Foresight in Dynamic Heterogeneous-Agent Economies

Bilal Islah* Bar Light†

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Abstract

We offer a novel framework for analyzing a wide range of large dynamic economies with heterogeneous agents and aggregate shocks, emphasizing the role of foresight. Our approach is flexible, encompassing previous frameworks for heterogeneous agent dynamic economies. Incorporating behaviorally motivated concepts such as bounded rationality and dynamic inconsistency, our framework simplifies the agents' optimization problem, resulting in a computationally tractable approach to solving for general equilibrium. We explore how foresight influences decision-making and equilibrium outcomes. Our comparative statics analysis reveals that foresight significantly affects the variation in endogenous equilibrium variables, distinguishing our findings from traditional risk aversion or precautionary savings effects. This variation stems from a feedback mechanism between individual decision-making and equilibrium variables, highlighting how increased foresight leads to greater non-stationarity in agents' decisions and, consequently, in aggregate state variables.

*University Mohammed VI Polytechnic
†Tel Aviv University

1 Introduction

In many important settings in economics, heterogeneous agents or firms make decisions over time, incurring rewards based on these decisions and economic variables such as market prices or total production. We consider a large dynamic economy where agents or firms are small and overlook their impact on key economic variables. We focus on the crucial case where each agent has an individual state (such as the agent's current wealth or the firm's current capacity) and there is an aggregate state that influences all the agents. A prominent example of such a model is Krusell-Smith type models (Krusell and Smith, 1998) where each agent has an individual state that corresponds to the agent's wealth and the aggregate state influences the productivity of the economy. Another example is Ericson-Pakes type dynamic oligopoly models (Ericson and Pakes, 1995) with many firms (Weintraub et al., 2008) where each firm has an individual state that corresponds to the firm's capacity or product's quality and the aggregate state influences the total demand for the firms' products.

These models involve complex interactions between individual decisions and aggregate states, where the actions of many agents or firms collectively influence the economic environment, which in turn affects individual decisions. A key challenge in these models is that the key economic variables are not stationary (as opposed to the stationary equilibrium concept in models without aggregate states such as Aiyagari (1994)). Uncertainty and external shocks (e.g., technological changes, changes in the aggregate demand) add layers of complexity to forecasting future economic variables, making it difficult to even make predictions on these economic variables. For example, in Krusell-Smith type models, agents make individual consumption-savings decisions that, among other factors, depend on observing the current period's interest rate and tracking the periods' interest rates. The interest rate generally depends on the entire distribution of individual savings decision. Thus, the agents need to track the evolution of agents' wealth and savings decisions for each future state.

The dynamic nature of these models, coupled with the necessity to account for an extensive array of potential future states and decisions, renders computational solutions demanding. Solving these models typically necessitates advanced numerical techniques and substantial computational power. Beyond the computational hurdles, the assumption that agents monitor the trajectory of crucial economic variables across any sequence of aggregate states places a significant presumption on the capacity of individual agents to foresee the future of key economic variables or predict the future distributions of other agents' individual states.

To tackle these challenges, the existing literature has focused on a moment-based approximation for solving large dynamic models with heterogeneous agents and aggregate shocks. This approach preserves the infinite-horizon tracking of economic variables but simplifies the computational task by focusing on a few moments of the agents' distribution rather than the entire distribution. Krusell and Smith (1998) concentrated on the first moment of the distribution, although subsequent studies and models have explored additional moments (Weintraub et al., 2010). This moment-based equilibrium approach is computationally tractable and has demonstrated some approximation capabilities, at least within certain models (Krusell and Smith, 1998).

While this approach enjoys popularity in solving large dynamic economies with aggregate shocks due to its tractability, it lacks direct motivation from the behavioral literature. In this paper, we introduce a novel method for solving large dynamic economies with aggregate shocks that combines tractability with a behaviorally motivated concept of computational complexity. Our method hinges on the concept of foresight, which specifies the number of periods over which agents can monitor the equilibrium evolution of key aggregate economic variables, such as market prices or aggregate production.

Leveraging our foresight framework, we introduce a behaviorally motivated conceptual framework that encompasses two pivotal aspects of heterogeneous agent models: a stationary scenario devoid of aggregate shocks (in the tradition of stationary equilibrium models (Aiyagari, 1994)) and a fully dynamic non-stationary equilibrium scenario with aggregate shocks (in the tradition of dynamic recursive equilibrium (Krusell and Smith, 1998)). This framework is not only computationally tractable, but also illuminates interesting comparative statics analysis for different degrees of of foresight.

Our foresight framework splits the agents' dynamic optimization problem into two parts: a finite-horizon problem where key economic variables such as prices or aggregate production are consistent with the agents' decisions, and a stationary infinite-horizon continuation that assumes these variables remain constant over time. As we adjust the foresight level regarding economic variables, our model reveals two main characteristics: agents have a limited information set on economic variables, embodying a form of bounded rationality, and agents exhibit dynamic inconsistency.

Our model draws upon a well-explored, behaviorally motivated body of literature on dynamic inconsistency and rational inattention in macroeconomics, e.g., Laibson (1997) and Gabaix (2024). Specifically, this literature underscores bounded rationality or cognitive costs that induce a tension between short-term and long-term horizons (Laibson, 1997) or that diminish agents' ability to monitor key state variables over an infinite-horizon (Gabaix, 2024). Importantly, we take inspiration from these behavioral insights that relate to the agent decision problem, and build on them to introduce a behaviorally motivated method to solve a wide range of dynamic heterogeneous agent general equilibrium models.

