

# Econ 512 Lab 4

## Extreme Value Theory

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### 1 Exercises

1. Modification of Tsay (2001), chapter 7, exercise 2. The file `d.csc9199.dat` contains the daily log returns of Cisco Systems stock from 1991 to 1999 with 2275 observations. Suppose you hold a long position of Cisco stock valued at \$1 million. Compute the Value-at-Risk (VaR) of your position, with 95% confidence intervals if possible, for the next trading day using probabilities  $p = 0.01$  and  $p = 0.005$  and the following methods
  - (a) Assume that the log returns are normally distributed
  - (b) Use a GARCH(1,1) model with a conditional Gaussian distribution
  - (c) Use a GARCH(1,1) model with a conditional Student-t distribution, where you estimate the degrees of freedom.
  - (d) Use the unconditional sample quantile of the log returns (historical simulation)
  - (e) Use the extreme value theory for the maximum of the negative daily log returns. Use quarterly blocks to estimate the GEV distribution and use equation (7.26) from Tsay to compute the VaR.
  - (f) Use the extreme value theory for the exceedances over thresholds (peaks over thresholds) based on the negative daily log returns. Use the empirical mean excess plots created by the **S+FinMetrics** function `meplot` to determine the appropriate threshold value for the generalized Pareto distribution (GPD) and then fit the model by maximum likelihood. Examine the sensitivity of the estimated shape parameter  $\xi$  to variations in the threshold using the **S+FinMetrics** function `shape`. Compute the VaR and 95% confidence intervals using the **S+FinMetrics** function `riskmeasures`. Plot the 95% confidence using the functions `tailplot` and `gdp.q`. Finally, examine the sensitivity of the VaR estimates to the variations in the threshold using the function `quant`.

2. Combining GARCH and EVT. This exercise guide you through the process of combining GARCH with EVT along the lines described in McNeil and Frey (2000), “Estimation of Tail-Related Risk Measures for Heteroskedastic Financial Time Series: An Extreme Value Approach,” *Journal of Empirical Finance*. See also Bingcheng Yan’s class handout. For this exercise, use the data on Cisco log returns from the previous exercise, and assume you hold a long position of Cisco stock valued at \$1 million. The basic model is of the form

$$X_t = \mu_t + \sigma_t Z_t$$

where  $X_t$  is the negative return on Cisco stock (loss variable),  $\mu_t$  is the conditional mean,  $\sigma_t$  is the conditional scale parameter and  $Z_t$  is *iid* random variable with zero mean and unit variance. The conditional 1-step ahead distribution of  $X_t$  is

$$\begin{aligned} F_{X_{t+1}|I_t}(x) &= \Pr\{\mu_{t+1} + \sigma_{t+1}Z_{t+1} \leq x|I_t\} \\ &= F_Z\left(\frac{x - \mu_{t+1}}{\sigma_{t+1}}|I_t\right) \end{aligned}$$

For  $0.95 \leq q < 1$ , the conditional quantile for  $X_t$  is computed using

$$x_t^q = \mu_{t+1} + \sigma_{t+1}z_q$$

where  $z_q$  is the  $q$ th quantile of the distribution of  $Z_t$ .

- (a) Fit an AR(1)-GARCH(1,1) model, with Gaussian errors, to the negative log-returns on Cisco stock:

$$\begin{aligned} X_t &= \mu_t + \sigma_t Z_t = \phi X_{t-1} + \varepsilon_t \\ \mu_t &= \phi X_t, \quad \varepsilon_t = \sigma_t Z_t \\ \sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned}$$

and extract the estimated standardized residuals  $\hat{Z}_t = \hat{\varepsilon}_t / \hat{\sigma}_t$ .

- (b) Fit a GPD to the estimated standardized residuals  $\hat{Z}_t$ .
- (c) Using the GPD model, estimate the quantile  $z_q$  and the conditional mean  $E[Z||Z > z_q]$  for  $q = 0.01, 0.005$ .
- (d) Using the estimates of  $z_q$  and  $E[Z||Z > z_q]$  compute the quantile and conditional mean estimates for  $X_t$  using

$$\begin{aligned} \hat{x}_{t+1,q} &= \hat{\mu}_{t+1} + \hat{\sigma}_{t+1}\hat{z}_q \\ \hat{E}[X_{t+1}|X_{t+1} > x_{t+1,q}] &= \hat{\mu}_{t+1} + \hat{\sigma}_{t+1}\hat{E}[Z|Z > z_q] \end{aligned}$$

- (e) From the quantiles and conditional means computed previously, compute the  $VaR_q$  and  $ES_q$  for a \$1 million long position in Cisco stock.
- (f) Compare the results with those found in exercise 1.