# Part II

# Statistics

SENSEI

# Ваш Sensei

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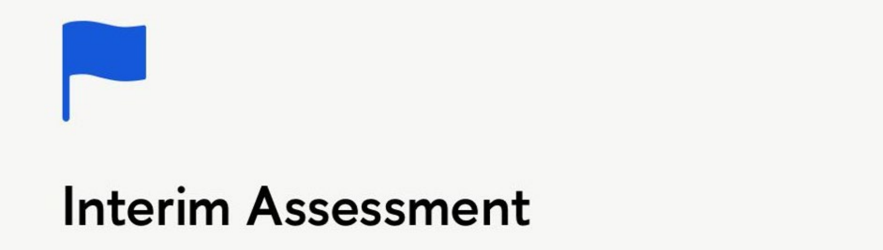
# Total score



Interim Assessment

- -2024/2025 2nd semester

$$Score = 0{,}09 \cdot Activity + 0{,}11 \cdot Home\ assignments + 0{,}1 \cdot (CR1 + CR2 + CR3 + CR4) + 0{,}06 \cdot Project + 0{,}34 \cdot Final\ exam$$

# Chapter 1. Graphical Display and Descriptive Statistics

> *The greatest moments are those when you see the result pop up in a graph or in your statistics analysis - that moment you realize you know something no one else does and you get the pleasure of thinking about how to tell them.*
> *Emily Oster*

## Where Data comes from

Random variables and their probability distributions we've talked so far are *theoretical* things. In the matter of fact usually we are not given the true distribution of a variable. For example if X is a monthly salary of a Russian citizen in 2017, no one is to provide us with the true distribution of that variable. So now we are switching to *practice* and start work with real data.

To get knowledge on a salary, first we need to gather data on X. Each observed value of X is called an **observation** and denoted **$x_i$**. For example, if Masha' sister salary is 40 000 roubles, then $x_{sister}$=40 000 is one of the observations. If we get information on salary for all Russian citizens, we would have **population of X** - all existent observations $x_i$, i=1,2,..., N, where N is the size of the population – total number of citizens. If so, we would be able to construct the true probability distribution of X: calculate probabilities (relative frequencies) for each value of X. This is called a **population distribution**.

However, in many cases it would require too large amount of time and money to get information about the whole population. Then, we only take a small but representative part of it to analyze X. This is what we call **a sample of X** - a set of observations $x_i$, i=1,2, ... n, where n is called the **sample size**, n<<N. For example, we can use the data on salaries of 1000 randomly chosen citizens - a sample of size 1000 - to analyze monthly salary in Russia. Based on such data you can construct the **sample distribution** of X – observed relative frequencies for different values of X. In statistics we usually work with samples, as population data is rarely available. You will learn more on sampling in chapter 2.

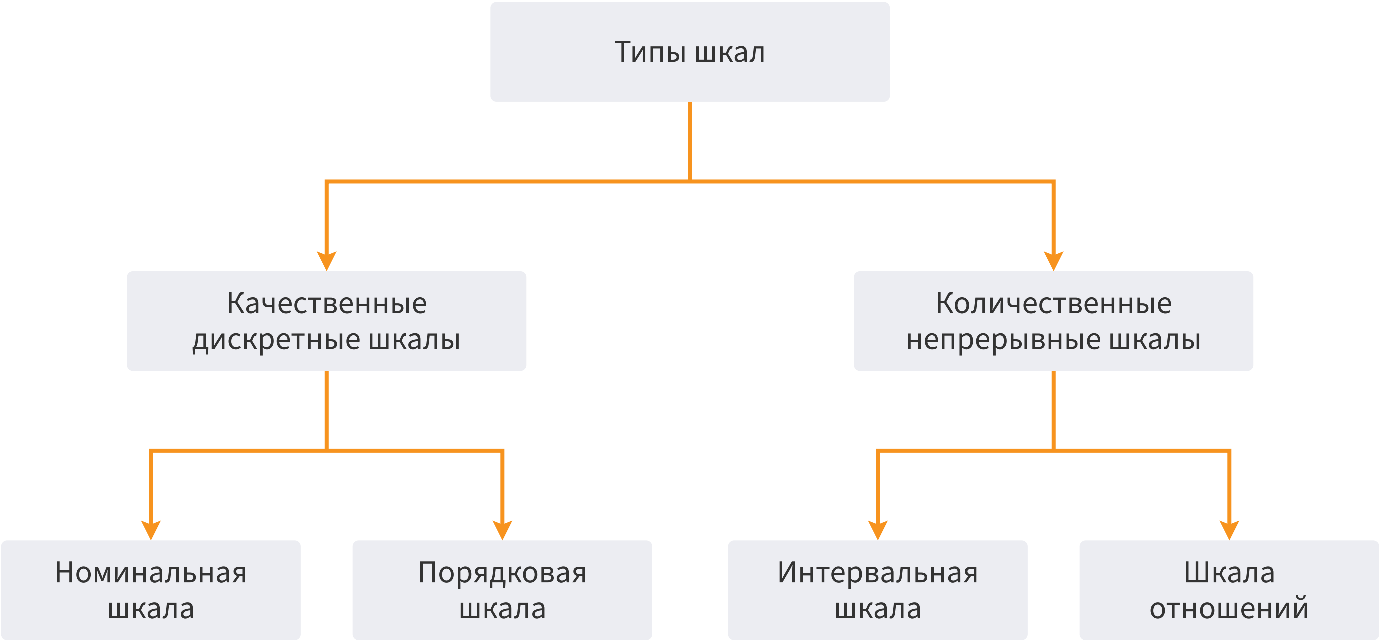
To summarize when you are looking at a dataset it could contribute the whole population or be only a sample taken from a population. Further that would matter since analysis for those would be different.

### Types of data

According to the basic classification data can be quantitative or qualitative.

| **Quantitative** data<br>**$x_i$** are **numbers** | **Qualitative** data<br>**$x_i$** are **not numbers** |
|---|---|

Another name for quantitative data is numerical. It could be discrete (*number of assignments submitted*) or continuous (*measured level of temperature*). Qualitative data is non-neumerical or categorical. Examples are: yes/no; eye color; student's name.

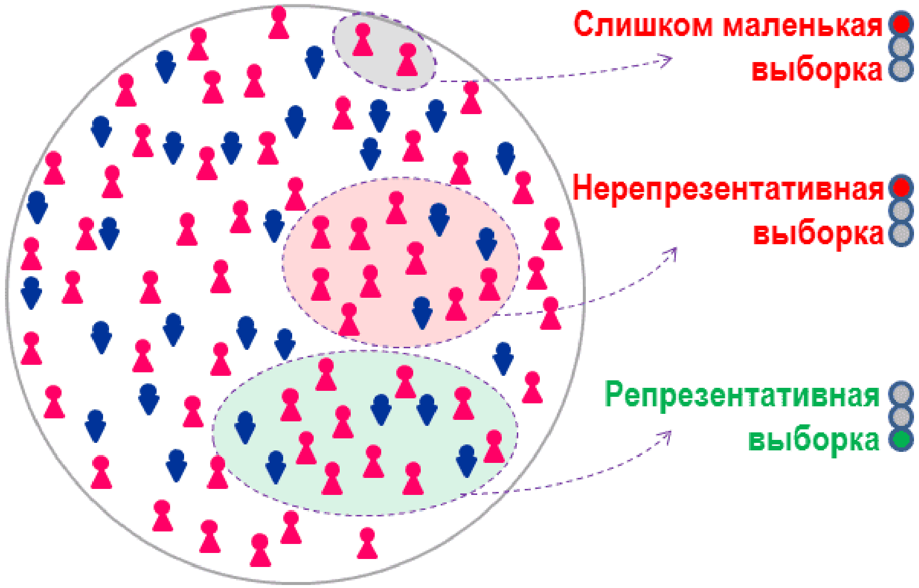


1 **Номинальная шкала** (только категории, без порядка)
📌 *Пример*: Цвет глаз (синий, зеленый, карий), пол (м/ж), марка автомобиля.
2 **Порядковая шкала** (есть порядок, но без точных интервалов)
📌 *Пример*: Оценки в отзывах (плохо, средне, хорошо), уровни образования (бакалавр, магистр, PhD).
3 **Интервальная шкала** (разница между значениями имеет смысл, но нет абсолютного нуля)
📌 *Пример*: Температура в °C или °F (0°C ≠ отсутствие тепла), IQ (нет абсолютного нуля интеллекта).
4 **Шкала отношений** (есть ноль, можно сравнивать в пропорциях)
📌 *Пример*: Вес (0 кг = отсутствие веса), рост, доход (можно сказать, что 200$ — это в 2 раза больше, чем 100$).

Sample – выборка (n = 3)

Population – ген. совокупность (N = ? – устал считать)

Генеральная совокупность включает ♂ - 1/3 и ♀ -2/3



Слишком маленькая выборка 🔴⚪⚪

Нерепрезентативная выборка 🔴⚪⚪

Репрезентативная выборка ⚪⚪🟢

How to proceed work with gathered data? The raw dataset usually looks like a messy set of values which does not allow to make sense of what is observed. It would be much easier to work with data if we organized it somehow. For example we can provide a quick impression of it from its visual appearance using **graphical representation** or we can summarize data using quantitative characteristics called **descriptive statistics**.

The choice of a particular instrument depends, above all, on the type of data you work with. There are different methods of graphical representation available for both qualitative and quantitative data. Most of descriptive statistics can be calculated for quantitative data only.

Graphical representation and descriptive statistics are the two ways to meaningfully describe and analyze the observed distribution of a random variable whether it is based on **population** or on **sample** data.

In general also Statistics consists of two branches, Descriptive and Inferential Statistics. First just summarises and describes the set while second make further generalized conclusions.

## Section 1. Graphical Representation of Data

One said you can't do anything you could not picture yourself doing.
*-unknown*

Graphs are used to provide ~~quick~~ visual representation of data. Immediate impression is useful to notice any patterns in data such as shape, location of center or dispersion. It allows making fast conclusions from the first glance.

**Graphical methods** of **representing data** include:

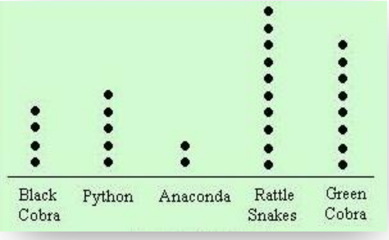
1. Dot plot
2. Bar chart
3. Steam-and-leaf plot
4. Histogram (labeling frequency or relative frequency)
5. Cumulative frequency plot
6. Boxplot (discussed in the descriptive statistics section of this chapter)

### Dot Plot

Dot plot represents the number of observations in each category by plotting the corresponding number of dots. It could be applied to all types of data, both qualitative and quantitative.

Dot plot for qualitative data

The dataset presented below shows number of snakes of different species in some zoo.

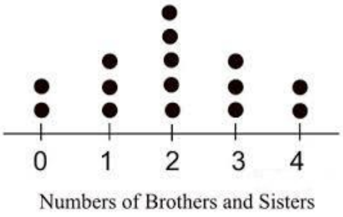


Types of snakes in the Zoo

From this graph for instance you can say that there are 2 anacondas in this Zoo.

Dot plot for quantitative data

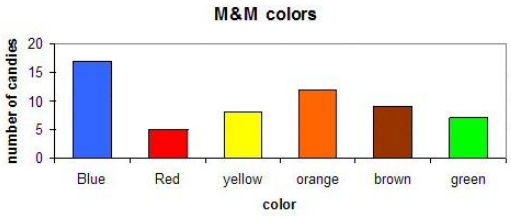
Dataset was received in the following way: 15 people were asked if they have siblings and if yes, how many. The dotplot below pictures the result.



### Bar Chart

Bar chart shows the number of observations in different categories through the **heights** of corresponding bars. Sizes of categories can be measured in absolute (number of observations) or relative terms (proportions of all observations).

The bar chart below shows the distribution of colors of M&M candies in some chosen pack.



## Stem and Leaf Plot

**EXAMPLE 'SPRINTERS'**

Masha is preparing for an athletic competition in HSE. In order to evaluate her chances Masha gathered data about results of 20 university athletes (in seconds).
She has got the following dataset: 10.17, 10.23, 10.25, 10.28, 10.31, 10.32, 10.34, 10.35, 10.41, 10.44, 10.45, 10.46, 10.49, 10.52, 10.55, 10.64, 10.68, 10.69, 10.71, 11.

Below is the stem and leaf plot representing this data. The graph took its name from the way it looks – like a tree – leaves are growing on the ~~on the base of~~ stem ~~"grow".~~

**Time for 100-meters Sprint**

| *stem* | *leaves* |
|---|---|
| 10.0 | |
| 10.1 | 7 |
| 10.2 | 3 5 8 |
| 10.3 | 1 2 4 5 |
| 10.4 | 1 4 5 6 9 |
| 10.5 | 2 5 |
| 10.6 | 4 8 9 |
| 10.7 | 1 |
| 10.8 | |
| 10.9 | |
| 11.0 | 0 |

key: 10.7|1 = 10.71 seconds

stem unit: 1.00
leaf unit: 0.01
The graph contains the so-called stem-values (on the left) and leaf values (on the right). They are separated by a vertical line. Each leaf represents one observation which can be read as the sum of stem and leaf value. For example here stem is measured in unites and leaves are measured in hundredths. For example notation 10.2|3 means 10.23 = 10.2+0.03.

Note that even if some stems have no leaves, you could not skip them and have to plot on the graph, like 10.8 here.

Also notice that the number of leaves equals the number of observations. Therefore if you have repeated numbers like 42, 42, 45 you cannot skip any of them, In this example you have to write it as 4| 2  2  5.

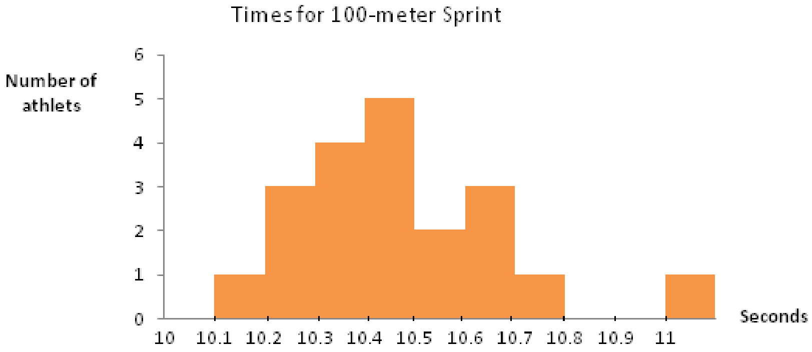
The main advantage of stem and leaf plot over many other types of graph is that it shows the dataset without loss of information, since you can recover the value of each particular observation. This is not possible with the histogram for example.

