Frying Pan Mechanism Project Report

İslam AYDOĞMUŞ

Abstract

In this project, our goal is to design a mechanism that can do the same movement as a chef cooks in a frying pan. The main action is based on mixing the daily food put into an average pan in a way that can be thrown up slowly from the end of the pan and lowered back by turning it upside down. Our design is based on a four-bar mechanism, provided it can be operated by human power or a small engine. The purpose of this task is to teach us how to analyze a movement, how many ways we find a solution to a problem, and how to apply the knowledge we have learned in our mechatronic system design lessons to real life.

Contents

1 Introduction

The main procedure we followed while making this frying pan shaking mechanism was like following:
We firstly realised the movement by capturing a video of the shaking motion with a camera. We decided to use body guidance synthesis technique to generate the motion of the frying pan.

Figure 1: A frame from the captured motion. Special thanks to the Assist. Mertcan KOÇAK for helping us capturing this motion.

The motion analysed by a program called *Tracker* and gathered the position and angle data from the video. We seen from video we captured. The inputs then are fed into the polynomial approximation of our objective function. The results were construction parameters we wanted to obtain. Lastly we simulated the mechanism in *SolidWorks* and observed its motion.

2 Kinematic Synthesis

2.1 About Mechanism

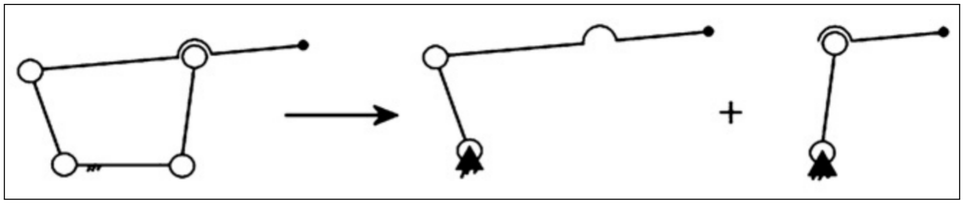

Figure 2: Separating four-bar mechanism into two serial manipulators. Figure is from [1].

We want our precision points to have both coordinates and orientation of our end-effector, namely coupler link, so that we can realise the analyzed motion by a simple four bar mechanism. In order to realise the four bar we want, we can make a trick to separate the four-bar into two 2 DoF[1] serial manipulators as can be seen in Figure 2.

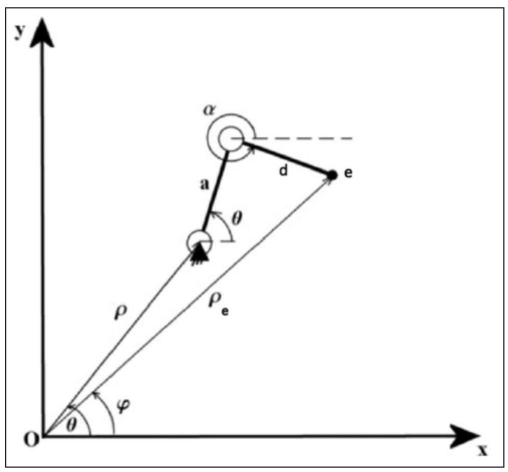

Figure 3: Construction parameters and variables of serial manipulator. Figure is from [1].

We have *construction parameters* of $\{a,\ d,\ \rho,\ \theta_1\}$ and *variables* of $\{x_e,\ y_e,\ \theta_1\}$ for serial manipulator. Determining construction parameters and variables are important because the method we're giving values to variables by *precision points* and try to obtain construction parameters to, of course, construct the mechanism that will supply us the desired motion.

2.2 Obtaining Objective Function

We start obtaining objective function by defining *loop closure equation* for 2 DoF serial manipulator;

$$\vec{\rho} + \vec{a} + \vec{d} = \vec{e} \tag{1}$$

[1]Degrees of Freedom

. . . and then separating x and y components of Eqn. (1):

$$\rho\cos\theta_1 + a\cos\beta + d\cos\alpha = x_e$$
$$\rho\sin\theta_1 + a\sin\beta + d\sin\alpha = y_e$$

Since we don't want β to be in our objective function, we eliminate it by gathering terms that include β in left-hand side of equation:

$$a\cos\beta = x_e - \rho\cos\theta_1 - d\cos\alpha \tag{2}$$
$$a\sin\beta = y_e - \rho\sin\theta_1 - d\sin\alpha \tag{3}$$

Now we can eliminate β by squaring both equations (2) and (3) and summing them:

$$a^2(\underbrace{\sin\beta + \cos\beta}_{=1}) = x_e^2 + y_e^2 + \rho^2(\underbrace{\sin\theta_1 + \cos\theta_1}_{=1}) + d^2(\underbrace{\sin\alpha + \cos\alpha}_{=1}) - 2\rho x_e\cos\theta_1 - 2\,d\,x_e\cos\alpha$$
$$+\,2\,\rho\,d\,\cos\alpha\,\cos\theta_1 - 2\,\rho\,y_e\,\sin\theta_1 - 2\,d\,y_e\,\sin\alpha + 2\,\rho\,d\,\sin\alpha\,\sin\theta_1$$

Lastly we can reconstruct the equation as:

$$0 = -a^2 + \rho^2 + d^2 + x_e^2 + y_e^2 - 2\,\rho x_e\,\cos\theta_1 - 2\,\rho\,y_e\,\sin\theta_1 + 2\,\rho\,d\,\sin\alpha\,\sin\theta_1 + 2\,\rho\,d\,\cos\alpha\,\cos\theta_1$$
$$-\,2\,d\,(x_e\,\cos\alpha + y_e\,\sin\alpha) \tag{4}$$

Finally, Eqn. (4) is our objective function.

2.3 Polynomial Approximation

Objective polynomial is a specifically arranged version of objective function. Main form we're using are;

$$P_0f_0 + P_1f_1 + P_2f_2 + P_3f_3 + P_4f_4 + P_5f_5 - F = 0 \tag{5}$$

. . . where

$$
\begin{aligned}
P_0 &= \frac{-a^2 + \rho^2 + d^2}{2\,d}, & f_0 &= 1\\
P_1 &= \frac{1}{d}, & f_1 &= \frac{x_e^2 + y_e^2}{2}\\
P_2 &= \rho\,\cos\theta_1, & f_2 &= \cos\alpha\\
P_3 &= \rho\,\sin\theta_1, & f_3 &= \sin\alpha\\
P_4 &= -\frac{\rho\,\cos\theta_1}{d}, & f_4 &= x_e\\
P_5 &= -\frac{\rho\,\sin\theta_1}{d}, & f_5 &= y_e\\
F &= x_e\,\cos\alpha + y_e\,\sin\alpha
\end{aligned}
$$

Note that P_i terms only contains construction parameters while f_i terms only contains variables.

2.4 Non-linear Terms in Polynomial Approximation

Even though we have 6 polynomial constants, we are only allowed to determine 4 precision points since we have only 4 construction parameters. So we have dependent coefficients (or nonlinear parameters) of;

$$\begin{aligned} P_4 &= -P_1 P_2 \\ P_5 &= -P_1 P_3 \end{aligned} \tag{6}$$

We're going to address P_4 and P_5 as λ_1 and λ_2 respectively and construct nonlinear equations as:

$$P_i = l_i + m_i\,\lambda_1 + n_i\,\lambda_2 \tag{7}$$

... for $i = 1, 2, \ldots, 5$.

Now we substitute these parameters into equation (5);

$$\begin{aligned} &(l_0 + m_0\,\lambda_1 + n_0\,\lambda_2)\,f_0 + (l_1 + m_1\,\lambda_1 + n_1\,\lambda_2)\,f_1 + (l_2 + m_2\,\lambda_1 + n_2\,\lambda_2)\,f_2 + (l_3 + m_3\,\lambda_1 + n_3\,\lambda_2)\,f_3 \\ &\qquad + \lambda_1\,f_4 + \lambda_2\,f_5 - F = 0 \end{aligned} \tag{8}$$

Now we can separate the equation (8) into its three components namely linear, first nonlinear and second nonlinear components;

$$\begin{aligned} l_0\,f_0 + l_1\,f_1 + l_2\,f_2 + l_3\,f_3 &= F \\ m_0\,f_0 + m_1\,f_1 + m_2\,f_2 + m_3\,f_3 &= -f_4 \\ n_0\,f_0 + n_1\,f_1 + n_2\,f_2 + n_3\,f_3 &= -f_5 \end{aligned}$$

... and then start to construct respective matrices:

$$\underbrace{\begin{bmatrix} f_0^1 & f_1^1 & f_2^1 & f_3^1 \\ f_0^2 & f_1^2 & f_2^2 & f_3^2 \\ f_0^3 & f_1^3 & f_2^3 & f_3^3 \\ f_0^4 & f_1^4 & f_2^4 & f_3^4 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} l_0 \\ l_1 \\ l_2 \\ l_3 \end{bmatrix}}_{\mathbf{L}} = \underbrace{\begin{bmatrix} F^1 \\ F^2 \\ F^3 \\ F^4 \end{bmatrix}}_{\mathbf{F}}$$

$$\mathbf{A} \cdot \underbrace{\begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \end{bmatrix}}_{\mathbf{M}} = \underbrace{\begin{bmatrix} -f_4^1 \\ -f_4^2 \\ -f_4^3 \\ -f_4^4 \end{bmatrix}}_{\mathbf{f_4}} \qquad \mathbf{A} \cdot \underbrace{\begin{bmatrix} n_0 \\ n_1 \\ n_2 \\ n_3 \end{bmatrix}}_{\mathbf{N}} = \underbrace{\begin{bmatrix} -f_5^1 \\ -f_5^2 \\ -f_5^3 \\ -f_5^4 \end{bmatrix}}_{\mathbf{f_5}}$$

Where superscript indicates the value of the parameter at the respective precision point. Now we can calculate values of **L**, **M** and **N** by substituting precision points and calculating below equations;

$$\begin{aligned} \mathbf{L} &= \mathbf{A^{-1}} \cdot \mathbf{F} \\ \mathbf{M} &= \mathbf{A^{-1}} \cdot \mathbf{f_4} \\ \mathbf{N} &= \mathbf{A^{-1}} \cdot \mathbf{f_5} \end{aligned}$$

Since we now know matrices $\mathbf{L}$, $\mathbf{M}$ and $\mathbf{N}$; we can find λ_1 and λ_2 by solving substituting equations (7) into equations (6);

$$\begin{aligned}
\lambda_1 &= -(l_1 + m_1\,\lambda_1 + n_1\,\lambda_2)\,(l_2 + m_2\,\lambda_1 + n_2\,\lambda_2) \\
\lambda_2 &= -(l_1 + m_1\,\lambda_1 + n_1\,\lambda_2)\,(l_3 + m_3\,\lambda_1 + n_3\,\lambda_2)
\end{aligned} \tag{9}$$

...so we'll obtain results;

$$\begin{aligned}
\lambda_1 &= \{\lambda_{11}, \lambda_{12}, \lambda_{13}\} \\
\lambda_2 &= \{\lambda_{21}, \lambda_{22}, \lambda_{23}\}
\end{aligned} \tag{10}$$

...since we had third degree polynomial in above equations.

