Electromagnetics Cheatsheet

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Multi-Variable Calculus

Vector Algebra

- Thm 0: $\vec{A} = \langle A_i \hat{a}_i, A_j \hat{a}_j, A_k \hat{a}_k \rangle$, $||\vec{A}|| = \sqrt{A_i^2 + A_j^2 + A_k^2}$
- Thm 1: $||c\vec{A}|| = |c| ||\vec{A}||$
- Thm 2: $\vec{a} = \frac{\vec{A}}{||\vec{A}||}$ a:a is unit vector
- Thm 3: Dot Product $\vec{A} \cdot \vec{B} = A_i B_i + A_j B_j + A_k B_k$
- Thm 4: Dot Product Properties

$$\vec{a} \cdot \vec{b} = (\vec{b} \cdot \vec{a})$$

$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$(\vec{a} + \vec{b}) \cdot \vec{c} = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$$

$$\vec{a} \cdot \vec{a} = ||\vec{A}||^2$$

- Thm 5: Angle $\vec{a} \cdot \vec{b} = ||\vec{a}|| ||\vec{b}|| \cos \theta, \ \theta : 0 < \theta < \pi$
- Thm 6: Orthogonality $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$
- Thm 7: Component (Scalar) comp $_{\vec{a}}\vec{b} = ||\vec{b}|| \cos \theta = \frac{\vec{a} \cdot \vec{b}}{||\vec{a}||}$
- Thm 8: Projection (Vector) $\operatorname{proj}_{\vec{a}} \vec{b} = \operatorname{comp}_{\vec{a}} \vec{b} \cdot \frac{\vec{a}}{||\vec{a}||} = \frac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}} \vec{a}$
- Thm 9: \perp Projection (Vector) oproj $_{\vec{a}}\vec{b} = \vec{b} \text{proj}_{\vec{a}}\vec{b}$
- Thm 10: Cross Product $\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_i & A_j & A_k \\ B_i & B_j & B_k \end{vmatrix}$
- Thm 11: Cross Product Properties

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

$$(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$$

$$(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$$

$$\vec{a} \times \vec{a} = 0$$

- Thm 11: Angle $\vec{a} \cdot \vec{b} = ||\vec{a}|| ||\vec{b}|| \sin \theta$, $\theta : 0 < \theta < \pi$
- Thm 12: Parallelity $\vec{a} \parallel \vec{b} \iff \vec{a} \times \vec{b} = 0$
- Thm 13: $\vec{c} = \vec{a} \times \vec{b} : \vec{c} \perp \vec{a}, \vec{b}$
- Thm 14: Area of parallelogram $||\vec{a} \times \vec{b}||$
- Thm 15: Area of triangle $\frac{1}{2}||\vec{a} \times \vec{b}||$
- Thm 16: Triple Product (Scalar)

$$\vec{A} \times \vec{B} \cdot \vec{C} = \vec{A} \cdot \vec{B} \times \vec{C} \begin{vmatrix} A_i & A_j & A_k \\ B_i & B_j & B_k \\ C_i & C_j & C_k \end{vmatrix}$$

Vector Geometry

Coordinates

• Cartesian:

$$\vec{A} = \langle A_x \hat{a}_x, A_y \hat{a}_y, A_z \hat{a}_z \rangle \\ -\infty \langle x, y, z \rangle$$

• Cylindrical:

$$\begin{split} \vec{A} = < A_{\rho} \hat{a}_{\rho}, A_{\phi} \hat{a}_{\phi}, A_{z} \hat{a}_{z} > \\ 0 \leq \rho < \infty, 0 \leq \phi < 2\pi, -\infty < z < \infty \end{split}$$

• Spherical:

$$\vec{A} = \langle A_r \hat{a}_r, A_\theta \hat{a}_\theta, A_\phi \hat{a}_\phi \rangle$$

$$0 < r < \infty, 0 < \theta < \pi, 0 < \phi < 2\pi$$

Transformations

Cylindrical \iff Spherical

$$\begin{bmatrix} \mathbf{A}_r \\ \mathbf{A}_{\theta} \\ \mathbf{A}_{\phi} \end{bmatrix} = \begin{bmatrix} \sin(\theta) & 0 & \cos(\theta) \\ \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A}_{\rho} \\ \mathbf{A}_{\phi} \\ \mathbf{A}_z \end{bmatrix}$$
$$\begin{bmatrix} \mathbf{A}_{\theta} \\ \mathbf{A}_{z} \end{bmatrix} \begin{bmatrix} \sin(\theta) & \cos(\theta) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A}_r \\ \mathbf{A}_r \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{A}_{\rho} \\ \mathbf{A}_{\phi} \\ \mathbf{A}_{z} \end{bmatrix} = \begin{bmatrix} \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \\ \cos(\theta) & -\sin(\theta) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A}_{r} \\ \mathbf{A}_{\theta} \\ \mathbf{A}_{\phi} \end{bmatrix}$$

