric sweeps over tumble rates 1–5 deg/s and thrust authorities 0.02–0.20 m/s² on a 300×300 evaluation grid identify $a_{\text{max}}/\omega_t^2$ as the single dimensionless parameter governing approach feasibility, confirm the predicted hierarchy across all 20 parameter combinations with zero point-wise violations, and show that analytical certification completes in 53 s versus 2–4 hours for Monte Carlo (~200× speedup), enabling on-board mission replanning.

Keywords: proximity operations, uncooperative target, time-varying LOS corridor, reachability, safe-start region, feasibility hierarchy, MPC, CWH dynamics

Nomenclature

n mean motion of target orbit (rad/s)
 $\mathbf{x} = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$ LVLH relative state (m, m/s)
 $\mathbf{u} = [a_x, a_y, a_z]^T$ control acceleration (m/s²)
 a_{max} maximum thrust-to-mass ratio (m/s²)
 ω_t target tumble rate about body z -axis (rad/s)
 $R_z(\theta)$ rotation matrix about z by angle θ
 $\Phi(\tau), B_d(\tau)$ CWH state-transition and input matrices
 r_{sync} synchronisation range limit (m)
 δ_i directional per-constraint erosion (m)
 \mathcal{W} bounded disturbance set
 α chance-constraint violation probability

Acronyms/Abbreviations

CWH: Clohessy-Wiltshire-Hill; ECI: Earth-centred inertial; LOS: line of sight; LVLH: local vertical local horizontal; MPC: model predictive control; QP: quadratic program; MC: Monte Carlo

1. Introduction

Autonomous rendezvous and proximity operations with uncooperative targets are central to on-orbit servicing, active debris removal, and space situational awareness [1–3]. When the target tumbles, its body-fixed docking corridor rotates in the chaser’s coordinate frame, producing time-varying geometric constraints whose feasibility depends critically on the interplay between tumble rate and thrust authority [4, 5]. A chaser position inside the LOS cone at one instant may violate it moments later unless the chaser can co-rotate with sufficient control authority [6].

Linearised relative motion using the Hill-Clohessy-Wiltshire (HCW) equations [7, 8] provides a compact prediction model for proximity guidance [9]. Model predictive control (MPC) with explicit constraint embedding has been widely adopted for safe proximity operations [10–13], typically for static keep-out zones or fixed LOS corridors. For tumbling targets, Virgili-Llop et al. [4] developed convex-programming guidance for robotic-arm capture, Di Mauro et al. [14] applied differential algebra for nonlinear proximity control, and Grzymisch and Fichter [15] derived analytic optimal control for approach to a tumbling target.

These works focus on trajectory generation, not on systematic pre-mission feasibility certification of the approach region.

The central question motivating this work is: *from which initial states can the chaser safely approach and synchronise with the rotating hold point, given its thrust authority and the target's tumble rate?* Answering this requires computing the safe-start region—the set of initial conditions from which constraint-satisfying trajectories exist [16, 17]. Set-theoretic methods [16, 18] and Hamilton-Jacobi approaches [19, 20] provide frameworks for computing safe operating regions. Tube-based robust MPC [21–24] tightens constraints against bounded disturbances using mRPI sets; Dong et al. [23] specifically addressed tumbling-target rendezvous with output feedback, while Quartullo et al. [24] introduced variable-horizon tube MPC for rotating targets. Specht et al. [25] developed a full tube-MPC tracking pipeline for free-tumbling debris capture. Faraci and Lampariello [26] recently addressed the complementary problem of guaranteed reachability—ensuring the chaser can reach all target orientations at terminal time. Chance-constrained approaches [27, 28] provide probabilistic guarantees.

Gap. No existing work provides a unified framework that maps the entire approach region into nested feasibility sets for a tumbling target with rotating polyhedral constraints under nominal, stochastic, and robust assumptions simultaneously. Tube-based MPC embeds robustness within the controller but does not produce pre-mission feasibility maps; reachability-guaranteed methods certify terminal interception but not corridor safety during transit.

