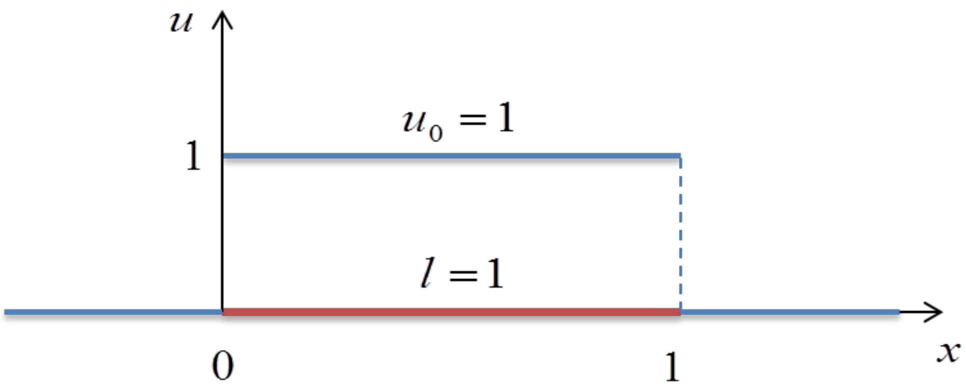
Task №2

(Solution of the one-dimensional homogeneous heat equation)

Statement of the problem.



A rod with the length of $l = 1$ at the initial time moment $t_0 = 0$ has the temperature equals to $u_0 = 1$. The ambient temperature is maintained at 0.

Initial condition: $u(x, 0) = u_0$.

Boundary condition: $u(0, t) = u(l, t) = 0$.

It is necessary to solve the one-dimensional homogeneous heat equation in the form

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2},$$

whose finite difference approximation has the form

$$\frac{u_i^{n+1} - u_i^n}{\tau} = k\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{h^2}. \qquad (1)$$

The task:

1) Obtain the temperature distribution along the rod at time *T = 0,1*, using the following parameters: *k = 1, h = 0,02; dt = 0,0002* (see note 1). Display the temperature values in 11 (including edge) points, i.e. at the ends of small segments of 0.1 in length. These temperatures must be obtained by different processes.

2) Compare with the exact solution (solved in the same program):

$$u(x,t)=\frac{4u_0}{\pi}\sum_{m=0}^{\infty}\frac{exp\left(-k\pi^2(2m+1)^2\frac{t}{l^2}\right)}{2m+1}\sin\left(\frac{\pi(2m+1)x}{l}\right)$$

3) On the one Cartesian coordinate plane, build 3 graphs of the speedup *S* dependence on the number of processes *p*, where *p = 1,2,3,…,8-12* for the number of points equal to 2000, 10 000, 50 000 (see note 2). For the same number of processes, build graphs of the efficiency *E(p) = S / p*. Make the conclusion.

4) Think about how to organize the transfer of messages between processes using blocking send/receive functions so, that the total transmission time at the end of each time step will be proportional to *O(1)* (not *O(p)*). Implement both options.

Notes:

1) The time step must satisfy the Courant condition (Courant–Friedrichs–Lewy indeed).

2) The beginning of the graphs should lie almost on a straight line. Courant condition must be satisfied. To ensure that the counting time is reasonable, the finite time *T* can be reduced (say, *T* can be equal to 10^{-4}).