Probability Cheatsheet

Compiled by Eigen-π (https://www.linkedin.com/in/islamibr29/).
 Please share comments, suggestions, and errors at http://github.com/islamibr/today_i_learned.

Last Updated January 13, 2024

Prerequisites

- Binomial Expansion

$$\sum_{n=0}^{\infty} a^n = 1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}$$

$$\sum_{n=1}^{\infty} a^n = a(1 + a + a^2 + a^3 + a^4 + \cdots) = \frac{a}{1-a}$$

$$\sum_{n=0}^{\infty} (n+1)a^n = 1 + 2a + 3a^2 + 4a^3 + \cdots = \frac{1}{(1-a)^2}$$

- Taylor Expansion

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

- Trig Identities

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$
$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$
$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$
$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos(\alpha+\pi) = -\cos(\alpha)$$
$$\cos(\alpha+2\pi) = \cos(\alpha)$$

- Fourier Transform

$g(t)$	$G(f)$
$A\,\mathrm{rect}\left(\frac{t}{T}\right)$	$AT\,\mathrm{sinc}(fT)$
$\mathrm{e^{-at}u(t), a > 0}$	$\frac{1}{a+j2\pi f}$
$\mathrm{e^{-a\lvert t\rvert}, a > 0}$	$\frac{2a}{a^2+(2\pi f)^2}$
$\mathrm{e^{-\pi t^2}}$	$e^{-\pi f^2}$
$\delta(t)$	1
1	$\delta(\mathrm{f})$
$\delta(t-t_0)$	$e^{-j2\pi t_0 f}$
$e^{j2\pi f_c t}$	$\delta(f-f_c)$
$\cos(2\pi f_c t)$	$[\delta(f-f_c)+\delta(f+f_c)]/2$
$\sin(2\pi f_c t)$	$[\delta(f-f_c)-\delta(f+f_c)]/2j$

Axioms of Probability

- Non negativity : ensures that probability is never negative.

$$\mathbb{P}[A] \geq 0$$

- Normalization : ensures that probability is never greater than 1.

$$\mathbb{P}[\Omega] = 1$$

- Additivity : allows us to add probabilities when two events do not overlap.

$$\mathbb{P}\left[\bigcup_{i=1}^{\infty} A_i\right] = \sum_{i=1}^{\infty} \mathbb{P}[A_i]$$

Contional probability

$$\mathbb{P}[A|B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$$

- Independence : Two events A and B are independent if :

$$\mathbb{P}[A|B] = \mathbb{P}[A]$$
$$\text{or} \qquad \mathbb{P}[A \cap B] = \mathbb{P}[A]\mathbb{P}[B]$$

- Note:

$$\text{Disjoint} \not\Leftrightarrow \text{Independent}$$

- Unions of Two Non-Disjoint Sets

$$\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B] - \mathbb{P}[A \cap B]$$

- Unions of Two Non-Disjoint Sets in Independent case

$$\mathbb{P}[A \cup B] = 1 - \mathbb{P}[A^c \cap B^c]$$

  If A and B are disjoint, then $A \cap B = \phi$. This only implies that $\mathbb{P}[A \cap B] = 0$.

- Law of Total Probability :

$$\mathbb{P}[A] = \sum_{i=1}^{n} \mathbb{P}[A|B_i]\mathbb{P}[B_i]$$

- Bayes' rule

$$\mathbb{P}[A|B] = \frac{\mathbb{P}[B|A]\mathbb{P}[A]}{\mathbb{P}[B]}$$

Series and Parallel Circuits

- Series devices:

$$\mathbb{P}[\text{Circuit } \textbf{operates}] = \mathbb{P}[\text{device 1 } \textbf{operates}] \times \mathbb{P}[\text{device 2 } \textbf{operates}] \times \cdots \times \mathbb{P}[\text{device n } \textbf{operates}]$$

- Parallel devices:

$$\mathbb{P}[\text{Circuit } \textbf{fails}] = \mathbb{P}[\text{device 1 } \textbf{fails}] \times \mathbb{P}[\text{device 2 } \textbf{fails}] \times \cdots \mathbb{P}[\text{device n } \textbf{fails}]$$