## Histogram

Histogram is a graph showing how observed values are distributed. Below you can see the histogram for the "sprinters" dataset. The height of each bar represents the number of observations (frequency) of values in the corresponding interval.

Therefore, to draw the histogram you should first divide data into **groups, or intervals**. Here we used interval of width 0.1. For example, Interval **[**10.1, 10.2) has one element - 10.17. This is reflected in its height, it equals 1.

This way we counted the number of observations in each interval: Interval **[**10.2, 10.3) has 3 elements - 10.23, 10.25 and 10.28, and so on for all the other intervals. **[**10.3, 10.4) – 4 elements, **[**10.4, 10.5) – 5 elements, **[**10.5, 10.6) – 2, **[**10.6, 10.7) – 1, **[**10.7, 10.8) – 1, **[**11, 11.01) – 1.



Note that those groups are usually not given in advance. So how to do it? We look at the whole range of values (from min to max) in our dataset and decide in how many **intervals** of equal length we will divide it.

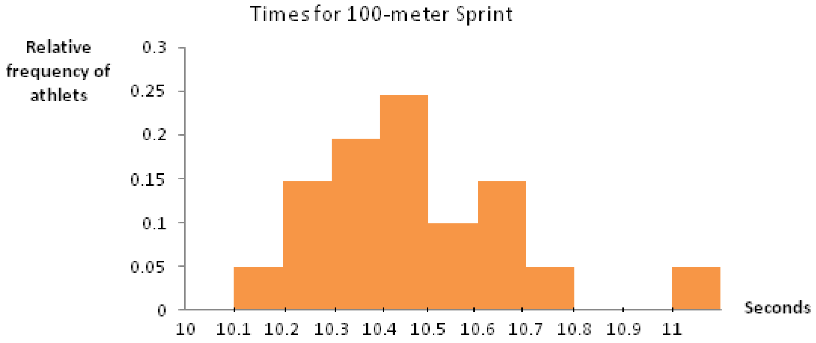
The most important that intervals always has to be of the **EQUAL LENGTH.**

How many intervals there would be is up to you, there is no strict rule for that. Keep in mind that too few number of intervals and thus columns makes the graph less informative, and too many – difficult to perceive. Usually optimal is 5 to 10. Our observations start from 10.17 to 11 seconds. Let us record on graph range 10.1 to 11.1 for accuracy and divide it in 10 intervals of length 0.10 seconds.

Then intervals would be (10.1,10.2), (10.2,10.3) ,..., (10.9,11), (11,11.1). Where to include observations that fall exactly on boarders is again up to you. Choose a unique rule - for instance to include them into the "right" interval. Thus we would get intervals **[**10.1, 10.2), **[**10.2, 10.3), ..., **[**10.9,11), **[**11,11.1).

Note that frequency can be measured in absolute terms (as presented above), as well as in relative terms. Recall from chapter 1 that relative frequency is the proportion of number of times some event occurred in n observations. Here it means proportion of athletes with results belonging to some interval to the total number of athletes.

For instance, there are 5 observations falling into the interval [10.4, 10.5), so the frequency on this interval is 5. **Frequency** answers the question **how many observations** fall into an interval. We can also find the relative frequency of interval [10.4, 10.5) as 5/20=0.25. **Relative frequency** shows the **proportion of values observations** into an interval. The histogram in relative frequencies is shown below.



Note that the shape of the histogram has not changed. This is always the case since to transfer to relative frequencies we simply divided heights of the columns (frequencies) by the number of observations n.

Histogram allows to see patterns of data distribution, such as symmetry.

Note that:

1) histogram can only be constructed for quantitative data,
2) Its bars stay close to each other unless there is a gap there (find details below).

If you have **qualitative** dataset analogous graph must be called a bar chart, discussed earlier. You can't use the term histogram in this case. It differs, except for the type of data used, by always having spaces between the bars. These spaces emphasize the idea that categories represent essentially different realizations of some quality which cannot be unambiguously put on axis of continuous values. Think of a bar chart for the students' university majors. Values on X-axis are department names (Economy, Medicine, Media, etc.), Y-Axis shows frequency. It is necessary to keep distance between categories to show there is no "value" between Media and Medical majors in university. Contrary, histogram columns are back to back. Space between them should be interpreted as a gap. Look at the histogram above: there is no spaces between columns except for one place, because there are no observations between 10.8 and 11.

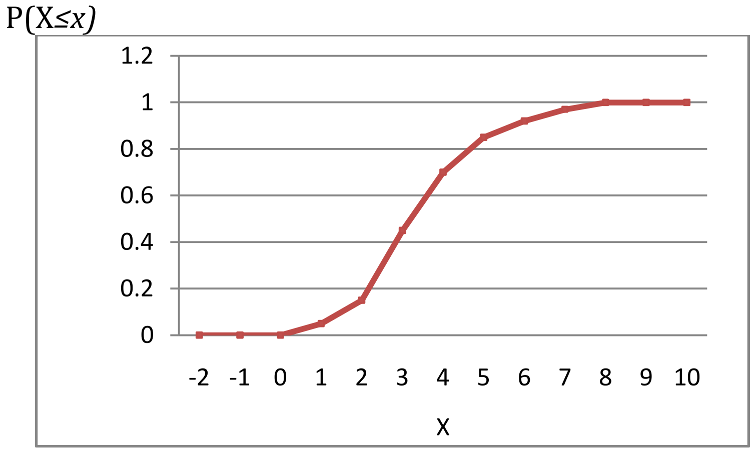
## 5. Cumulative frequency plot (ogive)

Please, recall that you are already familiar with the cumulative distribution and its graph from chapters 2 and 4. It shows the percentage of observations which do not exceed some value. For example below is the table derived in Chapter 2 for the “driving license” example.

| *x* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| *P(X=x)* | 0.05 | 0.1 | 0.3 | 0.25 | 0.15 | 0.07 | 0.05 | 0.03 |
| ***P(X≤x)*** | **0.05** | **0.15** | **0.45** | **0.7** | **0.85** | **0.92** | **0.97** | **1** |

The lowest raw in the table contains cumulative probabilities for the corresponding values of X. As you can see, cumulative probability of value x is the sum of its probability and probabilities of all values below x, e.g. $P(X\leq 3)=0.45=0.3+0.1+0.05$.

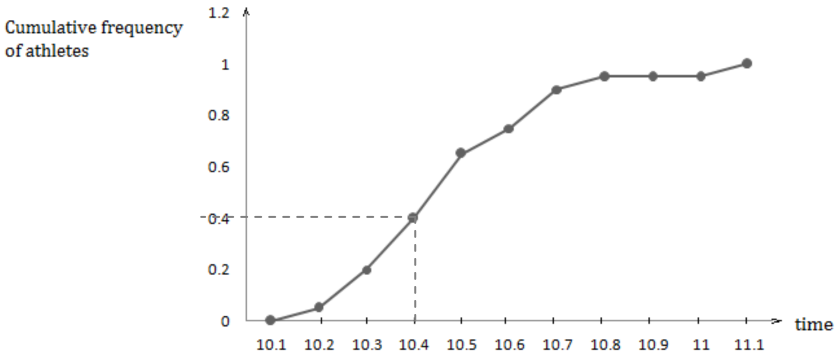
Plotting these values, you get the cumulative probability plot.



As you can see it is a non-decreasing function, which values increase from 0 to 1 as x increases from the value below minimum up to maximal value. At each possible value cumulative probability function raises by probability of this value. E.g. P(X≤3) is higher than P(X≤2) by exactly P(X=3).

Here it is constructed based on the known probability distribution. You can construct the same graph for the “sprinters” dataset. The only difference is that you don’t really have true probabilities for the times shown by sprinters, you only have observed values. Therefore you should construct the graph based on the **relative frequencies**. The resulting graph is called cumulative frequency plot.

**Cumulative frequency plot** or ogive shows the **accumulated** frequency up to a reference value. In other words it shows what percent of observations are no higher than each possible value. Cumulative frequency plot for the “sprinters dataset” is presented below.



How did we get it? Each value of cumulative frequency (y-coordinates) is the sum of frequencies of all values up to the corresponding value of x.
Look at the point (10.4,0.4). It means that 40% of sprinters run as fast as in 10.4 sec (their resulting times are no higher than 10.4).
How the value of 0.4 arised? Go back to the histogram. Check that to the LEFT of 10.4 there are three columns showing relative frequencies 0.05, 0.15 and 0.2 correspondingly. The total of 0,4 is the cumulative frequency for 10.4 sec.

Cumulative frequency plot can be used to find relative frequency of values above some value or relative frequency of values in some interval.
What percent of sprinters has shown time above 10.3 seconds? The point (10.3, 0.2) suggests that 0.2 of all athletes had resulting time less than 10.3 seconds. Thus, the other 1-0.2=80% of all athletes ran slower than 10.3 (shown higher time).
What percent of athletes had time between 10.3 and 10.4 seconds? As you might guess it is the difference of frequency below 10.4 sec and frequency below 10.3 sec, or 0.4 - 0.2=0.2. You can check that this answer is correct looking at the column between these values on the histogram.
Note that cumulative frequency plot is always a non-decreasing graph, which is quite obvious characteristic.

There is one more type of graph used in this course, called a box plot. It will be introduced in the second section of this chapter.

**Summary**

We've discussed several graphical representation approaches. As you have seen the same data could be represented on different graphs.

This topic seems quite simple, but don't get too relaxed. First, you should accurately choose the graph type according to the type of data you have. Second, graphs are drawn to reveal important and interesting features of data. Any graph simplifies the dataset emphasizing some patterns of it, and ignoring others. So, if a graph is not fitted accurately, it can be misleading. Careless choice and interpretation of a graph may result in wrong understanding of data.

When solving problems always make sure that your graph is clear and accurate leaving no doubt on what information it provides. Do not forget to label your graph. AP Statistics scoring guide harshly penalize for the lack of titles!

# Section 2. Descriptive Statistics

*All right everyone, line up alphabetically according to your height.*
*- Casey Stengel*

Just looking at a data set what can you say about its characteristics. For instance, check below the data set on number of home assignments submitted by students.

**EXAMPLE "NUMBER OF HOME ASSIGNMENTS"**

Masha decided to analyze how well her classmates cope with homework. For this she gathered data on the total number of home assignments on Statistics submitted by 25 students of her group during the 1st semester:
23, 45, 23, 44, 34, 56, 54, 12, 11, 44, 44, 31, 4, 30, 20, 49, 38, 48, 38, 40, 36, 41, 33, 47, 32.

Just looking at this data can you say how well students perform in general? Like what is the average number of assignments done, whether results are concentrated at some number or spread, or whether there is a gap between low and high numbers submitted. The dataset itself doesn't provide an obvious insight. For being able to make conclusions the observations need first to be put in order and then processed. So, *to get knowledge from data* you need some instruments. Descriptive statistics are those instruments.

**Descriptive statistic** is a formula to produce a single number (e.g. an average score) out of a set of numbers you have (e.g., a sample of students' scores). Using descriptive statistics you can analyze datasets easier and compare them with each other. For example, you can compare AP scores in Stats of 1st year ICEF students with those of LSE applicants from the university of Singapore.

We can divide descriptive statistics into three main categories. The first category describes the location of observed numbers along data set. They answer questions about the **center** of the dataset (mean, median and mode), its extreme values (minimum and maximum) and intermediate values (quartiles and percentiles).

The second category of descriptive statistics addresses the question of **variability** or spread among observations. It answers the questions of typical variability (standard deviation, variance), extreme variability (Range) and spread of middle observations (Interquartile range).

The third category of descriptive tools is used to describe the **shape** of data distribution (such as symmetry).

## Location of Observations along Dataset

### Center

Intuitively the first step in description of any dataset is finding its central or typical value.

## Mean

Mean is the arithmetic average of all observed values. It is the sum of all observations divided by their total number.

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Masha wants to find the mean number of submitted home assignments in her group: $\overline{X} = (23 + 45 + \cdots + 32)/25$. Check whether she has calculated the mean correctly: $\overline{X}$=35,08.

✓ *When mean is calculated on sample data it is called a* ***sample mean****. Contrary, population mean $\mu_{x=}$ E(X) introduced in Chapter 2 is based on population data. It requires the knowledge of the true probability distribution of X. You will learn more on this distinction in Chapter 9.*

## Median

Median is the number that **separates** lower half of the sample from the upper half. If all observations are ordered and divided into 2 equal parts the median would be the **border** between them, so that 50% of observations lie above and the other 50% lie below the median.

How to find the median?

First, you need to put all observations in ascending order, from lowest to highest and assign them numbers from 1 to n. As a result, the 1st observation is the minimum and the nth observation is the maximum.

If the number of observations is **odd**, the median is very easily found: it occurs in the middle of the list, just like the orange candy on the picture:

-

Formally if the number of observations is odd **the median** is the **observation** with the **number** $\frac{\mathbf{n+1}}{\mathbf{2}}$ in the list of sorted observations.

Thus, if you have 5 observations, median is equal to the 3rd observation, since $\frac{\mathbf{5+1}}{\mathbf{2}} = \mathbf{3}$.

However, if the number of observations is **even**, you have 2 observations in the middle. On the picture below those are the orange and the red candies. Median equals to the average of those two central observations.

Formally for even number of observations **median** is the **average of observations** with

**numbers** $\mathbf{\frac{n}{2}}$ and $\mathbf{\frac{n}{2}+1}$ in the list of sorted observations.

Then, if you have 6 observations, median equals to the arithmetic mean of the 3rd and the 4th observation.

