İZMİR KÂTİP ÇELEBİ UNIVERSITY

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System Identification Based Control Design for an Experimental Magnetic Levitation System

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1 Introduction

The Maglev setup serves as a simple model of devices, which are becoming more and more popular in recent years i.e. Maglev trains and magnetic bearings. Maglev trains are recently tested and some lines are already available as for example in Shanghai. Magnetic bearings are used in turbines for the same reason as Maglev trains are being built, which is low friction in the bearing itself. Already many turbines are used commercially where the rotating shaft is levitated with magnetic flux. Some other magnetic bearings applications include pumps, fans and other rotating machines.

The magnetic levitation systems are appealing for their additional possibility of active vibration damping. This can be done by various control algorithms implementations and without any modifications to the mechanical parts of the whole system. The Maglev unit allows for the design of different controllers and tests in real time using Matlab and Simulink environment.

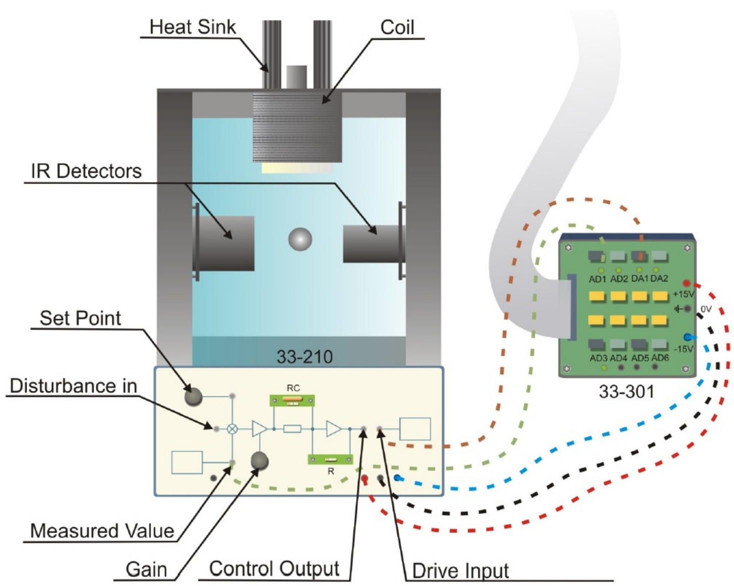

Figure 1: Maglev Mechanical Unit

1.1 Objective

Our goal in this experiment is to control the position of the metal ball. Robust adaptive controller was preferred to realize this position control. When using a robust controller, the control effort is very high because we do not have information about the system's internal dynamics. To reduce the control effort in robust adaptive controller, we have little knowledge about the system, and thanks to this knowledge, we can compensate for the modeling error and control the system with less control effort. The disadvantage that this brings us is more complex mathematics and if we do not know about the system, we cannot use an adaptive controller.

2 Methodology

2.1 Modelling

Magnetic Levitation model Every control project starts with plant modelling, so as much information as possible is given about the process itself. The mechanical-electrical model of Maglev is presented in below Figure 2.

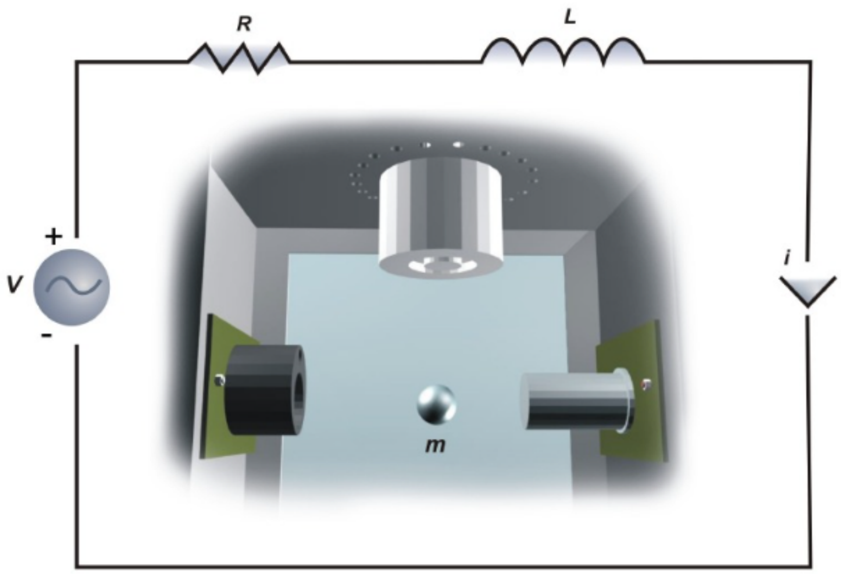

Figure 2: Maglev Phenomenological Model

Usually, phenomenological models are nonlinear, that means at least one of the states (i – current, x– ball position) is an argument of a nonlinear function. In order to present such a model as a transfer function (a form of linear plant dynamics representation used in control engineering), it has to be linearised. According to the electrical-mechanical diagram presented in Figure 3 the nonlinear model equations can be derived.