$$\begin{array}{l} r = \sqrt{\rho^2 + z^2}, \, \rho = rsin(\theta) \\ \theta = tan^{-1}(\frac{\rho}{z}), \, z = rcos(\theta) \\ \phi = \phi \end{array}$$

 $Cartesian \iff Cylindrical$

$$\begin{bmatrix} \mathbf{A}_{\rho} \\ \mathbf{A}_{\phi} \\ \mathbf{A}_{z} \end{bmatrix} = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ -\sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{A}_{x} \\ \mathbf{A}_{y} \\ \mathbf{A}_{z} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{A}_x \\ \mathbf{A}_y \\ \mathbf{A}_z \end{bmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{A}_\rho \\ \mathbf{A}_\phi \\ \mathbf{A}_z \end{bmatrix}$$

$$\rho = \sqrt{x^2 + y^2}, \ x = \rho \cos(\theta)
\phi = \tan^{-1}(\frac{y}{x}), \ y = \rho \sin(\theta)
\xrightarrow{x = x}$$

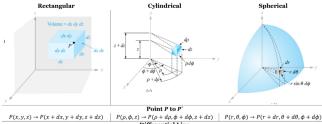
 $Cartesian \iff Spherical$

$$\begin{bmatrix} \mathbf{A}_r \\ \mathbf{A}_\theta \\ \mathbf{A}_\phi \end{bmatrix} = \begin{bmatrix} \sin(\theta)\cos(\phi) & \sin(\theta)\sin(\phi) & \cos(\theta) \\ \cos(\theta)\cos(\phi) & \cos(\theta)\sin(\phi) & -\sin(\theta) \\ -\sin(\phi) & \cos(\phi) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A}_x \\ \mathbf{A}_y \\ \mathbf{A}_z \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{A}_x \\ \mathbf{A}_y \\ \mathbf{A}_z \end{bmatrix} = \begin{bmatrix} \sin(\theta)\cos(\phi) & \cos(\theta)\cos(\phi) & -\sin(\phi) \\ \sin(\theta)\sin(\phi) & \cos(\theta)\sin(\phi) & \cos(\phi) \\ \cos(\theta) & -\sin(\theta) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{A}_r \\ \mathbf{A}_\theta \\ \mathbf{A}_\phi \end{bmatrix}$$

$$\begin{split} r &= \sqrt{x^2 + y^2 + z^2}, \ x = rsin(\theta)cos(\phi) \\ \theta &= cos^{-1}(\frac{z}{\sqrt{x^2 + y^2 + z^2}}), \ y = rsin(\theta)sin(\phi) \\ z &= rcos(\theta), \ \phi = tan^{-1}\frac{y}{x} \end{split}$$

Overview



Point P to P'				
$P(x, y, z) \rightarrow P(x + dx, y + dy, z + dz)$	$P(\rho, \phi, z) \rightarrow P(\rho + d\rho, \phi + d\phi, z + dz)$	$P(r, \theta, \phi) \rightarrow P(r + dr, \theta + d\theta, \phi + d\phi)$		
Differential Line				
 In x direction: dl_x = dx a_x 	 In ρ direction: dl_ρ = dρ a_ρ 	 In r direction: dl_r = dr a_r 		
 In y direction: dl_y = dy a_y 	\circ In ϕ direction: $\mathbf{dl}_{\phi} = \rho d\phi \mathbf{a}_{\phi}$	ο In θ direction: $dl_{\theta} = r d\theta a_{\theta}$		
 In z direction: dl_z = dz a_z 	 o In z direction: dl_z = dz a_z 	\circ In ϕ direction: $\mathbf{dl}_{\phi} = r \sin \theta d\phi \mathbf{a}_{\phi}$		
o $\mathbf{dl} = dx \mathbf{a_x} + dy \mathbf{a_y} + d_z \mathbf{a_z}$	o dl = $d\rho$ $\mathbf{a}_0 + \rho d\phi$ $\mathbf{a}_{\phi} + dz$ \mathbf{a}_{τ}	$0 dl = dr a_r + r d\theta a_\theta + r \sin\theta d\phi a_\phi$		