Masha wants to find the median for her dataset. First, she puts the observations in ascending order:

4, 11, 12, 20, 23, 23, 30, 31, 32, 33, 34, 36, 38, 38, 40, 41, 44, 44, 44, 45, 47, 48, 49, 54, 56

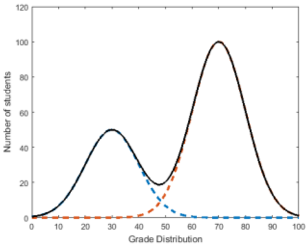
The number of observations n=25 is odd. Thus, median equals to the observation with number $\frac{25+1}{2}$ = 13 in the list. This way Masha finds that median=$x_{13}$ = 38.

## Mode

Mode is the observation with maximum frequency. It is the most *popular* observation. If everyone would wear red boots, they would be in fashion. It means 'red boots' are popular and this is why are the mode because of their maximum frequency.

In the above example mode equals 44 – the only number occurred three times.

Basically mode is not *exactly* the characteristic of center. There may be more than one mode in the distribution! If numbers of occurrences of several different elements are equal or almost equal, all of them can be viewed as modes.



It may happen that most frequent are high observations in the distribution. Then mode will not be a good characteristics of a center.

*The mode can be found both for quantitative and qualitative data. E.g., in the zoo example (distribution of snake species) rattle is the mode.*

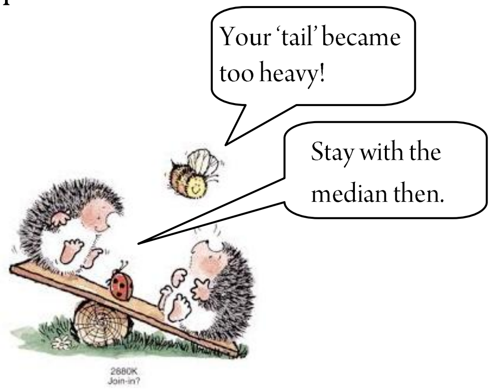
## Mean or Median: What is better?

Consider a dataset on monthly income of 29 employees working for some start-up project in Moscow (measured in thousands of roubles): 13, 15, 21, 21, 22, 25, 25, 25, 27, 28, 28, 30, 30 , 33, 34, 35, 35, 35, 39, 40, 40, 40, 41, 45, 45, 50, 55, 63, 1235. The latest observation is the average monthly income of the owner of the enterprize, he earns more than a million roubles per month.

Let's calculate mean and median income: $\overline{X}$=75, median=34. They differ substantially. Which one is a better estimate for a typical income in this dataset?

In this case median is better. 34 000 roubles seems to be a nice measure of center of this distribution. Mean is 75 000 roubles, while no one has income close to 75 000 roubles in this dataset and 75 000 cannot be viewed as a typical income.
This problem of mean as a measure of center arises in datasets which contain small number of highly extreme observations (very large or very small). Mean gives equal weight to all of the observations. Therefore even one millionaire in the dataset can substantially skew the value of mean income upwards. In the same way, if your sample contains a few dwarfs, then, the mean height will be skewed downwards. So, mean is **value-sensitive**, while median goes directly to the center of the list of ordered observations and returns the middle value.

This kind of difference is typical only for skewed distributions with the so-called "heavy tails". In this case median is a better measure of central value. For symmetric data distributions mean and median produce approximately equal numbers.



However even for skewed distributions mean is not a non-sense. It does not provide the value of typical observation in this case anymore, but represents other important information. For example comparing 2 start-up projects you may want to compare their income per worker to learn which one is more effective in terms of profit-to-labour balance. Then, you would prefer mean over median. Analogically economists use GDP per capita to compare welfare of countries. If you compare absolute values of GDP then large countries inevitably will be viewed as most wealthy. This does not take into account that large countries also have more citizens. Therefore total GDP is divided by population size so that the amount of wealth hypothetically available to a citizen in different countries can be meaningfully compared. Thus, median is better when you search for the central value of X, while mean is better as a measure of overall average value.

## Extreme values

**Minimum** and **Maximum** are extreme values of a dataset – the lowest and the highest observations.

## Quantiles

**Quantile** is a value that separates dataset. It is a value which part of the set *does not exceed* with fixed probability. Formal definition is the following:
$P(X \leq x_a) = p$
where $x_a$ is the quantile and p is the fixed probability. This fixed probability is set to name the quantile and is also called the level of a quantile.

If distribution is continuous $x_a$ is unique and is defined by:
$F(x_a)$ =p. This should remind you the cdf function because it is exactly it.
If the distribution of observation is empirical that is just a sample obtain from practice $x_a$ there are ways to calculate it which are analysed below.

Types of quantiles

Depending on how many parts quantile separates the set there exist different types of quantiles.
quartiles
percentiles
deciles

## Quartiles

Quartiles represent additional reference points for the position of dataset on the numerical axis. There are three quartiles: lower, upper and middle.

**Lower quartile LQ** is the value which separates the lower 25% of observations from the higher 75% of observations.

It is also called the 1st quartile $Q_1$ because it separates first (lower) quarter of data from the rest of observations.

In other words, if you put all observations in ascending order and divide them into 4 parts of equal size (quarters), lower quartile would be the value between the highest observation in the first quarter and the lowest observation in the second quarter. You can also view lower quartile as a median of the lower half of your observations.

**Upper quartile UQ** separates the lower 75% of observations from the higher 25% of observations.

It is also called the 3rd quartile $Q_3$ because it separates the lower three quarters of the data from the rest of observations.

It is the threshold between the third and fourth quarters of sorted observations. It can be viewed as the median of the upper half of observations.

Finally, second quartile is the value that separates the 2nd and the 3rd quarters or, equivalently, the lower and the upper halves of all observations. That coincides with the definition of median! Therefore, median is also sometimes called a 2nd quartile $Q_{2.}$ Median is the middle (second) quartile!

Q1
Med(Q2)
Q3

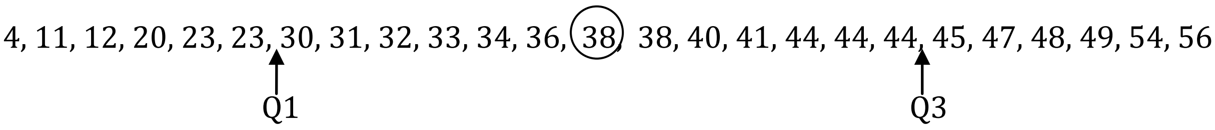
## How to calculate quartiles?

Masha wants to find lower and upper quartiles for her dataset on home assignments.

Method 1.

By this method quartiles are just the medians of first and second halves.

1. She puts the numbers in *ascending* order and finds the *median*. We've already shown that Median=$x_{13}$ = 38.
2. Median *divides the set into 2 parts*: to the right and to the left of it (the median itself should be excluded from both parts). Thus, the lower half is: 4, 11, 12, 20, 23, 23, 30, 31, 32, 33, 34, 36. The upper half is: 38, 40, 41, 44, 44, 44, 45, 47, 48, 49, 54, 56.
3. Masha finds *LQ* as the *median of the lower half* and *UQ* as the *median of the upper half* of observations. Both halves contain 12 observations, which is an even number. Therefore, median is between observations with number $\frac{12}{2}$ and $\frac{12}{2} + 1$. Thus, LQ and UQ lie between 6th and 7th observations in the lower and the upper datasets correspondingly. $LQ = \frac{x_6+x_7}{2} = \frac{23+30}{2} = 26.5$ and the upper quartile is $UQ = \frac{44+45}{2} = 44.5$.



4, 11, 12, 20, 23, 23, 30, 31, 32, 33, 34, 36, 38, 38, 40, 41, 44, 44, 44, 45, 47, 48, 49, 54, 56

*12+1+12 numbers in set in total*

Method 2.

There is another method of finding LQ and UQ which is based on the more general notion of percentile.

**Percentile** is the number which divides the dataset into 2 parts, so that the given proportion *p* of observations is below the percentile and the other *(1-p)* proportion is above the percentile. For example, for lower quartile, median and upper quartile p is equal to 0.25, 0.5 and 0.75 correspondingly, and thus LQ=25th percentile, median=50th percentile, and LQ=75th percentile. 30-percentile is the number which is above 30% of lower observations and below 70% of upper observations.

There is a general method for calculating the p-percentile.

1. Put observations in ascending order: 4, 11, 12, 20, 23, 23, 30, 31, 32, 33, 34, 36, 38, 38, 40, 41, 44, 44, 44, 45, 47, 48, 49, 54, 56.
2. Calculate the number $p \cdot (n+1)$, where n is the number of observations in the dataset. For example, to find LQ for the home assignment dataset: n=25, p=0.25 and the resulting number is 6,5 : p$\cdot (n+1) = 0.25 \cdot (25+1) = 6.5$.
3. Divide the number into integer part k and fractional part a. In our example 6.5=6+0.5. Thus, k=6, a=0.5. *k+a* indicates the location of the percentile in the sorted list of observations, e.g. 6.5 means that LQ is between 6nd and 7rd observations.
4. Find the percentile as $x_p = x_k + a \cdot (x_{k+1} - x_k)$. Thus, LQ= $x_{0.25} = x_6 + 0.5 \cdot (x_7 - x_6)$=23 + $0.5 \cdot (30 - 23) = 26.5$. Since LQ is between 6nd and 7rd observations, it equals $x_6$ plus some proportion $a$ of the distance between $x_7$ and $x_6$.

$$\boxed{\begin{array}{c} p \cdot (n+1) = k + a, \\ x_p = x_k + a \cdot (x_{k+1} - x_k) \end{array}}$$

In the same way for median: $p \cdot (n+1) = 0.5 \cdot 26 = 13$, k=13, a=0.
median=$x_{0.5} = x_{13} + 0 \cdot (x_{14} - x_{13}) = x_{13} = 38$.
For the UQ: $p \cdot (n+1) = 0.75 \cdot 26 = 19.5$, k=19, a=0.5.
UQ=$x_{0.75} = x_{19} + 0.5 \cdot (x_{20} - x_{19}) = 44 + 0.5 \cdot (45 - 44) = 44.5$.

✓ *Note, that different methods may sometimes produce slightly different values of quartiles. That happens when quartile is between two observations $x_{k+1}$ and $x_k$ (or a≠0). That's Ok. Quartile is the number which lies between the two quarters of observations. Strictly speaking, any number between $x_{k+1}$ and $x_k$ satisfies this definition. However, it is conventional to give a single number for a quartile, therefore you should apply any one method (remember to clearly explain it) to produce the answer.*

~~✓ *Note that there is one more similar statistical term quantile or q-quantile. It is the same as percentile, but gives the percentage of observations q in the decimal form: q∈[0,1]. Thus, q-quantile is 100q-percentile. Thus, 0.25-quantile is just the same as 25-percentile.*~~

## Variability of Observations

Once the center of X is defined, another important question is how large is the difference between different values of X. This is referred to as variability or spread among the numbers in the dataset. Do you remember the example about the choice among three assets with different risks (page 5 of Chapter 2)? It was shown that although mean result is important, another factor to take into account is variability of observations.

### Typical variability: Standard Deviation

**Variance** answers the question of how far do observations typically fall from its mean. It is the average squared deviation:

$$\boxed{\text{Var(X)} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

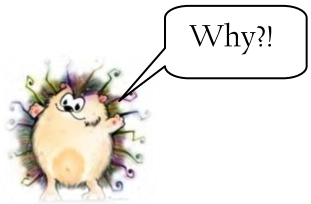
What does that mean?

Well, let's try to derive this formula together with Masha. Imagine, that you are trying to invent a formula to evaluate variability between observations. You might prefer to take a center as the reference point and measure the distance from each observed value $x_i$ to the center. Thus, you get a set of deviations of each of observation $i$ from the mean: $(x_i - \bar{X})$. Masha says that variability in X may well be measured by average deviation. What if we simply calculate the mean of deviations – sum of them divided by n? For a big observation deviation is positive (it lies above the mean $\bar{X}$), and for a small observation it is negative. Then, the simple average of all deviations would reduce to zero: $\frac{\sum(X_i - \bar{X})}{n} = 0$. So, it does not work this way.

In order to avoid this problem and get the value that would reflect the overall scale of deviations, we suggest to take each deviation squared: $\sum(X_i - \overline{X})^2$. Thus, each summand is now positive and the sum of deviations will not reduce to zero. Now, to get an average deviation the sum $\sum(X_i - \overline{X})^2$ should be divided by the number of observations. Well, that's true for the case when you have the data on the whole population. Then, you should use the formula for population variance: $\sigma^2 = Var(X) = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$. However, the formula provided in the frame above is for sample variance, and we divide by (n-1) instead of n.

Now it is important with what you are working, with the whole available population or only with part of it, a sample. Depending on it you will need to choose different formulas for variance and standard deviation.

| If you have the whole population: | If you have only sample: |
|---|---|
| $\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$ | $s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$ |
| $\sigma = \sqrt{\sigma^2}$ | $s = \sqrt{s^2}$ |



*No, hedgehog. We won't answer your question here.*

The explanation will be provided in Chapter 9. It happens that when true mean is unknown the second formula provides more accurate calculation.

***Additional intuition: why squared deviations?*** *The framed formula also gives different "weights" to deviations of different size, introducing a "penalty" to large deviations. What is meant by a penalty? Note, that deviation of ½ contributes ¼ to the variance value, while a deviation of 10 contributes the value of 100. Thus, the sum is very sensitive to large deviations, while allowing small ones to make only a tiny influence. This has a very important practical implication. In practice we don't care about small mistakes, which do not introduce significant risk for the accuracy of overall economic evaluation. However, big deviations may completely change the result and economic decisions based on it.*

Variance provides the result in squared units. E.g. for the "home assignments" dataset variance approximately equals 182 homeworks squared. The squared answer is very inconvenient for interpretation. **Standard deviation** solves this problem by taking the square root of Var:

$$\boxed{s_x = \sqrt{Var(X)} = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}}$$

You can also use the following formula for calculating the sample standard deviation:

$s^2 = \frac{\sum x^2 - n\bar{X}^2}{n-1}$

*Proof*

$\sum_{i=1}^{n}(X_i - \bar{X})^2 = \sum_{i=1}^{n}(X_i^2 - 2X_i\bar{X} + \bar{X}^2) = \sum_{i=1}^{n} X_i{}^2 - 2\bar{X}\sum_{i=1}^{n} X_i + n\bar{X}^2 = \sum_{i=1}^{n} X_i{}^2 - 2\bar{X} \cdot n\bar{X} + n\bar{X}^2 = \sum_{i=1}^{n} X_i{}^2 - n\,\bar{X}^2$. *Thus,* $\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1} = \frac{\sum x^2 - n\bar{X}^2}{n-1}$

Standard deviation is measured in the same units as the variable X itself. It can be easily interpreted. For example, standard deviation of the number of home assignments is approximately 13.49. we also know that $\bar{X} = 35.08$. It means that, while students submitted 35 home assignments on average (exact value 35.08), typical result was observed to vary from 35 by 13,5 assignments approximately.