Now we can solve for P_0, P_1, P_2 and P_3 with solutions from Eqns. (10). Two of these, reasonable, solutions will give us enough information to find construction parameters of the desired four-bar mechanism that can be seen in Fig. 2.

3 Calculations

PP	x_e (cm)	y_e (cm)	α
1	29.96	5.673	-9.1°
2	32.20	7.410	-6.9°
3	33.60	10.14	-3.1°
4	33.46	13.59	2.4°

Table 1: Precision Points used in the synthesis

We substituted precision points indicated into Table 1 in matrices $\mathbf{A}$, $\mathbf{F}$, $\mathbf{f}_4$ and $\mathbf{f}_5$ and calculated vectors $\mathbf{L}$, $\mathbf{M}$ and $\mathbf{N}$ by method we've discussed earlier.

$$\begin{bmatrix} l_0 \\ l_1 \\ l_2 \\ l_3 \end{bmatrix} = \begin{bmatrix} 69.3173 \\ 0.0344 \\ -57.6962 \\ -2.2400 \end{bmatrix} \qquad \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} -39.5363 \\ -0.0367 \\ 29.4086 \\ 15.1446 \end{bmatrix} \qquad \begin{bmatrix} n_0 \\ n_1 \\ n_2 \\ n_3 \end{bmatrix} = \begin{bmatrix} 43.2785 \\ -0.0014 \\ -54.5544 \\ -35.1047 \end{bmatrix} \tag{11}$$

Now we can use results of $\mathbf{L}$, $\mathbf{M}$ and $\mathbf{N}$ in (11) on Eqn. (9) to find the three pairs of answers on the form of (10).

$$\begin{aligned} \lambda_1 &= \{11.1469,\ 0.50908,\ 0.21908\} \\ \lambda_2 &= \{4.41488,\ -0.19867,\ -0.79306\} \end{aligned} \tag{12}$$

Finally we can substitute both of the results (12) and (11) into Eqn. (7) to obtain the values of P_0, P_1, P_2 and P_3 (Since we have three λ pairs, we will get three different possible result for each construction parameters).

$$P_0 = \frac{-a^2 + \rho^2 + d^2}{2\,d} = -180.3200 \qquad P_1 = \frac{1}{d} = -0.3809$$
$$P_2 = \rho\,\cos\theta_1 = 29.2666 \qquad P_3 = \rho\,\sin\theta_1 = 11.5914$$

So construction parameters for given set of equations are;

No.	a (cm)	d (cm)	ρ (cm)	θ_1
1	7.14	-2.63	31.48	21.61°
2	3.14	62.64	34.23	158.68°
3	17.59	36.46	30.00	105.44°

Table 2: Construction Parameters of the Mechanism

As we can clearly see, construction parameters that Table 2 no. 1 suggested aren't reasonable since we can't have negative link lengths. However this isn't a problem at all since we only needed two set of logical construction parameters to construct four-bar mechanism that can be seen in the Figure 2.

4 Results of Polynomial Synthesis

4.1 General Mechanism

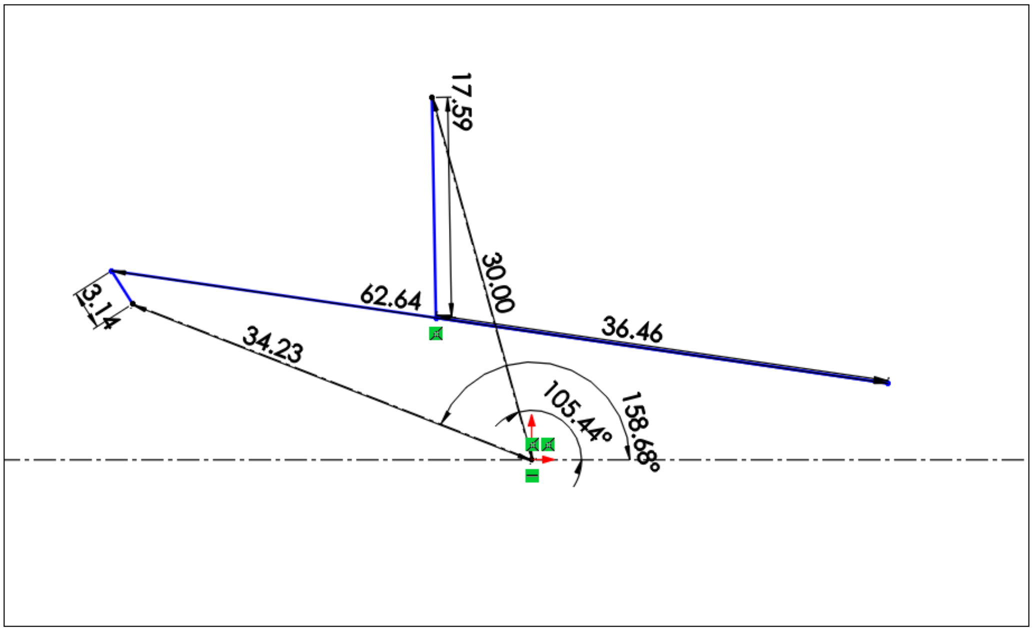

Figure 4: Mechanism constructed by construction parameters from Table 2.

Mechanism generated gave us a wonderful new perspective about how we can realise this motion. At the start of this project, we expected the ground links of the result to be closer to a four-bar mechanism like in Figure 2, but the mechanism generated uses one of the links to supply the circular motion of frying operation, other link supports the movement by swinging the mechanism like a cradle. This result not seems like important but it is so novel and so simple that, you cannot do much but enjoy.