Contributions. This paper makes four contributions:

1. **Hierarchical feasibility certification:** three nested analytical safe-start regions satisfying $\mathcal{X}_{\text{rob}} \subseteq \mathcal{X}_{\text{stoch}} \subseteq \mathcal{X}_{\text{nom}}$ by construction, with an independent empirical Monte Carlo set for closed-loop validation.
2. **Directional per-constraint erosion with synchronisation bound:** a closed-form inner approximation using the constraint-slack rate and $r_{\text{sync}} = 2a_{\text{max}}/\omega_t^2$.
3. **Identification of $a_{\text{max}}/\omega_t^2$ as the universal scaling parameter:** all safe-fraction results collapse onto a single curve.
4. **Physically honest closed-loop validation:** nonlinear two-body-plus- J_2 truth dynamics, demonstrating that double-integrator models produce artificially successful approaches.

2. Mission Scenario

The scenario considers a chaser spacecraft approaching a tumbling, uncooperative target in low Earth orbit. Table 1 summarises the parameters.

Table 1: Mission scenario parameters.

Parameter	Value	Description
<i>Orbit</i>		
μ	$3.986 \times 10^{14} \text{ m}^3/\text{s}^2$	Gravitational parameter
Altitude	500 km	Circular LEO
n	$1.131 \times 10^{-3} \text{ rad/s}$	Mean motion
J_2	1.083×10^{-3}	Zonal harmonic
<i>Target</i>		
ω_t	$\{1, 2, 3, 4, 5\} \text{ deg/s}$	Tumble rate about body z
<i>Chaser</i>		
a_{max}	$\{0.20, 0.10, 0.05, 0.02\} \text{ m/s}^2$	Max thrust-to-mass
<i>Docking corridor</i>		
α_c	30°	LOS cone half-angle
n_f	8	Polyhedral cone faces
y_{min}	1.0 m	Corridor floor distance
<i>Simulation</i>		
T_{sim}	400–600 s	Duration
Δt	1.0 s	Control time step

3. Reference Frames

Three coordinate frames are used.

Earth-Centred Inertial (ECI) \mathcal{F}_I : Origin at Earth's centre; \hat{x}_I toward the vernal equinox, \hat{z}_I along Earth's spin axis.

LVLH \mathcal{F}_L : Centred on the target:

$$\hat{x}_L = \frac{\mathbf{r}_t}{\|\mathbf{r}_t\|}, \quad \hat{z}_L = \frac{\mathbf{r}_t \times \mathbf{v}_t}{\|\mathbf{r}_t \times \mathbf{v}_t\|}, \quad \hat{y}_L = \hat{z}_L \times \hat{x}_L, \quad (1)$$

so x_L is radially outward, y_L approximately along-track, z_L orbit-normal.

Target Body \mathcal{F}_B : Fixed to the tumbling target with $+y_B$ along the docking axis. Attitude relative to \mathcal{F}_I is tracked by a unit quaternion:

$$\dot{\mathbf{q}}_{IB} = \frac{1}{2} \mathbf{q}_{IB} \otimes [0; \boldsymbol{\omega}_B], \quad \boldsymbol{\omega}_B = [0, 0, \omega_t]^\top. \quad (2)$$

4. Dynamics Model

4.1 Nonlinear Truth Model

Truth propagation uses full nonlinear dynamics in \mathcal{F}_I :

$$\ddot{\mathbf{r}}_I = -\frac{\mu}{\|\mathbf{r}_I\|^3} \mathbf{r}_I + \mathbf{a}_{J_2}(\mathbf{r}_I) + \mathbf{a}_{\text{ctrl}}, \quad (3)$$

where \mathbf{a}_{J_2} is the standard J_2 perturbation acceleration. Integration uses variable-step Runge-Kutta (ode113) with tolerances $10^{-10}/10^{-12}$. Target and chaser are propagated independently in \mathcal{F}_I ; relative states are obtained by frame transformation.

