

For any complex number  $|z| < 1$ :

$$(1 + z)^{-1} = \sum_{n=0}^{\infty} (-z)^n$$

In particular assuming  $|g| < 1$  and  $|\epsilon| < 1$ , so that  $|g^* \epsilon| < 1$ :

$$(1 + g^* \epsilon)^{-1} = \sum_{n=0}^{\infty} (-g^* \epsilon)^n$$

So,

$$\begin{aligned} \epsilon' &= \frac{\epsilon + g}{1 + g^* \epsilon} \\ &= (\epsilon + g) (1 + g^* \epsilon)^{-1} = (\epsilon + g) \sum_{n=0}^{\infty} (-g^* \epsilon)^n \\ &= \epsilon + g + \sum_{n=1}^{\infty} [(-g^*)^n \epsilon^{n+1} + g(-g^*)^n \epsilon^n] \end{aligned}$$

Taking expectation from both sides,

$$\langle \epsilon' \rangle = g + \langle \epsilon \rangle + \sum_{n=1}^{\infty} [(-g^*)^n \langle \epsilon^{n+1} \rangle + g(-g^*)^n \langle \epsilon^n \rangle]$$

So, in general,  $\langle \epsilon' \rangle = g$  **exactly** if  $\langle \epsilon^n \rangle = 0$  for all  $n \geq 1$ .

But this is, in fact, the case. Note that any particular ellipticity can be written in terms of its magnitude and angle in the complex plane:

$$\epsilon = \epsilon_0 e^{i\theta}, \quad 0 \leq \theta \leq 2\pi$$

Assume the that distribution of intrinsic ellipticities  $f(\epsilon_0, \theta)$  is uniform in the angle, this means that  $f(\epsilon_0, \theta) = f(\epsilon_0)$  (only depends on the radius). Then, the expectation  $\langle \epsilon^n \rangle$  over this distribution  $f$  becomes,

$$\begin{aligned}
\langle \epsilon^n \rangle &= \frac{1}{2\pi} \int_0^1 \int_0^{2\pi} \epsilon^n f(\epsilon_0, \theta) d\theta d\epsilon_0 = \frac{1}{2\pi} \int_0^1 \epsilon_0^n f(\epsilon_0) d\epsilon_0 \int_0^{2\pi} e^{in\theta} d\theta \\
&= \frac{1}{2\pi} \int_0^1 \epsilon_0^n f(\epsilon_0) \left( \frac{e^{in\theta}}{in} \Big|_0^{2\pi} \right) d\epsilon_0 \\
&= \frac{1}{2\pi in} \int_0^1 \epsilon_0^n f(\epsilon_0) (e^{i2\pi n} - 1) d\epsilon_0 = 0, \quad \text{for } n = 1, 2, 3 \dots
\end{aligned}$$