4.2 Performance at Precision Points

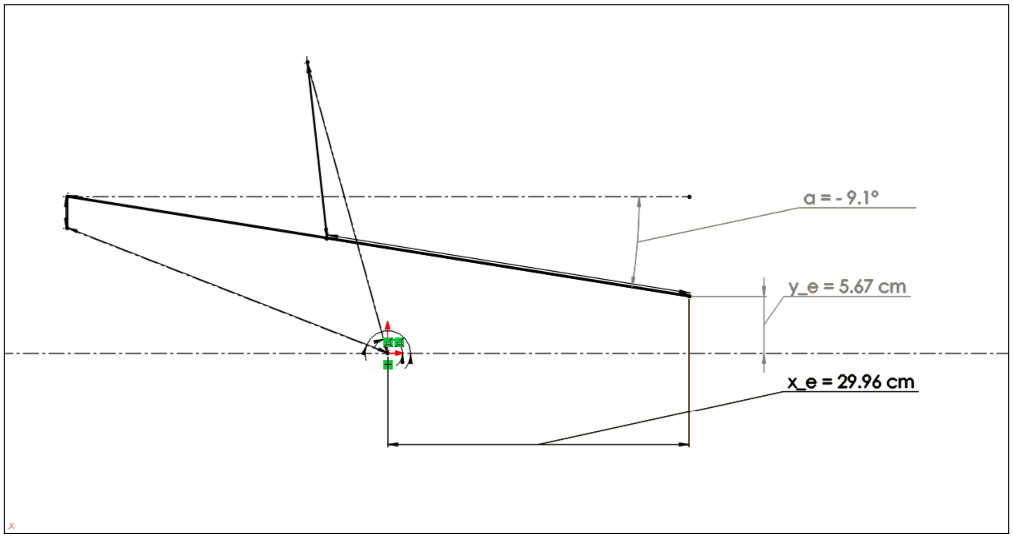

Figure 5: Mechanism at first precision Point

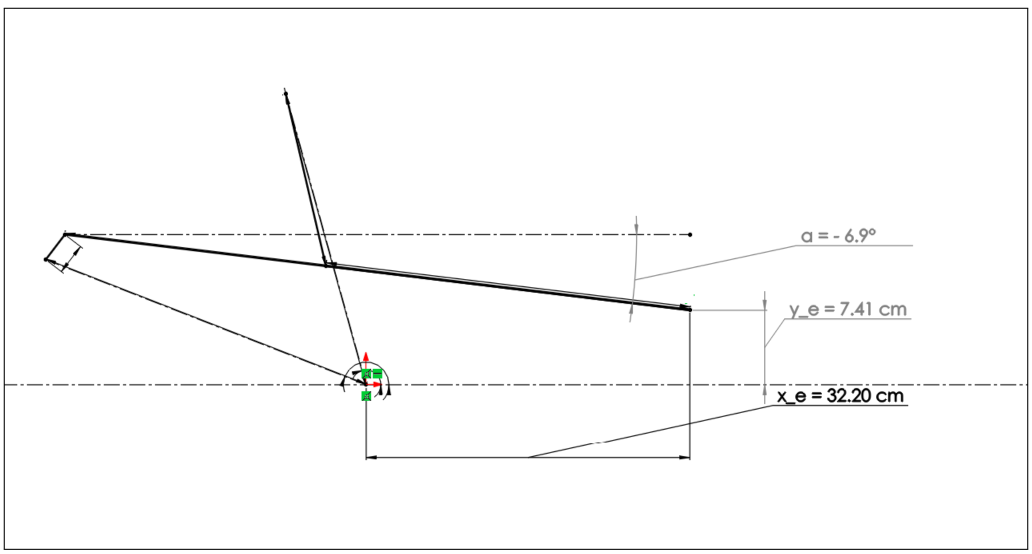

Figure 6: Mechanism at second precision Point

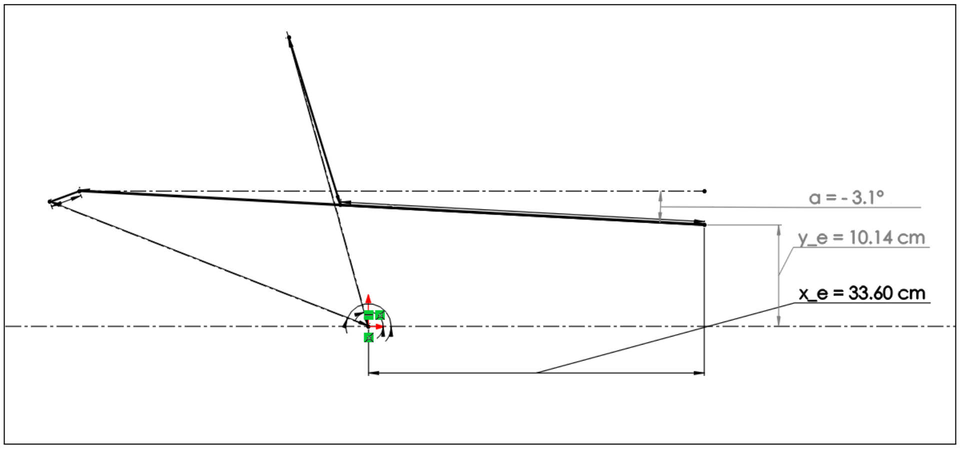

Figure 7: Mechanism at third precision Point

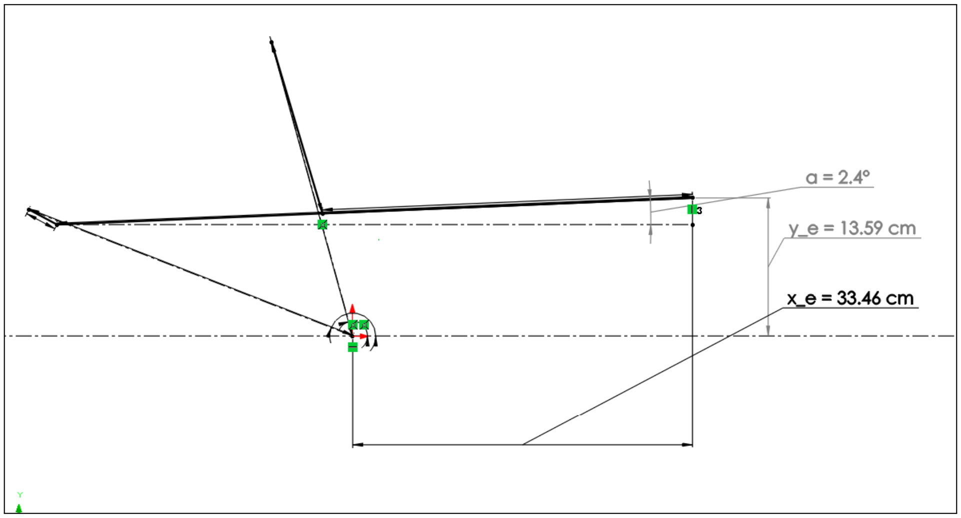

Figure 8: Mechanism at fourth precision Point

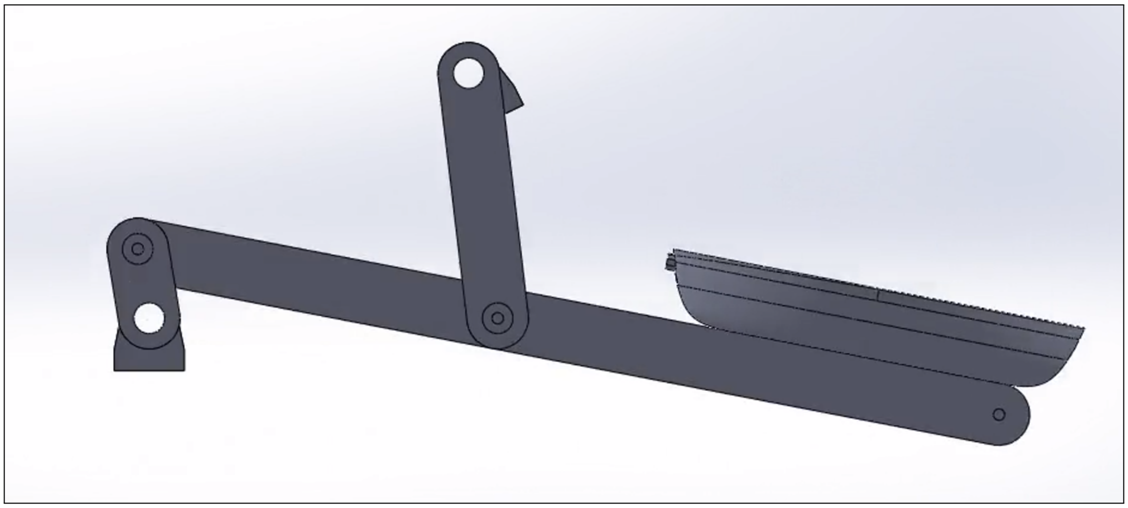

Figure 9: A very simple prototype of our results.

5 Final Design

5.1 Scaling the Mechanism

We used 3D printing to manufacture all of our mechanism. As you can see from Table 2 our construction parameters aren't suitable for 3D printing. We had 220x220x250 printing space in our hand so compromises had to be made.

Firstly we changed the pan with a new smaller one. The previous pan that you can see in Fig. 1 had 28 cm diameter. We changed this pan with smaller 18 cm diameter pan. For this reason we scaled every construction parameter by 18/28. Unfortunately this scaling weren't enough for our limited printing space so we changed the parameters until it fits into our limitations. Final construction parameters are:

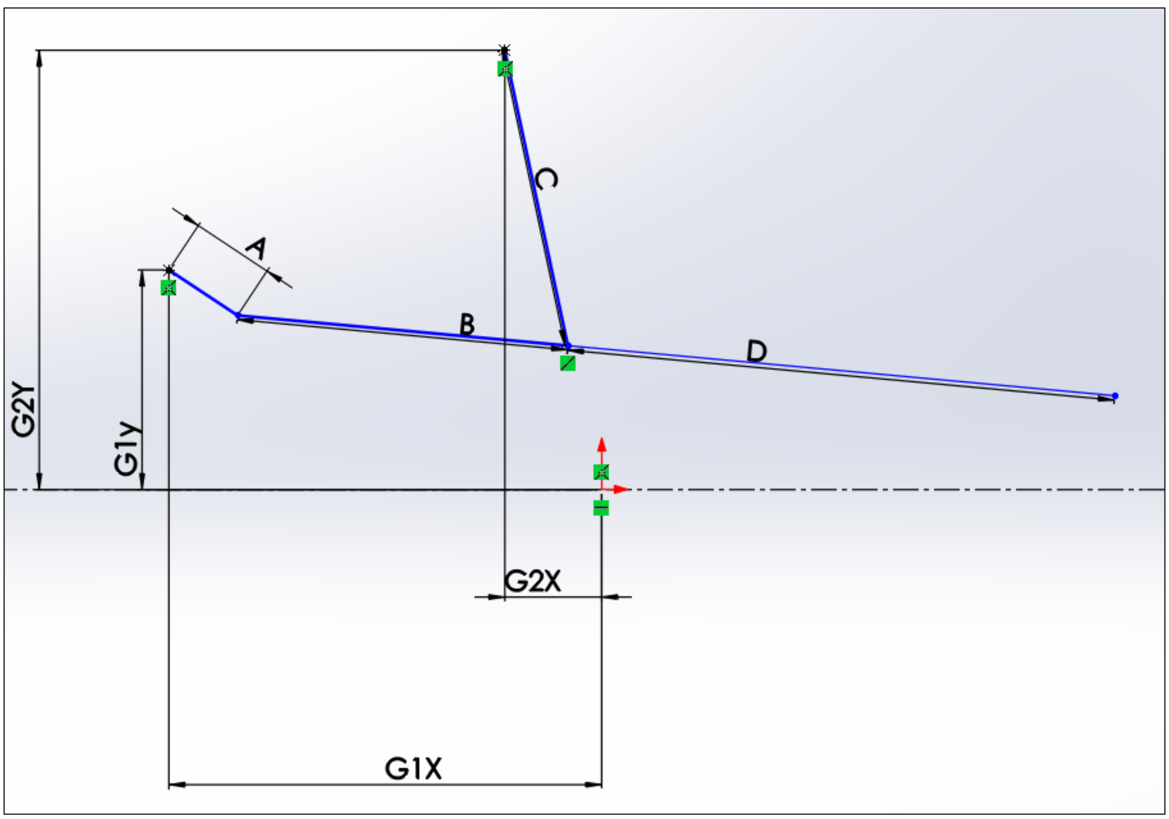

Figure 10: Final Construction Parameters

A (cm)	B (cm)	C (cm)	D (cm)	G1x (cm)	G1y (cm)	G2x (cm)	G2y (cm)
3.00	12.81	11.00	20.00	15.76	8.00	3.53	16.00

Table 3: Construction Parameters of the Mechanism that is indicated in Fig. 10.