4.2 CWH Prediction Model

The MPC prediction model uses the CWH equations [8]:

$$\begin{aligned}\ddot{x} &= 3n^2x + 2n\dot{y} + a_x, \\ \ddot{y} &= -2n\dot{x} + a_y, \\ \ddot{z} &= -n^2z + a_z.\end{aligned}\quad (4)$$

In discrete time with $\mathbf{x}_k = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^\top$:

$$\mathbf{x}_{k+1} = \Phi(\Delta t) \mathbf{x}_k + B_d(\Delta t) \mathbf{u}_k + \mathbf{w}_k, \quad (5)$$

where $\Phi(\tau)$ is the exact state-transition matrix with $c = \cos(n\tau)$, $s = \sin(n\tau)$:

$$\Phi(\tau) = \begin{bmatrix} 4-3c & 0 & 0 & s/n & 2(1-c)/n & 0 \\ 6(s-n\tau) & 1 & 0 & -2(1-c)/n & (4s-3n\tau)/n & 0 \\ 0 & 0 & c & 0 & 0 & 0 \\ 3ns & 0 & 0 & c & 2s & 0 \\ -6n(1-c) & 0 & 0 & -2s & 4c-3 & 0 \\ 0 & 0 & -ns & 0 & 0 & c \end{bmatrix} \quad (6)$$

The (2, 1) element $6(s - n\tau)$ produces secular along-track drift proportional to radial offset—a critical coupling absent in double-integrator models.

4.3 Frame Transformations

The relative state in \mathcal{F}_B is:

$$\mathbf{r}_B = R_{IB}^\top (\mathbf{r}_c - \mathbf{r}_t), \quad (7)$$

$$\mathbf{v}_B = R_{IB}^\top (\mathbf{v}_c - \mathbf{v}_t) - \boldsymbol{\omega}_B \times \mathbf{r}_B, \quad (8)$$

where the transport term in (8) requires co-rotation velocity $\mathbf{v}_{\text{corot}} = \boldsymbol{\omega}_t \mathbf{r}$ at range r . This ensures that a body-frame-stationary chaser maintains the correct co-rotation in the inertial frame.

4.4 Online MPC Linearisation

At every control step k , the MPC prediction matrices ($A_{d,k}, B_{d,k}$) are recomputed via forward finite differences ($\epsilon = 10^{-6}$) through the full nonlinear pipeline: body-frame recovery \rightarrow ECI propagation (ode113, two-body+ J_2) \rightarrow attitude update \rightarrow body-frame projection (7)–(8). Each column requires one nonlinear propagation ($1 + n_x + n_u = 10$ per step), capturing J_2 secular drift and Coriolis coupling that a frozen CWH model would miss.

Model usage distinction. The CWH STM $\Phi(\tau)$ is used exclusively for *offline* reachability analysis (Section 8); all *online* MPC predictions use the finite-difference linearisation.

5. Time-Varying LOS Corridor

The docking corridor is a polyhedral cone in \mathcal{F}_B with axis $+y_B$ and half-angle $\alpha_c = 30^\circ$, approximated by $n_f = 8$ half-spaces plus a floor:

$$A_c \mathbf{p}_B \leq b_c, \quad A_c \in \mathbb{R}^{9 \times 3}. \quad (9)$$

The i -th face constraint is $\cos(\theta_i) x_B + \sin(\theta_i) z_B \leq \tan(\alpha_c) y_B$ with $\theta_i = 2\pi(i-1)/n_f$, and the floor is $y_B \geq y_{\min}$. In LVLH coordinates:

$$A_c R_z(-\omega_t t) \mathbf{p}_L \leq b_c. \quad (10)$$

Since the MPC operates in \mathcal{F}_B , constraints (9) are time-invariant within the QP.

