ICEF Probability theory and Statistics

**Home assignment 6/1**

**Problem 1.** You have a coin with P(tail) = p. You test null hypothesis that the coin is fair by flipping it 5 times. If number of tails is 2 or 3 you do consider coin to be fair. Otherwise you conlude that coin is biased.

(a) Formulate the appropriate null hypothesis and alternative hypothesis.
(b) What is significance level of the test?
(c) Find power of the test if actual $p = 0.75$.

**Problem 2.** Let $X_1, X_2, \ldots, X_{10}$ be iid $N(\mu, 4)$. To test $H_0 : \mu = 50$ against $H_1 : \mu \neq 50$ we use the rule: reject $H_0$ if $|\overline{X} - 50| > 0.5$.

(a) What is the probability of type I error?
(b) What is the probability of type II error if $\mu = 51$?

**Problem 3.** (a) $H_0 : p = 0.75$, $\hat{p} = 0.8$, $n = 70$, $\alpha = 10\%$. Should this null hypothesis be rejected in favor of the two-sided alternative hypothesis at the given significance level $\alpha$? Clearly write alternative hypothesis $H_1$, what test you use and show your work.

(b) The public health ministry reports that 55% of adults regularly smoke. A doctor wants to verify this claim and finds out that in a sample of 80 of his patients 57 people smoke. Formulate the appropriate null hypothesis and the two-sided alternative hypothesis, and test the null hypothesis at the 1% significance level based on the available data.

**Problem 4.** A standard test of the null hypothesis $H_0 : p = 0.4$ against the two-sided alternative is performed on a sample of size $n = 80$. The significance level is 10%.

(a) For what values of the sample proportion $\hat{p}$ should the null hypothesis be rejected? Test null hypothesis for $\widehat{p} = 0.3$.
(b) What will be the power of the test if $p = 0.33$?

**Problem 5.** A certain type of seed has always grown to a mean height of $\mu = 8.5$ inches with a standard deviation $\sigma = 1$ inch. A sample of 100 seeds is grown under enriched conditions to see whether the mean height might be improved (standard deviation is not changed). If sample mean height will exceed a certain cutoff level $X_c$ then agricultural company will make a conclusion that mean height has increased. Significance level is chosen to be 5%.

(a) Formulate null and alternative hypotheses.
(b) Calculate the cut-off level $X_c$.
(c) If the sample actually turns out to have a mean height $\overline{x} = 8.8$ inches do you reject $H_0$?
(d) Compute type II error of this test $\beta$ as a function of actual $\mu$ i.e. $\beta = \beta(\mu)$. Plot this function.

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**Home assignment 6/2**

**Problem 1.** Should the following null hypotheses be rejected in favor of the two-sided alternative hypotheses at the given significance level $\alpha$? Clearly write what tests you use and show your work. Assume the population distribution is normal.

(a) $H_0 : \mu = 1$, $\overline{x} = 0.55$, $\sigma = 1.2$ is known, $n = 20$, $\alpha = 10\%$.

(b) $H_0 : \mu = 1$, $\overline{x} = 0.55$, $s = 1.2$, $n = 20$, $\alpha = 10\%$.

**Problem 2.** A standard test of the null hypothesis $H_0 \colon \mu = 12$ against the two-sided alternative is performed on a sample of size $n = 80$ from a normal population with standard deviation $\sigma = 3$. The significance level is 5%.

(a) For what values of the sample mean $\overline{x}$ should the null hypothesis be rejected?

(b) What will be the power of the test if $\mu = 14$?

**Problem 3.** The university administration thinks that students get on average equal grades in economics and mathematics exams. The following table shows the grades of 16 randomly selected students.

| Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Economics grade | 71 | 75 | 55 | 39 | 32 | 83 | 49 | 90 | 100 | 58 | 59 | 83 | 52 | 29 | 26 | 54 |
| Mathematics grade | 78 | 64 | 34 | 21 | 23 | 90 | 56 | 93 | 90 | 71 | 51 | 84 | 50 | 21 | 22 | 45 |

Formulate the appropriate null hypothesis and the two-sided alternative hypothesis, and test the null hypothesis at the 1% significance level based on the available data. Assume normality of grades distributions.