- Remember :

$$\mathbb{P}[\text{failure}] = 1 - \mathbb{P}[\text{success}]$$

Techniques of Counting

- Arranging n items in n places : number of ways

$$n!$$

- Permutations :

$$k = \frac{n!}{(n-k)!}$$

- Combinations :

$$k = \frac{n!}{k!\,(n-k)!}$$

	order	no order
replacement	n^r	${}^{n-r+1}C_r$
no replacement	${}^{n}P_r$	${}^{n}C_r$

Discrete Random Variables

- What are random variables?
  Random variables are mappings from events to numbers.
- probability mass function (PMF) of a random variable X is a function which specifies the probability of obtaining a number x.

$$p_X(x) = \mathbb{P}[X = x]$$

- Note that a PMF should satisfy the following condition

$$\sum_{x \in X(\Omega)} p_X(x) = 1$$

- Cumulative distribution function CDF :

$$F_X(x_k) = [X \leq x_k] = \sum_{l=-\infty}^{k} p_X(l)$$

- What is expectation?
  Expectation = Mean = Average computed from a PMF.

$$\mathbb{E}[X] = \mu = \sum_{x \in X(\Omega)} x p_X(x)$$

- Properties:

$$\mathbb{E}[g(X)] = \sum_{x \in X(\Omega)} g(X) p_X(x)$$

$$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$$

- What is variance?
  It is a measure of the deviation of the random variable X relative to its mean.

$$\begin{aligned} \mathrm{Var}[X] = \sigma^2 &= \mathbb{E}[(X-\mu)^2] \\ &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\ &= \mathbb{E}[X^2] - \mu^2 \end{aligned}$$

- Properties:

$$\mathrm{Var}[aX + b] = a^2 \mathrm{Var}[X]$$

- Coefficient of variance $= \dfrac{\sigma}{\mu}$

Special Discrete Random Variables

Bernoulli

(a coin-flip random variable)

- [sucess] $= p$, [failure] $= 1 - p = q$
- PMF :
$$p_X(0) = 1 - p \qquad p_X(1) = p$$
- Expectation:
$$\mathbb{E}[X] = p$$
- Variance:
$$\begin{aligned}\text{Var}[X] &= p(1-p)\\ &= pq\end{aligned}$$

Bionomial

(n times coin-flips random variable)

- [sucess] $= p$, [failure] $= 1 - p = q$
- PMF :
$$p_X(k) = kp^k q^{n-k}$$
- Expectation:
$$\mathbb{E}[X] = np$$
- Variance:
$$\begin{aligned}\text{Var}[X] &= np(1-p)\\ &= npq\end{aligned}$$
- Show that the binomial PMF sums to 1.:
Use the binomial theorem:
$$\begin{aligned}\sum_{k=0}^{n} p_X(k) &= \sum_{k=0}^{n} kp^k q^{n-k}\\ &= (p + (1-p))^n\\ &= 1\end{aligned}$$

Geometric

(Trying a binary experiment until we succeed random variable)

- [sucess] $= p$, [failure] $= 1 - p = q$
- PMF :
$$p_X(k) = \underbrace{(1-p)^{k-1}}_{k-1 \text{ failures}} \underbrace{p}_{\text{final success}}$$
- CDF:
$$1 - q^x$$
- Expectation:
$$\mathbb{E}[X] = \frac{1}{p}$$
- Variance:
$$\begin{aligned}\text{Var}[X] &= \frac{1-p}{p^2}\\ &= \frac{q}{p^2}\end{aligned}$$

Poisson

(For small p and large n where $\lambda = np$)

- $\lambda =$ the rate of the arrival
- PMF :
$$p_X(k) = e^{-\lambda}\frac{\lambda^k}{k!}$$
- Expectation:
$$\mathbb{E}[X] = \lambda$$
- Variance:
$$\text{Var}[X] = \lambda$$
- Show that the Poisson PMF sums to 1.:
Use the exponential series:
$$\begin{aligned}\sum_{k=0}^{\infty} p_X(k) &= \sum_{k=0}^{\infty} e^{-\lambda}\frac{\lambda^k}{k!}\\ &= e^{-\lambda}\underbrace{\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}}_{=e^{\lambda}}\\ &= 1\end{aligned}$$

