Reducing the Error of Digital Algorithms for Deductive Signal Processing Based on Their Multi-Stage Discrete Fourier Transform by the Difference Digital Filters

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***Abstract*–The issues of using digital methods of discrete Fourier transform of polyharmonic signals for their deductive processing by elementary devices of nanoelectronics or programmable logic devices are considered. Deductive processing of digital signals is performed on the basis of their discrete Fourier transform by adding and shifting time samples of these signals. Their digital difference filtering with integer difference coefficients provides such a discrete Fourier transform without performing multiplication operations. Unerroric of deductive processing of polyharmonic signals based on their discrete Fourier transform, which does not require multiplication operations, will reduce the error of such processing on elementary nanoelectronics devices and programmable logic devices.**

***Keywords*–deductive signal processing; difference digital filter; digital signal processing; discrete Fourier transform; programmable logic device.**

I. Introduction

The constant development of the nanoelectronics element base stimulates the advanced development of computational algorithms for converting digital signals by its elementary devices [1]. In some cases, when building these algorithms, you can use digital signal processing (DSP) methods on universal or specialized processors and programmable logic devices (PLD) [2-4]. Moreover, the limited hardware implementation capabilities of fast matrix multipliers make it necessary to develop new DSP algorithms without performing multiplication operations.

Deductive digital signals processing (DDSP) of based on their multi-stage discrete Fourier transform (MDFT) using digital difference filters (DDF) provides DSP only with addition and shift operations. The concept of DDSP means their recurrent representation and transformation by finite difference methods [5]. However, the triviality of elementary nanoelectronics devices actualizes the search for a solution to the problem of reducing the error of software and hardware implementation of MDFT computational algorithms for digital signals based on their non-recursive difference digital filtering [6]. And the integer value of the difference coefficients of DDF allows you to reduce hardware costs for PLD implementation of computational algorithms of DSP, since it provides an DDSP based on MDFT using the DDF without performing multiplication operations [5-9].

The effect of quantization of the difference coefficients of the DDF has a strong influence on the accuracy of digital filtering, and, consequently, on the error of the DDSP based on the MDFT using DDF. To reduce the error of this treatment and its unerroric. The concept of "unerroric" (from the Latin "errare") means the combined application of methods and procedures to reduce the error of methods and algorithms for processing information. The term "unerroric" in relation to DDSP based on MDFT refers to the active process of reducing error rate radio when differential is selected, the coefficients for DDF [10].

II. Purpose and Methods

The purpose of this report is to define and formalize the necessary and sufficient condition for reducing the error of DDSP based on MDFT using DDF. To achieve this goal, we used methods of directional search and comparative analysis of the parameters and results of MDFT based on differential digital filtering of different orders of difference.

III. Theoretical Basis

The theoretical basis for unerroric of DDSP on the basis of their MDFT is the method of finite differences to replace the n-th time samples $x(n{\cdot}T)$ digital signal with sampling period T to their n_l-s and increments of l-th order $x_{\mathrm{p}}(l,n_l{\cdot}T)$ by the formula (1) for n=0,1,2…N-1, n_l=0,1,2…N+l-1, l=1,2,3…L, and k-s coefficients $h^*(k)$ digital filtering of K-th order k_M-s differential coefficients $h_{\mathrm{p}}(M,k_M)$ digital differential filter of the K+M-th order and M-th order difference for k=0,1,2...K-1 and k_M =0,1,2... K+M-1, as shown in figure 1 [6-9]:

$$x(n\cdot T)=\sum_{n_0=0}^{n}\sum_{n_1=0}^{n_0}\ldots\sum_{n_l=0}^{n_{l-1}}\ldots\sum_{n_{L-1}=0}^{n_{L-2}}\sum_{n_L=0}^{n_{L-1}}x_{\mathrm{p}}(L,n_L\cdot T). \quad (1)$$

This replacement allows the algorithm J-point MDFT performing convolution k-th coefficients of the filter $h^*(j,k)$ and n-th time samples $x(n{\cdot}T)$ via the canonical formula digital