The simplest nonlinear model of the magnetic levitation system relating the ball position and the coil current i is the following:

$$m\ddot{x} = mg - k\frac{i^2}{x^2} \tag{1}$$

where k is a constant depending on the coil (electromagnet) parameters. To present the full phenomenological model a relation between the control voltage u and the coil current would have to be introduced analysing the whole Maglev circuitry. However Maglev is equipped with an inner control loop providing a current proportional to the control voltage that is generated for control purpose:

$$i = k_1 u \tag{2}$$

Equations (1) and (2) constitute a nonlinear model, which has been assembled in Simulink. The bound for the control signal is set to [-5V .. +5V]. Maglev is a SISO plant – single input single output (Figure 3). Position is the model output and voltage is the control signal.

Figure 3: Maglev Model For Position Control

3 Results

3.1 Dynamic Model

$$m\ddot{x} + f(x, \dot{x}) + f_d = \tau \tag{3}$$

where...

$$\begin{aligned} x, \ddot{x} &\rightarrow measurable \\ |m|_{i\rightarrow\infty} > 0 &\rightarrow unknown \\ f(x, \dot{(x)}) &\rightarrow uncertain \\ |f_d(t)|_{i\rightarrow\infty} &\leq \rho_b \end{aligned}$$

3.2 Error Definitions

$$e \triangleq x_d - x \rightarrow \text{Tracking Error} \tag{4}$$

$$r \triangleq \dot{e} + \alpha e \rightarrow \text{Auxiliary Error} \tag{5}$$

3.3 Open Loop Error System

Multiplying the time derivative of eqn (5) with m.

$$m\dot{r} = m\ddot{e} + m\alpha\dot{e} \tag{6}$$

substitute double time derivative eqn (4) with $\ddot{e}$.

$$m\dot{r} = m\ddot{x}_d - m\ddot{x} + \alpha m(r - \alpha e) \tag{7}$$

substitute eqn (3) with $m\ddot{x}$

$$m\dot{r} = m\ddot{x}_d + f(x, \dot{x}) + f_d(t) - \tau + \alpha m(r - \alpha e) \tag{8}$$

rearrange eqn (8) as...

$$m\dot{r} = N + f_d(t) - \tau \tag{9}$$

where $N \in \Re$ is an auxilary function defined as

$$N \triangleq m\ddot{x}_d + f(x, \dot{x}) + \alpha m(r - \alpha e) + \frac{1}{2}m\dot{r} \tag{10}$$

let N is modeled as

$$N = \phi^T\sigma(\bar{x}) + \epsilon(t) \tag{11}$$

where...

$$\begin{aligned}
&\bar{x} = [\ddot{x}_d \;\; x \;\; \dot{x}] \\
&\sigma : \text{Activation function (for this instance } tanh(x)) \\
&\epsilon : \text{Bounded modelling error} \\
&\phi : \text{Matrix of ideal weight terms (constant)}
\end{aligned}$$

ϕ can be adaptively compensated as

$$\tilde{\phi} \triangleq \phi - \hat{\phi} \tag{12}$$

$$\begin{aligned}
\tilde{\phi} \in \Re^{3x1} &: \text{Adaptive compensation error} \\
\hat{\phi} \in \Re^{3x1} &: \text{Adaptive compensation of } \phi
\end{aligned}$$

substitute eqn (11) in eqn (9)

$$m\dot{r} = \phi^T\sigma(\bar{x}) + \epsilon(t) - \tau - \frac{1}{2}\dot{m}r + f_d(t) \tag{13}$$

substitute ϕ with $(\tilde{\phi} + \hat{\phi})$ at eqn (13).

$$m\dot{r} = (\tilde{\phi}^T + \hat{\phi}^T)\sigma(\bar{x}) + \epsilon(t) - \tau - \frac{1}{2}\dot{m}r + f_d(t) \tag{14}$$

$$= \underbrace{\tilde{\phi}^T\sigma(\bar{x})}_{\text{uncertain}} + \underbrace{\hat{\phi}^T\sigma(\bar{x})}_{\text{known}} + \epsilon(t) - \underbrace{\tau}_{\text{bounded unknown}} - \frac{1}{2}\dot{m}r + f_d(t) \tag{15}$$

3.4 Control Design

Control input τ designed as

$$\tau = \hat{\phi}^T \sigma(\bar{x}) + \rho_b sgn[r] + g_c r + e \tag{16}$$

where ...

$$\rho_b \geq |\epsilon(t)|_{i\to\infty} + |f_d(t)|_{i\to\infty} : \text{constant gain}$$
$$g_c : \text{constant control gain} \in \Re^+$$

3.5 Closed Loop Control System

substitute eqn (16) into eqn (15).

