$$|\Psi_k\rangle = |0\rangle^{\otimes(k-1)} |\phi\rangle^{\otimes(n-k+1)} \qquad 1 \le k \le n$$ [2]

Hence in our case

$|\psi_1\rangle = |\phi\rangle_1 |\phi\rangle_2 |\phi\rangle_3 |\phi\rangle_4 |\phi\rangle_5$ $\quad 1/5$

$|\psi_2\rangle = |0\rangle_1 |\phi\rangle_2 |\phi\rangle_3 |\phi\rangle_4 |\phi\rangle_5$ $\quad$ ''

$|\psi_3\rangle = |0\rangle_1 |0\rangle_2 |\phi\rangle_3 |\phi\rangle_4 |\phi\rangle_5$ $\quad$ ''

$|\psi_4\rangle = |0\rangle_1 |0\rangle_2 |0\rangle_3 |\phi\rangle_4 |\phi\rangle_5$ $\quad$ ''

$|\psi_5\rangle = |0\rangle_1 |0\rangle_2 |0\rangle_3 |0\rangle_4 |\phi\rangle_5$ $\quad$ ''.

<u>Greedy strategy:</u>

For particle $s \in (1, \dots, 5)$ maximize

$$P_s = \sum_{k=1}^{5} \eta_k^{(s)} \langle \Psi_k | E_k^{(s)} | \Psi_k \rangle.$$

$$P_1 = \frac{1}{5} \sum_{k=1}^{5} \langle \psi_k | E_k^{(s)} | \psi_k \rangle.$$

$$= \frac{1}{5} \langle \psi_1 | E_1^{(s)} | \psi_1 \rangle + \frac{4}{5} \sum_{k=2}^{5} \langle \psi_k | E_k^{(s)} | \psi_k \rangle.$$

$$= P_\phi^{(s)} \langle \phi | \Pi_\phi^{(s)} | \phi \rangle + \frac{4}{5} \langle 0 | \Pi_0 | 0 \rangle.$$

$$\mathcal{P}_2^{c_+} = \eta_1^{(2)} \langle \varphi_1 | E_1^{(2)} | \varphi_1 \rangle + \eta_2^{(2)} \langle \varphi_2 | E_2^{(2)} | \varphi_2 \rangle +$$

$$\eta_3^{(2)} \langle \varphi_3 | E_3^{(2)} | \varphi_3 \rangle + \eta_4^{(2)} \langle \varphi_4 | E_4^{(2)} | \varphi_4 \rangle +$$

$$\eta_5^{(2)} \langle \varphi_5 | E_5^{(2)} | \varphi_5 \rangle .$$

$$= \eta_1^{(2)} \langle \phi | E_1^{(2)} | \phi \rangle + \eta_2^{(2)} \langle \phi | E_2^{(2)} | \phi \rangle +$$

$$\eta_3^{(2)} \langle 0 | E_3^{(2)} | 0 \rangle + \eta_4^{(2)} \langle 0 | E_4^{(2)} | 0 \rangle + \eta_5^{(2)} \langle 0 | E_5^{(2)} | 0 \rangle$$

$$= \langle \phi | \left\{ \eta_1^{(2)} E_1^{(2)} + \eta_2^{(2)} E_2^{(2)} \right\} | \phi \rangle +$$

$$\langle 0 | \left\{ \eta_3^{(2)} E_3^{(2)} + \eta_4^{(2)} E_4^{(2)} + \eta_5^{(2)} E_5^{(2)} \right\} | 0 \rangle .$$

Now define

$$P_\phi^{(2)} := \max \left\{ \eta_1^{(2)}, \eta_2^{(2)} \right\}$$

$$P_0^{(2)} := \max \left\{ \eta_3^{(2)}, \eta_4^{(2)}, \eta_5^{(2)} \right\}$$

Let $\bar{r}_s \in \{0,1\}^s$ denote the sequence of result up to and including s. Then the average probability of success is

$$P_s^G = \sum_{k=1}^{n} \sum_{\bar{r}_s} \eta_k \, p(\hat{k}(\bar{r}_s)|k) \leq \sum_{\bar{r}_s} \max_{\hat{k}(\bar{r}_s)} p(\hat{k}(\bar{r}_s), k)$$

where $\hat{k}(\bar{r}_s)$ is our decision rule. The inequality is saturated by maximum likelihood. i.e

$$\hat{k}(\bar{r}_s) = \underset{k}{\operatorname{argmax}} \; p(\bar{r}_s, k).$$

where
$$p(\bar{r}_s, k) = \frac{1}{N} p(\bar{r}_{s-1}|k) \langle \psi_k | E_k^{(s)} | \psi_k \rangle$$

$\therefore$

$$P_s^G = \sum_{\bar{r}_{s-1}} p(\bar{r}_{s-1}) \cdot P_s^G(\bar{r}_{s-1})$$

$$= \sum_{\bar{r}_{s-1}} p(\bar{r}_{s-1}) \max_{\{E_k^{(s)}\}} \sum_{k=1}^{N} \max_{k} p(k|\bar{r}_{s-1}) \langle \psi_k | E_k^{(s)} | \psi_k \rangle$$

The optimal measurement & guessing rule depend only on $\eta_k^{(s)}$ and <u>not</u> on the sequence of measurement outcomes.

At step s the average success probability at each step is

$$\sum_{k=1}^{n}\sum_{\bar{r}_s} p(\bar{r}_s, k)\,\delta_{k,\hat{k}(\bar{r}_s)} \leq \sum_{\bar{r}_s} \max_k p(\bar{r}_s,k)$$

with equality iff $\hat{k}(\bar{r}_s) = \underset{k}{\mathrm{argmax}}\; p(\bar{r}_s,k)$.

$$p(\bar{r}_s,k) = \frac{1}{n}\, p(\bar{r}_{s-1}|k)\,\langle\psi_k|E_k^{(s)}|\psi_k\rangle$$

Hence

$$P_s^G = \sum_{\bar{r}_{s-1}} p(\bar{r}_{s-1})\, P_s^G(\bar{r}_{s-1})$$

$$P_s^G(\bar{r}_{s-1}) = \max_{\{E_k^{(s)}\}} \sum_{\ell=1}^{n} \max_k p(k|\bar{r}_{s-1})\,\langle\psi_k|E_k^{(s)}|\psi_k\rangle.$$

$$\overset{1}{p}(H_k|r_1) = \frac{p(r_1|H_k)\,\eta_k}{p(r_1)}$$

$$\overset{2}{p}(H_k|r_1 r_2) = \frac{p(r_2\ |H_k)\,p(H_k|r_1)}{p(r_1 r_2)}$$

$$= \frac{p(r_2|H_k)\,p(r_1|H_k)\,\eta_k}{p(r_1)\,p(r_2)}$$

$$:= \frac{p(r_1 r_2|H_k)\,\eta_k}{p(r_1 r_2)}$$

$$p^{(3)}(H_k|r_3) = \frac{p(r_3|H_k)\,p^{(2)}(H_k|r_1 r_2)}{p(r_3)}$$

$$= \frac{p(r_3|H_k)\,p(r_2|H_k)\,p(r_1|H_k)\,\eta_k}{p(r_1)\,p(r_2)\,p(r_3)}.$$

This is why they leave it unormalized.