AEC3: ÁLGEBRA

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PROBLEMA 1

a)

$$\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}+\begin{pmatrix}1&-1&2\\0&3&-5\end{pmatrix}$$

$$=\begin{pmatrix}1+1&2+(-1)&3+2\\4+0&5+3&6+(-5)\end{pmatrix}$$

$$=\begin{pmatrix}2&1&5\\4&8&1\end{pmatrix}$$

B1)

$$3\begin{pmatrix}1&-2&3\\4&5&-6\end{pmatrix}$$

$$=\begin{pmatrix}3&-6&9\\12&15&-18\end{pmatrix}$$

B2)

$$-5\begin{pmatrix}1&-2&3\\4&5&-6\end{pmatrix}$$

$$=\begin{pmatrix}-5&10&-15\\-20&-25&30\end{pmatrix}$$

C)

$$2\begin{pmatrix}1&2&3\\4&5&6\end{pmatrix}-3\begin{pmatrix}3&0&2\\-7&1&8\end{pmatrix}$$

$$=\begin{pmatrix}2&4&6\\8&10&12\end{pmatrix}-\begin{pmatrix}9&0&6\\-21&3&24\end{pmatrix}$$

$$=\begin{pmatrix}-7&4&0\\29&7&-12\end{pmatrix}$$

PROBLEMA 2

$$3\begin{pmatrix}x&y\\z&w(x,y,z)\end{pmatrix}=\begin{pmatrix}x&6\\-1&2w(x,y,z)\end{pmatrix}+\begin{pmatrix}4&x+y\\z+w(x,y,z)&3\end{pmatrix}$$

$$\begin{pmatrix}3x&3y\\3z&3w(x,y,z)\end{pmatrix}=\begin{pmatrix}x+4&6+x+y\\-1+z+w(x,y,z)&2w(x,y,z)+3\end{pmatrix}$$

$$x = 2, z = 1, y = 4, w = 3$$

PROBLEMA 3

a)

$$\begin{pmatrix} 3 & 8 & -2.4 \end{pmatrix} \begin{pmatrix} 5 \\ -1 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \cdot 5 + 8(-1) + (-2.4) \cdot 6 \end{pmatrix}$$

$$= \begin{pmatrix} -7.4 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & 8 & 3.4 \end{pmatrix} \times \begin{pmatrix} 6 & 1 & -3.5 \end{pmatrix}$$

$$= \begin{pmatrix} 8(-3.5) - 3.4 \cdot 1 & 3.4 \cdot 6 - 1 \cdot (-3.5) & 1 \cdot 1 - 8 \cdot 6 \end{pmatrix}$$

$$= \begin{pmatrix} -31.4 & 23.9 & -47 \end{pmatrix}$$

c) AB

$$\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 & -4 \\ 3 & -2 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & -6 & 14 \\ 1 & 2 & -14 \end{pmatrix}$$

BA

Imposible

Para multiplicar dos matrices el número de columnas de la primera matriz debe ser igual al de filas en la segunda.

PROBLEMA 4

AB

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$$

BA

$$\begin{pmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 15 & -21 \\ 10 & -3 \end{pmatrix}$$

PROBLEMA 5

A1)

$$\begin{pmatrix} 1 & 3 & 5 \\ 6 & -7 & -8 \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & 6 \\ 3 & -7 \\ 5 & -8 \end{pmatrix}$$

A2)

$$\left(\begin{pmatrix} 1 & 3 & 5 \\ 6 & -7 & -8 \end{pmatrix}^T\right)^T$$

$$= \begin{pmatrix} 1 & 6 \\ 3 & -7 \\ 5 & -8 \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & 3 & 5 \\ 6 & -7 & -8 \end{pmatrix}$$

B1)

$$\begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & -1 \\ 0 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 1 \\ 1 & 26 \end{pmatrix}$$

B2)

$$\begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}^T \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 \\ 2 & -1 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 10 & -1 & 12 \\ -1 & 5 & -4 \\ 12 & -4 & 16 \end{pmatrix}$$

PROBLEMA 6

A1)

$$\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}^2$$

A2)

$$\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}^3$$

$$= \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}$$

$$= \begin{pmatrix} -7 & 30 \\ 60 & -67 \end{pmatrix}$$

B)

$$= f\left(\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} \right)$$

$$= 2 \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}^3 - 4 \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} + 5$$

PROBLEMA 7

La matriz que nos das es una matriz triangular superior. Para este tipo de matrices, el cálculo de la potencia n-ésima es bastante directo. La matriz A elevada a la n (A^n) será:

$$A^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$$

Esto es válido para cualquier número natural n. Por lo tanto, la entrada (1,2) de la matriz se multiplica por n.

PROBLEMA 8

A)

$$\begin{pmatrix} 3 & 5 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -3 & 5 \\ 0 & 1 & 2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 5 \\ 2 & -3 \end{pmatrix}$$

B)

$$= \frac{1}{\det \begin{pmatrix} 3 & 5 \\ 2 & 3 \end{pmatrix}} \begin{pmatrix} 3 & -5 \\ -2 & 3 \end{pmatrix}$$

$$= \frac{1}{-1} \begin{pmatrix} 3 & -5 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 5 \\ 2 & -3 \end{pmatrix}$$

PROBLEMA 9

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ -1 & -1 & 5 & 0 & 1 & 0 \\ 2 & 7 & -3 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -16 & -11 & 3 \\ 0 & 1 & 0 & \frac{7}{2} & \frac{5}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{array}\right]$$

$$= \begin{pmatrix} -16 & -11 & 3 \\ \frac{7}{2} & \frac{5}{2} & -\frac{1}{2} \\ -\frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

PROBLEMA 10

Si A es una matriz triangular superior de la forma:

$$A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 8 & -57 \\ 0 & 27 \end{pmatrix} = \begin{pmatrix} a^3 & 3a^2b \\ 0 & c^3 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & -\frac{19}{4} \\ 0 & 3 \end{pmatrix}$$

PROBLEMA 11

$$det \begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix} = 3 \cdot 6 - 2 \cdot 5 = 8$$

$$\det \begin{pmatrix} 2 & -3 \\ 4 & 7 \end{pmatrix} = 2 \cdot 7 - (-3) \cdot 4 = 26$$

$$\det \begin{pmatrix} 4 & -5 \\ -1 & -2 \end{pmatrix} = 4 \cdot (-2) - (-5)(-1) = -13$$

$$\det \begin{pmatrix} t-5 & 6 \\ 3 & t+2 \end{pmatrix} = (t-5)(t+2) - 6 \cdot 3 = t^2 - 3t - 28$$

PROBLEMA 12

$$det \begin{pmatrix} 2 & 3 & 4 \\ 5 & 4 & 3 \\ 1 & 2 & 1 \end{pmatrix} = 2 \cdot 4 + 5 \cdot 2 \cdot 4 + 3 \cdot 3 - 4 \cdot 4 - 2 \cdot 3 \cdot 2 - 5 \cdot 3 = 14$$

$$det \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -1 \\ 1 & 5 & -2 \end{pmatrix} = 1 \cdot 4 \cdot (-2) + 2 \cdot 5 \cdot 3 + (-2) \cdot (-1) - 3 \cdot 4 - (-5) - 2^3 = 9$$

$$det\begin{pmatrix} 1 & 3 & -5 \\ 3 & -1 & 2 \\ 1 & -2 & 1 \end{pmatrix} = -1 + 2 \cdot 5 \cdot 3 + 3 \cdot 2 - 5 - (-4) - 9 = 25$$