$$m\dot{r} = \tilde{\phi}^T \sigma(\bar{x}) + [\epsilon(t) + f_d(t) - \rho_b sgn[r]] - \frac{1}{2}\dot{m}r - g_c r - e \tag{17}$$

3.6 Stability analysis

Lyapunov function

$$V = \frac{1}{2}mr^2 + \frac{1}{2}e^2 + \frac{1}{2}\tilde{\phi}^T\tilde{\phi} \tag{18}$$

Time derivative of Eqn (18).

$$\dot{V} = rm\dot{r} + e\dot{e} + \frac{1}{2}\dot{m}r^2 + \tilde{\phi}^T\dot{\tilde{\phi}} \tag{19}$$

since..

$$\tilde{\phi} = \phi - \hat{\phi}$$
$$\dot{\tilde{\phi}} = -\dot{\hat{\phi}}$$

Eqn (19) becomes

$$\dot{V} = r\underbrace{m\dot{r}}_{(17)} + e\underbrace{\dot{e}}_{(5)} + \frac{1}{2}\dot{m}r^2 - \tilde{\phi}^T\dot{\hat{\phi}} \tag{20}$$

$$\downarrow$$

$$\dot{V} = r\underbrace{[\tilde{\phi}^T \sigma(\bar{x}) + [\epsilon(t) + f_d(t) - \rho_b sgn[r]] - \frac{1}{2}\dot{m}r - g_c r - e]}_{m\dot{r}} + e\underbrace{(r - \alpha e)}_{\dot{e}} - \tilde{\phi}^T\dot{\hat{\phi}} + \frac{1}{2}\dot{m}r^2 \tag{21}$$

After substituting eqn (21) becomes

$$\dot{V} = r\tilde{\phi}^T\sigma(\bar{x}) + r[\epsilon(t) + f_d(t) - \rho_b sgn[r]] - g_c r^2 - \alpha e^2 - \tilde{\phi}^T\dot{\hat{\phi}} \tag{22}$$

Since

$$r\tilde{\phi}^T\sigma(\bar{x}) = \tilde{\phi}^T r\sigma(\bar{x})$$

Final form of eqn (22) becomes

$$\dot{V} = \tilde{\phi}^T(r\sigma(\bar{x}) - \dot{\hat{\phi}}) + r[\epsilon(t) + f_d(t) - \rho_b sgn[r]] - g_c r^2 - \alpha e^2 \tag{23}$$

to remove adaptive compensation error of $\tilde{\phi}$, we should use the update rule of;

$$\dot{\hat{\phi}} = r\sigma(\bar{x}) \tag{24}$$

With update rule we can see from eqn 23 that for all values, $\dot{V}$ is negative or zero. Since we know that V is positive for all values from eqn 18, all of the terms that is in lyapunov function is going to decay to a value and wont increase. This means that system is stable and error e and r is going to zero if hyperparameters are chosen correctly.

3.7 Matlab Code

*The code given below is the code content in the function block that generates the tau input to be applied to the maglev simulation.

```
function [tau,phi_hat_1_dot,phi_hat_2_dot,phi_hat_3_dot,debug] = fcn(x_des
    ,xd_des,x_bar_1,x_bar_2,x_bar_3,phi_hat_1,phi_hat_2,phi_hat_3)
    x_bar = [x_bar_1;x_bar_2;x_bar_3];
    phi_hat = [phi_hat_1;phi_hat_2;phi_hat_3];

    %X_bar
    xdd_des = x_bar_1;
    x = x_bar_2;
    xdot = x_bar_3;

    %Hyperparameters
    alpha = 20;
    rho_b = 150;
    gc =    20;

    %Errors
    e = x_des - x;
    e_dot = xd_des - xdot;
    r = e_dot + alpha*e;

    sigmoid = tanh(x_bar);

    %Tau
    tau = phi_hat'*sigmoid + rho_b*tanh(r) + gc*r + e;

    %phi_hat defination
    phi_hat_1_dot = r*tanh(x_bar_1);%phi_hat_dot(1);
    phi_hat_2_dot = r*tanh(x_bar_2);%phi_hat_dot(2);
    phi_hat_3_dot = r*tanh(x_bar_3);%phi_hat_dot(3);
```

*Position input function block content shown as follows...

```
function [x_des,xd_des,xdd_des] = fcn(t)
%      x_des = 0.02;
%      xd_des = 0;
%      xdd_des = 0;
     x_des= 0.01*sin(t)+0.03;
     xd_des= 0.01*cos(t);
     xdd_des= -0.01*sin(t);
```

3.8 Simulation

The test phase should be checked in the simulation environment before being tested directly on the model. It is checked whether the tau value, which is the control input applied, creates the necessary control effort in the simulation environment. Thus, the response of the system according to the control effort we apply can be predicted.