Our comparative statics analysis demonstrates an economically significant channel that arises solely from altering the degree of foresight. Across a broad spectrum of simulations, we observe that a considerable portion of the variation in endogenous equilibrium variables is attributable to foresight. This phenomenon is distinct from mechanisms driven by risk aversion or precautionary savings behavior. Instead, it originates from a feedback loop between individual agent decision-making, which forecasts equilibrium variables, and the fact that equilibrium variables simultaneously reflect individual agents' decisions. We illustrate that as agents' capability to anticipate future equilibrium variables increases, they incorporate a higher degree of non-stationarity into their decisions. This, in turn, induces greater non-stationarity in the equilibrium state variables.

The framework of the rest of the paper is as follows. We first provide an illustrative example of how foresight mechanically functions in the model of Krusell-Smith. We then propose our framework of foresight in generalized terms. Then we provide a description of the computational methodology and the description of the case where we specifically simulate the results of two important economic heterogeneous agent model, Krusell-Smith and a capacity constraint dynamic oligopoly model. We conclude with a discussion of comparative statics that arise in increasing the degree of foresight and the observation of a source of variation tied directly to an equilibrium feedback channel via foresight.

2 Foresight

We first describe our approach for the popular Krusell-Smith model as an illustrative example before presenting the general case.

In the Krusell-Smith economy, agents face an exogenous binary aggregate shock, $\tilde{z}_t$, which delineates either an expansionary or recessionary aggregate state. Within heterogeneous agent models featuring aggregate shocks, such as the Krusell-Smith model, these exogenous aggregate states are crucial in determining the trajectory of other endogenous economic variables that are established in equilibrium. This process is depicted in Figure 1 as a binary tree, where endogenous aggregate variables in the Krusell-Smith economy, like the wealth distribution, $\tilde{x}_t$, and the interest rate, r_t, evolve based on the future histories of the aggregate shock z_t.

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In the case of the Krusell-Smith model, or what we refer to as infinite foresight, agents possess the capability to track every possible pair of $(\tilde{x}, r)$ across any future sequence of exogenous aggregate states. In the bounded foresight case we study in this paper, rather than having the capacity to track all future pairs of $(\tilde{x}, r)$, agents' ability is restricted to a finite number of periods for which they can follow the equilibrium evolution of $(\tilde{x}, r)$. Beyond this, agents assume a stationary pair of $(\tilde{x}, r)$ for all subsequent periods.

We now describe in detail the cases of infinite foresight, zero foresight, and one foresight.

2.0.1 Infinite Foresight

In the infinite scenario case, agents can track the evolution of the interest rates for any possible path of future exogenous aggregate states. We illustrate this in the tree in Figure 1. At period k, the agent has a foresight of the interest rate r_k along any string of aggregate states from period 1 to period k, $(z_1, \ldots, z_k)$. A typical equilibrium definition in this setting requires that these interest rates satisfy a consistency condition in the sense that the interest rate is consistent with agents' dynamic savings decisions and the evolution of the wealth distribution in the economy (Krusell and Smith, 1998).

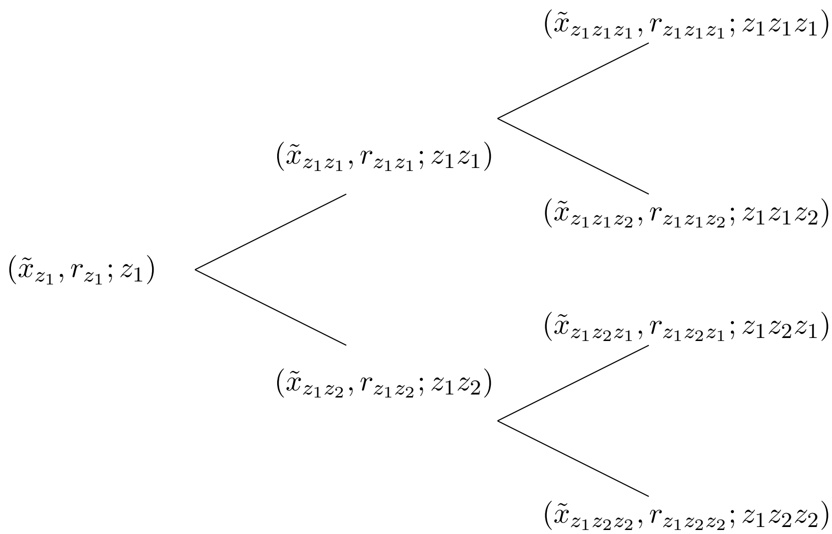

Figure 1: Infinite Foresight

2.0.2 Zero Foresight

In the zero foresight scenario, agents lack the ability to have foresight on any equilibrium aggregate variable for any future string of aggregate states. Instead, agents assume that the equilibrium aggregate variable they observe in the current period is the best guess for any future string of aggregate states.

This behavior leads to a stark dynamic inconsistency. Agents foresight of the next period does not take into account how a realization of an aggregate exogenous shock z_1z_1 or z_1z_2 can affect equilibrium aggregate quantities in those states (as in figure 2).