Now the new mechanism look like below figure in SolidWorks:

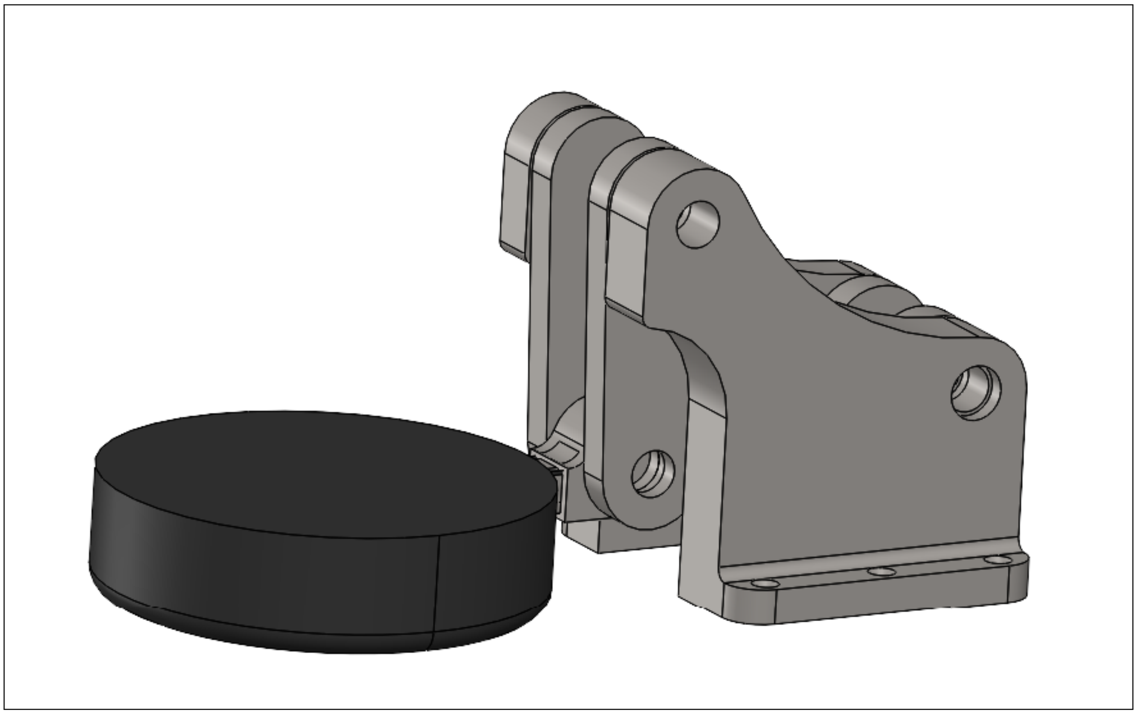

Figure 11: Designed mechanism.

6 Kinematic Analysis

6.1 Identifying the tasks

Our task for kinematic analysis is to find the acceleration of crucial points so that we can use that acceleration to find forces acting on the body via Newton's Equations. This allow us the find the required minimum torque for our crank to support the motion desired.

6.2 Obtaining Outputs in Terms of Inputs

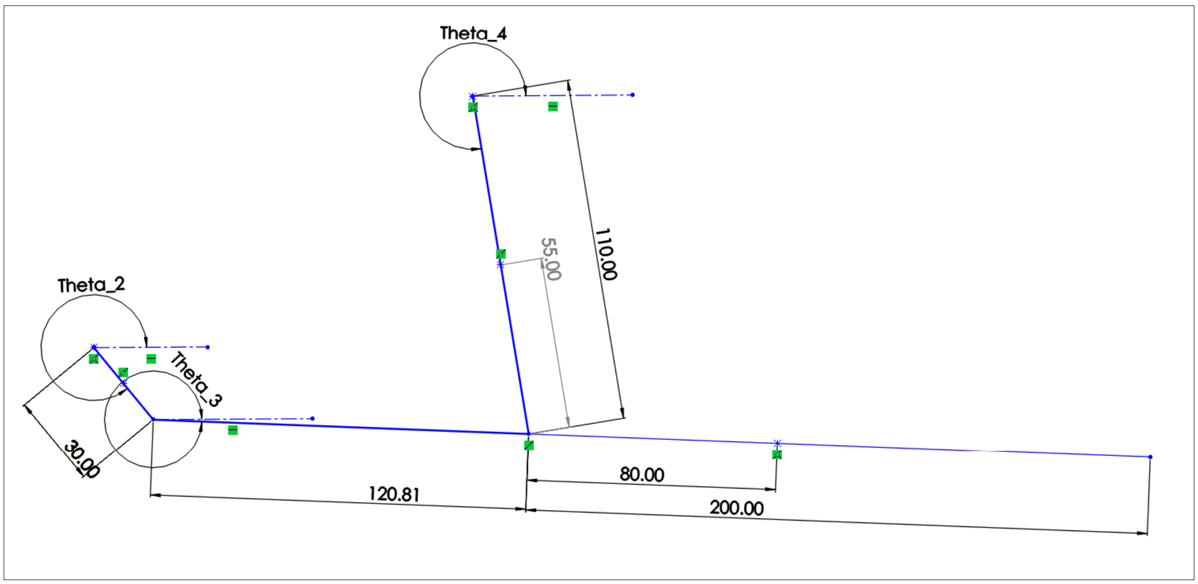

Figure 12: We logged θ_3 and θ_4 for given θ_2

Kinematic analysis of our mechanism includes finding θ_3 and θ_4 in terms of θ_2 that we want to become our only independent variable. We couldn't figured this values analytically sadly. But we figured out another approach, namely numeric approach. We logged all θ_3 and θ_4 values for given θ_2 values for every 2 degrees. Finally we had 180 values each for θ_3 and θ_4 values for given 180 θ_2 values.
We log the data every 2 degrees, since we need to find period T for our numerical derivation progress, we have to determine the angular velocity that we're rotating the crank. We predict that we can rotate the crank 1 rotation per second, so calculations came out as following;

$$\omega[n] = \frac{\theta[n+1] - \theta[n]}{T} \quad \alpha[n] = \frac{\omega[n+1] - \omega[n]}{T}$$

...where

$$T = \frac{1}{180}[sec]$$

6.3 Position Analysis

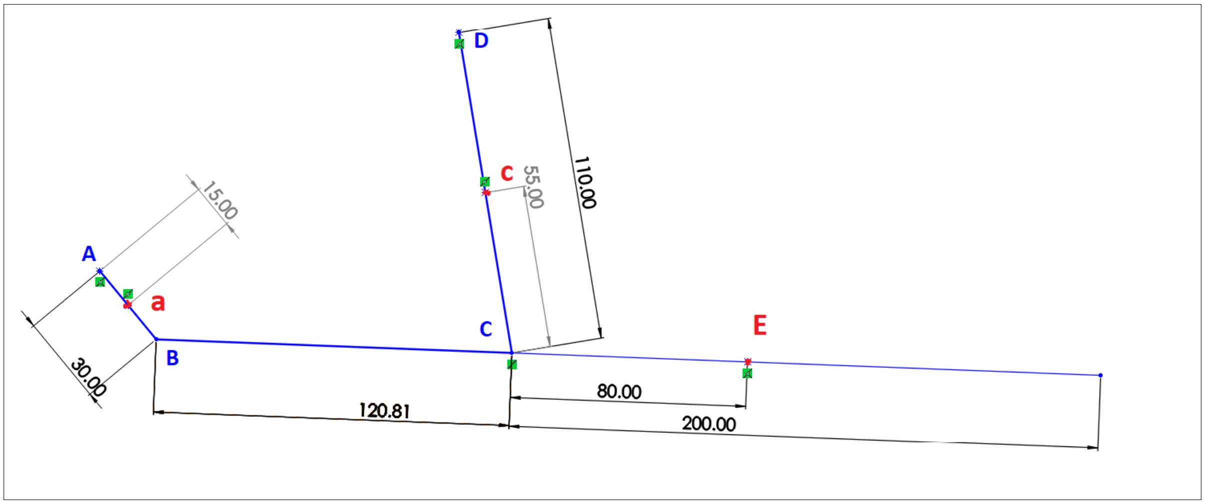

Figure 13: Positions of centers of masses of the 3 links (Units are millimeters).

We want to find position vectors of points a, E, and c which is center of mass of l_2, l_3, and l_3 respectively. So given vectors are constructed for these purposes:

$$\begin{aligned}
\boldsymbol{A} &= 0\,\hat{\boldsymbol{\imath}} + 0\,\hat{\boldsymbol{\jmath}} \\
\boldsymbol{a} &= 0.015 \cdot \cos\theta_2\,\hat{\boldsymbol{\imath}} + 0.015 \cdot \sin\theta_2\,\hat{\boldsymbol{\jmath}} \\
\boldsymbol{B} &= 0.03 \cdot \cos\theta_2\,\hat{\boldsymbol{\imath}} + 0.03 \cdot \sin\theta_2\,\hat{\boldsymbol{\jmath}} \\
\boldsymbol{C} &= 0.12081 \cdot \cos\theta_3\,\hat{\boldsymbol{\imath}} + 0.12081 \cdot \sin\theta_3\,\hat{\boldsymbol{\jmath}} + \boldsymbol{B} \\
\boldsymbol{c} &= 0.055 \cdot \cos\theta_4\,\hat{\boldsymbol{\imath}} + 0.055 \cdot \sin\theta_4\,\hat{\boldsymbol{\jmath}} + \boldsymbol{D} \\
\boldsymbol{E} &= 0.20081 \cdot \cos\theta_3\,\hat{\boldsymbol{\imath}} + 0.20081 \cdot \sin\theta_3\,\hat{\boldsymbol{\jmath}} + \boldsymbol{B}
\end{aligned}$$

6.4 Velocity Analysis

We want to find the velocity vectors for a, E, and c, so we're going to differentiate the position vectors in order to obtain velocity vectors:

$$\begin{aligned}
\dot{\boldsymbol{a}} &= -0.015 \cdot \sin\theta_2\,\dot{\theta}_2\,\hat{\boldsymbol{\imath}} + 0.015 \cdot \cos\theta_2\,\dot{\theta}_2\,\hat{\boldsymbol{\jmath}} \\
\dot{\boldsymbol{c}} &= -0.055 \cdot \sin\theta_4\,\dot{\theta}_4\,\hat{\boldsymbol{\imath}} + 0.055 \cdot \cos\theta_4\,\dot{\theta}_4\,\hat{\boldsymbol{\jmath}} \\
\dot{\boldsymbol{E}} &= (-0.06\,\sin\theta_2\,\dot{\theta}_2 - 0.32162\,\sin\theta_3\,\dot{\theta}_3)\,\hat{\boldsymbol{\imath}} + (0.06\,\cos\theta_2\,\dot{\theta}_2 + 0.32162\,\cos\theta_3\,\dot{\theta}_3)\,\hat{\boldsymbol{\jmath}}
\end{aligned}$$