- ✓ *Note that both s and Var(X) are non-negative.*
- ✓ *Note that* ***s*** *is called a* ***sample standard deviation*** *and* $s^2$ *is a sample variance. In Chapter 2 you've learned about population variance or variance of a random variable. It is the expected value of the squared deviation of X from its population mean* $m Var(X) = \sigma^2 = E[(X - \mu)^2]$*: . We've also shown that for the dataset containing the whole population you should use the formula* $Var(X) = \sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$*. Check that they are equal:* $\frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N} = E[(X - \mu)^2]$.Note that sample variance $s^2$ and population variance $\sigma^2$ are not the same. *You'll learn more on this distinction in Chapters 8 and 9.*

### Extreme variability: Range

What is the extreme (the largest possible) difference between the numbers in the dataset? It is the difference between the maximum and the minimum values, called Range:

$$\boxed{\text{Range} = X_{max} - X_{min}}$$

In our example Range is 56-4= 52

### Variability of middle observations: IQR

Interquartile range (IQR) is the difference between Upper and Lower quartiles. It answers the question: what is the **range of the middle 50%** of observations?

$$\boxed{\text{IQR} = \text{UQ} - \text{LQ}}$$

Let's calculate IQR for the homeworks dataset: IQR=44,5-26,5=18.

### Frequencies

So far we were working mostly with raw of observations. What if each observation is presented much more than one time. Recall that this is called frequency. Then more convenient will be to work with slightly different version of formulas provided above.

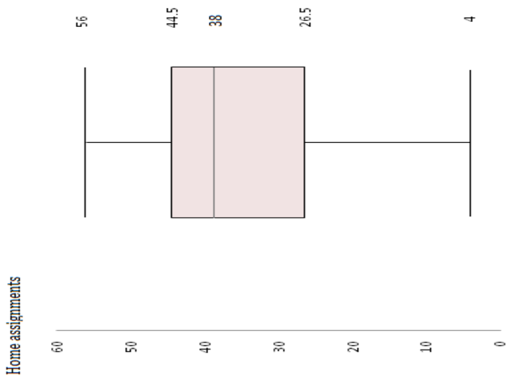
$$\bar{X} = \frac{\sum_{i}^{m} x_i f_i}{n}$$

## Boxplot

### A graph summarizing descriptive statistics:

There is one more graph, which represents important descriptive statistics visually. Below is the descriptive statistics on our dataset and the graph summarizing it.

Min=4
LQ=26,5
Med=38
UQ=44,5
Max=56
UF = 71,5.
LF = -0.5



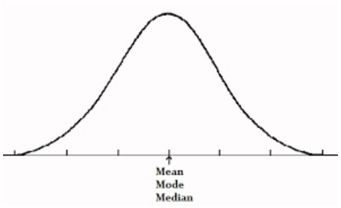
## Shape

As we've seen in the 1st section of this chapter, visual appearance of the distribution (histogram, dotplot, stemplot, …) is very informative. Therefore, when describing a dataset or comparing several datasets it is conventional to verbally describe the shape of distribution.

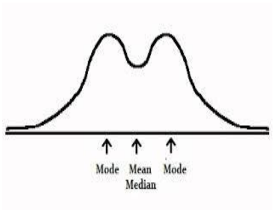
### Symmetric versus Skewed Shape

Distribution can be described as symmetric when the right half of a histogram looks as an approximate specular image of a left half:

Symmetric unimodal

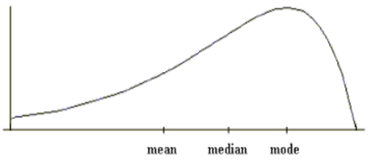


Symmetric bimodal

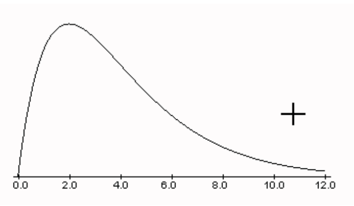


Otherwise, the distribution is described as non-symmetric.
Sometimes non-symmetric distributions are described as being skewed to the left or to the right. If the right tail of a histogram looks much longer and thinner, the distribution is skewed **to the right.** Contrary, when the left tails is longer and thinner, the distribution is skewed **to the left.**

Skewed to the left
Negatively skewed



Skewed to the right
Positively skewed



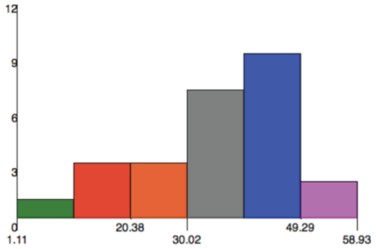
*Help. Term skewed could be thought of as 'stretched'. That means from which side distribution is more stretched, to this side is the skewedness. Another way to remember is distribution is skewed to the side from which distribution was hit to take this shape* ☺

Sometimes the direction of skewedness is not visually obvious. Then you can help yourself comparing mean and median. Median is always located at a point which divides the graph into two parts with equal areas. It coincides with what we visually perceive as a 'middle' of the dataset. Contrary, mean is value sensitive and it is always skewed to the extremely small or extremely large observations. Thus, for right skewed distribution $\overline{\mathrm{X}} >$ median and vice versa.

*If $\overline{X} >$ median significantly, the distribution is skewed to the right*
*If $\overline{X} <$ median significantly, the distribution is skewed to the left*

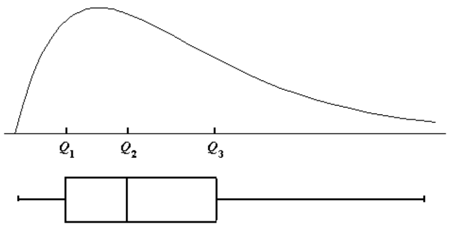
Thus, mean is located closer to the 'tail' of a skewed distribution and lies in the direction of the skewness.

Let's draw a histogram, for home assignment example:



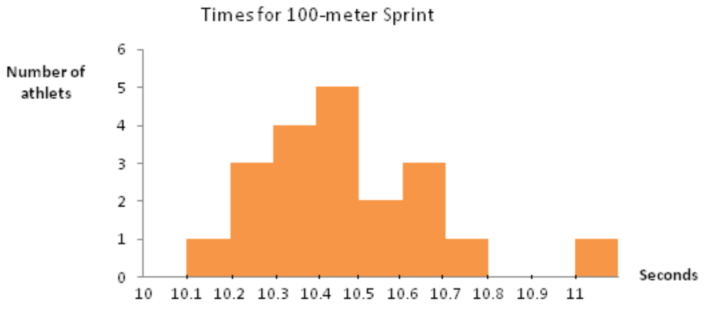
It can be seen that the distribution is skewed to the left. In such a case it must be true that $\overline{\mathrm{X}} < Med$. This is exactly the case since As we've calculated $\overline{\mathrm{X}} = 35.08$ and median=38.

### How to "read" shape from a boxplot



## Gaps

If the distribution contains an interval with no observations inside, it is said to have a gap in that interval. As you can see the "time of sprinters" example, there is a gap on the [10,8;11) interval.



Note that gaps can only be seen on histograms, stem and leaf plots and dotplots. Boxplot does not reveal this feature of data.

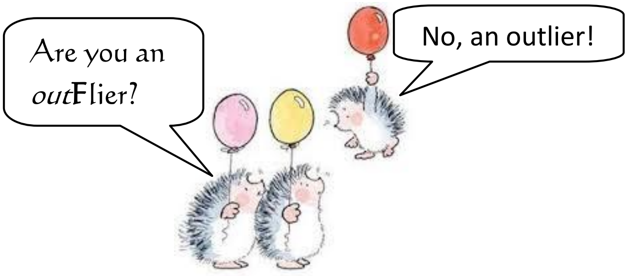
## Outliers

Outlier is an observation lying far from other observations in a data set. For example, in the dataset on IQ levels outliers would be produced by extremely smart people or by complete idiots. The former have IQ levels far above the others' results, the letter – far below. What are the *cut* points to define who is stupid enough (or smart enough) to be an outlier? Here are the 'fences' for that:

Lower fence: LF = LQ - 1,5·IQR
Upper fence: UF = UQ + 1,5·IQR

An outlier is an observation with value above the upper fence: $X_i$> UQ+1,5·IQR
or below the lower fence: $X_i$< LQ-1,5·IQR

On graphs it is usually clearly seen which observations are potential candidates for outliers if such exists. They stand away from other observations. Then however you should formally check them if they satisfy common criteria.



Are there outliers in the home assignments dataset?
First, let's calculate the fences:
Lower fence: LF=LQ-1,5·IQR=26,5-1,5·18 = -0,5
Upper fence: UF=UQ+1,5·IQR=44,5+1,5·18 =71,5.
Since there are no observations outside the interval [-0,5;71,5] the dataset contains no outliers.

You can also check that there is an outlier in the dataset on incomes of participants of a startup. Salary of the business owner is an outlier.

**Is it necessary to check the presence of outliers in any dataset before solving the problem?**
Not necessarily. You should first ask yourself the following questions:
1) Is it clearly stated in the conditions of a problem that you need to check the presence of outliers?
2) Do you suspect that there is an outlier in the dataset? For instance, in HOME ASSIGNMENTS example you are not expected to check that and it's Ok if you don't check. However, if you asked to describe the distribution of incomes in the STARTUP EXAMPLE it is highly required that you notice a single outstanding observation and check that it is an outlier.

The presence of outliers may strongly affect descriptive statistics, so if there are outliers, we need to be careful.
What to do if you have an outlier?

1. An outlier may indicate an error in records (e.g. someone misrecorded 75 as 750). In this case just exclude outlier from the dataset.
2. An outlier may indicate specific features of data. In this case what to do with it depends on how descriptive statistics are going to be used. First, you can make calculations separately of outlier and then mention that there was such an observation. Alternatively, you can include it in your calculations and after that provide careful interpretation of the resulting numbers.

For example in the startup case both answers are acceptable:

1) excluding a millionaire from the dataset we get $\bar{X} \approx 33.571$. We can use it keeping in mind that an extremely high value of income was also observed.

2) The mean of 74 is highly skewed towards the value of outlier: $X_{29}$=1235.

## Relative location of elements

### Z-scores

So far we've talked about methods of description of a dataset. Sometimes we face an opposite problem: to characterize an observation in the context of its environment or dataset. Z-scores and percentiles of observations serve this goal.

Z-score measures position of a point in the distribution. It is **the number of standard deviations** by which the observation stands away from the mean.
Precisely z-score states by how many standard deviations a particular value is situated far from the mean (is higher than the mean). For example $z_i$=1 states that the value $x_i$ is 1 standard deviation greater than mean (to the right of it) and z-score z=-1 says that the value is 1 standard deviation below mean (to the left of it).

$$\boxed{\text{z-score: } z_i = \frac{x_i - \mu}{\sigma}}$$

Note that when population mean $\mu$ and population standard deviation $\sigma$ are not known, you can use sample statistics in the formula instead: $z_i = \frac{x_i - \bar{X}}{s}$ .
The procedure of converting the set of Xi's to the set of Zi's is called **standartisation**. This is why in Chapter 5 Z is called a standard normal variable.

*Intuition:*
$x_i - \bar{X}$ is the distance from mean. Dividing by s converts it to the number of standard deviations.
Positive $z_i$ indicates that observation $x_i$ exceeds the mean, while negative $z_i$ implies that $x_i$ is smaller than the mean.

You can also do the reverse procedure. Given $z_i$ you can find the initial observation $x_i$:

$$\boxed{x_i = \mu + z_i\sigma}$$

### Compare two observations using Z-score.

Standartisation (convertation to z-scores) allows to compare observations from different populations.

EXAMPLE "FRIENDS FROM SUMMER SCHOOL"

Masha has got two friends from the LSE Summer School, Greta from Germany and Junko from Japan. It happened that after the seminars they started discussing which girl is taller. Greta (171 cm) turned out to be slightly taller than Junko (170cm), but the latter was arguing that she was the tallest in her class in Tokyo!

What is the right way to compare the girls' height?

**Solution**:
It would not be correctly to compare heights of girls in absolute terms, because in general Japanese people tend to be relatively shorter compared to people of other nations.
Please, pay attention: These are two different variables!!!
It is not like comparing two values of the same variable – these are two values of different variables: one is measuring the height of a person from Japanse population, while the other – from a German population. Therefore, it is important not only to compare the girls' height values, but also to take their relative position in the distributions from which they are taken from. One reasonable way to do this is using *z*-scores.

The women's height in Germany and Japan have the parameters:E(G)=168, Var(G)=$15^2$ and E(J)=163, Var(J)=$10^2$.
Then z-scores of the girls are equal to:

- German girl Greta: $z_G = \frac{171 - 168}{15} = 0.2$
- Japanese girl Junko: $z_J = \frac{170 - 163}{10} = 0.7$

z-scores are positive, indicating that both girls are higher than an average girl in each of the countries. But the z-scores also indicate the volume of that difference. Since $z_J$ is by 0.5 of standard deviation (calculated as 0.7-0.2) higher than $z_G$, we can say that Junko is **relatively** higher than Greta, taking into account her **initial** national **conditions.**

## Inverse Percentile

An alternative way to characterize the position of an observation in the population is to find the p of its percentile. It is the **proportion** of observations in the population which **lie below** the observation of interest.

Suppose that height is normally distributed. Then, : G~N(168, $15^2$) and J~N(163, $10^2$).
Let's compare percentiles of the girls' height:

$P(G \leq 171)=P(z<0,2) \approx 0,579$ 58-th percentile

$P(J \leq 170)=P(z<0,7) \approx 0,758$ 76-th percentile

Thus, Greta is taller than 58% of girls in her country, while Junko is taller than 76% of girls in her country. Again, we come to the conclusion, that Junko is relatively taller.