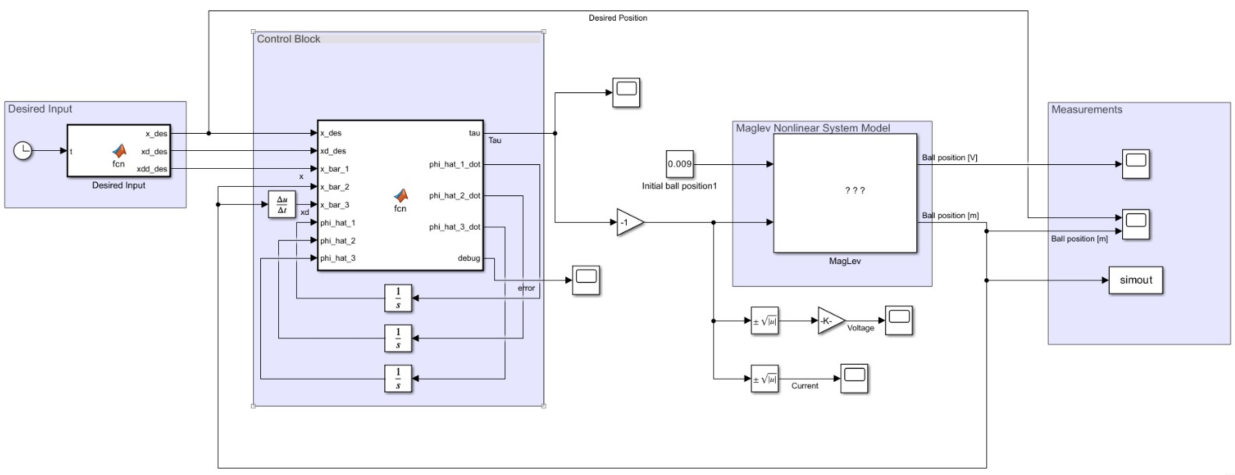

Figure 4: Matlab Simulink Maglev System Model

As seen in figure 4 above, it is simulated on the model created in the matlab environment. When the simulation was run, the errors and unexpected results were found and eliminated. At first, such results were expected. After the simulation runs correctly, the results as in the figures given below are obtained. Two kinds of inputs are given in the simulation. First a fixed position input is given and then sinusoidal input.

