We quickly derive expressions for numerical second partial derivatives. Let $f(\alpha,\beta)$ be a function with two parameters α,β. Consider the numerical first partial derivative of f with respect to α using some step size $\Delta\alpha$:

$$\frac{\partial f}{\partial \alpha}=\frac{f(\alpha+\Delta\alpha)-f(\alpha-\Delta\alpha)}{2\Delta\alpha}$$

how about the second partial $\partial_\alpha^2 f$?

$$\begin{aligned}
\frac{\partial^2 f}{\partial^2 \alpha} &= \frac{\partial}{\partial \alpha}\left(\frac{\partial f}{\partial \alpha}\right)\\
&= \frac{\partial}{\partial \alpha}\left(\frac{f(\alpha+\Delta\alpha,\beta)-f(\alpha-\Delta\alpha,\beta)}{2\Delta\alpha}\right)\\
&= \frac{1}{2\Delta\alpha}\left(\frac{\partial}{\partial \alpha}f(\alpha+\Delta\alpha,\beta)-\frac{\partial}{\partial \alpha}f(\alpha-\Delta\alpha,\beta)\right)\\
&= \frac{1}{2\Delta\alpha}(\frac{f(\alpha+\Delta\alpha+\Delta\alpha,\beta)-f(\alpha+\Delta\alpha-\Delta\alpha,\beta)}{2\Delta\alpha}-\\
&\frac{f(\alpha-\Delta\alpha+\Delta\alpha,\beta)-f(\alpha-\Delta\alpha-\Delta\alpha,\beta)}{2\Delta\alpha})\\
&= \frac{f(\alpha+2\Delta\alpha,\beta)+f(\alpha-2\Delta\alpha,\beta)-2f(\alpha,\beta)}{4\Delta\alpha^2}
\end{aligned}$$

We can similarly obtain an expression for mixed partials:

$$\begin{aligned}
\frac{\partial^2 f}{\partial \alpha \partial \beta} &= \frac{\partial}{\partial \beta}\left(\frac{\partial f}{\partial \alpha}\right)\\
&= \frac{\partial}{\partial \beta}\left(\frac{f(\alpha+\Delta\alpha,\beta)-f(\alpha-\Delta\alpha,\beta)}{2\Delta\alpha}\right)\\
&= \frac{1}{2\Delta\alpha}\left(\frac{\partial}{\partial \beta}f(\alpha+\Delta\alpha,\beta)-\frac{\partial}{\partial \beta}f(\alpha-\Delta\alpha,\beta)\right)\\
&= \frac{1}{2\Delta\alpha}(\frac{f(\alpha+\Delta\alpha,\beta+\Delta\beta)-f(\alpha+\Delta\alpha,\beta-\Delta\beta)}{2\Delta\beta}-\\
&\frac{f(\alpha-\Delta\alpha,\beta+\Delta\beta)-f(\alpha-\Delta\alpha,\beta-\Delta\beta)}{2\Delta\beta})\\
&= \frac{f(\alpha+\Delta\alpha,\beta+\Delta\beta)+f(\alpha-\Delta\alpha,\beta-\Delta\beta)-f(\alpha+\Delta\alpha,\beta-\Delta\beta)-f(\alpha-\Delta\alpha,\beta+\Delta\beta)}{4\Delta\alpha\Delta\beta}
\end{aligned}$$