You **MUST BE ABLE TO REPRODUCE** even being drunk

## Comparison of distributions

full score strategy

How to compare several datasets (the very typical problem in our course).

Compare:
1. Center
2. Spread
3. Shape

+ gaps/clusters/outliers and other peculiar features

Make conclusions in terms of the problem (which asset is better, which group of students is more successful etc.

**Summary**

dot plots can be used for any type of data
stem and leaf plots are only for quantitative data
histograms are for quantitative data
bar charts are for qualitative data
size of intervals in histogram should be equal

To find all descriptive statistics:
Stat-Calc-1Var

**To find E(X), $\sigma$, E(X$^2$):**

1. Put values of X into List 1, the corresponding probabilities into List 2
2. CALC->SET.

| | | |
|---|---|---|
| 1 Var | XList | List1 |
| 1 Var | Freq | List2 |

->EXIT

3. 1VAR *Now you have results:*

| | |
|---|---|
| E(X) | $\bar{X} = \sum x$ |
| $\sigma$ | $\sigma_x$ |
| E(X$^2$) | $\sum x^2$ |

To find P(X=k) and P(X≤k) for a binomial random variable X~B(n,p):
DIST->BINM

➔ Bpd (for P(X=k) ) *means binomial probability dist.*

➔ Bcd (for P(X≤k) ) *means binomial cumulative dist.*

| | |
|---|---|
| Data | *:choose "Variable"* |
| x | :*insert the number* k |
| Numtrial | :*insert number of trials* n |
| P | :*insert probability of success* p |

➔ EXE

**Top secret information**

**A**wesome

**G**uys

**S**taff

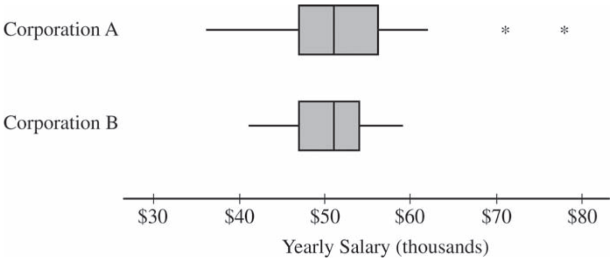
F.B.I.
WE ARE WATCHING YOU
Your activism to close the SOA endangers the status quo and is therefore subject to
COUNTERTERRORISM SURVEILLANCE

Since $\sigma_X$ and $\sigma_Y$ are constants they can be put inside the expectation operator: $\rho_{XY} = \frac{E[(X-E(X))(Y-E(Y)]}{\sigma_X \cdot \sigma_Y} = E\left(\frac{(X-E(X))(Y-E(Y)}{\sigma_X \cdot \sigma_Y}\right) = E\left(\frac{X-E(X)}{\sigma_X} \cdot \frac{Y-E(Y)}{\sigma_Y}\right) = E(X_{st} \cdot Y_{st}) = E((X_{st} - 0) \cdot (Y_{st} - 0)) = Cov(X_{st}, Y_{st})$. Then, you can also view correlation as the covariance between “standardized” versions of X and Y. Term “standardized” mean, that they have 0 expectation and variance equal to 1.

## Sample AP Problems with solutions

**Problem 1.** AP 2015 N1

Two large corporations, A and B, hire many new college graduates as accountants at entry-level positions. In 2009 the starting salary for an entry-level accountant position was $36,000 a year at both corporations. At each corporation, data were collected from 30 employees who were hired in 2009 as entry-level accountants and were still employed at the corporation five years later. The yearly salaries of the 60 employees in 2014 are summarized in the boxplots below.



(a) Write a few sentences comparing the distributions of the yearly salaries at the two corporations.
(b) Suppose both corporations offered you a job for $36,000 a year as an entry-level accountant.

(i) Based on the boxplots, give one reason why you might choose to accept the job at corporation A.
(ii) Based on the boxplots, give one reason why you might choose to accept the job at corporation B.

**Solution:**
(a) The median salary is approximately the same for both corporations.
The range of the salaries in Corporation A is greater than in Corporation B. Interquartile range is also slightly higher for Corporation A.
Distributions of salaries for both Corporations are quite symmetric.
Corporation A has two outliers, namely two highest salaries. Corporation B has no outliers.

(b)
(i) 5 years after, at least several highest salaries are higher at Corporation A than at Corporation B. If I choose the offer from Corporation A, I might be able to make a higher salary in future. As could be seen from the graph there is probably higher possibility for career growth at Corporation A.

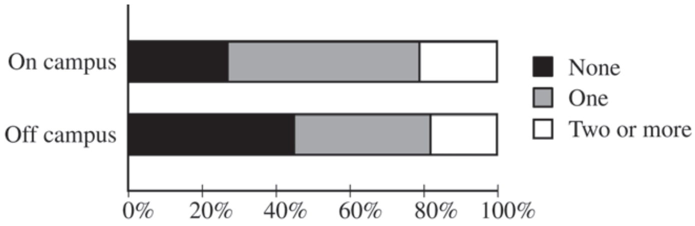
(ii) 5 years after, the minimum salary is higher at Corporation B rather than at Corporation A. It looks like at Corporation A some people are still making the starting salary $36,000 and never received a raise. Thus the promotion at Corporation B is probably more secured. At Corporation A in contrast there exists a possibility never to receive a raise in the salary.

**Problem 2.** AP 2014 N1 b

An administrator at a large university is interested in determining whether the residential status of a student is associated with level of participation in extracurricular activities. Residential status is categorized as on campus for students living in university housing and off campus otherwise. A simple random sample of 100 students in the university was taken, and each student was asked the following two questions.

- Are you an on campus student or an off campus student?
- In how many extracurricular activities do you participate?

The responses of the 100 students are summarized in the segmented bar graph shown.



Write a few sentences summarizing what the graph reveals about the association between residential status and level of participation in extracurricular activities among the 100 students in the sample.

**Solution**:
The graph reveals that on campus residents in this sample are more likely in general to participate in extra-curricular activities than off campus residents.

On campus residents have a greater proportion who participate in one activity (on campus: 0.515, off campus: 0.373) and a smaller proportion who participate in no extracurricular activities (on campus: 0.273, off campus: 0.448) than off campus residents. The proportions who participate in two or more extra-curricular activities are similar between the two groups but slightly greater for on campus residents (on campus: 0.212, off campus: 0.179).

**Problem 3.** AP 2014 N4 a
As part of its twenty-fifth reunion celebration, the class of 1988 (students who graduated in 1988) at a state university held a reception on campus. In an informal survey, the director of alumni development asked 50 of the attendees about their incomes. The director computed the mean income of the 50 attendees to be $189,952. In a news release, the director announced, "The members of our class of 1988 enjoyed resounding success. Last year's mean income of its members was $189,952!"
(a) What would be a statistical advantage of using the median of the reported incomes, rather than the mean, as the estimate of the typical income?

**Solution:**
The median is less affected by skewness and outliers than the mean. With a variable such as income, a small number of very large incomes could dramatically increase the mean but not the median. Therefore, the median would provide a better estimate of a typical income value.

**Problem 4.** AP 2013 N1 a
An environmental group conducted a study to determine whether crows in a certain region were ingesting food containing unhealthy levels of lead. A biologist classified lead levels greater than 6.0 parts per million (ppm) as unhealthy. The lead levels of a random sample of 23 crows in the region were measured and recorded. The data are shown in the stem-plot below.

**Lead Levels**

| | |
|---|---|
| 2 | 8 |
| 3 | 0 |
| 3 | 5 8 8 |
| 4 | 1 1 2 |
| 4 | 6 8 8 |
| 5 | 0 1 2 2 3 4 |
| 5 | 9 9 |
| 6 | 3 4 |
| 6 | 6 8 |

Key: 2|8 = 2.8 ppm

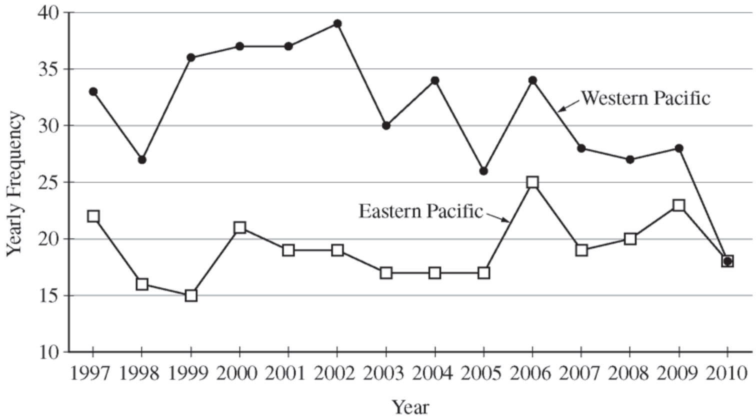
(a) What proportion of crows in the sample had lead levels that are classified by the biologist as unhealthy?

**Solution**:
Four of the 23 crows in the sample had a lead level greater than 6.0 ppm. Therefore, the proportion of crows in the sample that were classified as unhealthy is 4/23 ≈ 0.174

**Problem 5.** AP 2013 N6

Tropical storms in the Pacific Ocean with sustained winds that exceed 74 miles per hour are called typhoons. Graph A below displays the number of recorded typhoons in two regions of the Pacific Ocean—the Eastern Pacific and the Western Pacific—for the years from 1997 to 2010.

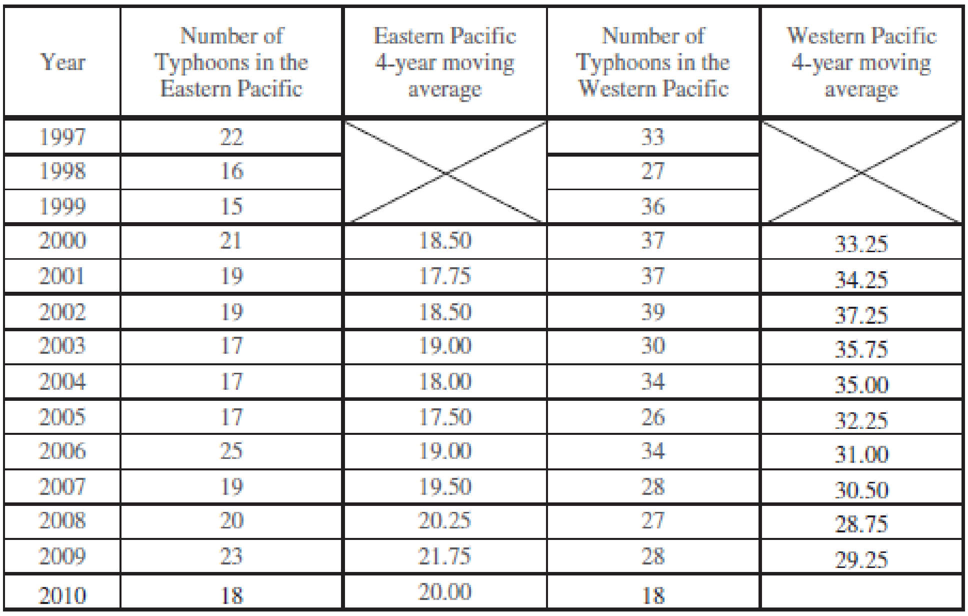
GRAPH A



(a) Compare the distributions of yearly frequencies of typhoons for the two regions of the Pacific Ocean for the years from 1997 to 2010.

(b) For each region, describe how the yearly frequencies changed over the time period from 1997 to 2010.

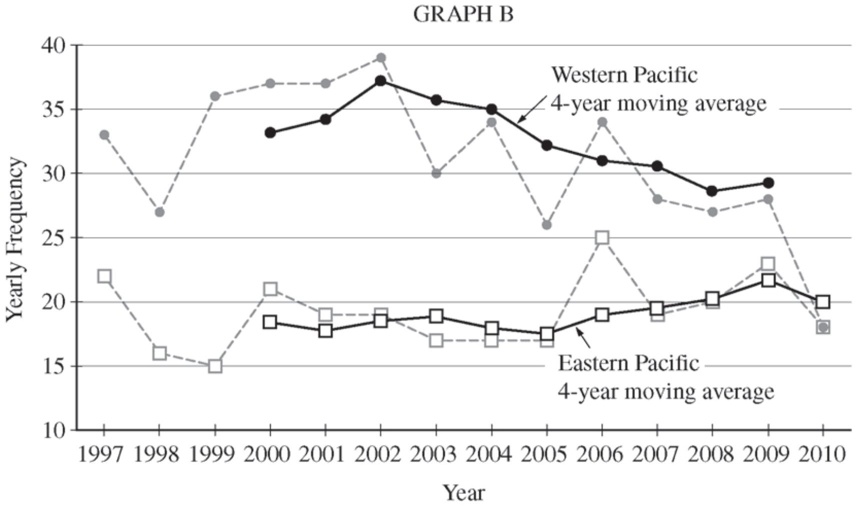
A moving average for data collected at regular time increments is the average of data values for two or more consecutive increments. The 4-year moving averages for the typhoon data are provided in the table below. For example, the Eastern Pacific 4-year moving average for 2000 is the average of 22, 16, 15, and 21, which is equal to 18.50.



| Year | Number of Typhoons in the Eastern Pacific | Eastern Pacific 4-year moving average | Number of Typhoons in the Western Pacific | Western Pacific 4-year moving average |
|---|---|---|---|---|
| 1997 | 22 | | 33 | |
| 1998 | 16 | | 27 | |
| 1999 | 15 | | 36 | |
| 2000 | 21 | 18.50 | 37 | 33.25 |
| 2001 | 19 | 17.75 | 37 | 34.25 |
| 2002 | 19 | 18.50 | 39 | 37.25 |
| 2003 | 17 | 19.00 | 30 | 35.75 |
| 2004 | 17 | 18.00 | 34 | 35.00 |
| 2005 | 17 | 17.50 | 26 | 32.25 |
| 2006 | 25 | 19.00 | 34 | 31.00 |
| 2007 | 19 | 19.50 | 28 | 30.50 |
| 2008 | 20 | 20.25 | 27 | 28.75 |
| 2009 | 23 | 21.75 | 28 | 29.25 |
| 2010 | 18 | 20.00 | 18 | |

(c) Show how to calculate the 4-year moving average for the year 2010 in the Western Pacific. Write your value in the appropriate place in the table.