In the Krusell-Smith model, the scenario of zero foresight depicts a situation where agents are aware that the economy could enter either a good or a bad state in the next period. However, they lack precise knowledge regarding how this new state will affect the interest rate, leading them to rely on their best estimate for the next period's interest rate, which is the rate from the current period. The next period's interest rate may differ from the current interest rate as the economy

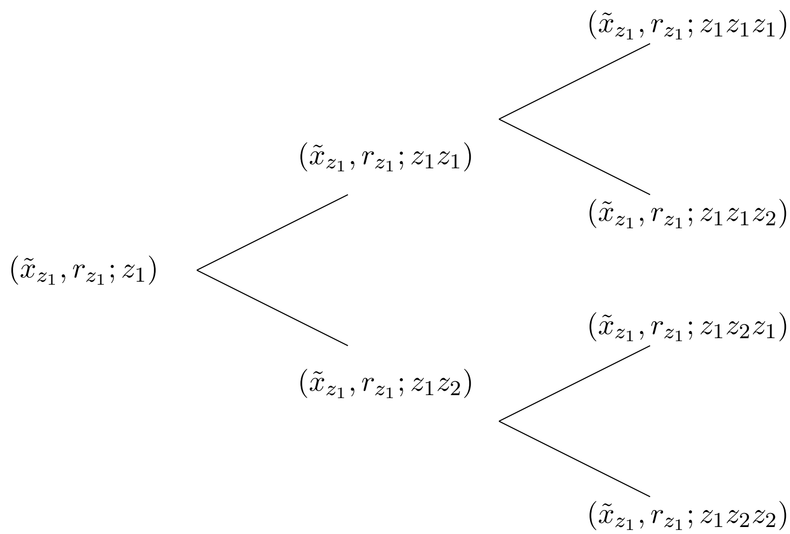

Figure 2: Zero Foresight

transitions into a different state. Agents, in the next period, will, once again, operate under the assumption that the current interest rate will persist into future periods.

2.0.3 One Foresight

In the one foresight scenario agents have the ability to forecast endogenous equilibrium aggregate state variables for one future period of aggregate shocks. What this means is slightly nuanced. It does not mean that the realized one period aggregate state variables are consistent with the forecast, but instead that from the perspective of the current period, the next periods aggregate state variable is an equilibrium result of an agent who beyond the first period lacks any additional foresight.

More simply, this means at time t_1 agents are solving for savings decisions one period forward (at t_{z1z1} and t_{z1z2} that are consistent with a zero-period foresight beyond that. This is then taken into account in agents saving decisions at t_1. So jointly, these are equilibrium savings decisions from the perspective of t_1.