6.5 Acceleration Analysis

Finally we want to obtain the acceleration vectors for a, E, and c, so we're going to differentiate the velocity vectors in order to obtain velocity vectors:

$$\begin{aligned}\ddot{\boldsymbol{a}} = (-0.015\,\sin(\theta_2(t))\,\frac{\partial^2}{\partial t^2}\,\theta_2(t) - 0.015\,\cos(\theta_2(t))\left(\frac{\partial}{\partial t}\,\theta_2(t)\right)^2)\,\hat{\boldsymbol{\imath}} \\ + (0.015\,\cos(\theta_2(t))\,\frac{\partial^2}{\partial t^2}\,\theta_2(t) - 0.015\,\sin(\theta_2(t))\left(\frac{\partial}{\partial t}\,\theta_2(t)\right)^2)\,\hat{\boldsymbol{\jmath}}\end{aligned}$$

$$\begin{aligned}\ddot{\boldsymbol{c}} = (-0.055\,\sin(\theta_4(t))\,\frac{\partial^2}{\partial t^2}\,\theta_4(t) - 0.055\,\cos(\theta_4(t))\left(\frac{\partial}{\partial t}\,\theta_4(t)\right)^2)\,\hat{\boldsymbol{\imath}} \\ + (0.055\,\cos(\theta_4(t))\,\frac{\partial^2}{\partial t^2}\,\theta_4(t) - 0.055\,\sin(\theta_4(t))\left(\frac{\partial}{\partial t}\,\theta_4(t)\right)^2)\,\hat{\boldsymbol{\jmath}}\end{aligned}$$

$$\ddot{\boldsymbol{E}} = (-0.06\,\cos(\theta_2(t))\,\sigma_2 - 0.32162\,\cos(\theta_3(t))\,\sigma_1 - 0.06\,\sin(\theta_2(t))\,\sigma_4 - 0.32162\,\sin(\theta_3(t))\,\sigma_3)\,\hat{\boldsymbol{\imath}} + (0.06\,\cos(\theta_2(t))\,\sigma_4 - 0.32162\,\sin(\theta_3(t))\,\sigma_1 - 0.06\,\sin(\theta_2(t))\,\sigma_2 + 0.32162\,\cos(\theta_3(t))\,\sigma_3)\,\hat{\boldsymbol{\jmath}}$$

...where

$\sigma_1 = \left(\frac{\partial}{\partial t}\,\theta_3(t)\right)^2$

$\sigma_2 = \left(\frac{\partial}{\partial t}\,\theta_2(t)\right)^2$

$\sigma_3 = \frac{\partial^2}{\partial t^2}\,\theta_3(t)$

$\sigma_4 = \frac{\partial^2}{\partial t^2}\,\theta_2(t)$

6.6 Results

We put the 180 position values we logged and put them into derived formulas at the previous sections. Raw results are as following;

As we can see from Fig. 14 and Fig. 15 our α data were to noisy. To fix this we applied discrete low-pass filter to angular acceleration and used these values.

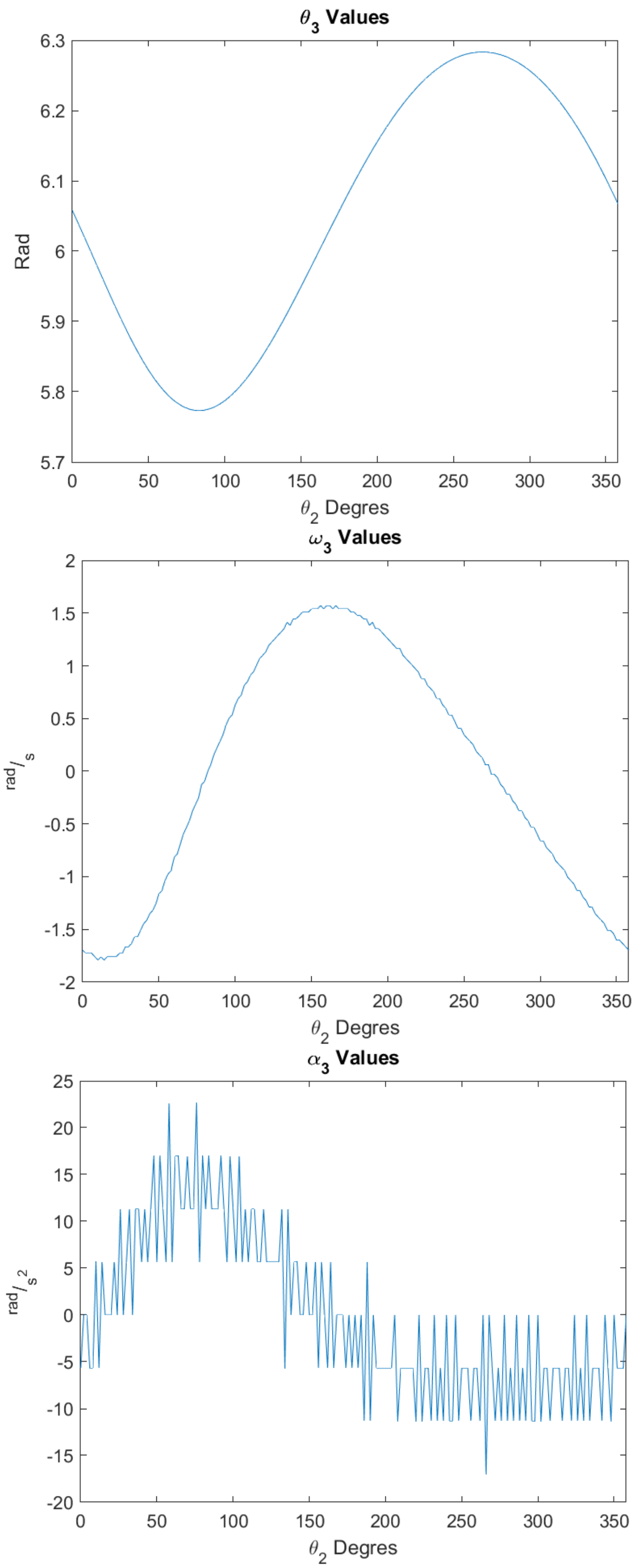

Figure 14: Numerical derivation of ω_3 and α_3 from θ_3.

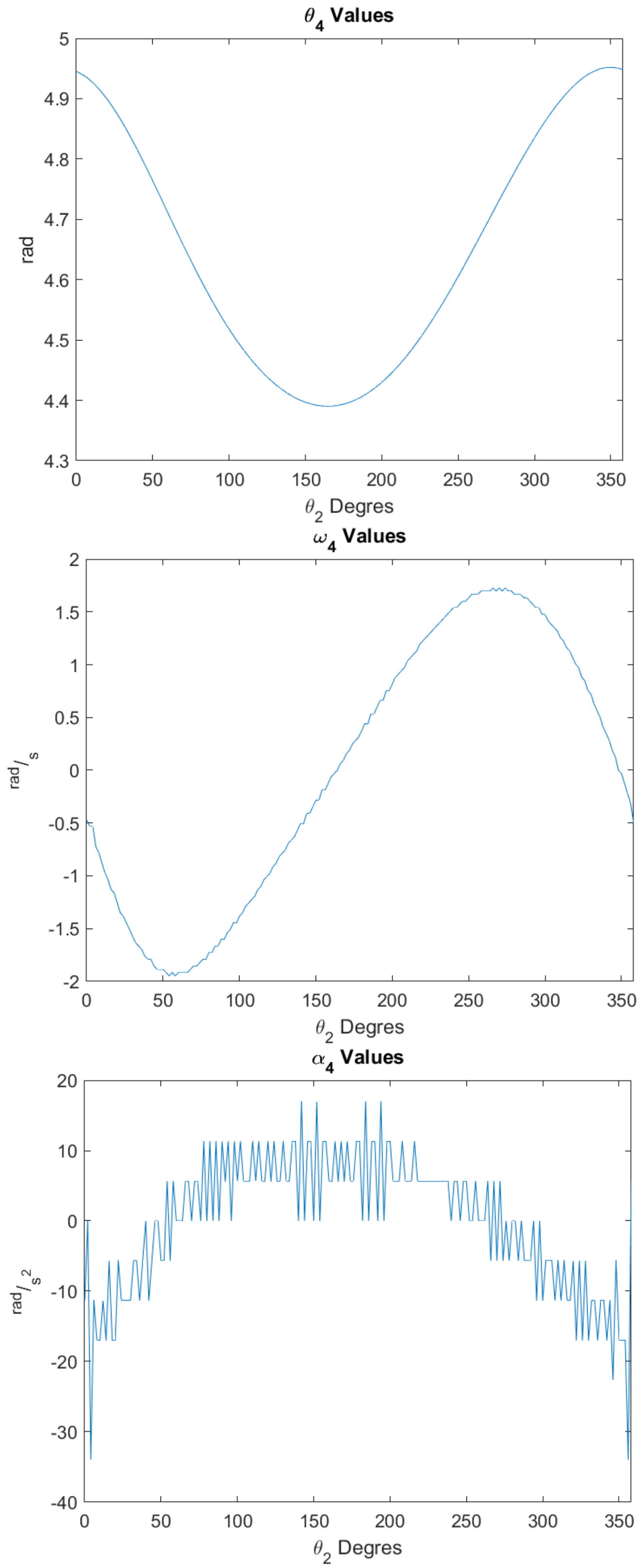

Figure 15: Numerical derivation of ω_4 and α_4 from θ_4.