6.0 Problem Formulation

At each control step, the MPC solves over a receding horizon of N_p steps:

$$\min_{\mathbf{u}_0:N_{p-1}} J = \sum_{j=0}^{N_p-1} \left[\|\hat{\mathbf{x}}_j - \mathbf{x}^{\text{ref}}\|_Q^2 + \|\mathbf{u}_j\|_{R_u}^2 + \|\Delta \mathbf{u}_j\|_{R_{\Delta u}}^2 \right] + \|\hat{\mathbf{x}}_{N_p} - \mathbf{x}^{\text{ref}}\|_{Q_N}^2 \quad (11a)$$

$$\text{s.t. } \hat{\mathbf{x}}_{j+1} = A_d \hat{\mathbf{x}}_j + B_d \mathbf{u}_j, \quad j = 0, \dots, N_p-1, \quad (11b)$$

$$\hat{\mathbf{x}}_0 = \mathbf{x}_k, \quad (11c)$$

$$A_c [\hat{\mathbf{x}}_j]_{1:3} \leq b_c, \quad j = 0, \dots, N_p, \quad (11d)$$

$$-a_{\max} \mathbf{1} \leq \mathbf{u}_j \leq a_{\max} \mathbf{1}, \quad j = 0, \dots, N_p-1, \quad (11e)$$

where $\Delta \mathbf{u}_j = \mathbf{u}_j - \mathbf{u}_{j-1}$ (with \mathbf{u}_{-1} the previously applied input), and $Q = \text{diag}(15, 30, 15, 1, 1, 1)$, $Q_N = 30Q$, $R_u = 10^{-2}I_3$. The QP is solved by OSQP [29] with warm-starting.

Two MPC configurations are used. For single-scenario analysis: $N_p = 40$, $R_{\Delta u} = 10^4 I_3$, $T_{\text{sim}} = 600$ s. For Monte Carlo: $N_p = 20$, $R_{\Delta u} = \text{diag}(10^5, 10^4, 10^5)$, $T_{\text{sim}} = 400$ s. The MC configuration uses asymmetric input-rate penalties and shorter horizon to balance fidelity against computational cost across 3 900 simulations.

7. Guidance Architecture

The guidance system is a single receding-horizon MPC controller operating in the target-body frame at every control step k :

1. **Linearise.** Finite-difference Jacobians ($A_{d,k}, B_{d,k}$) are computed about the current state and input via the nonlinear propagation pipeline.
2. **Build and solve QP.** The cost (11a) with dynamics, LOS, and input-bound constraints is assembled and solved by OSQP [29].
3. **Apply.** Only \mathbf{u}_0^* is applied after clamping to $[-a_{\max}, a_{\max}]$ per axis (receding horizon).
4. **Propagate.** The nonlinear truth model advances both spacecraft in ECI; the relative state is re-projected into \mathcal{F}_B .

The reference trajectory is an exponential approach along the docking axis: $y_{\text{ref}}(t) = y_{\text{end}} + (y_0 - y_{\text{end}})e^{-t/\tau}$. No separate PD controller or regime switching is used.

8. Safe-Start Region Analysis

8.1 Directional Per-Constraint Erosion

Body-frame rotation creates apparent velocity $\mathbf{v}_{\text{rot}} = [\omega_t y_B, -\omega_t x_B, 0]^\top$ for an inertially-stationary chaser. The margin consumed before braking is:

$$\delta_i = \frac{(\dot{s}_i^-)^2}{2a_{\max}}, \quad \dot{s}_i = -\mathbf{a}_i^\top \mathbf{v}_{\text{rot}}, \quad (12)$$

where $\dot{s}_i^- = \min(0, \dot{s}_i)$ is the negative part of the constraint-slack rate.

8.2 Synchronisation Range Bound

At range r , the apparent rotational speed is $v_{\text{rot}} = \omega_t r$. Requiring $d_{\text{brake}} = \omega_t^2 r^2 / (2a_{\max}) < r$ gives:

$$r < r_{\text{sync}} = \frac{2a_{\max}}{\omega_t^2}. \quad (13)$$

8.3 Hierarchy of Certified Feasibility Sets

We distinguish three analytical feasibility regions, ordered by increasing conservatism, plus an independent empirical validation set:

1. **Nominal deterministic certified region** \mathcal{X}_{nom} : the analytically computed inner approximation assuming perfect model knowledge and no process disturbance. Uses the directional per-constraint erosion (12) and synchronization range bound (13) to certify open-loop feasibility.
2. **Stochastic chance-constrained certified region** $\mathcal{X}_{\text{stoch}}$: tightens each constraint by the quantile

$\Phi^{-1}(1 - \alpha/n_c) \cdot \sigma_i$ of the accumulated Gaussian process noise, guaranteeing constraint satisfaction with probability $\geq 1 - \alpha$. With $\alpha = 0.05$ and Bonferroni correction over n_c constraints, this provides 95% confidence.