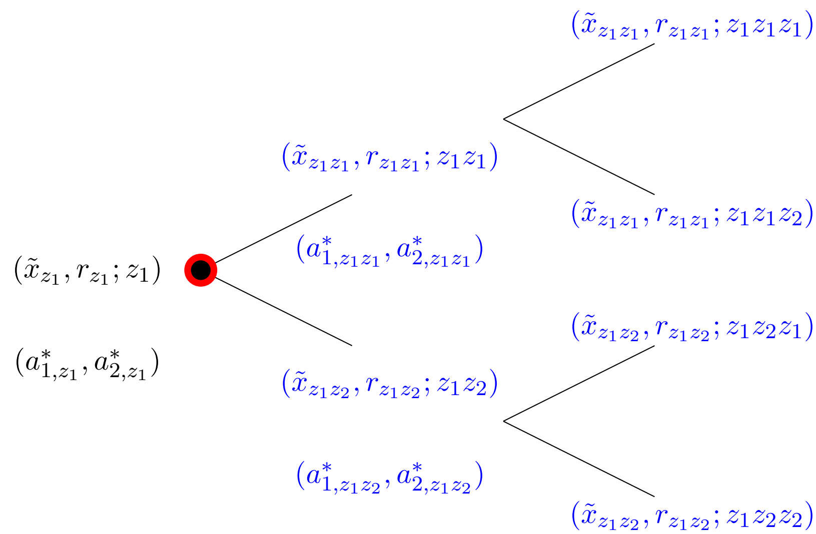

Figure 3: One Period Foresight

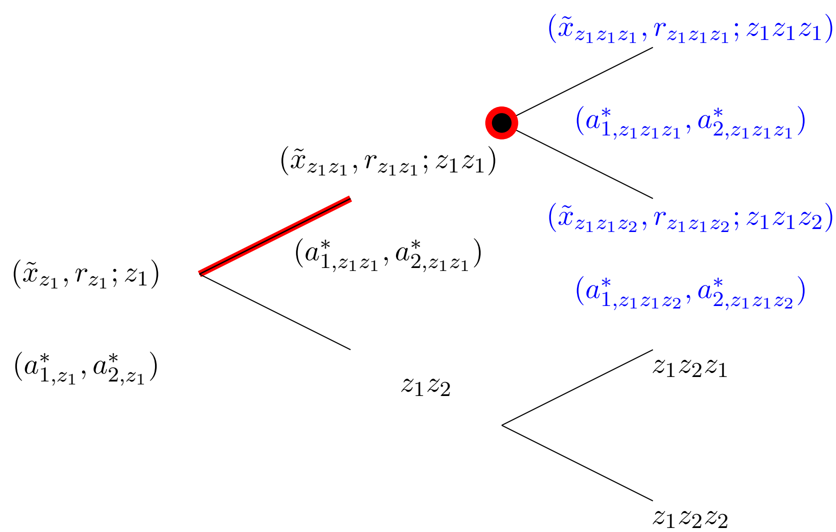

Figure 4: One Period Foresight

Dynamic inconsistency results when moving forward one period in the model. While the forecast aggregate states that are one period forward were the result of the constraint that savings decisions in t_1 took into the account of forecast t_2 savings decisions that required market clearing equilibrium, they don't reflect that each realized period moving forward gains a new period of foresight from the perspective of the previous period. Hence, new equilibrium savings in t_2 that realize will differ from those forecast in t_1. This is because in t_1, foresight allows for t_1 and t_2 to be taken into account but not t_3 whereas in t_2, t_3 is now part of the equilibrium savings decisions that affect t_2 savings decisions. Figure 3 and 4, illustrates how a^* evolves from one period to the next in terms of the equilibrium constraint impact foresight. Noteably, in figure 3, the agent assumes that $(\tilde{x}_{z_1 z_1}, r_{z_1 z_1})$ remains stationary in state $z_1 z_1 z_1$ (2-periods ahead of the agents current period and 1-period ahead of $z_1 z_1$. Whereas, in figure 4, the realized state of $z_1 z_1$, the agent assumes they can now forecast the equilibrium state variables $(\tilde{x}_{z_1 z_1 z_1}, r_{z_1 z_1 z_1)})$ in state $z_1 z_1 z_1$ as they now have gained one period of foresight.

3 General Model

In this section we describe the general large dynamic economy model with aggregate shocks we study in this paper.

Time. The game is played in discrete time. We index time periods by $t = 1, 2, \ldots$.

Agents. There is a continuum of ex-ante identical agents of measure 1. We use i to denote a particular agent.

States. There are individuals states for each agent and an aggregate state that is common to all agents. The individual state of agent i at time t is denoted by $x_{i,t} \in X$ where X is a separable metric space. The aggregate state at time t is denoted by $z_t \in Z$ where Z is a finite set. We let $\mathcal{P}(X)$ be the set of all probability measures on X and $\mathcal{B}(X)$ be the Borel sigma-algebra on X. We denote by $s_t \in \mathcal{P}(X)$ the probability measure that describes the distribution of agents' states at time t. We refer to s_t as the *population state* of time t.

Actions. The action taken by agent i at time t is denoted by $a_{i,t} \in A$ where $A \subseteq \mathbb{R}^q$. The set of feasible actions for a agent in state x is given by a compact set $\Gamma(x) \subseteq A$.

States' dynamics. The individual state of an agent evolves in a Markov fashion.

Agent i's state $x_{i,t}$ at time t depends on the past history only through the aggregate state z_{t-1}; the individual state of agent i at time $t-1$, $x_{i,t-1}$; the population state at time $t-1$, s_{t-1}; and the action taken by agent i at time $t-1$, $a_{i,t-1}$.

If agent's i's state at time $t-1$ is $x_{i,t-1}$, the agent takes an action $a_{i,t-1}$ at time $t-1$, the population state at time t is s_t, the aggregate state at time t is z_t, and $\zeta_{i,t}$ is agent i's realized idiosyncratic random shock at time t, then agent i's next period's state is given by

$$x_{i,t} = w(z_t, x_{i,t-1}, a_{i,t-1}, s_t, \zeta_{i,t}\ ) \ .$$

We assume that $\zeta_{i,t}$ are I.I.D random variables that take values on a compact separable metric space E that have a law q. The aggregate states evolve in a Markovian fashion according to a finite Markov chain with a transition matrix P. We call $w : Z \times X \times A \times \mathcal{P}(X) \times E \to X$ the transition function.

Payoff. In a given time period, if the aggregate state is z, the state of agent i is x_i, the population state is s, and the action taken by agent i is a_i, then the single period payoff to agent i is $\pi(z, x_i, a_i, s)$. The agents discount their future payoff by a discount factor $0 < \beta < 1$. Thus, a agent i's infinite horizon payoff is given by: $\sum_{t=0}^{\infty} \beta^t \pi(z_t, x_{i,t}, a_{i,t}, s_t)$.

Information structure. Let $Z^t := \underbrace{Z \times \ldots \times Z}_{t \text{ times}}$ be the space of all finite aggregate shock histories of length t. Every history of aggregate shocks $z^k \in Z^k$ has an associated population state $s_k(z^k)$ in period k. For $t \in \mathbb{N}$ we define

$$H^{t,t+N}(z_t) = (s_t(z_t), s_{t+1}(z^{t,t+1}), \ldots, s_{t+N}(z^{t,t+k}), \ldots) \tag{1}$$

where $z^{t,t+k} \in Z^{k+1}$ for $k = 1, \ldots$ is an history of aggregate shocks of length $k+1$ with an initial state z_t and for each history $z^{t,t+k}$ with $k > N-1$ such that its projection to the first N periods is exactly $z^{t,t+N}$ we have $s_{t+k}(z^{t,t+k}) = s_{t+N}(z^{t,t+N})$. $H^{t,t+N}(z_t)$ contains the information that an agent has about future population states. More precisely, it contains all the possible future population states in the next N-periods when the current aggregate shock is z_t and the population states after period N are fixed. An agent with N-degrees of foresight has the information structure $H^{t,t+N}(z_t)$ in period t, and thus, the agent has a prediction on the possible population states for every future history of length $k \leq N$. In our examples, the population state corresponds to an economic variable of interest (e.g., the interest rate or the average capacity in the economy). Hence, an agent with N-degrees of foresight has information on the possible economic variables for the future N periods and assume they are constant afterwards. For example, an agent with 0-degrees foresight, assumes that the economic variables are fixed and equals the current economic variable.