Continuous Random Variables

- probability density function (PDF) is a continuous version of a PMF, we integrate PDF to compute the probability
$$[a \le X \le b] = \int_a^b f_X(x)\,dx$$
- Note that a PMF should satisfy the following condition
$$\int_\Omega f_X(x)\,dx = 1$$
- Note :
$$[X = \text{certian point}] = 0$$
- Cumulative distribution function CDF :
$$F_X(x_k) = [X \le x] = \int_{-\infty}^{x} f_X(t)dt$$
- Note:
$$\text{CDF} = \int \text{PDF}$$
$$\text{PDF} = \frac{d}{dx}\text{PDF}$$
- Expectation (Mean):
$$\mathbb{E}[X] = \mu = \int_\Omega x\, f_X(x)dx$$
- Properties:
$$\mathbb{E}[g(X)] = \mu = \int_\Omega g(X)\, f_X(x)dx$$
$$\mathbb{E}[aX + b] = a\mathbb{E}[X] + b$$
- Mode: the peak of the PDF
How to find the mode from PDF:
  - Find a point c such that $f_X(c)$ is maximized by differentiation (and test the edges of the interval).

  How to find the mode from CDF:
  - Continuous: Find a point c such that $F_X(c)$ has the steepest slope.
  - Discrete: Find a point c such that $F_X(c)$ has the biggest gap in a jump.
- Median: (a point c that separates the PDF into two equal areas)
$$[x < c] = [x > c] = 0.5$$
$$F_X(c) = 0.5$$
- Note : Symmetric distribution is a distribution in which Median = Mean
- Percentiles:
To get the α percentile, find the value c at which
$$F_X(c) = \alpha$$
- Variance:
$$\begin{aligned}\text{Var}[X] = \sigma^2 &= \mathbb{E}[(X-\mu)^2]\\ &= \int_\Omega (x-\mu)^2 f_X(x)dx\\ &= \mathbb{E}[X^2] - \mathbb{E}[X]^2\\ &= \mathbb{E}[X^2] - \mu^2\end{aligned}$$
- Properties:
$$\text{Var}[aX + b] = a^2\text{Var}[X]$$

Special Continuous Random Variables

Uniform

- PDF :
$$f_X(x) = \begin{cases}\frac{1}{b-a} & a \le x \le b\\ 0 & \text{otherwise}\end{cases}$$
- CDF :
$$F\frac{x-a}{b-a}$$
- Expectation:
$$\mathbb{E}[X] = \frac{a+b}{2}$$
- Variance:
$$\text{Var}[X] = \frac{(a-b)^2}{12}$$

Exponential

- What is the origin of exponential random variables?
  - An exponential random variable is the *interarrival* time between two consecutive Poisson events.
- PDF :
$$f_X(x) = \begin{cases}\lambda e^{-\lambda x} & x \ge 0\\ 0 & \text{otherwise}\end{cases}$$
- CDF :
$$F_X(x) = \begin{cases}0 & x < 0\\ 1 - e^{-\lambda x} & x \ge 0\end{cases}$$
- Expectation:
$$\mathbb{E}[X] = \frac{1}{\lambda}$$

- Variance:

$$\text{Var}[X] = \frac{1}{\lambda^2}$$

- Memorylessness property:

$$[T < t + m | T > t] = [T < m] = F_X(m)$$

- Starting from poisson distribution, derive an expression of PDF of exponential random variable

We assume that N is Poisson with a parameter λt or any duration t :

$$[N = n] = e^{-\lambda t}\frac{(\lambda t)^n}{n!}$$

Let T be the interarrival time between two events

$$[T > t] = [\text{interarrival time} > t] = [\text{no arrival in t}]$$

$$= [N = 0] = e^{-\lambda t}\frac{(\lambda t)^0}{0!} = e^{-\lambda t}$$

$$\text{since, } [T > t] = 1 - F_T(t)$$

$$\therefore F_T(t) = 1 - e^{-\lambda t}$$

$$f_T(t) = \frac{d}{dx}F_T(t) = \lambda e^{-\lambda t}$$

Erlange-k

(A generalization of the exponential distribution is the length until r counts occur in a Poisson process.)