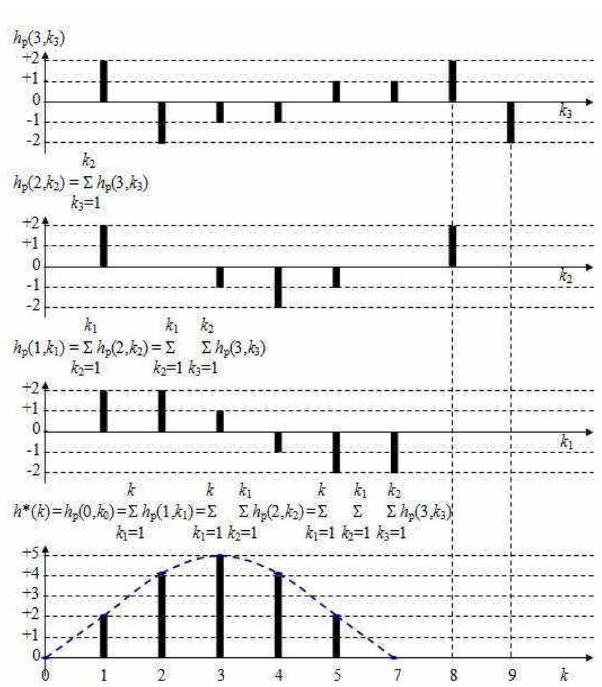

Fig. 1 – Replacement of k -th coefficients $h^*(k)$ of K-th order digital filtration with k_m-th difference coefficients $h_p(m,k_m)$ of $K+m$-th order digital difference filtration and m-th order of difference at $k=0,1,2\ldots K-1$, $k_m=0,1,2\ldots K+m-1$, $m=1,2,3\ldots M$

filtering with finite impulse response by successive summation of low-bit k_M -th differential coefficients of M-th order difference $h_p(j,M,k_M)$ and low-bit n_L increments L-th order $x_p(L,n_L\cdot T)$ for $j=1,2,3\ldots J$, $k=0,1,2\ldots K-1$, $k_M=0,1,2\ldots K+M-1$, $n=0,1,2\ldots N-1$ and $n_L=0,1,2\ldots N+L-1$ [5-6]. The MDFT algorithm has a pyramid-type structure. The pyramid structure of the MDFT algorithm is a sequence of DSP stages, each of which consists of computational procedures for dividing the spectrum of a digital signal into bands using the methods CORDIC (Coordinate Rotation Digital Compute) and difference digital filtering of time samples of this signal [9]. At the first stage, the digital signal spectrum is divided into three bands. At each subsequent stage, the number of band spectra is three times greater than at the previous stage. The multi-stage structure of the MDFT algorithm allows adapting it to changes in time in the frequency resolution of the digital signal spectrum, which makes it possible to improve the quality of DDSP by significantly reducing hardware costs when implementing DDF on PLD [4-6].

IV. Practical Significance and Novelty

The practical significance of the research is to determine the necessary and sufficient conditions for reducing the error of DDSP based on MDFT using DDF, when their hardware and software implementation on elementarynanoelectronics devices and on PLD.

The novelty of the research results lies in the formalization of this condition. The reliability of the research results is confirmed by their compliance with the results of well-known developments of digital signal processing methods.

V. Theoretical Background of the Research

MDFT has a pyramidal structure that ensures the recurrence of spectral analysis of a polyharmonic digital signal at all stages of such a Fourier transform. It is an alternation of computational algorithms of CORDIC and difference digital filtering. Using DDF with integer difference coefficients of digital filtering allows replacing each multiplication operation in the MDFT algorithm with a set of addition and shift operations [6,8-9].

DDF can and should improve the quality of MDFT-based DDSP by reducing the error of such differential digital filtering with integer difference coefficients. This error is determined only by the effects of rounding the processed numbers in case of possible register overflows during multiple addition operations. In addition, the effect of quantization of the coefficients of such digital filtering has a strong influence on its accuracy [10].

Therefore, to reduce the error of DDSP based on MDFT using DDF, it is necessary and sufficient to determine the acceptable ratios of the values of the orders of its DDF and the orders of their difference [6, 10]. Moreover, errors in rounding the results of arithmetic operations on digital signal counts are similar to the noise of quantization of this signal. But the theoretical analysis of the total noise due to quantization effects is always very difficult, since there are many sources of rounding noise, since they are all located in different parts of the software or hardware implementation of MDFT algorithm [10].

Thus, it is unlikely that a General analysis of the value of the DDSP error, which is caused by quantization effects and depends on the structure of the computational algorithm and the way numbers are represented in digital computing devices, is possible. At the same time, reducing the error of DDSP based on MDFT using DDF when developing its computational algorithms for integer DDF difference coefficients can and should provide unerroric of such processing by methods of comparative analysis and directed iteration of MDFT results using these DDF of different orders at different orders of difference in order to reduce hardware costs for software and hardware implementation of computational algorithms based on such MDFT.