More generally, an *information structure* for an agent with N-degrees of foresight is a function $H^{t,t+N}$ from an history of aggregate shocks into a vector of possible population states that describes the information that an agent has about future population states given that history of aggregate shocks. With slight abuse of notation, for $\tau \in \mathbb{N} \cup \{0\} := \mathbb{N}_0$ a period t, and a string $(z_t, \ldots, z_{t+\tau})$, define $H^{t,t+N} : Z^{\tau+1} \rightarrow \mathcal{P}(X)^{\infty}$ by

$$H^{t,t+N}(z_t, \ldots, z_{t+\tau}) = (s_t(z_t), \ldots, s_{t+\tau}(z_t, \ldots, z_\tau), \ldots, s_{t+k}\left(z^{t,t+k}\right), \ldots)$$

where $z^{t,t+k} \in Z^{k+1}$ for $k = \tau + 1, \ldots$ is an history of aggregate shocks of length $k+1$ with an initial string $(z_t, \ldots, z_{t+\tau})$ and for each history $z^{t,t+k}$ with $k > N - 1$ such that its projection to the first N periods is exactly $z^{t,t+N}$ we have $s_{t+k}(z^{t,t+k}) = s_{t+N}(z^{t,t+N})$. Thus, with slight abuse of notation, we define H to any history of aggregate states where $H^{t,t+N}(z_t, \ldots, z_{t+\tau})$ means that the initial string of aggregate states is $(z_t, \ldots, z_{t+\tau})$.

Note that the information structures $H^{t,t+N}(z_t, \ldots, z_{t+\tau})$ and $H^{t,t+N}(z_t, \ldots, z_{t+\tau'})$ contain the same population states for any two histories $(z_t, \ldots, z_{t+\tau})$ and $(z_t, \ldots, z_{t+\tau'})$ such that $\tau' \geq \tau \geq N$ and the projection of $(z_t, \ldots, z_{t+\tau'})$ to the first $t + \tau$ periods is $(z_t, \ldots, z_{t+\tau})$.

For ease of notation, we often drop the subscripts i and t and denote a generic transition function by $w(x, a, s, \zeta)$ and a generic payoff function by $\pi(x, a, s)$.

We endow $\mathcal{P}(X)$ with the weak topology. We say that $s_n \in \mathcal{P}(X)$ converges weakly to $s \in \mathcal{P}(X)$ if for all bounded and continuous functions $f : X \rightarrow \mathbb{R}$ we have

$$\lim_{n \to \infty} \int_X f(x) s_n(dx) = \int_X f(x) s(dx) \quad .$$

For the rest of the paper, we assume the following conditions on the primitives of the model:

Assumption 1 *(i) π is bounded and (jointly) continuous. w is continuous.*

(ii) X is compact.

(iii) The correspondence $\Gamma : X \rightarrow 2^A$ is compact-valued and continuous.[1]

[1] By continuous we mean both upper hemicontinuous and lower hemicontinuous.

3.1 The Agents' Problem

Let $X^k := \underbrace{X \times \ldots \times X}_{k \text{ times}}$. Given an information structure in period t of a N-degrees of foresight agent $H^{t,t+N}(z_t)$, a nonrandomized strategy σ starting from period t is a sequence of (Borel) measurable functions $(\sigma_t, \sigma_{t+1}, \ldots,)$ such that $\sigma_k : (Z \times X)^{k-t+1} \to A$ and $\sigma_k(z_t, x_t, \ldots, x_k) \in \Gamma(x_k)$ for all $k = t, t+1, t+2, \ldots$. That is, a strategy σ assigns a feasible action to every finite string of states.

When a agent uses a strategy σ, the agent's information structure is $H^{t,t+N}$, the initial aggregate state is $z = z_t$ and the agent's initial state is $x = x_t \in X$, then the agent's expected present discounted payoff is

$$V_\sigma^N(x, H^{t,t+N}(z)) = \mathbb{E}_\sigma \left(\sum_{\tau=t}^{\infty} \beta^{\tau-t} \pi(z_t, x_t, a_t, s_t) \right)$$

where $\mathbb{E}_\sigma$ denotes the expectation with respect to the probability measure that is generated by the strategy σ.[2] Denote

$$V^N(x, H^{t,t+N}(z)) = \sup_\sigma V_\sigma^N(x, H^{t,t+N}(z)).$$

That is, $V^N(x, H^{t,t+N})$ is the maximal expected payoff that the agent can have when the initial state is x and his information structure is $H^{t,t+N}$. We call V the *value function* and a strategy σ attaining it *optimal*.

Standard dynamic programming arguments show that in period t, the dynamic optimization problem for an agent with N-degrees of foresight, $N \in \mathbb{N}_0$ can be solved recursively, and we have

$$\begin{aligned} V^N(x, H^{t,t+N}(z_t)) = &\max_{a \in \Gamma(x)} \pi(z_t, x, a, s_t(z_t)) \\ &+ \beta \int_E \sum_{z_i} P(z_t, z_i) V^N(w(z_t, x, a, s_t(z_t), \zeta), H^{t,t+N}(z_t, z_i)) q(d\zeta) \end{aligned}$$

where $z_t \in Z$ is the aggregate shock at time t and

$$\begin{aligned} V^N(x, H^{t,t+N}(z_t, \ldots, z_\tau)) = &\max_{a \in \Gamma(x)} \pi(z_\tau, x, a, s_{t+\tau}(z_t, \ldots, z_\tau)) \\ &+ \beta \int_E \sum_{z_i} P(z_\tau, z_i) V^N(w(z_\tau, x, a, s_{t+\tau}(z_t, \ldots, z_\tau), \zeta), H^{t,t+N}(z_t, \ldots, z_\tau, z_i)) q(d\zeta). \end{aligned}$$

is the value function of a agent given the history $(z_t, \ldots, z_\tau)$.