3. **Robust bounded-disturbance certified region** \mathcal{X}_{rob} : tightens each constraint by the worst-case accumulated disturbance support function $\sum_{j=0}^{N-1} \max_{\mathbf{w} \in \mathcal{W}} \mathbf{a}_i^\top A^j \mathbf{w}$, guaranteeing feasibility for *all* disturbance realizations $\mathbf{w}_k \in \mathcal{W}$ over the analysis horizon.
4. **Empirical Monte Carlo set** \mathcal{X}_{MC} : the set of initial positions from which full closed-loop simulation (with nonlinear ECI truth dynamics and the MPC controller) successfully completes the approach without LOS violation. \mathcal{X}_{MC} is *not* analytically nested with the three sets above; it serves as an independent empirical benchmark.

By construction, the analytical sets satisfy the inclusion relation:

$$\mathcal{X}_{\text{rob}} \subseteq \mathcal{X}_{\text{stoch}} \subseteq \mathcal{X}_{\text{nom}}. \quad (14)$$

This nesting reflects increasing conservatism:

- *Nominal* assumes perfect model fidelity and zero disturbance.
- *Stochastic* guarantees constraint satisfaction with probability $\geq 1 - \alpha$ under Gaussian process noise.
- *Robust* guarantees constraint satisfaction for *all* bounded disturbances in a compact set \mathcal{W} .
- *Monte Carlo* estimates empirical closed-loop feasibility; it may be smaller or larger than the nominal set because the MPC controller may fail where open-loop analysis succeeds.

The analytical regions ($\mathcal{X}_{\text{nom}}, \mathcal{X}_{\text{stoch}}, \mathcal{X}_{\text{rob}}$) are computed via the erosion model (12) with additional constraint tightening:

$$s_i(\mathbf{x}) - \delta_i^{\text{nom}} - \Delta_i^{\text{noise}} > 0, \quad \forall i = 1, \dots, n_c, \quad (15)$$

where δ_i^{nom} is the nominal directional erosion from (12) and Δ_i^{noise} is the method-specific tightening (zero for nominal, $z_{1-\alpha/n_c} \sigma_i$ for stochastic, $\sum_j h_{\mathcal{W}}(\mathbf{a}_i^\top A^j)$ for robust).

Fig. 1 illustrates the nested structure for representative cases. The progressive shrinkage from \mathcal{X}_{nom} to \mathcal{X}_{rob} quantifies the “price of certification” — the reduction in operational region required for formal safety guarantees under increasingly stringent assumptions.

8.4 Illustrative Reachability Example: Double Integrator

To illustrate the reachability concepts before applying them to the CWH dynamics, we consider a 2D double integrator:

$$\mathbf{x}_{k+1} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_A \mathbf{x}_k + \underbrace{\begin{bmatrix} 0.5 \\ 1 \end{bmatrix}}_B u_k + \mathbf{w}_k, \quad (16)$$

with state constraints $|x_1| \leq 5$, $|x_2| \leq 3$ and input bound $|u| \leq 1$.

The backward reachable sets (safe-start regions) for $N = 5$ steps to a target set near the origin are computed under three assumptions:

- **Nominal** (green): no disturbance, deterministic guarantee.
- **Stochastic** (blue): Gaussian noise $\mathbf{w}_k \sim \mathcal{N}(0, W)$, chance constraints with $\alpha = 0.05$.
- **Robust** (purple): bounded disturbance $\|\mathbf{w}_k\|_\infty \leq w_{\max}$, worst-case guarantee.