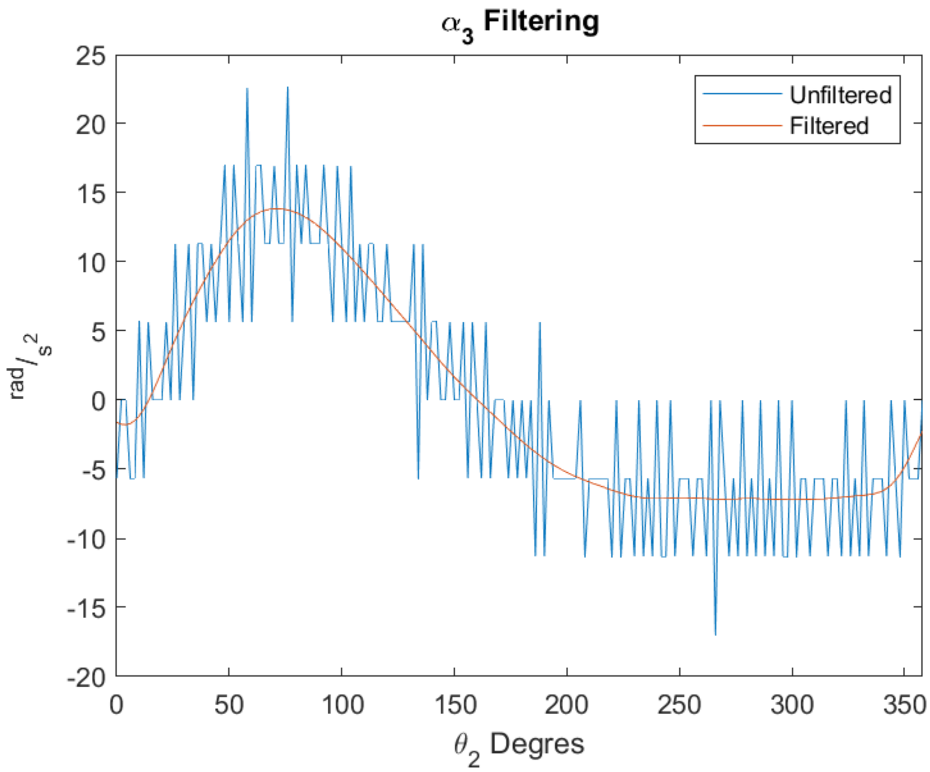

Figure 16: Low-pass filtered version of α_3.

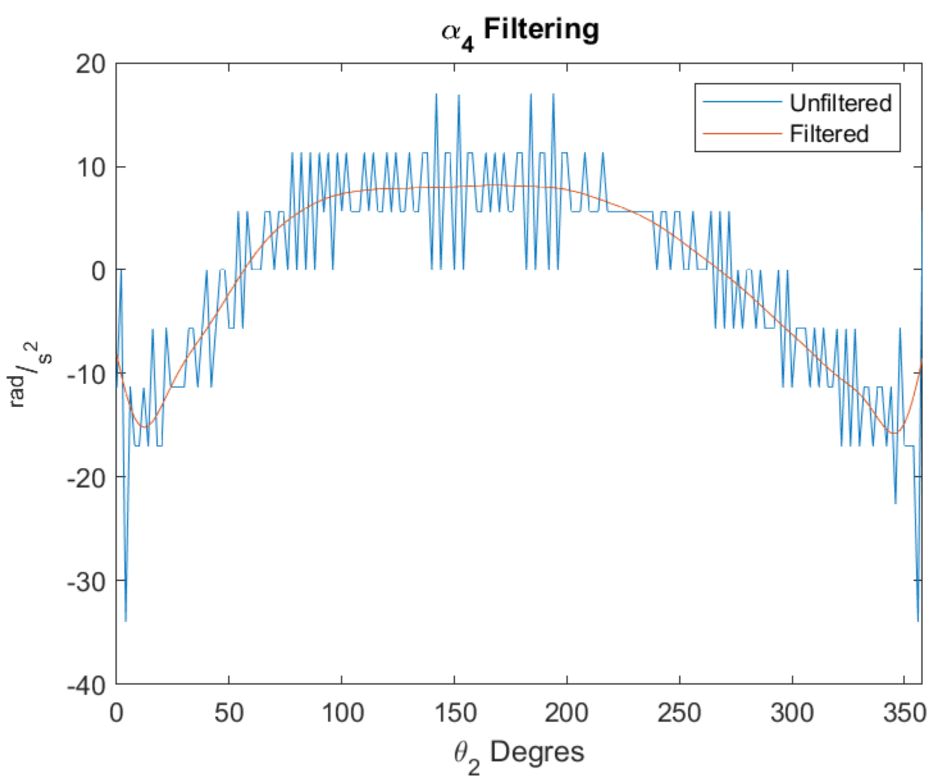

Figure 17: Low-pass filtered version of α_4.

7 Dynamic Force Analysis

7.1 Identifying Task

We found accelerations of crucial points in Kinematic Analysis section. Now with this knowledge, we can build dynamic force equations. First we have to make decide our values.

Properties	l_2	l_3	l_4
Mass center(mm)	15.00	200.81	55.00
Mass (grams)	30	1500	80
Moment of inertia (kg·m)	25.8e-6	6.1e-3	0.4e-3

Table 4: Properties.

7.2 Finding Moments of Inertia

We'll assume that all the links are rectangle and use the following formula for finding the moment of inertia with respect to it's gravitational center.

$$I_z = \frac{1}{12} m \cdot (h^2 + w^2) \tag{13}$$

...where
m = Mass [kg]
h = Planar height of the rectangle [m]
w = Planar width of the rectangle [m]

Then we can use *Parallel Axis Theorem* to find the moment of inertia with respect to rotational center.

$$I = I_c + m \cdot d^2 \tag{14}$$

...where
I = Moment of inertia of the body $[kg \cdot m^2]$
I_c = Moment of inertia about the center $[kg \cdot m^2]$
d = Distance between the two axes [m]

7.2.1 Crank Link

Sizes of our crank link are 75x45 mm. So our values become;

Properties	Value
m (kg)	0.030
h (m)	0.075
w (m)	0.045
d (m)	0.015

Table 5: Crank variables.

...so moment of inertia of one crank link becomes $25.88 \times 10^{-6}\ kg \cdot m^2$. Since we have 2 cranks in each side, we can multiply it by 2 and use that value instead.

7.2.2 Coupler Link (Pan)

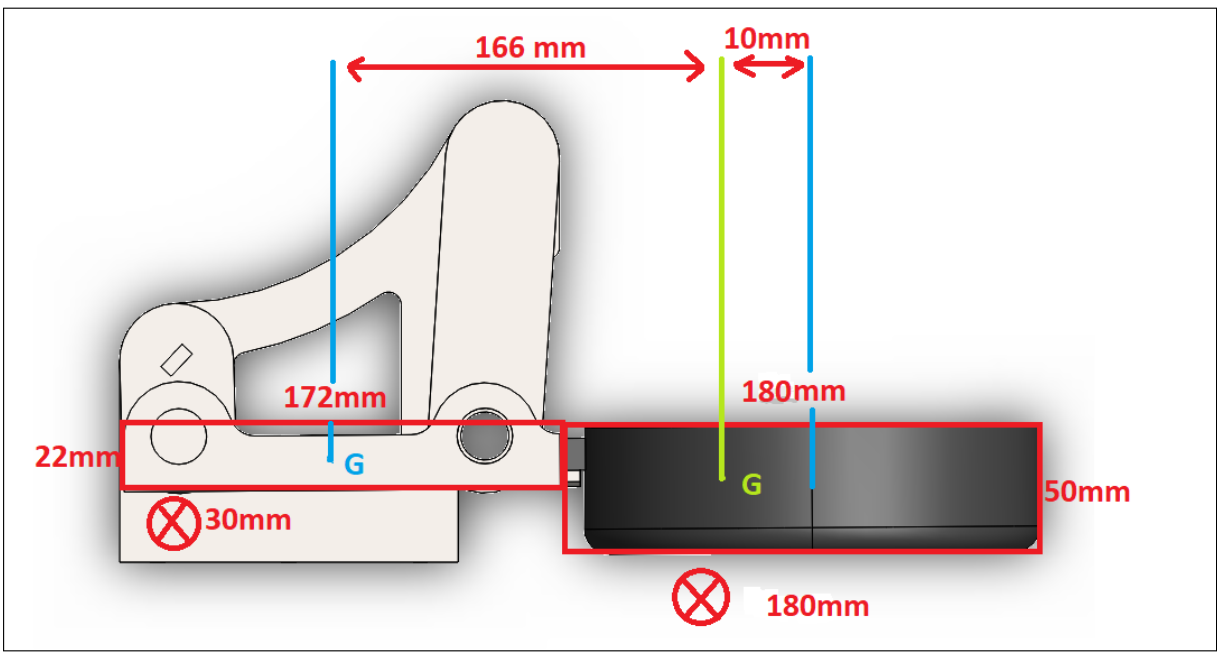

Figure 18: Values we're going to use for coupler.

We can see the values for our calculations in Fig. 18. If we put these values into their place in equations 7.2 and 7.2 we get the value of $6.1 \times 10^{-3}\ kg \cdot m^2$.

7.2.3 Rocker Link

Sizes of our crank link are 155x45 mm. So our values become;

Properties	Value
m (kg)	0.080
h (m)	0.155
w (m)	0.045
d (m)	0.055

Table 6: Crank variables.

...so moment of inertia of one crank link becomes $0.404 \times 10^{-3}\ kg \cdot m^2$. Since we have 2 rockers in each side, we can multiply it by 2 and use that value instead.

7.3 Free Body Diagrams

Free body diagrams of our 4 link will give us appropriate relations, and supply us the required equations in order to find the unknowns. Mechanism can be inspected at Fig. 13.

7.3.1 Crank Link

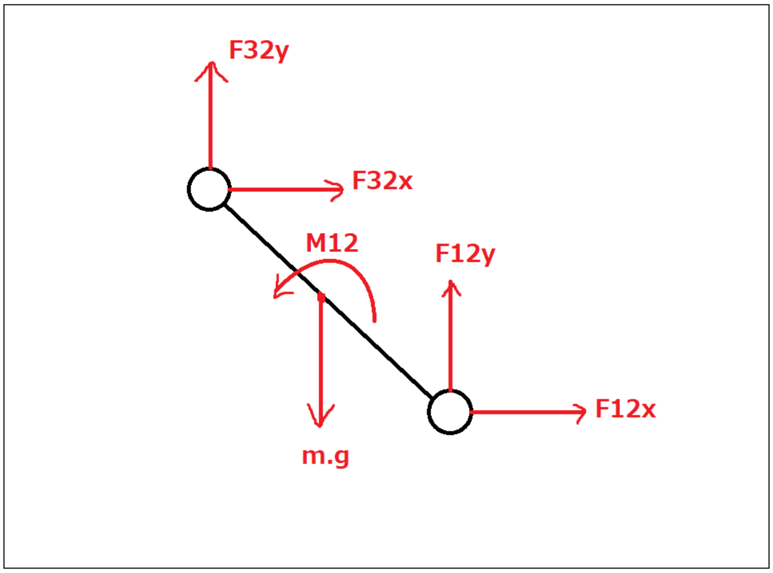

Figure 19: FBD of crank link.