- o $\mathbf{dl} = dx \, \mathbf{a_x} + dy \, \mathbf{a_y} + d_z \, \mathbf{a_z}$ o $\mathbf{dl} = d\rho \, \mathbf{a_p} + \rho \, d\phi \, \mathbf{a_\phi} + dz \, \mathbf{a_z}$ Differential Area

 o Constant x: $\mathbf{dS_x} = dy \, dz \, \mathbf{a_x}$ o Constant ρ : $\mathbf{dS_\phi} = \rho \, d\phi \, dz \, \mathbf{a_\phi}$
- o Constant x: $\mathbf{dS_x} = dy \, dz \, \mathbf{a_x}$ o Constant ρ : $\mathbf{dS_p} = \rho \, d\phi \, dz \, \mathbf{a_p}$ o Constant t: $\mathbf{dS_p} = dx \, dz \, \mathbf{a_p}$ o Constant ϕ : $\mathbf{dS_p} = d\rho \, dz \, \mathbf{a_p}$ o Constant z: $\mathbf{dS_z} = dx \, dy \, \mathbf{a_z}$ o Constant z: $\mathbf{dS_z} = \rho \, d\phi \, d\rho \, \mathbf{a_z}$ o Constant z: $\mathbf{dS_z} = \rho \, d\phi \, d\rho \, \mathbf{a_z}$ o Constant z: $\mathbf{dS_z} = \rho \, d\phi \, d\rho \, \mathbf{a_z}$ o Constant z: $\mathbf{dS_z} = \rho \, d\phi \, d\rho \, \mathbf{a_z}$ o Constant z: $\mathbf{dS_z} = \rho \, d\phi \, d\rho \, \mathbf{a_z}$
- $_{\theta}\,,\,A_{\phi}\hat{a}_{\phi}> \qquad \begin{array}{c|c} \text{Differential Volume} \\ dv=dx\,dy\,dz & dv=\rho\,d\rho\,d\phi\,dz & dv=r^2\sin\theta\,\,dr\,d\theta\,d\phi \\ \end{array}$

Vector Calculus

• Thm 0: Green

$$\oint_{\partial D} \vec{F} \cdot d\vec{r} = \iint_{D} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dx \, dy$$

• Thm 1: Gauss

$$\iiint_V \nabla \cdot \vec{F} \, dV = \oint_S \vec{F} \cdot d\vec{S}$$

• Thm 2: Stokes

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$$

Nabla

Cartesian:

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

Cylindrical:

$$\nabla = \frac{\partial}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial}{\partial \phi} \hat{\phi} + \frac{\partial}{\partial z} \hat{z}$$

Spherical:

$$\nabla = \frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \phi}\hat{\phi}$$

Div

Cartesian:

$$\nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

Cylindrical:

$$\nabla \cdot \vec{D} = \frac{1}{\rho} \frac{\partial \rho D_{\rho}}{\partial \rho} + \frac{1}{\rho} \frac{\partial D_{\phi}}{\partial \phi} + \frac{\partial D_{z}}{\partial z}$$

Spherical:

$$\nabla \cdot \vec{D} = \frac{1}{r^2} \frac{\partial r^2 D_r}{\partial r} + \frac{1}{r sin \theta} \frac{\partial sin \theta D_{\theta}}{\partial \theta} + \frac{1}{r sin \theta} \frac{\partial D_{\phi}}{\partial \phi}$$

Curl

Cartesian:

$$\nabla \times \vec{H} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

Cylindrical:

$$\nabla \times \vec{H} = \frac{1}{\rho} \begin{vmatrix} a_{\phi} & a_{\phi} & a_{z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_{\rho} & \rho H_{\phi} & H_{z} \end{vmatrix}$$

Spherical:

$$\nabla \times \vec{H} = \frac{1}{r^2 sin\theta} \begin{vmatrix} a_r & a_\theta & a_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_\rho & rH_\theta & rsin\theta H_\phi \end{vmatrix}$$

Notes

$$abla \cdot D = 0 \\
Divergent-free \\
Source-free \\
Solenoid$$
 $abla \cdot X + H = 0 \\
Curl-free \\
Irrational \\
Conservative$

- Gradient is Vector indicates the encreasing direction.
- Div is Scalar indicates sources (+ve) and sinks (-ve).
- Curl is Vector indicates rotation ccw (+ve) and cw (-ve).
- Div of Curl = 0, Curl of Grad = 0, Div of Grad = Laplace.
- Closed integral doesn't depends on path in case of curl-free.