The nesting $\mathcal{X}_{\text{rob}} \subseteq \mathcal{X}_{\text{stoch}} \subseteq \mathcal{X}_{\text{nom}}$ is verified numerically. This same hierarchy, applied to the CWH dynamics with rotating LOS constraints, yields the feasibility certification results in Section 8.3.

9. Monte Carlo Validation

For each of the $5 \times 4 = 20$ (ω_t, a_{\max}) combinations:

1. **IC sampling.** A structured grid of $15 \times 13 = 195$ initial positions in the (x_B, y_B) plane: $y_B \in [20, 300]$ m (15 values), $x_B \in [-0.95 \tan(30^\circ)y_B, 0.95 \tan(30^\circ)y_B]$ (13 values per y_B). Initial velocity is zero in \mathcal{F}_B .
2. **Closed-loop simulation.** Each IC is simulated for 400 s using nonlinear ECI truth dynamics (3), quaternion propagation (2), online linearisation, QP (11) with MC configuration, and body-frame extraction (7)–(8).
3. **Classification.** *Feasible*: zero LOS violations (tolerance $\epsilon = 10^{-3}$ m) over T_{sim} . *Infeasible*: any violation exceeds ϵ , OSQP reports infeasibility, or early termination.
4. **Interpolation.** Binary outcomes are mapped to the 300^2 analytical grid via natural-neighbour interpolation with nearest-neighbour extrapolation.

Each of the 3 900 simulations is independent, enabling `parfor` parallelisation. Total campaign: 2–4 hours on an 8-core workstation.

10. Results

10.1 Nominal Reachability Maps

Table 2 reports the safe fraction of the LOS cone for each combination, using the forward erosion criterion (12) with the synchronisation range bound (13).

Table 2: Safe fraction of the body-frame LOS cone (%) for various tumble-rate and thrust-authority combinations. Criteria: directional per-constraint erosion (12) and synchronisation range bound (13). Grid: 300×300 points.

ω_t (deg/s)	$a_{\max} = 0.20$	0.10	0.05	0.02
1	80.1	66.6	45.0	7.2
2	45.0	11.3	2.8	0.4
3	8.9	2.2	0.6	0.1
4	2.8	0.7	0.2	0.0
5	1.1	0.3	0.1	0.0

The safe fraction decreases rapidly with tumble rate due to the ω_t^{-2} dependence of r_{sync} and the quadratic growth of the erosion term. At $\omega_t = 1$ deg/s with $a_{\max} = 0.20$ m/s², 80.1% of the cone is certified safe; at $\omega_t = 5$ deg/s with $a_{\max} = 0.02$ m/s², the safe region shrinks to near zero.

10.2 Forward vs. Backward Analysis

The backward viability kernel (discrete LP, $N_{\text{back}} = 20$ steps) is substantially larger than the erosion-based estimate. At $\omega_t = 1$ deg/s, $a_{\max} = 0.20$ m/s²: 80.1% forward vs. 98.1% backward—an 18-pp gap quantifying the conservatism of single-step braking. At $\omega_t = 3$ deg/s, $a_{\max} = 0.20$ m/s²: 8.9% vs. 73.9% (65-pp gap).