(d) Graph B below shows both yearly frequencies (connected by dashed lines) and the respective 4-year moving averages (connected by solid lines). Use your answer in part (c) to complete the graph.



(e) Consider graph B.

(i) What information is more apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?

(ii) What information is less apparent from the plots of the 4-year moving averages than from the plots of the yearly frequencies of typhoons?

**Solution:**

(a) The Western Pacific Ocean had more typhoons than the Eastern Pacific Ocean in all but one of these years. The average seems to have been about 31 typhoons per year in the Western Pacific Ocean, which is higher than the average of about 19 typhoons per year in the Eastern Pacific Ocean. The Western Pacific Ocean also saw more variability (in number of typhoons per year) than the Eastern Pacific Ocean; for example, the range of the frequencies for the Western Pacific is about 21 typhoons and only 10 typhoons for the Eastern Pacific.

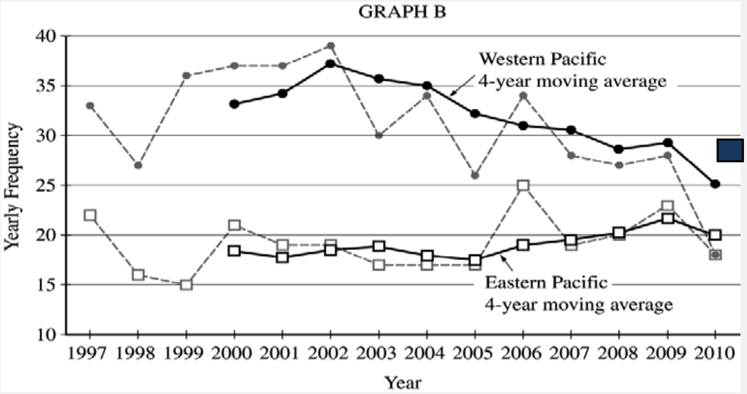
(b) The Western Pacific Ocean had a decreasing trend in number of typhoons per year over this time period, especially from about 2001 through 2010. In contrast, the Eastern Pacific Ocean was fairly consistent in the number of typhoons per year over this time period, with a slight increasing trend in the later years from 2005 through 2010.

(c) The four-year moving average for the year 2010 in the Western Pacific Ocean is:

$\frac{28+27+28+18}{4}$ = 25.25

The value is written in the lowest right empty box of the table.

(d)

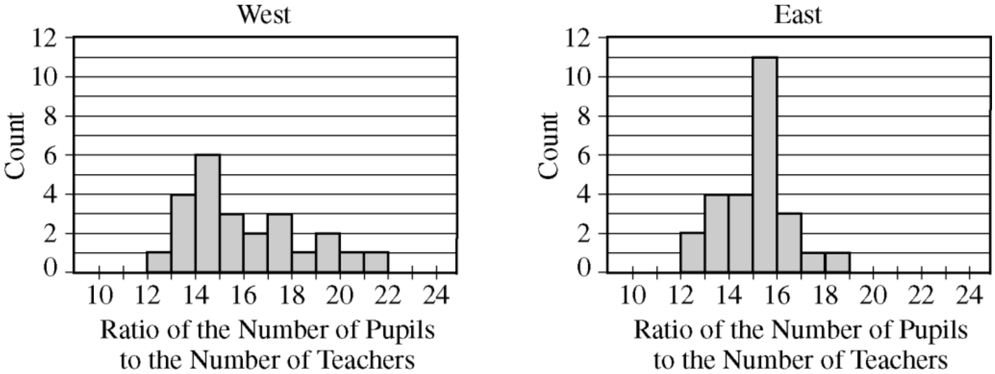


(e) (i) The overall trends across this time period were more apparent with the moving averages than with the original frequencies. The moving averages reduce variability, making more apparent the overall decreasing trend in number of typhoons in the Western Pacific Ocean and the slight increasing trend in the number of typhoons in the Eastern Pacific Ocean.

(ii) The year-to-year variability in number of typhoons is less apparent with the moving averages than with the original frequencies.

**Problem 6.** AP 2011B N1

Records are kept by each state in the United States on the number of pupils enrolled in public schools and the number of teachers employed by public schools for each school year. From these records, the ratio of the number of pupils to the number of teachers (P-T ratio) can be calculated for each state. The histograms below show the P-T ratio for every state during the 2001–2002 school year. The histogram on the left displays the ratios for the 24 states that are west of the Mississippi River, and the histogram on the right displays the ratios for the 26 states that are east of the Mississippi River.



(a) Describe how you would use the histograms to estimate the median P-T ratio for each group (west and east) of states. Then use this procedure to estimate the median of the west group and the median of the east group.

(b) Write a few sentences comparing the distributions of P-T ratios for states in the two groups (west and east) during the 2001–2002 school year.

(c) Using your answers in parts (a) and (b), explain how you think the mean P-T ratio during the 2001–2002 school year will compare for the two groups (west and east).

**Solution:**

(a) The median is the value with half of the P-T ratios at or below it and half of the values at or above it. For *n* observations in a group, use $\frac{n+1}{2}$ to find the position of the median in the ordered list of observations. For states west of the Mississippi (*n* = 24) the median falls between the 12th and 13th value in the ordered list, and both the 12th and 13th values fall in the interval 15–16. For states east of the Mississippi (*n* = 26) the median falls between the 13th and 14th value in the ordered list, and both of these values also fall in the interval 15–16.

From the histogram, cumulative frequencies for the two groups are shown in the table below.

| Interval | West | East |
|---|---|---|
| 12-13 | 1 | 2 |
| 13-14 | 1+4=5 | 2+4=6 |
| 14-15 | 1+4+6 = 11 | 2+4+4=10 |
| 15-16 | 1+4+6+3=14 | 2+4+4+11=21 |

Thus, the median P-T ratio for both groups is at least 15 students per teacher and at most 16 students per teacher.

(b)The shapes of the two histograms are different. The histogram for states that are west of the Mississippi River is unimodal and skewed to the right, whereas the histogram for states that are east of the Mississippi River is unimodal and nearly symmetric. As noted in part (a), the medians of the two distributions are about the same, between 15 and 16 for both distributions. The histograms also show that there is more variability in the P-T ratios for states that are west of the Mississippi River. Although the greatest and least values for each group are not known, the range can be approximated. The range for the west is at most 22 - 12 = 10, and the range for the east is at most 19 -12 = 7.

(c) The medians of the two distributions are about the same, as determined in part (a). The distribution of P-T ratios for states that are west of the Mississippi River is skewed to the right, indicating that the mean will probably be higher than the median. The rough symmetry for the east group indicates that the mean will be close to the median. Thus, the mean for the west group will probably be greater than the mean for the east group.

**Problem 7.** AP 2011 №1 bc

A professional sports team evaluates potential players for a certain position based on two main characteristics, speed and strength.

(a) *Topic: Normal Distribution*

(b) Strength is measured by the amount of weight lifted, with more weight indicating more desirable (greater) strength. From previous strength data for all players in this position, the amount of weight lifted has a mean of 310 pounds and a standard deviation of 25 pounds, as shown in the table below.

| | Mean | Standard Deviation |
|---|---|---|
| Amount of weight lifted | 310 pounds | 25 pounds |

Calculate and interpret the $z$-score for a player in this position who can lift a weight of 370 pounds.

(c) The characteristics of speed and strength are considered to be of equal importance to the team in selecting a player for the position. Based on the information about the means and standard deviations of the speed and strength data for all players and the measurements listed in the table below for Players A and B, which player should the team select if the team can only select one of the two players? Justify your answer.

| | Player A | Player B |
|---|---|---|
| Time to run 40 yards | 4.42 seconds | 4.57 seconds |
| Amount of weight lifted | 370 pounds | 375 pounds |

**Solution:**

(b) The *z*-score for a player who can lift a weight of 370 pounds is z-score = $\frac{370-310}{25}$ = 2.4. The *z*-score indicates that the amount of weight the player can lift is 2.4 standard deviations above the mean for all previous players in this position, so this player is quite a good strong.

(c) Because the two variables — time to run 40 yards and amount of weight lifted — are recorded on different scales, it is important not only to compare the players' values but also to take into account the standard deviations of the distributions of the variables. One reasonable way to do this is with *z*-scores.

The *z*-scores for the 40-yard running times are as follows:

Player A: z = $\frac{4.42-4.60}{0.15}$ = - 1.2

Player B: z = $\frac{4.57-4.60}{0.15}$ = - 0.2

The *z*-scores for the amount of weight lifted are as follows:

Player A: z = $\frac{370-310}{25}$ = 2.4

Player B: z = $\frac{375-310}{25}$ = 2.6

The z-scores indicate that both players are faster than average in the 40-yard running time and both are well above average in the amount of weight lifted. Player A is better in running time, and Player B is better in weight lifting. But the z-scores also indicate that the difference in their weight lifting (a difference of 0.2 standard deviation) is quite small compared with the difference in their running times (a difference of 1.0 standard deviation). Therefore, Player A is the better choice, because Player A is much faster than Player B and only slightly less strong.

**Problem 8.** AP 2011 №2

The table below shows the political party registration by gender of all 500 registered voters in Franklin Township.

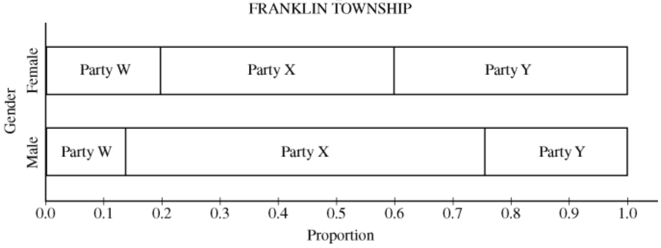
PARTY REGISTRATION–FRANKLIN TOWNSHIP

| | Party W | Party X | Party Y | Total |
|---|---|---|---|---|
| Female | 60 | 120 | 120 | 300 |
| Male | 28 | 124 | 48 | 200 |
| Total | 88 | 244 | 168 | 500 |

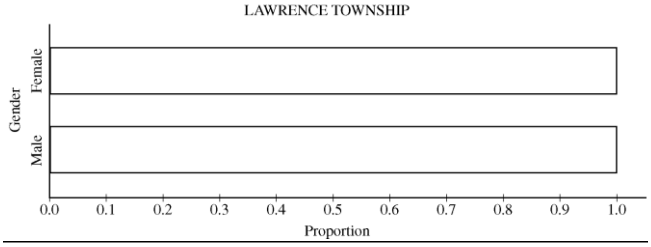
(a) *Topic: 2DRV*

(b) *Topic: 2DRV*

(c) One way to display the data in the table is to use a segmented bar graph. The following segmented bar graph, constructed from the data in the party registration–Franklin Township table, shows party-registration distributions for males and females in Franklin Township.



In Lawrence Township, the proportions of all registered voters for Parties W, X, and Y are the same as for Franklin Township, and party registration is independent of gender. Complete the graph below to show the distributions of party registration by gender in Lawrence Township.



**Solution:**

Let W,X,Y denote belonging to parties W,X and Y correspondingly.

Let M={Male}

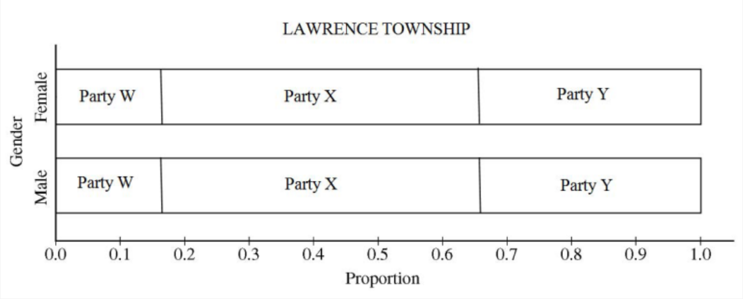
**(c)** The marginal proportions of voters registered for each of the three political parties (without regard to gender) are given below.

P(W)=$\frac{88}{500}$ = 0,176

P(X)= $\frac{244}{500}$ = 0,488

P(Y)= $\frac{168}{500}$ = 0,336

Because party registration is independent of gender in Lawrence Township, the proportions of males and females registered for each party must be identical to each other and also identical to the marginal proportion of voters registered for that party. Using the order Party W, Party X, and Party Y, the graph for Lawrence Township is displayed below.



**Problem 9.** AP 2010 №6

Hurricane damage amounts, in millions of dollars per acre, were estimated from insurance records for major hurricanes for the past three decades. A stratified random sample of five locations (based on categories of distance from the coast) was selected from each of three coastal regions in the southeastern United States. The three regions were Gulf Coast (Alabama, Louisiana, Mississippi), Florida, and Lower Atlantic (Georgia, South Carolina, North Carolina). Damage amounts in millions of dollars per acre, adjusted for inflation, are shown in the table below.

HURRICANE DAMAGE AMOUNTS IN MILLIONS OF DOLLARS PER ACRE

| | Distance from Coast | | | | |
|---|---|---|---|---|---|
| | < 1 mile | 1 to 2 miles | 2 to 5 miles | 5 to 10 miles | 10 to 20 miles |
| Gulf Coast | 24.7 | 21.0 | 12.0 | 7.3 | 1.7 |
| Florida | 35.1 | 31.7 | 20.7 | 6.4 | 3.0 |
| Lower Atlantic | 21.8 | 15.7 | 12.6 | 1.2 | 0.3 |

(a) Sketch a graphical display that compares the hurricane damage amounts per acre for the three different coastal regions (Gulf Coast, Florida, and Lower Atlantic) and that also shows how the damage amounts vary with distance from the coast.

(b) Describe differences and similarities in the hurricane damage amounts among the three regions.