Static Electromagnetism

Analogy Between Capacitance and Inductor

Maxwell Equations

	Integral Form	Point Form
Kirchhoff's Voltage Law	$\oint \vec{E} \cdot d\vec{L} = 0$	$\nabla \times \vec{E} = 0$
Ampere's Current Law	$\oint \vec{H} \cdot d\vec{\vec{L}} = \int \vec{J} \cdot d\vec{S} = I_{enc}$	$\nabla \times \vec{H} = J$
Gauss's Law for E-Field	$\oint \vec{D} \cdot d\vec{S} = \int \rho dV$	$\nabla \cdot \vec{D} = \rho_v$
Gauss's Law for H-Field	$\oint \vec{B} \cdot d\vec{S} = 0$	$\nabla \cdot \vec{B} = 0$

Analogy Between Electric and Magnetic

Electric Field Intensity \vec{E} $[\frac{V}{m}]$	Magnetic Field Intensity \vec{H} $\left[\frac{A}{m}\right]$
Permittivity $\epsilon = \epsilon_0 \epsilon_r \left[\frac{F}{m} \right]$	Permeability $\mu = \mu_0 \mu_r \left[\frac{H}{m} \right]$
Coulomb's Law $dE = \frac{dQ}{4\pi\epsilon R^2}\vec{a}_r \Longleftrightarrow E = \int \frac{dQ}{4\pi\epsilon R^2}\vec{a}_r$	Biot-Savart Law $dH = \frac{Id\vec{L} \times \vec{a}_r}{4\pi\epsilon R^2} \Longleftrightarrow H = \int \frac{Id\vec{L} \times \vec{a}_r}{4\pi\epsilon R^2}$
Kirchhoff Voltage Law $ \oint \vec{E} \cdot d\vec{L} = 0 $ $ \nabla \times \vec{E} = 0 $	Ampere Current Law $ \oint \vec{H} \cdot d\vec{L} = I_{enc} $ $ \nabla \times \vec{H} = J $
Electric Flux Density $\vec{D} = \epsilon \vec{E} \; [\frac{C}{m^2}]$	Magnetic Flux Density $\vec{B} = \mu \vec{H} \; [\frac{\mathrm{Wb}}{m^2}]$
Electric Flux Lines $\psi = \oint \vec{D} \cdot d\vec{S} \ [C]$	$\begin{array}{ll} \text{Magnetic Flux Lines} \\ \phi = \int_S \vec{B} \cdot d\vec{S} \text{ [Wb]} \end{array}$
Gauss Law for E-Field $ \oint \vec{D} \cdot d\vec{S} = Q_{enc} $ $ \nabla \cdot \vec{D} = \rho_v $	Gauss Law for H-Field $ \oint \vec{B} \cdot d\vec{S} = 0 $ $ \nabla \cdot \vec{B} = 0 $
Stored Electric Energy $W_E = \frac{1}{2} \int \vec{D} \cdot \vec{E} dv$ $\vec{D} \cdot \vec{E} = \epsilon E^2 = \frac{D^2}{\epsilon}$	Stored Magnetic Energy $W_E = \frac{1}{2} \int \vec{B} \cdot \vec{H} dv$ $\vec{B} \cdot \vec{H} = \mu H^2 = \frac{B^2}{\mu}$ $W_E = \int \vec{B} \cdot d\vec{H}$
Uniform E-Flux Lines $\psi = \vec{D} \cdot \vec{A} = Q_{enc}$	Uniform H-Flux Lines $\psi = \vec{B} \cdot \vec{A}$
Electric Force $F = qE$	Magnetic Force $F = q(v \times B)$

All Differential Elements

• Electric Field

$$dE = \begin{cases} \frac{\rho_L dl}{4\pi R^2 \epsilon} \vec{a}_r & \text{Line,} \\ \\ \frac{\rho_S dS}{4\pi R^2 \epsilon} \vec{a}_r & \text{Surface,} \\ \\ \\ \frac{\rho_V dV}{4\pi R^2 \epsilon} \vec{a}_r & \text{Volume.} \end{cases}$$