[2]The probability measure is uniquely defined (see Bertsekas and Shreve (1996) for details).

Let $g^N(x, H^{t,t+N}(\cdot))$ be an action that attains the maximum of the equation above. That is, $g^N(x, H^{t,t+N}(z_t, \ldots, z_\tau))$ is an optimal action for agent if the history of aggregate states is $(z_t, \ldots, z_\tau)$ and the agent's individual state is x.

3.2 N-Bounded Foresight Equilibrium

In this section we define a N-bounded foresight equilibrium (N-BFE).

For any history $z^{t,t+k} \in Z^{k+1}$ define $s(z^{t,t+k}, z_i, \cdot)$ to be the probability measure on $\mathcal{P}(X)$ that describes the agents' individual states distribution given the history of aggregate shocks $(z^{t,t+k}, z_i)$ and the agents optimal decisions. That is,

$$s(z^{t,t+k}, z_i, D) = \int_X \int_E \sum_{z_i} P(z_{t+k}, z_i) \cdot \tag{2}$$

$$1_D \left(w(z_{t+k}, x, g^N(x, H^{t+N}(z^{t,t+k})), s(z^{t,t+k}), \zeta) \right) q(d\zeta) s(z^{t,t+k}, dx) \tag{3}$$

for any $D \in \mathcal{B}(X)$.

We now define an N-BFE. A N-BFE consists of optimal strategies for agents that solve the agents' problem we defined in Section 3.1 and that the individual states distributions are consistent with the agents' strategies.

Definition 1 *Fix a period t, an initial aggregate shock z_t, an initial population state $s(z_t, \cdot)$, and $N \in \mathbb{N}_0$. An N-BFE is given by an information structure $H^{t,t+N}$, policy functions g^N, population states s such that*

(i) Optimality: Given the information structure $H^{t,t+N}$, the agents decisions are optimal. That is, g^N solves the agents problem as described in Section 3.1.

(ii) Consistency: For any history $z^{t,t+k}$ with $k \leq N$, $s(z^{t,t+k}, \cdot)$ is the population state given the history $z^{t,t+k}$ and the agents' optimal decisions as defined in Equation (2).

Note that if there are no aggregate shocks, and the distribution s is stationary we retrieve the standard stationary equilibrium that is popular in a wide range of large dynamics economies models (e.g., Aiyagari (1994) and Acemoglu and Jensen (2015)). On the other hand, if $N = \infty$ so the agents' foresight is not bounded, we retrieve the rational recursive equilibrium (e.g., Krusell and Smith (1998)).

4 Simulation

In this section, we present results that illustrate how our algorithm compares to the Krusell-Smith model and a dynamic oligopoly model.

4.1 Krusell-Smith

We run the foresight algorithm with zero, one and two periods of foresight and simulate along a path of aggregate shocks for 500 periods (equivalent to 125 years, where each period is calibrated to a quarter as in the standard Krusell-Smith model).

A key statistic we focus on is the standard deviation of the mean of aggregate capital over time. In the Krusell-Smith model, aggregate capital is the distribution of individual savings and there is a one-to-one relationship with the equilibrium interest rate, a key state variable in the individual's optimization.

In table 1, we present the standard deviation of mean capital along a single simulated path of aggregate shocks $(z_1, ..., z_{500})$. Increasing just a couple periods of foresight from zero to two, shows that variability in mean capital rapidly begins to approach the variability of infinite foresight (the benchmark Krusell-Smith case). The standard deviation of mean capital over the simulated period increases monotonically toward the Krusell-Smith value of 0.485 in table 1. The second row of the table shows that the mean of the per period standard deviation of capital (or in other words, the standard deviation of individual savings is relatively stable), even though the mean of savings is more variable over time. In table 2, we show that these results hold also on a larger Monte-Carlo simulation of 1000 periods and 1000 paths.

The plot in figure 5 shows a single simulated path of aggregate shocks and demonstrates the sensitivity of mean equilibrium capital to aggregate shocks is greatest in the Krusell-Smith case, and that each period of foresight appears to increase that sensitivity. In particular, the plot highlights the response to consecutive good or bad states in the model.

In figure 6, we expand on this sensitivity to aggregate shocks driving foresight. In a model where there is no variability in aggregate shocks, there is a stationary equilibrium capital distribution, as in Aiyagari (1994). We can observe something approaching this behavior when there are repeated aggregate shocks of the same type. We show the plot of mean capital on zero and one foresight

Table 1: Single Path - 500 Periods

	Zero	**One**	**Two**	**Krusell-Smith/∞**
$\sigma_{\bar{K}}$	0.263	0.321	0.382	0.485
$\bar{\sigma}_K$	2.62	2.62	2.61	2.71

Table 2: 1000 Paths - 1000 Periods

	Zero	**One**	**Two**	**Krusell-Smith/∞**
$\sigma_{\bar{K}}$	0.265	0.324	-	0.483
$\bar{\sigma}_K$	2.609	2.603	-	2.703

relative to the Krussel-Smith model when there are consecutive sequences of good or bad shocks[3]. From the plot, we see that greater foresight shows greater deviation away from a stationary mean capital level. This is consistent with the plot of figure 5 of a single path simulation. That after a series of consecutive shocks, the mean capital level is more responsive with greater foresight.