This link yields us these equations:

$$F_{12,x} + F_{32,x} = m_2\, A_{G_2,x} \tag{15}$$

$$-m_2\, g + F_{12,y} + F_{32,y} = m_2\, A_{G_2,y} \tag{16}$$

$$0.015\,(-cos\theta_2\,\hat{\boldsymbol{\imath}} - sin\theta_2\,\hat{\boldsymbol{\jmath}}) \times (F_{12,x}\,\hat{\boldsymbol{\imath}} + F_{12,y}\,\hat{\boldsymbol{\jmath}}) + 0.015\,(cos\theta_2\,\hat{\boldsymbol{\imath}} + sin\theta_2\,\hat{\boldsymbol{\jmath}}) \times (F_{32,x}\,\hat{\boldsymbol{\imath}} + F_{32,y}\,\hat{\boldsymbol{\jmath}}) + M_{12} = I_2\,\alpha_2 \tag{17}$$

Or equivalently:

$$-0.015\,F_{12,y}\,cos\theta_2 + 0.015\,F_{12,x}\,sin\theta_2 + 0.015\,F_{32,y}\,cos\theta_2 - 0.015\,F_{32,x}\,sin\theta_2 + M_12 = I_2\,\alpha_2 \tag{18}$$

7.3.2 Coupler Link

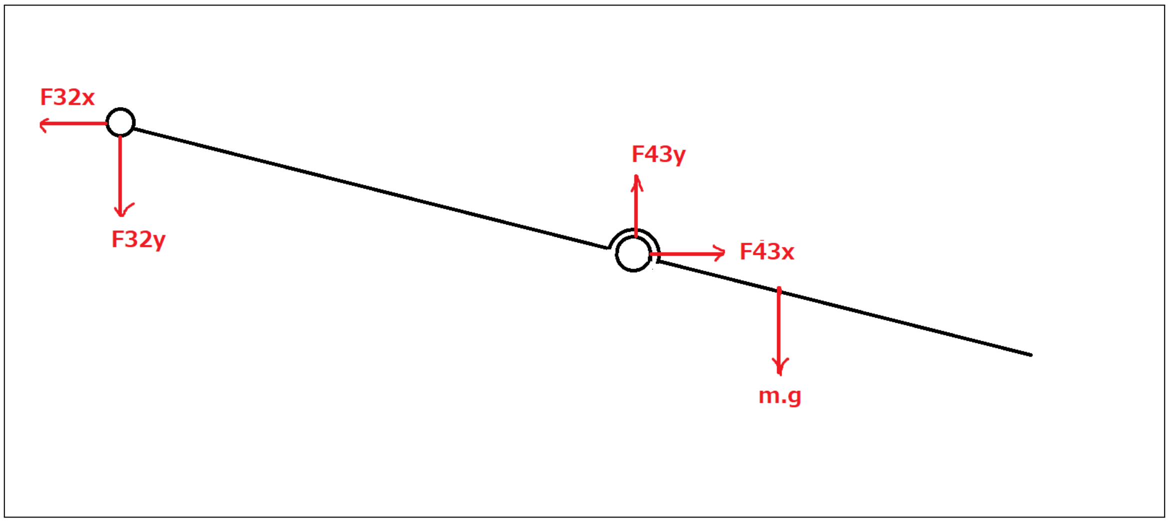

Figure 20: FBD of coupler link.

This link yields us these equations:

$$-F_{32,x} + F_{43,x} = m_3\, A_{G_3,x} \tag{19}$$

$$-m_3\, g - F_{32,y} + F_{43,y} = m_3\, A_{G_3,y} \tag{20}$$

$$0.08\,(-cos\theta_3\,\hat{\boldsymbol{\imath}} - sin\theta_3\,\hat{\boldsymbol{\jmath}}) \times (F_{43,x}\,\hat{\boldsymbol{\imath}} + F_{43,y}\,\hat{\boldsymbol{\jmath}}) + 0.20081\,(-cos\theta_3\,\hat{\boldsymbol{\imath}} - sin\theta_3\,\hat{\boldsymbol{\jmath}}) \times (-F_{32,x}\,\hat{\boldsymbol{\imath}} - F_{32,y}\,\hat{\boldsymbol{\jmath}}) = I_3\,\alpha_3 \tag{21}$$

Or equivalently:

$$-0.08\,F_{43,y}\,cos\theta_3 + 0.08\,F_{43,x}\,sin\theta_3 + 0.20081\,F_{32,y}\,cos\theta_3 - 0.20081\,F_{32,x}\,sin\theta_3 = I_3\,\alpha_3 \tag{22}$$

7.3.3 Rocker Link

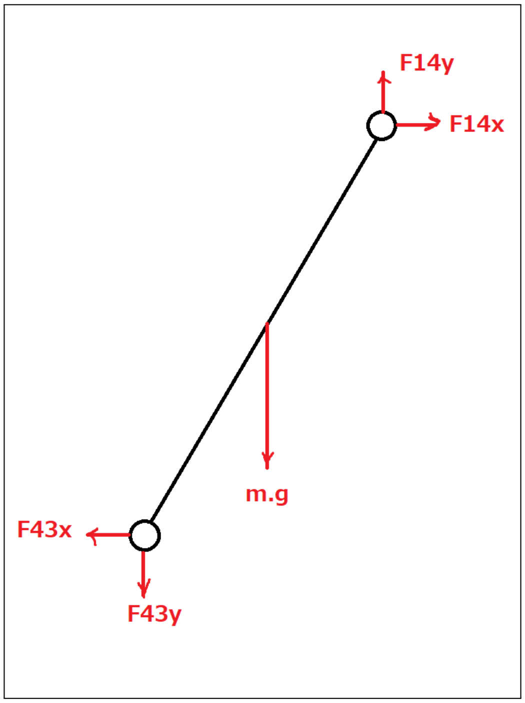

Figure 21: FBD of rocker link.

This link yields us these equations:

$$-F_{43,x} + F_{14,x} = m_4\, A_{G_4,x} \tag{23}$$

$$-m_4\, g - F_{43,y} + F_{14,y} = m_4\, A_{G_4,y} \tag{24}$$

$$\begin{aligned} &0.055\,(-cos\theta_4\,\hat{\boldsymbol{\imath}} - sin\theta_4\,\hat{\boldsymbol{\jmath}}) \times (F_{14,x}\,\hat{\boldsymbol{\imath}} + F_{14,y}\,\hat{\boldsymbol{\jmath}}) \\ &\qquad + 0.055\,(cos\theta_4\,\hat{\boldsymbol{\imath}} + sin\theta_4\,\hat{\boldsymbol{\jmath}}) \times (-F_{43,x}\,\hat{\boldsymbol{\imath}} - F_{43,y}\,\hat{\boldsymbol{\jmath}}) = I_4\,\alpha_4 \end{aligned} \tag{25}$$

Or equivalently:

$$-0.055\,F_{14,y}\,cos\theta_4 + 0.055\,F_{14,x}\,sin\theta_4 - 0.055\,F_{43,y}\,cos\theta_4 + 0.055\,F_{43,x}\,sin\theta_4 = I_4\,\alpha_4 \tag{26}$$

8 Calculations

Since we're calculating for relatively high number of instances, we could speed up the calculating process by deriving coefficient matrix of our own. From 9 equations we derived from free body diagrams, we can derive following equation:

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \sigma_3 & -\sigma_4 & -\sigma_3 & \sigma_4 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{20081\,\sin(\theta_3)}{100000} & \frac{20081\,\cos(\theta_3)}{100000} & \frac{2\,\sin(\theta_3)}{25} & -\frac{2\,\cos(\theta_3)}{25} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \sigma_2 & \sigma_1 & \sigma_2 & \sigma_1 & 0 \end{pmatrix} \cdot \begin{pmatrix} F_{12,x} \\ F_{12,y} \\ F_{32,x} \\ F_{32,y} \\ F_{43,x} \\ F_{43,y} \\ F_{14,x} \\ F_{14,y} \\ M_{12} \end{pmatrix}$$

$$= \begin{pmatrix} m_2\,A_{G2,y} \\ m_2\,A_{G2,y} + m_2\,g \\ I_2\,\alpha_2 \\ m_3\,A_{G3,y} \\ m_3\,A_{G3,y} + m_3\,g \\ I_3\,\alpha_3 \\ m_2\,A_{G4,y} \\ m_4\,A_{G4,y} + m_4\,g \\ I_4\,\alpha_4 \end{pmatrix} \tag{27}$$

where

$\sigma_1 = -0.055\,\cos(\theta_4)$

$\sigma_2 = 0.055\,\sin(\theta_4)$

$\sigma_3 = 0.055\,\sin(\theta_2)$

$\sigma_4 = 0.055\,\cos(\theta_2)$

With Eqn. 8 we can find unknowns for each of our variables.
Let's revise what our unknowns represent. $F_{12,x}$ and $F_{12,y}$ is components of bearing response at crank link, namely A joint in Fig. 13. $F_{32,x}$ and $F_{32,y}$ is components of bearing response at joint B on coupler link. $F_{43,x}$ and $F_{43,y}$ is components of bearing response at joint C on coupler link. $F_{14,x}$ and $F_{14,y}$ is components of bearing response at joint D on rocker link, And last but certainly not the least, the M_12 is the minimum torque we need to supply to rotate the mechanism at 60 RPM. With this knowledge we can check out our results.

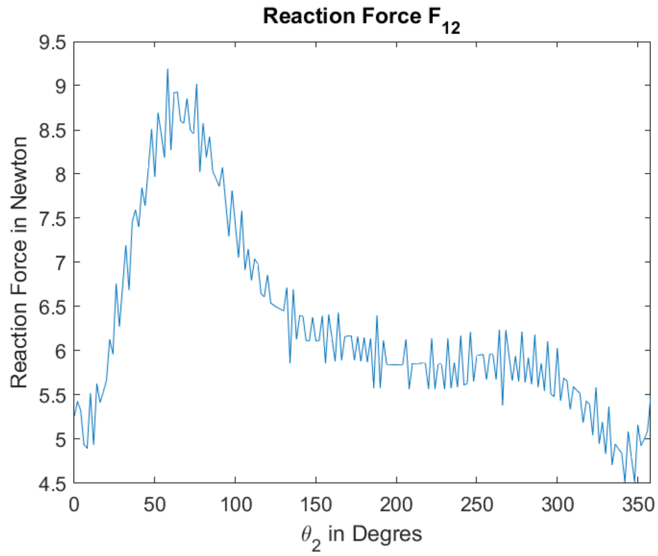

Figure 22: Magnitude of F_{12}.