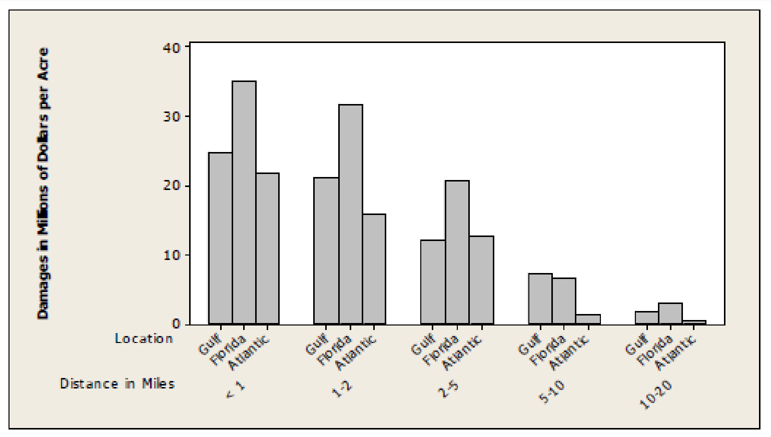
Because the distributions of hurricane damage amounts are often skewed, statisticians frequently use rank values to analyze such data.

(c) In the table below, the hurricane damage amounts have been replaced by the ranks 1, 2, or 3. For each of the distance categories, the highest damage amount is assigned a rank of 1 and the lowest damage amount is assigned a rank of 3. Determine the missing ranks for the 10-to-20-miles distance category and calculate the average rank for each of the three regions. Place the values in the table below.

ASSIGNED RANKS WITHIN DISTANCE CATEGORIES

| | Distance from Coast | | | | | |
|---|---|---|---|---|---|---|
| | < 1 mile | 1 to 2 miles | 2 to 5 miles | 5 to 10 miles | 10 to 20 miles | Average Rank |
| Gulf Coast | 2 | 2 | 3 | 1 | | |
| Florida | 1 | 1 | 1 | 2 | | |
| Lower Atlantic | 3 | 3 | 2 | 3 | | |

**Solution:**
(a)



(b) In all three regions (Gulf Coast, Florida, Lower Atlantic) the hurricane damage amounts tend to decrease as distance from the coast increases. For almost all given distances from the coast, the Florida region has the largest damage amounts. Also, for any given distance, the Gulf Coast and Lower Atlantic regions have similar damage amounts but with the Lower Atlantic damage amounts generally smaller.

(c) For the “10 to 20 miles” distance category: The Florida region has the most damage (3.0 million dollars per acre) and so has rank 1. The region with the second-most damage is the Gulf Coast (1.7 million dollars), obtaining rank 2. The Lower Atlantic region has the least damage (0.3 million dollars) and so has rank 3. The last columns of the table should be filled in as follows:

| | 10 to 20 miles | Average Rank |
|---|---|---|
| Gulf Coast | 2 | 2.0 |
| Florida | 1 | 1.2 |
| Lower Atlantic | 3 | 2.8 |

The average ranks are computed for: the five Gulf Coast damage ranks $\frac{2+2+3+1+2}{5}$ = 2.0, the five Florida damage ranks $\frac{1+1+1+2+1}{5}$ = 1.2 and the five Lower Atlantic damage ranks $\frac{3+3+2+3+3}{5}$ = 2.8.

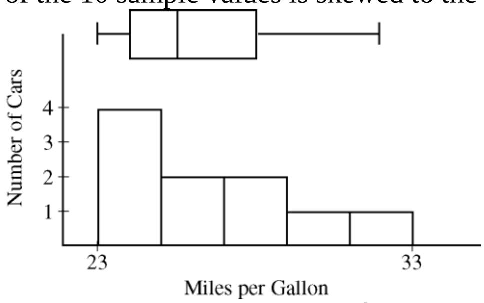
**Problem 10.** AP 2009 N2 a
A tire manufacturer designed a new tread pattern for its all-weather tires. Repeated tests were conducted on cars of approximately the same weight traveling at 60 miles per hour. The tests showed that the new tread pattern enables the cars to stop completely in an average distance of 125 feet with a standard deviation of 6.5 feet and that the stopping distances are approximately normally distributed.

(a) What is the 70th percentile of the distribution of stopping distances?

**Solution:**
Let $X$ denote the stopping distance of a car with new tread tires where $X$ is normally distributed with a mean of 125 feet and a standard deviation of 6.5 feet. The $z$-score corresponding to a cumulative probability of 70 percent is $z = 0.52$. Thus, the 70th percentile value can be computed as:
$x=\mu+z\sigma = 125 + 0.52(6.5) = 128.4$ feet

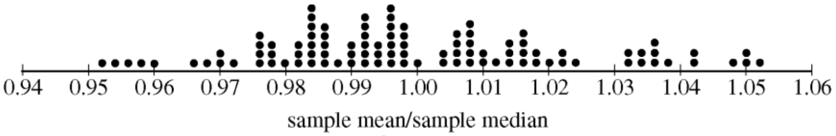
**Problem 11.** AP 2009 №6
A consumer organization was concerned that an automobile manufacturer was misleading customers by overstating the average fuel efficiency (measured in miles per gallon, or mpg) of a particular car model. The model was advertised to get 27 mpg. To investigate, researchers selected a random sample of 10 cars of that model. Each car was then randomly assigned a different driver. Each car was driven for 5,000 miles, and the total fuel consumption was used to compute mpg for that car.

One condition for conducting a one-sample *t*-test in this situation is that the mpg measurements for the population of cars of this model should be normally distributed. However, the boxplot and histogram shown below indicate that the distribution of the 10 sample values is skewed to the right.



(b) One possible statistic that measures skewness is the ratio $\frac{\text{sample mean}}{\text{sample median}}$. What values of that statistic (small, large, close to one) might indicate that the population distribution of mpg values is skewed to the right? Explain.

(c) Even though the mpg values in the sample were skewed to the right, it is still possible that the population distribution of mpg values is normally distributed and that the skewness was due to sampling variability. To investigate, 100 samples, each of size 10, were taken from a normal distribution with the same mean and standard deviation as the original sample. For each of those 100 samples, the statistic $\frac{\text{sample mean}}{\text{sample median}}$ was calculated. A dotplot of the 100 simulated statistics is shown below.



In the original sample, the value of the statistic $\frac{\text{sample mean}}{\text{sample median}}$ was 1.03. Based on the value of 1.03 and the dotplot above, is it plausible that the original sample of 10 cars came from a normal population, or do the simulated results suggest the original population is really skewed to the right? Explain.

(d) The table below shows summary statistics for mpg measurements for the original sample of 10 cars.

| Minimum | Q1 | Median | Q2 | Maximum |
|---|---|---|---|---|
| 23 | 24 | 25.5 | 28 | 32 |

Choosing only from the summary statistics in the table, define a formula for a different statistic that measures skewness.
What values of that statistic might indicate that the distribution is skewed to the right? Explain.

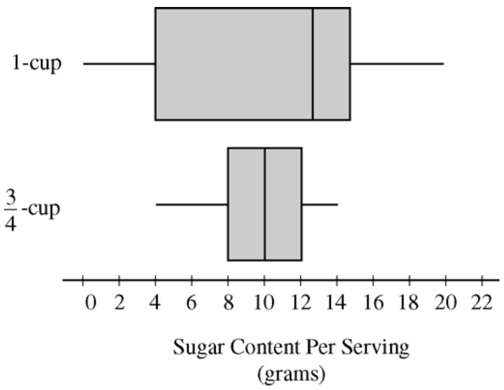
**Solution:**
(b) If the distribution is right-skewed, one would expect the mean to be greater than the median. Therefore the ratio $\frac{\text{sample mean}}{\text{sample median}}$ should be large (at last greater than 1).

(c) Because we are testing for right-skewness, the estimated *p*-value will be the proportion of the simulated statistics that are greater than or equal to the observed value of 1.03. The dotplot shows that 14 of the 100 values are more than 1.03. Because this simulated *p*-value (0.14) is larger than any reasonable significance level, we do not have convincing evidence that the original population is skewed to the right and conclude that it is plausible that the original sample came from a normal population.

(d) One possible statistic is $\frac{Maximum\ - Median}{Median\ - Minimum}$
If the distribution is right-skewed, one would expect the distance from the median to the maximum to be larger than the distance from the median to the minimum; thus the ratio should be greater than 1.

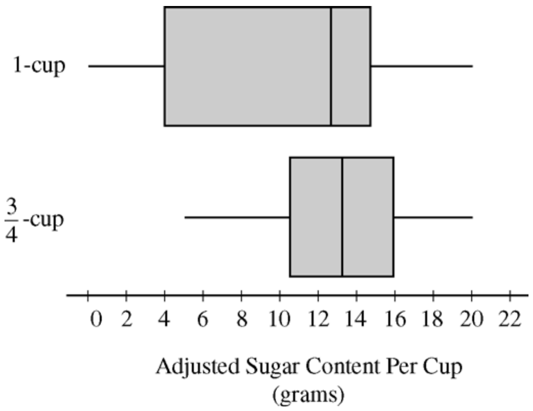
**Problem 12.** AP 2008 №1
To determine the amount of sugar in a typical serving of breakfast cereal, a student randomly selected 60 boxes of different types of cereal from the shelves of a large grocery store. The student noticed that the side panels of some of the cereal boxes showed sugar content based on one-cup servings, while others showed sugar content based on three-quarter-cup servings. Many of the cereal boxes with side panels that showed three-quarter-cup servings were ones that appealed to young children, and the student wondered whether there

might be some difference in the sugar content of the cereals that showed different-size servings on their side panels. To investigate the question, the data were separated into two groups. One group consisted of 29 cereals that showed one-cup serving sizes; the other group consisted of 31 cereals that showed three-quarter-cup serving sizes. The boxplots shown below display sugar content (in grams) per serving of the cereals for each of the two serving sizes.



(a) Write a few sentences to compare the distributions of sugar content per serving for the two serving sizes of cereals.

(b) After analyzing the boxplots on the preceding page, the student decided that instead of a comparison of sugar content per recommended serving, it might be more appropriate to compare sugar content for equal-size servings. To compare the amount of sugar in serving sizes of one cup each, the amount of sugar in each of the cereals showing three-quarter-cup servings on their side panels was multiplied by 4/3. The bottom boxplot shown below displays sugar content (in grams) per cup for those cereals that showed a serving size of three-quarter-cup on their side panels.



(b) What new information about sugar content do the boxplots above provide?
(c) Based on the boxplots shown above on this page, how would you expect the mean amounts of sugar per cup to compare for the different recommended serving sizes? Explain.

**Solution:**
(a) The cereals that list a serving size of one cup have a median sugar amount larger than the median for the cereals that list a serving size of three-quarters of a cup. There is more variability (larger range and larger IQR) for the one-cup cereals. The shapes of the two distributions differ. The distribution of sugar content for three-quarter-cup cereals is reasonably symmetric: notice that the median is in the middle of the box. The distribution of sugar content for one-cup cereals is clearly skewed to the left (skewed toward the lower values): notice that the median is pulled to the right side of the central box closer to the third quartile.

(b) The distribution of sugar content in the cereals that list one-cup serving sizes remains the same as in part (a) because no transformations were applied to this distribution. There is a noticeable shift toward higher sugar content for the cereals that list three-quarter-cup servings after the transformation was applied to this distribution. The two types of cereals (one-cup and three-quarter-cup) now have similar medians, and the two distributions now show similar maximum values. In addition, the variability in the sugar content for cereals with a three-quarter-cup serving size increased by a factor of 4/3after the transformation was applied to the data in this distribution.

(c) Judging from the boxplots in part (b), we would expect the mean amounts of sugar per serving to be different. By the symmetry of the boxplot for the three-quarter-cup cereals, we would expect the mean and median to be similar. Because the boxplot for the one-cup cereals is skewed to the left, we would expect the mean to be lower than the median. Thus, because both types of cereal have similar medians, we would expect the mean amount of sugar per cup for cereals with a one-cup serving size to be lower than the mean amount of sugar per cup for cereals with a three-quarter-cup serving size.

**Problem 13.** AP 2008B N6 d

The nerves that supply sensation to the front portion of a person's foot run between the long bones of the foot. Tight-fitting shoes can squeeze these nerves between the bones, causing pain when the nerves swell. This condition is called Morton's neuroma. Because most people have a dominant foot, muscular development is not the same in both feet. People who have Morton's neuroma may have the condition in only one foot or they may have it in both feet.

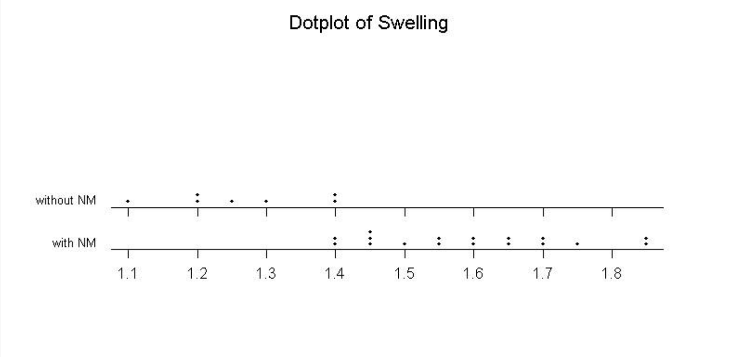
Investigators selected a random sample of 12 adult female patients with Morton's neuroma to study this disease further. The data below are measurements of nerve swelling as recorded by a physician. A value of 1.0 is considered "normal," and 2.0 is considered extreme swelling. The population distribution of the swelling measurements is approximately normal for adult females who have Morton's neuroma.

| Dominant Foot | Swelling in Dominant Foot | Swelling in Nondominant Foot | Foot with Neuroma |
|---|---|---|---|
| Left | 1.40 | 1.10 | Left |
| Left | 1.55 | 1.25 | Left |
| Left | 1.65 | 1.20 | Left |
| Left | 1.55 | 1.40 | Both |
| Left | 1.70 | 1.40 | Left |
| Left | 1.85 | 1.50 | Both |
| Right | 1.45 | 1.20 | Right |
| Right | 1.65 | 1.30 | Right |
| Right | 1.60 | 1.40 | Right |
| Right | 1.70 | 1.45 | Both |
| Right | 1.85 | 1.45 | Both |
| Right | 1.75 | 1.60 | Both |

(d) The nerve swelling measurement is used to indicate whether a foot has Morton's neuroma. Use the 24 measurements of nerve swelling to suggest a criterion for diagnosing Morton's neuroma. Justify your suggestion graphically.