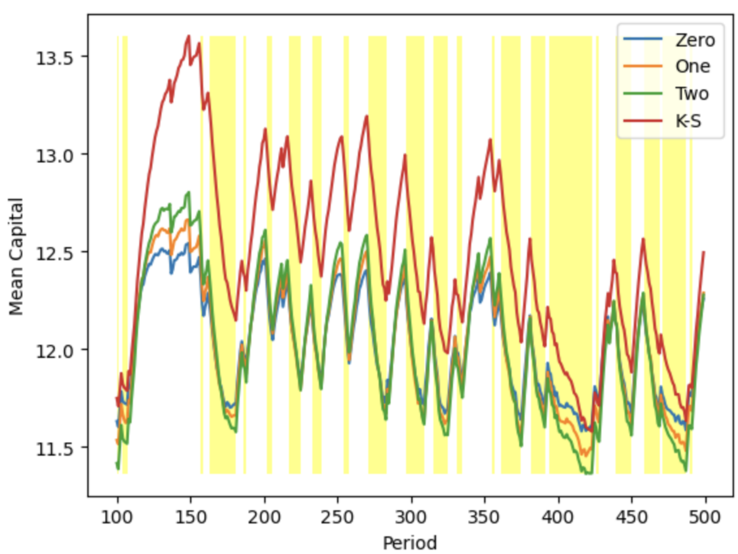

Figure 5: The yellow indicates recessionary periods.

We interpret this result in the following sense. Foresight alters individual savings decisions, in particular due to the incorporation of additional knowledge of variability in equilibrium capital and hence interest rates. The infinite horizon scenario assumes that individuals can track the mean of

[3]We simulate the model 1000 times for 1000 periods, so while the likelihood of 50 consecutive good shocks is low, we are able to observe sufficiently many to obtain an estimate of the mean capital after 50 consecutive shocks.

aggregate capital in all future agggregate states and that it can take on different values in each state, this induces an equilibrium response of from individuals that has a feedback into generating variability in equilibrium capital. When the ability to foresee different future values of mean capital is diminished, individuals optimize without knowledge of greater variability, hence the equilibrium response is diminished (individuals assume a stationary value of mean capital in the periods beyond where they have foresight). Hence foresight induces a feedback mechanism in generating variability.

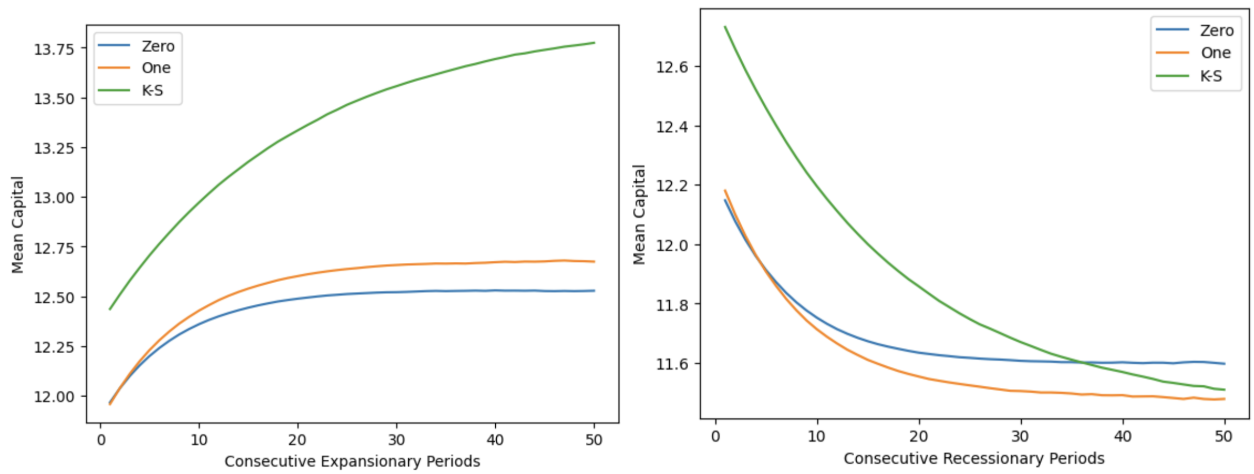

Figure 6

We view this as a new channel for explaining variability in heterogeneous agent models. Interestingly, we also demonstrate that foresight need not stem from behavior induced by precautionary savings, but can interact in any intertemporal settings where there exists a simultaneous connection between the evolution of endogenous equilibrium aggregate state variables and individual choice variables.

4.2 Dynamic Oligopoly - Capacity Constraint

We demonstrate the adaptiveness of foresight in the context of Ericson-Pakes type dynamic oligopoly models. Specifically, we simulate a capacity constraint model with aggregate productivity shocks where there is a moment-based Markov Equilibrium approximation Ifrach and Weintraub (2017). In this context, firms track moments of the distribution of firm capacity. For this particular case, there is a linear relationship between the first moment of firm capacity and equilibrium prices via the inverse demand function.

The firm's value function is composed of a linear single-period profit function determined by an exogenous aggregate demand shock z_t, individual capacity x_i, the mean aggregate capacity $\bar{x}$ and a linear cost of investment as follows: $\pi(\tilde{x}_t, x_{i,t}, z_t) - c(i_t)$ where $\pi = z * (a - b * \bar{x}) * x_i$ and $c(i) = c * i$.

$$V(\tilde{x}_t, x_{i,t}, z_t) = \max_i \pi(\tilde{x}_t, x_{i,t}, z_t) - c(i_t) + \beta * V(\tilde{x}_{t+1}, x_{i,t+1}, z_t)$$

Crucial to this model, is that firms understand the evolution of $\tilde{x}$, the distribution of firm capacities[4]. Under a moment-based Markov Equilibrium, firms track moments of $\tilde{x}$ in order to determine the transition dynamics of $\tilde{x}$, and in this case tracking the first moment.