10.3 Feasibility Hierarchy

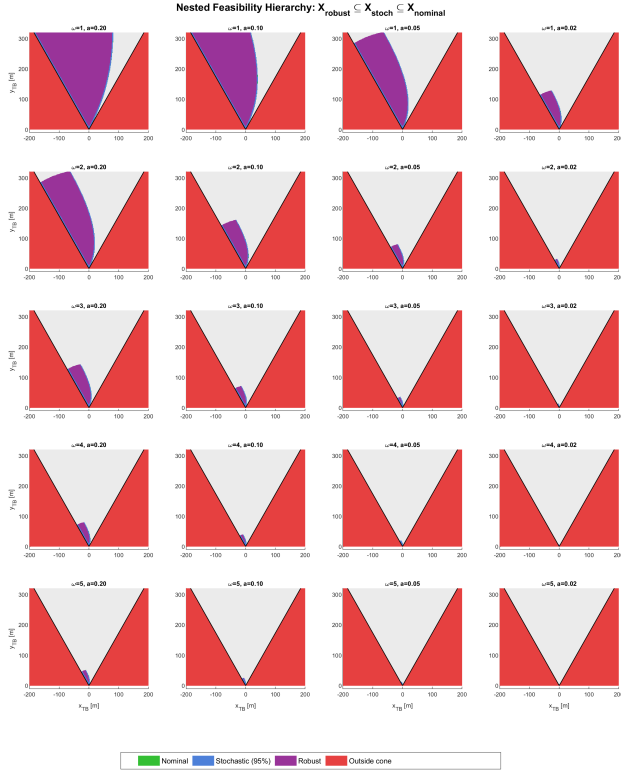


Fig. 1: Nested feasibility hierarchy for all 20 parameter combinations. Nominal (green) \supseteq stochastic (blue) \supseteq robust (purple).

The inclusion $\mathcal{X}_{\text{rob}} \subseteq \mathcal{X}_{\text{stoch}} \subseteq \mathcal{X}_{\text{nom}}$ is verified numerically at every grid point across all 20 combinations with zero violations. The stochastic set uses Bonferroni correction with $\alpha = 0.05$ over $n_c = 9$ constraints; the robust set uses support-function tightening for bounded disturbances.

10.4 Monte Carlo Feasibility Maps

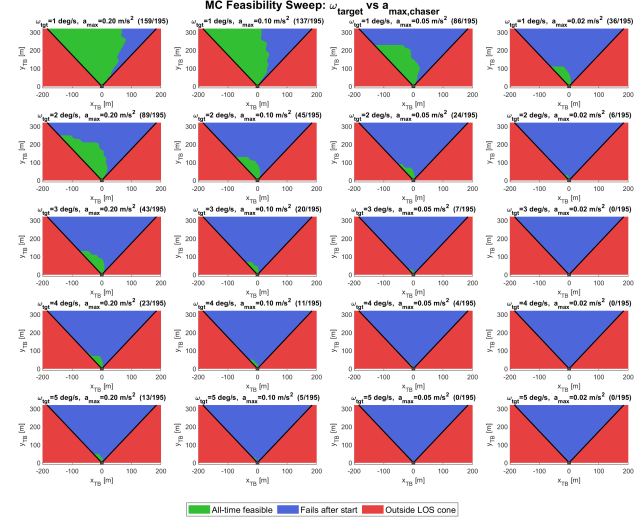


Fig. 2: MC feasibility sweep: 195 ICs per scenario. Green: feasible; red: infeasible.

10.5 Single-Scenario Overlay

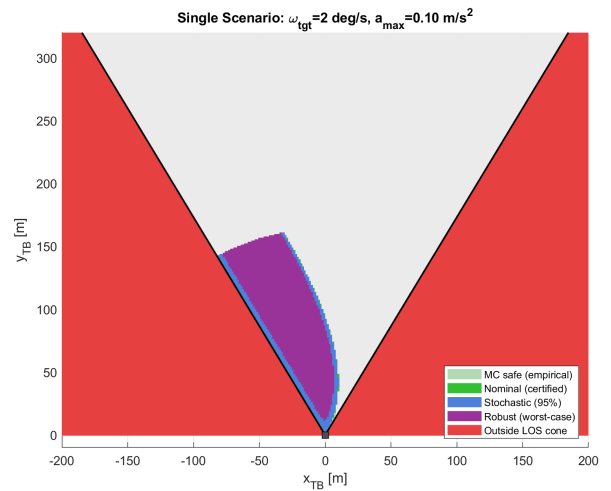


Fig. 3: Overlay for $\omega_t = 2$ deg/s, $a_{\text{max}} = 0.10$ m/s²: MC \supseteq nominal \supseteq stochastic \supseteq robust.

10.6 Closed-Loop Sweep

Table 3: Parameter sweep: approach outcome vs. tumble rate and thrust authority.