- PDF :

$$f_X(x) = \frac{\lambda^k e^{-\lambda x} x^{k-1}}{(k-1)!}$$

- CDF :

$$\int_{-\infty}^{X'} \frac{\lambda^k e^{-\lambda x} x^{k-1}}{(k-1)!} dx$$

- Expectation:

$$\mathbb{E}[X] = \frac{k}{\lambda}$$

- Variance:

$$\text{Var}[X] = \frac{k}{\lambda^2}$$

Gamma

- PDF :

$$f_X(x) = \frac{1}{\beta^r \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$$

α : Shape parameter
β : Scale parameter

- Expectation:

$$\mathbb{E}[X] = \alpha\beta$$

- Variance:

$$\text{Var}[X] = \alpha\beta^2$$

- Starting from gamma distribution, derive an expression of PDF for erlang-k random variable

$$f_X(x) = \frac{1}{\beta^r \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$$

Substitute $\alpha = k$ and $\beta = \dfrac{1}{\lambda}$

$$f_X(x) = \frac{\lambda^k e^{-\lambda x} x^{k-1}}{\Gamma(k)}$$

If k is an integer, X has an Erlang distribution.

$$f_X(x) = \frac{\lambda^k e^{-\lambda x} x^{k-1}}{(k-1)!}$$

- Exponential distribution is a special case of Gamma distribution with $\alpha = 1$ and $\beta = \dfrac{1}{\lambda}$
- Chi-Squared distribution χ^2 is a special case of Gamma distribution with $\alpha = v/2$ and $\beta = 2$
it is a important distribution in statistics, also called as number of degrees of freedom

Gaussian

- We write

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

- PDF :

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

- Expectation:

$$\mathbb{E}[X] = \mu$$

- Variance:

$$\text{Var}[X] = \sigma^2$$

Standard Gaussian

- We write

$$Z \sim \mathcal{N}(0, 1)$$

- Conversion from Gaussian to Standard Gaussian

$$Z = \frac{X - \mu}{\sigma}$$

- PDF :

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

- CDF:

$$\Phi(z) = [Z < z]$$
$$[Z > z] = 1 - \Phi(z)$$
$$\Phi(-z) = 1 - \Phi(z)$$

- Expectation:

$$\mathbb{E}[X] = \mu = 0$$

- Variance:

$$\text{Var}[X] = \sigma^2 = 1$$

Moment generating function

- MGF:

$$M_X(t) = \mathbb{E}[e^{tX}]$$

- r$^{\text{th}}$ moment:

$$\mathbb{E}[X^r] = \frac{d^r}{dt^r} M_X(t)\Big|_{t=0}$$

Joint Discrete Probability

- $$f_{XY}(x, y) = [X = x, Y = y]$$

- Properties :

$$\sum_X \sum_Y f_{XY}(x, y) = 1$$

- Marginal PMF :

$$f_X(x) = \sum_Y f_{XY}(x, y)$$

$$f_Y(x) = \sum_X f_{XY}(x, y)$$

- Independence : X and Y are independent if

$$\underbrace{f_{XY}(x, y) = f_X(x) \times f_Y(y)}_{\text{for all values of x and y}}$$

also if :

$$f_{X|Y} = f_X$$
$$f_{Y|X} = f_Y$$

- Conditional probability:

$$f_{Y|X} = \frac{f_{XY}(x, y)}{f_X(x)}$$

$$f_{X|Y} = \frac{f_{XY}(x, y)}{f_Y(y)}$$

Joint Continuous Probability

$$[\Lambda] = \iint_\Lambda f_{XY}\ dx\ dy$$

for any event $\Lambda \subseteq \Omega_X \times \Omega_Y$

Properties :

$$\iint_A f_{XY}(x, y)\ dA = 1$$

Marginal PMF :

$$f_X(x) = \int_Y f_{XY}(x, y)\ dy$$

$$f_Y(x) = \int_X f_{XY}(x, y)\ dx$$

Independence : X and Y are independent if

$$f_{XY}(x, y) = f_X(x) \times f_Y(y)$$

also if :

$$f_{X|Y} = f_X$$
$$f_{Y|X} = f_Y$$

Conditional probability:

$$f_{Y|X} = \frac{f_{XY}(x, y)}{f_X(x)}$$

$$f_{X|Y} = \frac{f_{XY}(x, y)}{f_Y(y)}$$

Expectation, Covariance and Correlation

- Discrete:

$$\mathbb{E}[g(x,\,y)] = \sum_X \sum_Y g(x,\,y)\, f_{XY}(x,\,y)$$

- Continuous:

$$\mathbb{E}[g(x,\,y)] = \int_X \int_Y g(x,\,y)\, f_{XY}(x,\,y)dx\,dy$$

- Properties:

$$\mathbb{E}[x+y] = \mathbb{E}[x] + \mathbb{E}[y]$$

- if x and y are independent:

$$\mathbb{E}[xy] = \mathbb{E}[x]\mathbb{E}[y]$$

- Covariance:

$$\begin{aligned}\mathrm{Cov}(X,\,Y) = \sigma_{XY} &= \mathbb{E}[(X-\mu_X)(Y-\mu_Y)] \\ &= \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]\end{aligned}$$