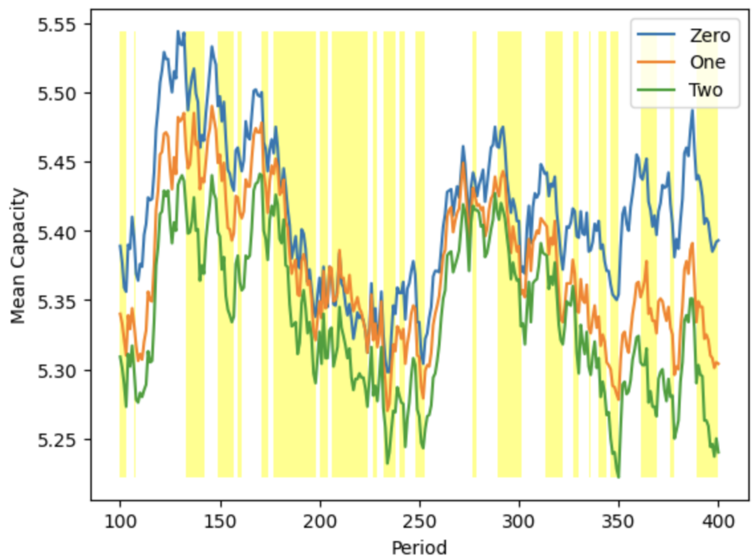

Figure 7: The yellow indicates recessionary periods

We compute the foresight equilibrium dynamics in the case of zero, one and two periods of foresight. Similarly to the Krusell-Smith simulation, we observe a greater amount of variability in the endogenous aggregate state variable, in this case mean firm capacity. Specifically, we observe that mean capacity is more responsive to good and bad aggregate demand shocks with greater foresight. Figure 7 demonstrates the variability under the simulation of a single path of 400 periods. Along with figure 8, these figures showcase a greater responsiveness to consecutive aggregate shocks with greater degrees of foresight, leading to greater variability in equilibrium prices. Table 3 and 4 show that the variance of mean aggregate capacity over a simulated path of aggregate shocks is increasing in the degree of foresight.

[4]Firm investment in period t, i_t, determines the transition probabilities of $x_{i,t} \rightarrow x_{i,t+1}$

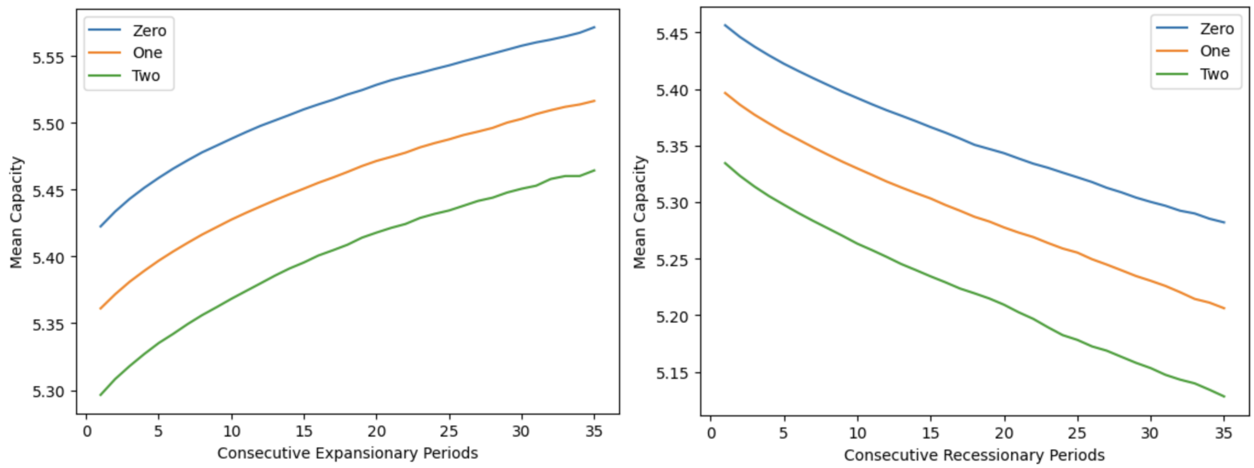

Figure 8

In this case, agent decisions do not stem from risk-aversion or precautionary-savings consumption-smoothing motives. Instead, firms adjust their investment decisions to reflect their ever-more variable prediction of aggregate capacity in order to better match the transition dynamics of their own individual state capacity. Foresight gives the firm greater insight on the evolution of the distribution of future aggregate capacity and how it differs from the current aggregate state capacity. Hence the firm adjusts its investment decisions more variably which once again induces a feedback between firms gaining insight of variation of the aggregate state and optimal policy decisions inducing greater variability.

Table 3: Single Path - 500 Periods

	Zero	**One**	**Two**
$\sigma_{\bar{x}}$	0.075	0.079	0.087
$\bar{\sigma}_x$	3.46	3.40	3.34

Table 4: 1000 Paths - 1000 Periods

	Zero	**One**	**Two**
$\sigma_{\bar{x}}$	0.087	0.093	0.101
$\bar{\sigma}_x$	3.46	3.40	3.35

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