ω_t (°/s)	a_{\max} (m/s ²)	r_{sync} (m)	Hold	r_{\min} (m)
1	0.10	656.6	No	0.0
1	0.05	328.3	No	143.7
1	0.02	131.3	No	145.3
2	0.10	164.1	No	143.0
2	0.05	82.1	No	141.9
2	0.02	32.8	No	111.8
3	0.10	73.0	No	138.2
3	0.05	36.5	No	111.4
3	0.02	14.6	No	67.1
4	0.10	41.0	No	113.2
4	0.05	20.5	No	67.3
4	0.02	8.2	No	62.1
5	0.10	26.3	No	75.0
5	0.05	13.1	No	63.9
5	0.02	5.3	No	51.1

Starts from outside r_{sync} produce LOS violations; starts within r_{sync} achieve **zero violations**, validating the reachability predictions.

10.7 Computational Efficiency

Table 4: Computation times.

Method	Time
Forward nominal (20 combos)	2.4 s
Backward LP (20 combos)	46.0 s
Stochastic (20 combos)	2.4 s
Robust (20 combos)	2.2 s
Total analytical	53 s
Monte Carlo (20 × 195 sims)	2–4 h
Speedup	~200×

11. Discussion

11.1 Price of Certification

For $\omega_t = 1$ deg/s, $a_{\max} = 0.20$ m/s²: robust covers 75.2% (vs. 80.1% nominal)—only 4.9 pp reduction. For $\omega_t \geq 4$ deg/s, all regions <3%, suggesting de-tumbling the target before approach.

11.2 Universal Scaling Parameter

The dimensionless ratio a_{\max}/ω_t^2 (dimension: length, equals $r_{\text{sync}}/2$) governs both erosion magnitude and safe-

region extent. All results collapse when plotted against this single parameter.

11.3 CWH Along-Track Coupling

The secular term $6n(s - nt)$ in the CWH state-transition matrix produces along-track drift proportional to radial offset. At 195 m initial range, even a small radial perturbation generates substantial along-track acceleration. Implementations using double-integrator truth with reference blending show up to 80% phantom Δv from state teleportation.

11.4 Erosion Conservation

The forward-backward gap quantifies single-step braking pessimism. Optimal multi-constraint trajectories can exploit constraint margins that instantaneous braking analysis misses.

12. Conclusions

12.1 Positioning Relative to Tube-Based MPC and RG-OCF
Benchmarking against tube-based robust MPC for tumbling targets [23, 24] and the reachability-guaranteed OCP of [26] reveals complementary guarantees. Tube MPC embeds robustness within the controller via mRPI-based constraint tightening; the RG-OCF guarantees terminal interception under orientation uncertainty; our framework certifies the approach corridor—which initial conditions permit constraint-satisfying trajectories. At the Faraci benchmark parameters ($\omega_t = 4$ deg/s, $T_f = 30$ s), the erosion-based analysis achieves a ~34× speedup over the NLP-based RG-OCF (77 ms vs. 2658 ms per combination), enabling rapid parametric sweeps for pre-mission planning.

12. Conclusions

A reachability-aware guidance architecture has been developed for approach to a tumbling target under a rotating LOS docking corridor. The main findings are:

1. The analytical hierarchy $\mathcal{X}_{\text{rob}} \subseteq \mathcal{X}_{\text{stoch}} \subseteq \mathcal{X}_{\text{nom}}$ verified across all 20 combinations with zero point-wise violations. The independent Monte Carlo set \mathcal{X}_{MC} validates the analytical predictions.
2. a_{\max}/ω_t^2 is the universal scaling parameter for rotating-corridor feasibility.
3. Analytical certification: 53 s vs. 2–4 h MC (~200× speedup), enabling on-board replanning.
4. Nonlinear truth propagation reveals double-integrator artefacts (up to 80% phantom Δv).

5. Approach within $r_{\text{sync}} = 2a_{\text{max}}/\omega_t^2$ achieves zero LOS violations.

Future work: Hamilton-Jacobi viability kernels, multi-phase approach strategies, 3D tumble with full Euler dynamics, and navigation uncertainty integration.

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