• Electric Potential

$$dV = \begin{cases} \frac{\rho_L dl}{4\pi R \epsilon} \vec{a}_r & \text{Line,} \\ \frac{\rho_S dS}{4\pi R \epsilon} \vec{a}_r & \text{Surface,} \\ \frac{\rho_Y dV}{4\pi R \epsilon} \vec{a}_r & \text{Volume.} \end{cases}$$

• Magnetic Field

$$dH = \frac{Id\vec{L} \times \vec{a}_r}{4\pi R^2}$$

Inductor

Inductance $L = \frac{N\Phi}{I} = \frac{NBA}{I}$

Relative Permeability

 $\mu_r = \frac{L}{L_0} = \frac{B}{B_0}$

Magnetic Dipole Moment

 $\vec{m} = IA\vec{a}_n$

Magnetization

 $\vec{M} = (\mu_r - 1)\vec{H}$

 $\vec{M} = \frac{I_m}{I}$

Magnetic Susceptibility

 $\chi_m = \mu_r - 1$

Energy Stored in Inductor

 $W_H = \frac{1}{2}LI^2$

DC Magnetic Field Intensity

 $H = \frac{NI}{I} = \frac{I_c}{I}$

Magnetic Boundary Conditions

Capacitance $C = \frac{Q}{V} = \frac{\epsilon A}{I}$

Capacitance

Relative Permittivity $\epsilon_r = \frac{C}{C_0} = \frac{Q}{Q_0}$

Electric Dipole Moment $\vec{p} = Q\vec{x}$

Polarization
$$\vec{P} = \epsilon_o(\epsilon_r - 1)\vec{E}$$

$$\vec{P} = \frac{\epsilon_r - 1}{\epsilon_r}\vec{D} = \frac{n\vec{p}}{V}$$

 ${\bf Electric\ Susceptibility}$ $\chi_e = \epsilon_r - 1$

Energy Stored in Capacitor $W_E = \frac{1}{2}CV^2$ $\nabla \cdot \vec{D} = \rho_v$

DC Electric Field Intensity $E = \frac{V}{I}$

Electric Boundary Conditions Conductors $E_t = 0, D_N = \epsilon E_N = \rho_s : E_N = -\nabla V$

$$\begin{aligned} & \text{Dielectric} \\ E_{t1} &= E_{t2}, D_{n1} = D_{n2} \\ \epsilon_1 E_{n1} &= \epsilon_2 E_{n2} \\ \frac{tan\theta_1}{tan\theta_2} &= \frac{\epsilon_1}{\epsilon_2} \end{aligned}$$

Common Formulas

• Electric Field due Point Charge

$$\vec{E} = \frac{Q}{4\pi\epsilon R^2} \vec{a}_R$$

• Electric Potential due Point Charge

$$V = \frac{Q}{4\pi\epsilon R}$$

• Electric Field due Infinite Line Charge

$$\vec{E} = \frac{\rho_l}{2\pi\epsilon\rho} \vec{a}_\rho$$

• Electric Field due Infinite Plane Charge

$$\vec{E} = \frac{\rho_s}{2\epsilon} \vec{a}_n$$

• Magnetic Field due Infinite Line Current

$$\vec{H} = rac{I}{2\pi
ho} \vec{a}_{\phi}$$

Other Formulas

• Charge

$$Q = \begin{cases} \int \rho_L dL & \text{Line,} \\ \int \rho_S dS & \text{Surface,} \\ \int \rho_V dV & \text{Volume.} \end{cases}$$

- Electric Flux Density $\vec{D} = \frac{d\psi}{ds} \vec{a}_{\psi}$
- Electric Work $W = -Q \int \vec{E} \cdot d\vec{L} = QV$
- Electric Potential $V_{AB} = V_A V_B = -\int_{R}^{A} \vec{E} \cdot d\vec{L}$
- Electric Potential with Field $\vec{E} = -\nabla V$
- Poisson Equation $\nabla^2 V = \frac{-\rho}{2}$
- Laplace Equation $\nabla^2 V = 0$
- Electric Energy $U = \frac{1}{2} \sum QV = \frac{1}{2} \int \rho_v V dv$
- Force in Conductors

$$\vec{F} = m\vec{a} = -e\vec{E}, \vec{a} = \frac{\vec{v}_d}{\tau}, \vec{v}_d = -\frac{e\tau}{m}\vec{E} = -\mu_e\vec{E}$$