- Variance:

$$\mathbb{V}[X+Y] = \mathbb{V}[X] + \mathbb{V}[Y] + 2\mathrm{Cov}(X,\,Y)$$

- if x and y are independent:

$$\mathbb{V}[X+Y] = \mathbb{V}[X] + \mathbb{V}[Y]$$

- Correlation Coefficient:

$$\rho_{X,Y} = \frac{\mathrm{Cov}(X,\,Y)}{\sigma_X \sigma_Y}$$

CLT and Rayleigh Distribution

CLT

$$\lim_{n\to\infty} P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < x\right) = \Phi(x)$$

$$\overline{\mathrm{X}} = \frac{1}{\mathrm{n}}\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X_i} \sim \mathrm{Normal}\left(\mu; \frac{\sigma^2}{\mathrm{n}}\right)$$

Rayleigh

$$R = \sqrt{X^2+Y^2}$$

$$f_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, \quad r > 0$$

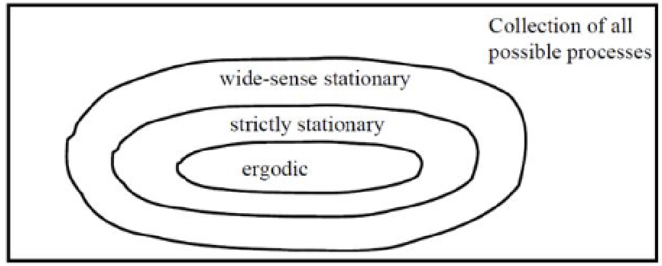
$$\mathrm{F_R(r)} = 1 - \mathrm{e}^{-\mathrm{r}^2/2\sigma^2}, \quad \mathrm{r} \geq 0$$

$$E(R) = \mu_R = \sqrt{\frac{\pi}{2}}\sigma$$

$$V(R) = \frac{4-\pi}{2}\sigma^2$$

$$E\left(R^2\right) = 2\sigma^2$$

Random Processes



Strict Stationary, not depends on time but on difference τ

- Expectation:

$$\mu_X(t) = \mathbb{E}[X(t,\,A)] = \sum_A X(t,\,A) f_A(a)$$

$$\mu_X(t) = \mathbb{E}[X(t,\,A)] = \int_A X(t,\,A) f_A(a)\,dA$$

- Auto-correlation function:

$$R_{XX}(t, t+\tau) = \mathbb{E}[X(t)X(t+\tau)]$$

- Auto-covariance function:

$$\begin{aligned}\mathrm{Cov}_{XX}(t, t+\tau) &= R_{XX}(t, t+\tau) - \mu_X(t)\mu_X(t+\tau) \\ &= \mathbb{E}[X(t)X(t+\tau)] - \mathbb{E}[X(t)]\mathbb{E}[X(t+\tau)]\end{aligned}$$

- Auto-correlation Coefficient Function:

$$\rho_{XX}(t, t+\tau) = \frac{\mathrm{Cov}_{XX}(t, t+\tau)}{\sigma_X(t)\sigma_X(t+\tau)}$$

- Wide Sense Stationary Process WSSP:

$$\text{Expectation} = \text{Constant, Not depend on time}$$

$$R_{XX}(t, t+\tau) = \mathbb{E}[X(t)X(t+\tau)] = R_{XX}(\tau)$$
$$\text{depend on time difference only}$$

- Average power for WSSP:

$$R_{XX}(\tau = 0) = \mathbb{E}[X(t)X(t+0)] = \mathbb{E}[X^2(t)]$$

- Properties of WSSP:
  - Power Spectral Density $S_{XX}(f) = \mathcal{F}\{R_{XX}(\tau)\}$
  - Average Power $= R_{XX}(0) = \int_{-\infty}^{\infty} S_{XX}(f)df$
  - $R_{XX}(\tau) = R_{XX}(-\tau)$ (Even function)
  - $R_{XX}(\tau)$ is maximum at the origin.
- Properties of Strict Stationary Process SSP:
  - $\mu_X = 0$
  - $R_{XX}(t, t+\tau) = R_{XX}(\tau)$
  - $\rho_{XX}(t, t+\tau) = \frac{\mathrm{Cov}_{XX}(\tau)}{\mathrm{Cov}_{XX}(0)}$
  - $\mathrm{Cov}_{XX}(\tau) = R_{XX}(\tau) - \mu_X^2$
  - IID = Independent Identical distributions
- Properties of White Noise:
  - $R_{XX}(\tau) = N_0\delta(\tau)$
  - $S_{XX}(f) = N_0$
  - For the zero-mean signals $N_0 = \sigma_X^2$
- Cross-Correlation Function:
  - $R_{XY}(t, t+\tau) = E(X(t)Y(t+\tau))$
  - $R_{XY}(t, t+\tau) \neq R_{YX}(t, t+\tau)$
  - If $R_{XY}(t, t+\tau) = 0$ the X and Y are orthogonal.
  - If X and Y are independent, then $R_{XY}(t, t+\tau) = C$
- Cross-Covariance Function:
  - $\mathrm{Cov}_{XY}(t, t+\tau) = R_{XY}(t, t+\tau) - \mu_X(t)\mu_Y(t+\tau)$
  - If $\mathrm{Cov}_{XY}(t, t+\tau) = 0$ the X and Y are Uncorrelated.
- The time-averaged mean:

$$\langle X(t)\rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X(t)dt$$

- The time-averaged auto-correlation function:

$$\langle X(t)X(t+\tau)\rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X(t)X(t+\tau)dt$$

- The WSS Signal is Called "Ergodic in its mean" if:

$$E(X(t)) = \langle X(t)\rangle$$

- The WSS Signal is Called "Ergodic in its auto-correlation function" if:

$$R_{XX}(\tau) = \langle X(t)X(t+\tau)\rangle$$

Fast-View: Table of Distributions

Distribution	PMF/PDF and Support	Expected Value	Variance	MGF
Bernoulli Bern(p)	$P(X=1)=p$ $P(X=0)=q=1-p$	p	pq	$q+pe^t$
Binomial Bin(n,p)	$P(X=k)=\binom{n}{k}p^k q^{n-k}$ $k\in\{0,1,2,\dots n\}$	np	npq	$(q+pe^t)^n$
Geometric Geo(p)	$P(X=k)=q^{k-1}p$ $k\in\{1,\ 2,\ \dots\}$	$1/p$	q/p^2	$\frac{pe^t}{1-qe^t}$, $qe^t<1$
Poisson Pois(λ)	$P(X=k)=\frac{e^{-\lambda}\lambda^k}{k!}$ $k\in\{0,\ 1,\ 2,\ \dots\}$	λ	λ	$e^{\lambda(e^t-1)}$
Uniform Unif(a,b)	$f(x)=\frac{1}{b-a}$ $x\in(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
Normal $\mathcal{N}(\mu,\sigma^2)$	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}$ $x\in(-\infty,\infty)$	μ	σ^2	$e^{t\mu+\frac{\sigma^2t^2}{2}}$
Exponential Expo(λ)	$f(x)=\lambda e^{-\lambda x}$ $x\in(0,\infty)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda-t}$, $t<\lambda$
Erlang-k Erlang(λ,k)	$f(x)=\frac{\lambda^x x^{k-1}e^{-\lambda x}}{(k-1)!}$ $x\in(0,\infty)$	$\frac{k}{\lambda}$	$\frac{k}{\lambda^2}$	$\left(\frac{\lambda}{\lambda+t}\right)^k$, $t<\lambda$
Gamma Gamma(a,λ)	$f(x)=\frac{1}{\Gamma(a)}(\lambda x)^a e^{-\lambda x}\frac{1}{x}$ $x\in(0,\infty)$	$\frac{a}{\lambda}$	$\frac{a}{\lambda^2}$	$\left(\frac{\lambda}{\lambda-t}\right)^a$, $t<\lambda$
Chi-Square χ^2_n	$\frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2}$ $x\in(0,\infty)$	n	$2n$	$(1-2t)^{-n/2}$, $t<1/2$
Rayleigh R(r)	$\frac{r}{\sigma^2}e^{-r^2/2\sigma^2}$ $r\in(0,\infty)$	$\sqrt{\frac{\pi}{2}}\sigma$	$V(R)=\frac{4-\pi}{2}\sigma^2$	-