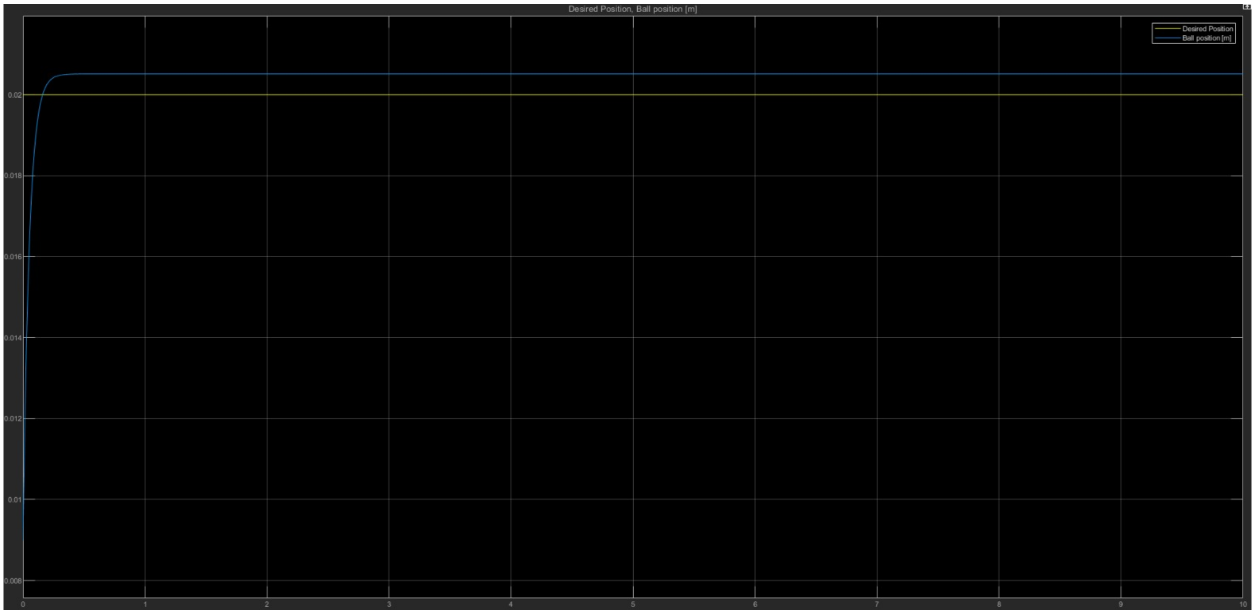

Figure 5: Input and output graph of fixed ball position

Figure 6: Voltage graph of fixed ball position

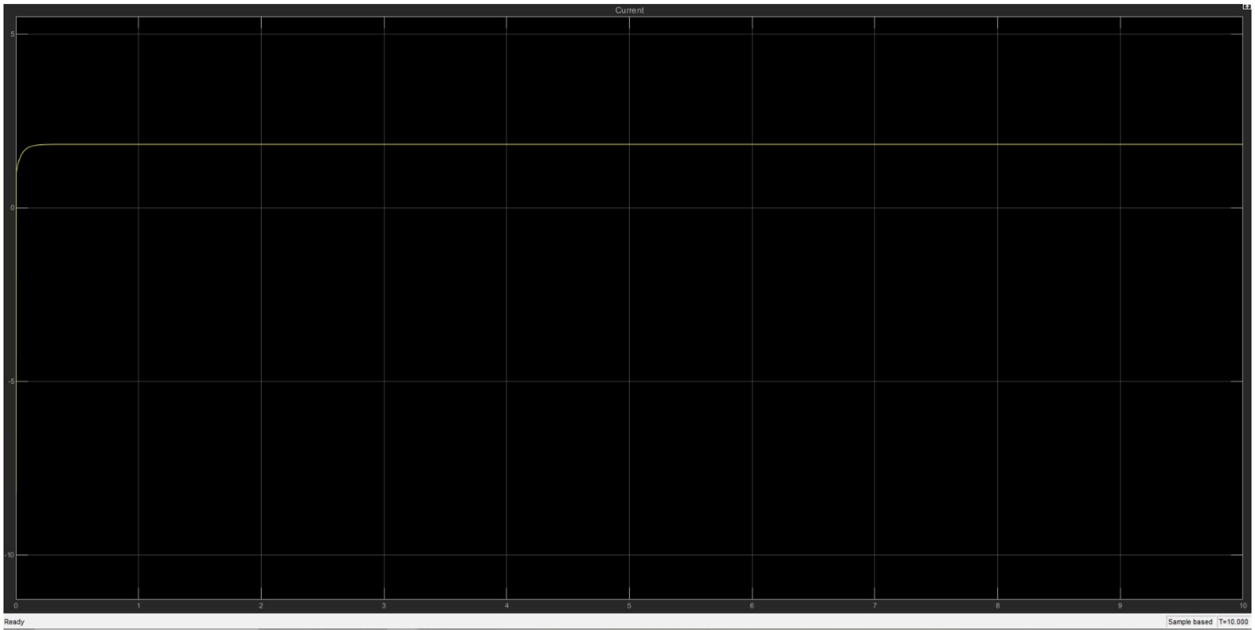

Figure 7: Current graph of fixed ball position

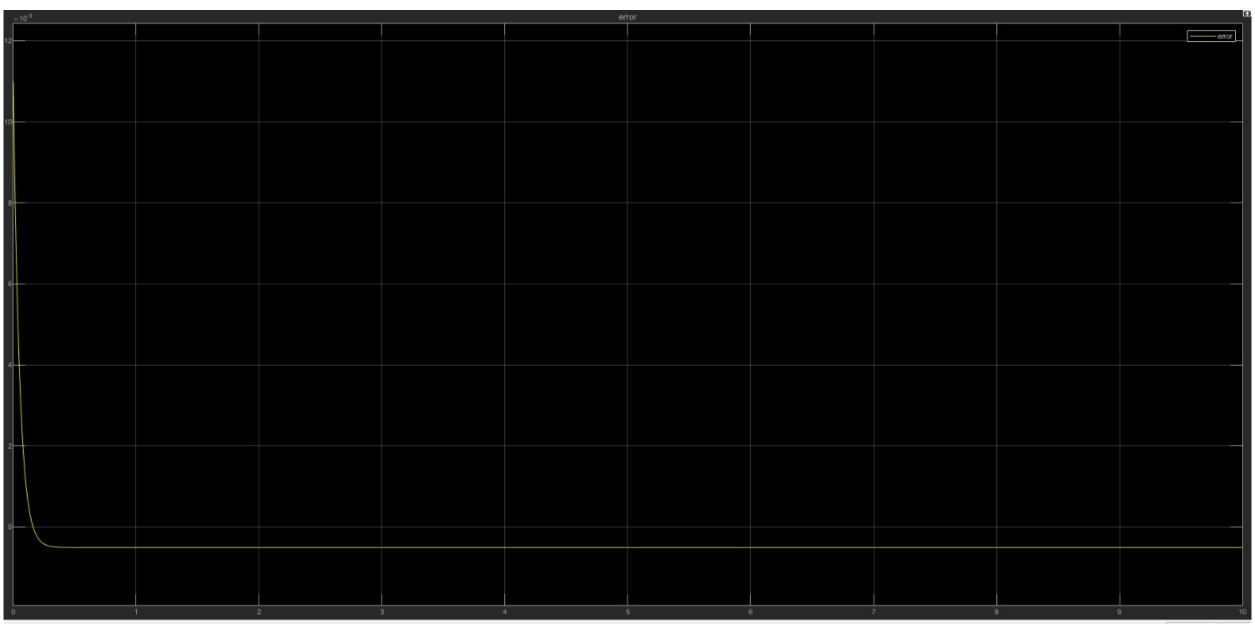

Figure 8: Error graph of fixed ball position

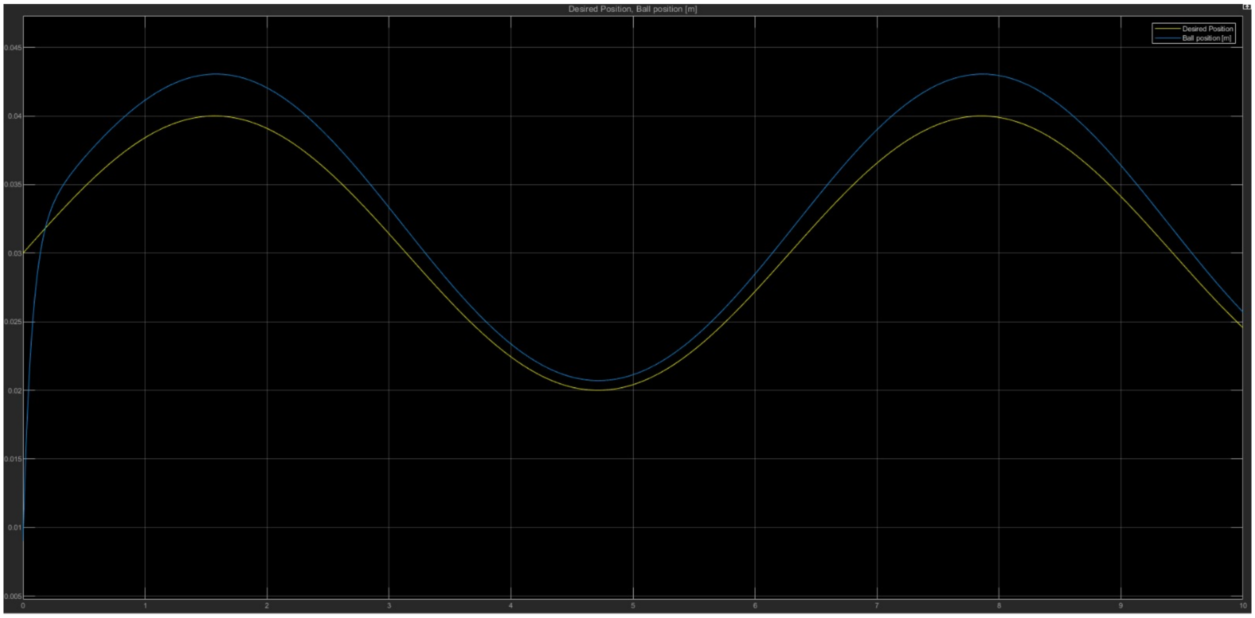

Figure 9: Input and output graph of sinusoidal ball position

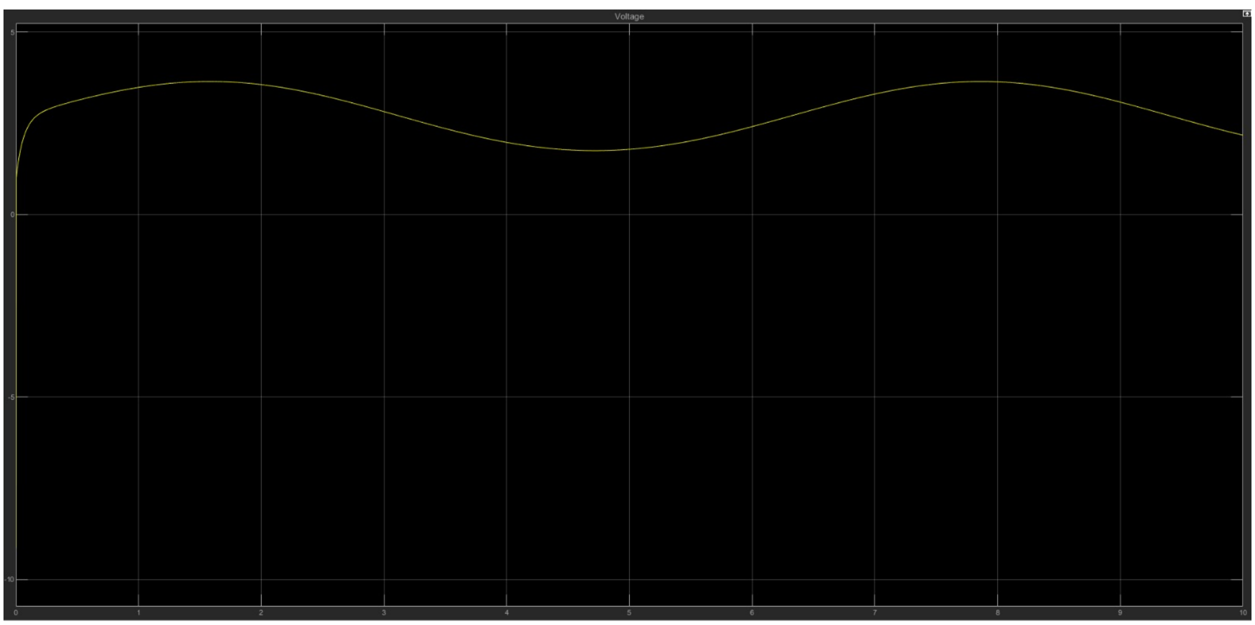