- Current and Current Density $\vec{J} = \sigma \vec{E} = \rho_e \vec{v}_d = -\mu_e \rho_e \vec{E}$
- Current $I = \int \vec{J} \cdot d\vec{S}$
- $n(\text{electron/m}^3) = \frac{\rho(\text{Density}) N_A(\text{Avogadro's number})}{M_a(\text{atomic mass})}$
- Electron Density $\rho_e = en$
- Ohm's Law $R = \frac{V}{I} = \frac{\int \vec{E} \cdot d\vec{L}}{\int \sigma \vec{E} \cdot d\vec{S}}$
- Continuity of Current $\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \iff \oint \vec{J} \cdot d\vec{S} = -\frac{dQ}{dt}$
- Current Elements due to Magnet

$$F = -I \oint \vec{B} \times d\vec{L}, F = I(\vec{L} \times \vec{B}) \ \ \text{for straight conductor}$$

- Torque due to Magnet $\vec{T} = \vec{R} \times \vec{F} = \vec{R} \times (I(\vec{L} \times \vec{B}))$
- Torque due to Magnetic dipole $\vec{T} = \vec{m} \times \vec{B}$
- Current due to electron spin $I = e \frac{\omega}{2\pi}$
- Faraday's Law $V = NA\frac{dB}{dt}$

Serious Notes

- Ch.1
 - Convert from any system to Cartesian then covert back if you have points and need vector between.
 - For Cartesian Indicates by three planes.
 - For Cylindrical Indicates by two planes and Cylinder.
 - For Spherical Indicates by Sphere, Convex and plane.
 - Magnitude is the same in all systems.
- Ch.2

$$-\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$
$$-\int \frac{xdx}{(a^2+x^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$
$$-\int \frac{dx}{a^2+x^2} = \frac{1}{a}tan^{-1}(\frac{x}{a})$$

- Ch.3
 - Length with D is changeable anywhere but isn't for Q
 - Flux lines = Q[C]
 - Flux density $[C/m, C/m^2C/m^3]$

• Ch.4

- Potential and Work are independent of path.
- $\nabla V=-\vec{E}$ is normal to equipotential surface and V is max when $d\vec{L}$ in the same diraction.
- Work is done by system if it is (-ve).
- Workdone for putting first charge = 0

• Ch.5

- Charge on the surface of conductor is equal any where and no charge inside so $\vec{E}=0$.
- Charging the inside charging the surface by negative Q.

• Ch.6

- After adding dielectric material we get:

$$Q = Q_o + Q_p$$

$$\vec{D} = \vec{D}_o + \vec{P}$$

$$\epsilon_o \epsilon_r \vec{E} = \epsilon_o \vec{E}_o + \vec{P}$$

$$\vec{P} = \epsilon_o (\epsilon_r - 1) \vec{E}$$

- Dielectric have charges on surface and inside.
- Dipole direction form (-ve) to (+ve).
- $-\ {\bf Q'}={\bf Q}$ after removing the dielectric and battery.
- Electric Field inside dielectric < External Field.

• Ch.7

- After adding dielectric material we get:

$$\phi = \phi_o + \phi_m$$

$$\vec{B} = \vec{B}_o + \vec{B}_m$$

$$\mu_o \mu_r \vec{H} = \mu_o \vec{H}_o + \mu_o \vec{M}$$

$$\vec{M} = (\mu_r - 1) \vec{H}$$

- The magnetization forms surface current.
- Magnetization direction is normal.
- Cylindrical $\vec{r} \times \vec{\phi} = \vec{z}, \vec{\phi} \times \vec{z} = \vec{r}, \vec{z} \times \vec{r} = \vec{\phi}$
- Spherical $\vec{r} \times \vec{\theta} = \vec{\phi}, \vec{\theta} \times \vec{\phi} = \vec{r}, \vec{\phi} \times \vec{r} = \vec{\theta}$
- Toroid Coil is same as Normal Coil as $l_{\rm coil} = 2\pi r_{\rm toroid}$
- No monopole existence.
- Magnetic Field rotate the electrons in circular motion between electric field accelerate/decelerate electrons.
- $-\,$ Workdone by Magnetic Field on proton >0
- As $\nabla \cdot \vec{B}=0$, So B has no sink/start or source/end and As $\nabla \times \vec{E}=0$ So, E has no rotations.