Since we have 2 bearings that forms joint A in our design, we can divide max value with two to find required minimum bearing radial strength. Which is approximately 10 divided by 2 equals to 5.

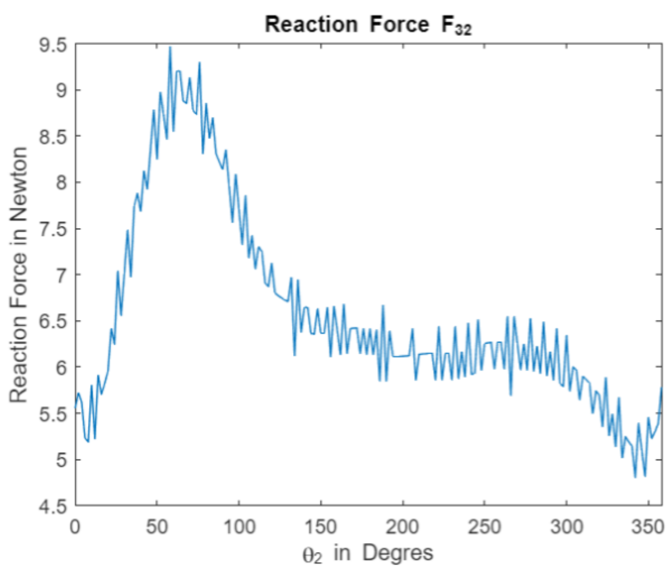

Figure 23: Magnitude of F_{32}.

Since we have 2 bearings that forms joint B in our design, we can divide max value with two to find required minimum bearing radial strength. Which is approximately 10 divided by 2 equals to 5 Newton. Note that F_{12} and F_{32} seems very close to each other. This is because they construct the couple force that builds M_{12} so it's reasonable them to be equal.

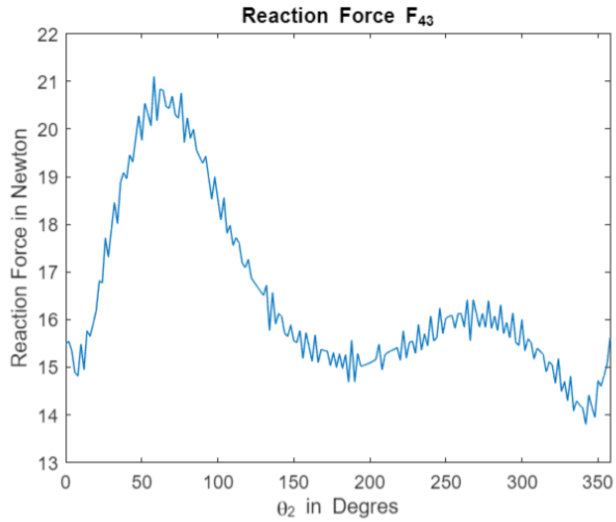

Figure 24: Magnitude of F_{43}.

Since we have 2 bearings that forms joint C in our design, we can divide max value with two to find required minimum bearing radial strength. Which is approximately 22 divided by 2 equals to 11 Newton.

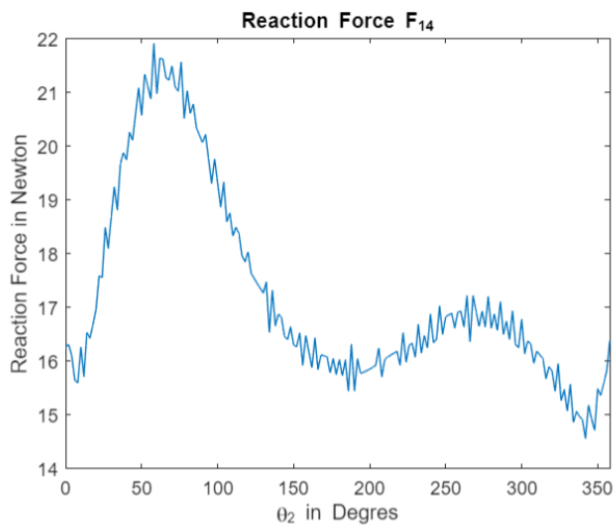

Figure 25: Magnitude of F_{14}.

Since we have 2 bearings that forms joint D in our design, we can divide max value with two to find required minimum bearing radial strength. Which is approximately 22 divided by 2 equals to 11 Newton. Note that F_{43} and F_{13} seems very close to each other. This is because they construct the two force element of link 4 so it's reasonable them to be equal again.

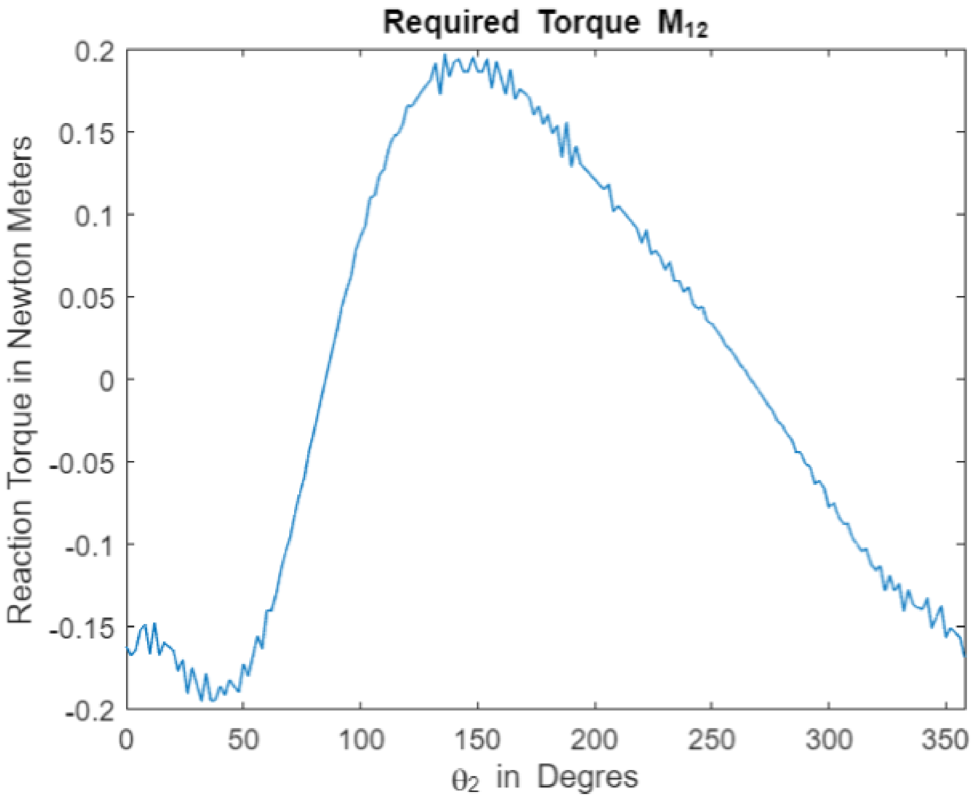

Figure 26: Magnitude of M_{12}.

Finally the most important value. As we can see from the figure 0.2 N.m torque minimum can supply this movement. This actually approximately equals to 20 N.cm torque which most of the hobby servos can supply.

9 Manufacturing

9.1 Printing the Parts

The first thing we did during the production phase was to make the designed parts removable from the 3D printer. The parts were optimized considering the asymmetries in the parts and the printing technique of the printer. The fill rate of the parts was adjusted to be 50 percent, and after a few trials and errors, the first piece designed as a panhandle was produced with a few minor errors after 10 hours of printing. Those minor errors were later fixed manually.

Figure 27: Modified panhandle

The second part that was pressed was the rocker of the mechanism, and this link is the main arm that carries the pan. Since this mechanism was designed symmetrically, we had to print two of some parts. This piece was one of the pieces that we had to print two. The printing process took 8 hours per piece.

Figure 28: Rocker of the mechanism.

After the long pieces were produced, the first piece printed was the short arm of the mechanism. In the symmetrical version of this part, they were not completely symmetrical as there was a shaft hole of the motor, or to crank mechanism. After an entire printing process of 6 hours, these two pieces were in our hands.

Figure 29: Crank links of the mechanism.

In order for the printed parts to be joined to each other, we first had to put bearings in their bearing holes. Except for the first printed part, the bearings of the other parts were put into place without much trouble. When placing the bearing on the first part, the bearings did not fit properly due to an error that we thought was caused by the manual processing of the part. During the installation of the bearings, our first part cracked and considerably reduced the part's durability. We solved this problem by partially melting part of the piece and pouring glue into the cracks.

Figure 30: Ground structure of the mechanism.

Since the printing time of the remaining pieces exceeds 20 hours, we plan to print them after the submission date of this report, before the mechanism is presented to our instructor.

10 Discussions and Conclusion

In this study one DoF frying pan shaker mechanism are constructed for automating shaking process. Essential feature of this mechanism is it's mobility. Since mechanism is one DoF, we can control all the process by on single DC motor. This just don't eliminate the need of control but also reduces overall electrical cost.

We captured pan shaking motion by filming the motion and analysed the motion by Tracker software. Body guidance synthesis procedure was followed with 4 precision points which we gathered from tracker. Then this results are adjusted so that we can reduce the cost of manufacturing and fit the printing space we had in our hand. We designed a new gripper for our pan, 4 links and 2 ground to realise this mechanism.

Main problem we encountered when designing the mechanism was long printing processes. The problems we couldn't anticipate become very hard to fix since we couldn't just print a better version all over again. Some of the links were modified with heated skewer. Bearings were really tight fits so we had to sand the bearing slots a little bit to avoid cracking. Lastly we have to connect the ground to a heavy block in order to avoid shaking but heavy blocks aren't cheap and since we exceed our budged already we aren't sure we can get a suitable ground block.

References

[1] Erkin Gezgin, Pyung Chang, and Ahmet Akhan. “Synthesis of a Watt II six-bar linkage in the design of a hand rehabilitation robot”. In: *Mechanism and Machine Theory* 104 (Oct. 2016), pp. 177–189. DOI: `10.1016/j.mechmachtheory.2016.05.023`.