Figure 10: Voltage graph of sinusoidal ball position

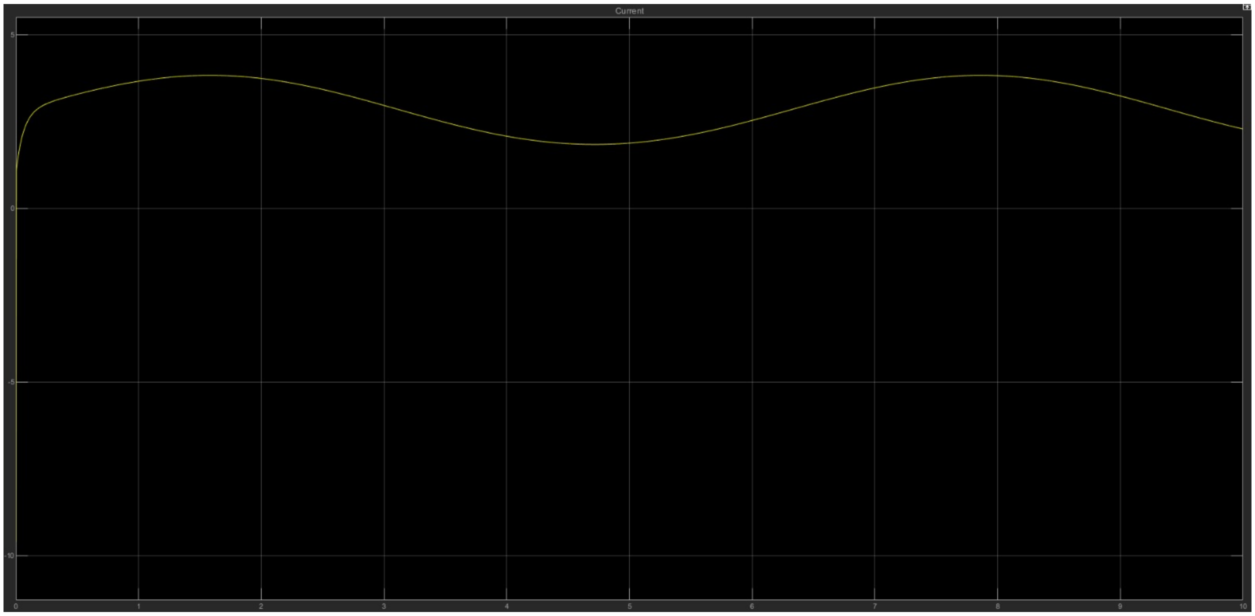

Figure 11: Current graph of sinusoidal ball position

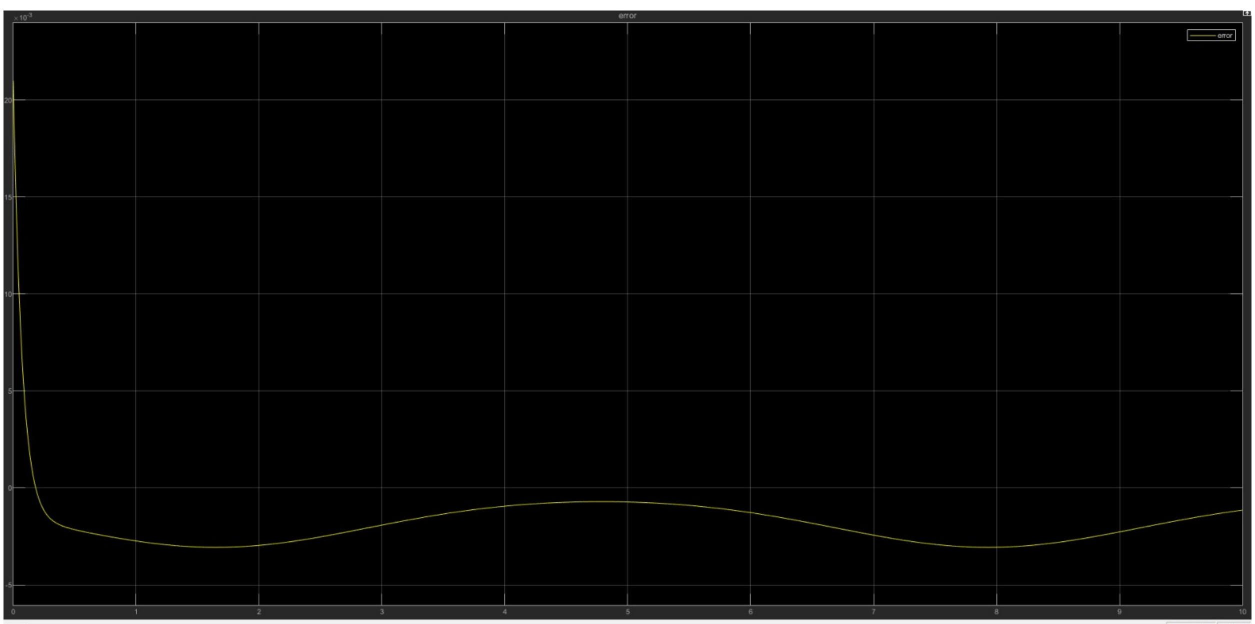

Figure 12: Error graph of sinusoidal ball position

4 Discussion

Our simulations shows that controller properly stabilize the system and decrease the tracking error. This means that the calculations are correct and application is possible.