VI. The Analysis of the Research Results

MDFT digital methods provide the calculation of j-th statistics (estimates) $y(j,n\cdot T)$ coefficients of the J-point recurrent discrete Fourier transform of the digital signal $x(n\cdot T)$ by J-band filtering of the n_L-th increments of the L-th order $x_p(L, n_L\cdot T)$ of its n-th time samples based on their digital difference filtration $K+M$-th order with k_M-th difference coefficients of the M-th order of the difference $h_p(j,M,k_M)$ using the formula (2) at

j=1,2,3…J, k=0,1,2…K-1, k_M=0,1,2…$K+M$-1, m=1,2,3…M и n=0,1,2…N-1, $N \geq L$ [6]:

$$
\begin{aligned}
&y(j,n\cdot T)=\sum_{k=0}^{K-1}h^*(j,k)\cdot x(n\cdot T-k\cdot T)=\\
&=\sum_{k=0}^{K+M-1}\sum_{k_0=0}^{k}\sum_{k_1=0}^{k_0}\dots\sum_{k_M=0}^{k_{M-1}}h_{\mathrm{p}}(j,M,k_M)\cdot x(n\cdot T-k_M\cdot T)=\\
&=\sum_{k=0}^{K+M-1}\sum_{k_0=0}^{k}\sum_{k_1=0}^{k_0}\dots\\
&\dots\sum_{k_M=0}^{k_{M-1}}\sum_{n_0=0}^{n}\sum_{n_1=0}^{n_0}\dots\sum_{n_L=0}^{n_{L-1}}h_{\mathrm{p}}(j,M,k_M)\cdot x_{\mathrm{p}}(L,n_L\cdot T-k_M\cdot T).
\end{aligned}
\tag{2}
$$

The results of DDSP unerroric using digital pyramid-type MDFT methods with DDF of different orders and different orders of difference with integer difference coefficients are shown in figure 1.

Choosing the ratio of DDF orders and their difference orders reduces the error of deductive processing of a polyharmonic digital signal based on MDFT, which does not require multiplication operations, to 3% compared to the error of DSP based on fast Fourier transform (FFT) with a minimum number of multiplication and addition operations, the pulse characteristic of which, having the form of (*Sin*[*x*]/*x*), is displayed by a bold dotted line in figure 1. Moreover, the hardware cost of such a DSP exceeds the hardware cost of deductive processing of the same signal. Unerroric DDSP based on MDFT using DDF allows you to select the order of difference of coefficients used by DDF with the permissible error of such a conversion based on the acceptability of the ratio of the required number of difference coefficients of DDF of the specified order for the selected order of difference and the required number of additional addition operations to perform MDFT using DDF with such coefficients, as shown in figure 1, where the MDFT error does not exceed 3% of the FFT error.

A necessary and sufficient condition for reducing the error of an MDFT-based DDSP using DDF without performing arithmetic multiplication operations is the triviality of the values of integer DDF difference coefficients of a given order and a given order of difference. This condition can and should be met by setting the ratio of values of DDF orders with integer difference coefficients and their difference orders, which is provided by a minimum number of additional addition operations for MDFT digital signals using these DDF to achieve the permissible error level of DDSP based on MDFT without performing multiplication operations in comparison with the error level of the corresponding digital FFT algorithm with a minimum number of multiplication operations.

The formalization of such conditions allows to determine the value of the order K and the order of the difference M for DDF, in which case it's integer differential coefficients have only a trivial value, provides the formula (2) convolution of the n_L-th increment L-th order $x_{\mathrm{p}}(L, n_L \cdot T - k_M \cdot T)$ and k_M-th differential coefficient of M-th order difference $h_{\mathrm{p}}(j,M,k_M)$ without multiplying by the formula (3) for j=1,2,3…J, k_M=0,1,2…$K+M$-1, n_L=0,1,2…$N+L$-1:

$$
\begin{aligned}
&h_{\mathrm{p}}(j,M,k_M)\cdot x_{\mathrm{p}}(L,n_L\cdot T-k_M\cdot T)=\\
&=\begin{cases}x_{\mathrm{p}}(L,n_L\cdot T-k_M\cdot T), \text{ if } h_{\mathrm{p}}(j,M,k_M)=1;\\ 0, \text{ if } h_{\mathrm{p}}(j,M,k_M)=0;\\ -x_{\mathrm{p}}(L,n_L\cdot T-k_M\cdot T) \text{ if } h_{\mathrm{p}}(j,M,k_M)=-1.\end{cases}
\end{aligned}
\tag{3}
$$

VII. Conclusion

The variability of constructing the MDFT computational algorithm using a high-order difference DDF is due to the possibility of choosing such trivial sets of integer coefficients of difference digital filtering that differ both in the order of this filtration and in the order of its difference. Unerroric of deductive signal processing by digital methods MDFT allows reducing hardware costs for software and hardware implementation of computational algorithms for converting digital signals on elementary devices of nanoelectronics or PLD. The decision by the developers of such algorithms on the validity of the ratio of the orders of DDF used and the orders of their difference, ensuring a balance between the number of multiplication operations required for MDFT digital signals, and the number of addition operations required for MDFT without performing multiplication operations, allows you to meet the necessary and sufficient condition for reducing the error of DDSP based on MDFT using DDF without performing arithmetic multiplication operations.

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