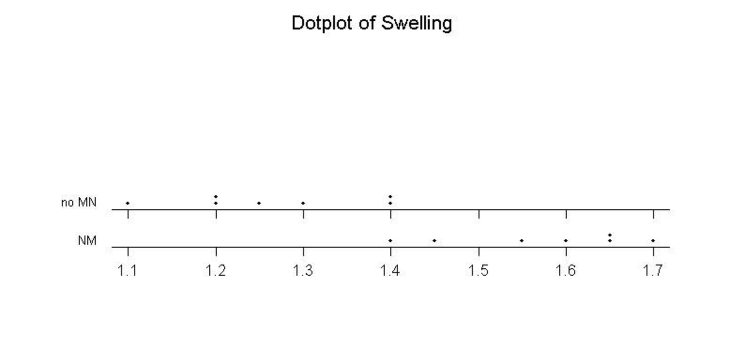
**Solution:**

(d) There are more than one possible variants of the answer
Variant 1.



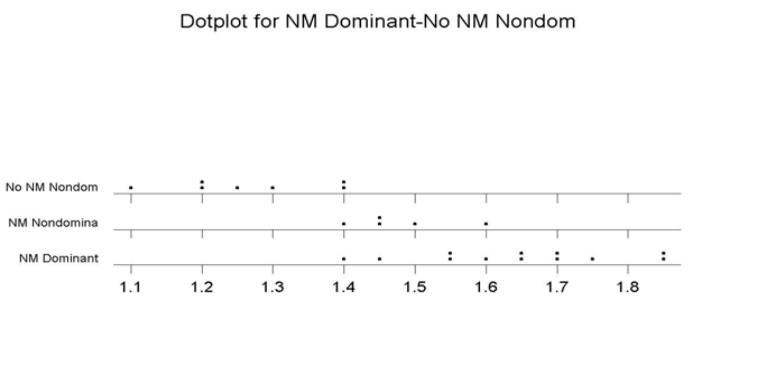
Separate the 24 swelling measurements into two groups—the 17 feet with MN and the 7 feet without MN. Construct a plot that displays the two groups, such as stacked dotplots or a back-to-back stemplot. The plot below suggests that a swelling measurement of about 1.4 or higher would be a reasonable criterion for Morton's neuroma.

Variant 2.



Plot swelling measurements for only the seven individuals who do not have MN in both feet, plotting the measurements for their MN feet and their non-MN feet. The plot below suggests that a swelling measurement of about 1.4 or higher would be a reasonable criterion for MN.

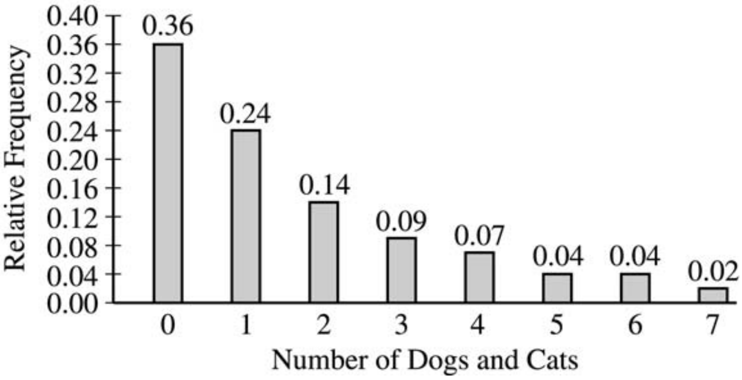
Variant 3.



Plot the 12 measurements of MN in the dominant foot, the 5 measurements of MN in the nondominant foot, and the 7 measurements of no MN in the nondominant foot. (There are no individuals in the sample who do not have MN in the dominant foot.) The plot below suggests that a swelling measurement of about 1.4 or higher would be a reasonable criterion for MN.

**Problem 14.** AP 2007 Form B №2

The graph below displays the relative frequency distribution for *X*, the total number of dogs and cats owned per household, for the households in a large suburban area. For instance, 14 percent of the households own 2 of these pets.

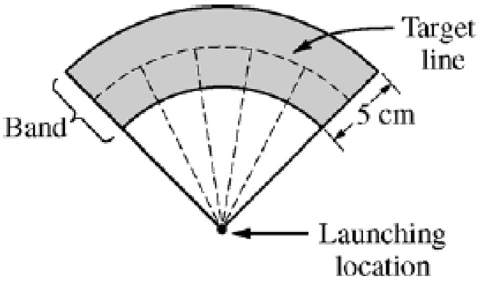


(a) According to a local law, each household in this area is prohibited from owning more than 3 of these pets. If a household in this area is selected at random, what is the probability that the selected household will be in violation of this law? Show your work.

**Solution**:

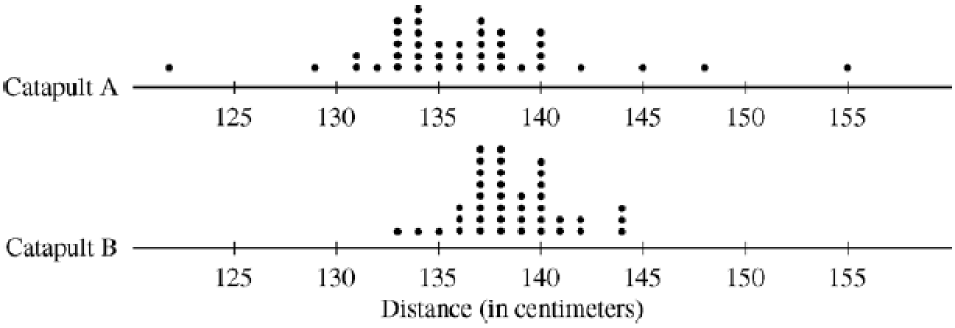
P(X>3) = 0.07 + 0.04 + 0.04 + 0.02 = 0.17

**Problem 15.** AP 2006 №1
Two parents have each built a toy catapult for use in game at an elementary school fair. To play the game the students will attempt to launch Ping-Pong balls from the catapults so that the balls land within a 5-centimeter band. A target line will be drawn through the middle of the band, as shown in the figure below. All points on the target lie are equidistant from the launching location.



If a ball lands within the shaded band, the student will win a prize.

The parents have constructed the two catapults according to slightly different plans. They want to test these catapults. Under identical conditions, the parents launch 40 Ping-Pong balls from each catapult and measure the distance that the ball travels before landing. Distances to the nearest centimeter are graphed in the dotplots below.



(a) Comment on any similarities and any differences in the two distributions of distances traveled by balls launched from catapult A and catapult B.

**Solution:**
(a) Both distributions of distances are rather symmetric with high concentration in center and lighter tails. However the median of the distribution for catapult A(≈136 cm) is lower than the median of the distribution for catapult B (≈138 cm). Also there is much more variability in the distances traveled by balls launched with catapult A than with catapult B. For catapult A there exist some distances to be called potential outliers while there are no such for catapult B.

**Problem 16.** AP 2005 №1
The goal of a nutritional study was to compare the caloric intake of adolescents (молодой человек, юноша, подросток) living in rural areas of the United States with the caloric intake of adolescents living in urban areas of the United States. A random sample of ninth-grade students from one high school in a rural area was selected. Another random sample of ninth graders from one high school in an urban area was also selected. Each student in each sample kept records of fall me food  consumed in one day.

The back-to-back box template below displays the number of calories of food consumed build kilogram of body weight for a student on the day that the day

(a) All right if you sentences comparing the distribution of the daily caloric intake Who is the distribution with

(b) Is it reasonable to generalize the findings of the study tool students in the United States

(c) Researchers who want to conduct the simvastatin similar study are debating which of the following two plants to use
Plan I. Have each student in the standing record all the food she consumed in one day . Then search is what computer the number of colors of food consume per kilogram of body fortune for Monday
Seven deep unit

Assuming that the students keep accurate records which plan would be better meet the goal of the study justify your answer

**Problem 17.** AP 2005 №6
Lead, found in some plants, is a neurotoxin that can be especially harmful to the developing brain and nervous system of children. Children frequently put their hands in their mouth after touching painted surfaces, and this is the most common type of exposure to lead.
A study was conducted to investigate whether there were differences in children's exposure to lead between suburban day-care centers and urban day-care centers in one large city. For this study, researchers used a random sample of 20 children in suburban day-care centers. Ten of these 20 children were randomly selected to play outside; the remaining 10 children played inside. All children had their hands wiped clean before beginning their assigned one-hour play period either outside or inside. After the play period ended, the amount if lead in micrograms (mcg) on each child's dominant hand was recorded.

The mean amount of lead on the dominant hand for the children playing inside was 3.75 mcg, and the mean amount of lead for the children playing inside was 5.65 mcg. A 95 percent confidence interval for the difference in the mean amount of lead after one hour inside versus one hour outside was calculated to be (-2.46, -1.34).

A random sample of 18 children in urban day-care centers in the same large city was selected. For this sample, the same process was used, including randomly assigning children to play inside or outside. The data for the amount (in mcg) of lead in each child's dominant hand are shown in the table below.

| Urban day-care centers | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Inside | 6 | 5 | 4 | 4 | 4.5 | 5 | 4.5 | 3 | 5 |
| Outside | 15 | 25 | 18 | 14 | 20 | 13 | 11 | 22 | 20 |

(a) *Topic:*
(b)
(c)

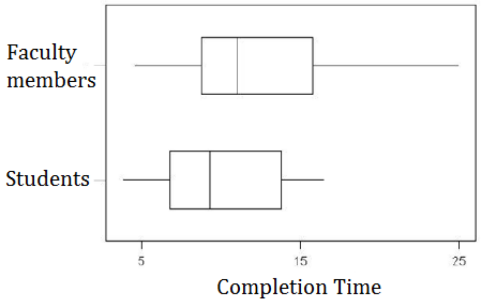
**Problem 18.** AP 2005 Form B №1

**Problem 19.** AP 2004 №1

**Problem 20.** AP 2002 Form B №5
At a school field day, 50 students and 50 faculty members each completed and obstacle course. Descriptive statistics for the completion times (in minutes) for the two groups are shown below.

| | Students | Faculty Members |
|---|---|---|
| Mean | 9.9 | 12.09 |
| Median | 9.25 | 11 |
| Minimum | 3.75 | 4.5 |
| Maximum | 16.5 | 25 |
| Lower quartile | 6.75 | 8.75 |
| Upper quartile | 13.75 | 15.75 |

(a) Use the same scale to draw boxplots for the completion times for students and for faculty members.
(b)
**Solution:**
(a)



(b)

**Problem 21.** AP 2001 №1
The summary statistics for the number of inches of rainfall in Los Angeles for 117 years, beginning in 1877, are shown below.

| n | mean | median | trmean | stdev | se mean |
|---|---|---|---|---|---|
| 117 | 14.941 | 13.070 | 14.416 | 6.747 | 0.624 |

| min | max | Q1 | Q3 |
|---|---|---|---|
| 4.850 | 38.180 | 9.680 | 19.250 |

(a) Describe a procedure that uses these summary statistics to determine whether there are outliers.
(b) Are there outliers in these data? ____
Justify your answer based on the procedure that you described in part (a).
(c) The news media reported that in a particular year, there were only 10 inches of rainfall. Use the information provided to comment on this reported statement.

**Solution:**

(a) An outlier considered to be any value that is 1.5·IQR below the lower quartile or 1.5·IQR above the upper quartile. Thus, using this summary statistics, if:
min < Q1 - 1.5·IQR => there is at least one outlier on the low side of these data
max > Q3 + 1.5·IQR => there is at least one outlier on the high side of these data

*Alternative answer:* An outlier is any observation that is more than 2(or 3) standard deviations away from the mean.

(b) IQR = Q3 – Q1 = 19.250 - 9.680 = 9.57,

1.5·IQR = 1.5·9.57 = 14.355

Bound for outlier on the low side: Q1 - 1.5·IQR = 9.680 - 14.355 = **- 4.675**
Bound for outlier on the high side: Q3 + 1.5·IQR = 19.25 + 14.355 = **33.605**

**Problem 22.** AP 2001 №6

**Problem 23**. AP 2000 №3.

Five hundred randomly selected middle aged men and five hundred randomly selected young adult men were rated on a scale from 1 to 10 on their physical flexibility, with 10 being the most flexible. Their rating appear in the frequency table below. For example, 17 middle aged-men has a flexibility rating of 1.

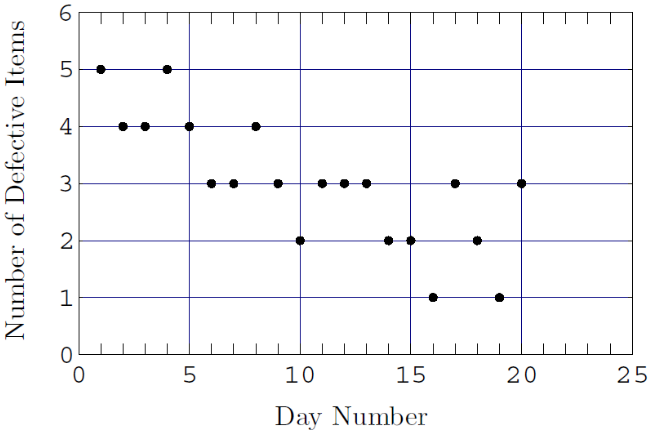
| **Physical Flexibility Rating** | **Frequency of Middle-Aged Men** | **Frequency of Young Adult Men** |
|---|---|---|
| 1 | 17 | 4 |
| 2 | 31 | 17 |
| 3 | 49 | 29 |
| 4 | 71 | 39 |
| 5 | 70 | 54 |
| 6 | 87 | 69 |
| 7 | 78 | 83 |
| 8 | 54 | 93 |
| 9 | 34 | 73 |
| 10 | 9 | 39 |

(a) Display the data graphically so that the flexibility of middle-aged men and young adult men can be easily compared.

(b) Based on an examination of your graphical display, write a few sentences comparing the flexibility of middle-aged men with the flexibility of young adult men.

**Problem 24**. AP 1998 №2.

A plot of the number of defective items produced during 20 consecutive days at a factory is shown below.



(a) Draw a histogram that shows the frequencies of the number of defective items.

(b) Give one fact that is obvious from the histogram but is not obvious from the scatterplot.

(c) Give one fact that is obvious from the scatterplot but is not obvious from the histogram.