**Problem 4.** The summary statistics for 2 independent datasets from the same population with a normal distribution are as follows:

| | Sample size | Sample mean | Sample standard deviation |
|---|---|---|---|
| $x$ data | 13 | 4.4 | 1.2 |
| $y$ data | 18 | 5.1 | 1.4 |

(a) Compute the mean and the variance of the combined dataset $x_1, \ldots, x_{13}, y_1, \ldots, y_{18}$.

(b) Test null hypothesis that population mean equals 5.4 against 2-sided alternative at significance levels 1%, 5%, 10%.

**Problem 5.** Population has the following distribution:

| $X$ | 20 | -10 | 0 |
|---|---|---|---|
| P | $2p$ | $p$ | $1 - 3p$ |

Let $X_1, X_2, \ldots, X_{50}$ be a sample from this distribution.

(a) Find all possible values of parameter $p$.

(b) Find population mean, population variance and approximate sampling distribution of sample mean $\overline{X}$.

(c) Test null hypothesis $H_0 : p = 0.2$ against alternative $H_1 : p \neq 0.2$ at 10% significance level if data is given in aggregated form $\sum_{i=1}^{50} x_i = 180$.

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**Home assignments 6/3**

**Problem 1.** A test for the null hypothesis that the population proportion is equal to 0.7 against the two-sided alternative was performed on a sample of size 80. The sample proportion turned out to be 0.79.

(a) What is the value of the $z$-statistic for this test?
(b) What is the $p$-value of the test?
(c) Interpret the obtained $p$-value.
(d) Test null hypothesis at 5% and 10% significance levels. Use the obtained $p$-value to answer this question.

**Problem 2.** (a) A sample of size $n = 25$ was selected from a population with normal population distribution. Its sample mean $\overline{x} = 144$ and sample standard deviation $s = 12$. Perform a test of the null hypothesis $H_0 : \mu = 150$ against the two-sided alternative hypothesis at the 5% significance level.

(b) A coin was flipped 120 times and landed heads-up 70 times. What is the $p$-value of the null hypothesis that this coin is symmetric? Can the hypothesis be rejected at the 10% significance level?

**Problem 3.** A restaurant owner claims that his restaurant has on average 100 visitors per day. According to financial records, during the last 50 days there were on average 96 visitors per day with sample standard deviation 15 visitors.

Formulate the appropriate null hypothesis and the two-sided alternative hypothesis, and test the null hypothesis at the 1%, 5%, 10% significance levels based on the available data. Find p-value of the test (use T.DIST function in Excel or scipy.stats.t.cdf function in Python.). Assume that number of visitors per day is normally distributed.

**Problem 4.** Equipment which fills soft drink bottles, when operating correctly, fills on average, 18 ounces ml of the soft drink in a bottle. A random sample of nine bottles from a single production run yielded the following content volume (in ounces):

17.3, 18.25, 19.1, 18.5, 17.75, 20.1, 18.4, 19.05, 19.1, 17.8.

Assuming that the population distribution is normal,

(a) Test at the 10% significance level the null hypothesis that the equipment is operating correctly.
(b) Construct 90% confidence interval for the average amount of soft drink in a bottle and compare it with acceptance region.

**Problem 5.** A physicist made 20 measurements of the temperature of an object and obtained sample standard deviation of 1.1. He knew that his thermometer had a measurement error which is normally distributed with zero mean. Physicist wants to test a hypothesis that measurement error has variance 1.

(a) Test null hypothesis at 10% significance level.
(b) Find power of the test when true variance of measurement error $\sigma^2 = 1.3$ (use CHISQ.DIST function in Excel or scipy.stats.chi2.cdf function in Python).
(c) How your test of variance $\sigma^2$ would change if you know that true temperature is $60^\circ$ and $\widehat{\sigma^2} = 1.25$ where $\widehat{\sigma^2} = \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$?