Selection of proper control gains of g_c, ρ_b and α is essential for system to become stable. The control gains of $\alpha = 20$, $g_c = 20$ and ,$\rho_b = 150$ are selected for this application.

The control gain τ is realized to be $-i^2$ for $m(x)$ to be positive for all values of x. The

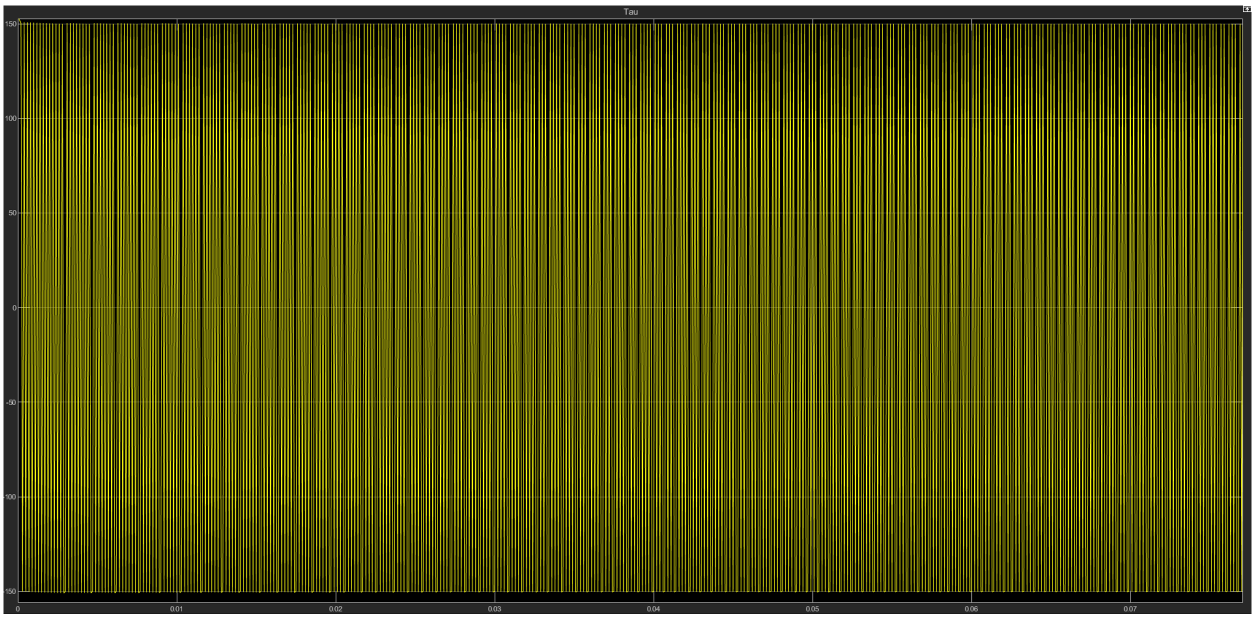

Figure 13: Tau with $\rho_b sign(r)$ term.

$\rho_b sign(r)$ term in the controller made tau discontinuous as can be seen in figure 13 and that would probably couldn't realised in real experimental system. So we used $tanh(r)$ instead of $sign(r)$ to preserve the continuity of the control gain τ. To $tanh(r)$ to become more like a $sign(r)$ function, we, in some sense, compressed the function along x axis by factor of 2 by using $tanh(2r)$ instead of $tanh(r)$. That made the control gain τ more stable by improving steady state error a little bit.

On the next semester we aim to apply this controller design into experimental MAGLEV system that is present in our lab