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Answer 1

a-)

- Let's say the statement is P(n).
- Basis is n=1. For n=1, $2^3 - 3$=5 is divisible by 5. So, base step P(1) is true.
- Inductive hypothesis: Assume that P(k) is true for k≥1.
- It means that $2^{3k} - 3^k$ divisible by 5. We can say 2^{3k}-3^k=5a, $\exists a \in Z$. (eqn. 1)
- Then, we look for if the statement holds for n=k+1.
- If we substitute k+1 into the statement, it becomes 2^{3k+3} - 3^{k+1}. It is equal to 8.2^{3k} - 3.3^k. (eqn. 2)
- From (eq. 1) we know that $2^{3k} = 5a + 3^k$. Substitute 2^{3k} into eqn. 2 gives 40a+5.3^k. This can be written as 5(8a+3^k) which is divisible by 5.
- So, P(k+1) is true also.
- By induction, the statement is true for all integers n ≥ 1.

b-)

- Let's say the statement is Q(n).
- Basis is n=2. For n=2, $4^2 - 7.2 - 1 > 0$ holds. i.e, Q(2) is true.
- Inductive hypothesis: Assume that Q(k) is true for k≥2.
- It means that $4^k - 7.k - 1 > 0$ holds. We can infer that $4^k > 7k + 1$
- Then, multiplying each side by 4 gives $4^{k+1} > 28k + 4$
- Then, substracting 7k from each side gives $4^{k+1} - 7k - 8 > 21k - 4$
- We know that $21k - 4 > 0$ because k≥2.
- It means that we can say $4^{k+1} - 7k - 8 > 0$.
- This shows that Q(k+1) ($4^{k+1} - 7(k + 1) - 1 > 0$) is true.
- By induction, the statement holds for all integers n≥ 2.

Answer 2

a-)

- Bit strings of length 10 have 7 1's :$\frac{10!}{7!.3!}$ = 120
- Bit strings of length 10 have 8 1's :$\frac{10!}{8!.2!}$ = 45
- Bit strings of length 10 have 9 1's :$\frac{10!}{9!.1!}$ = 10
- Bit strings of length 10 have 10 1's :$\frac{10!}{10!.0!}$ = 1
- Answer is 120+45+10+1 = 176

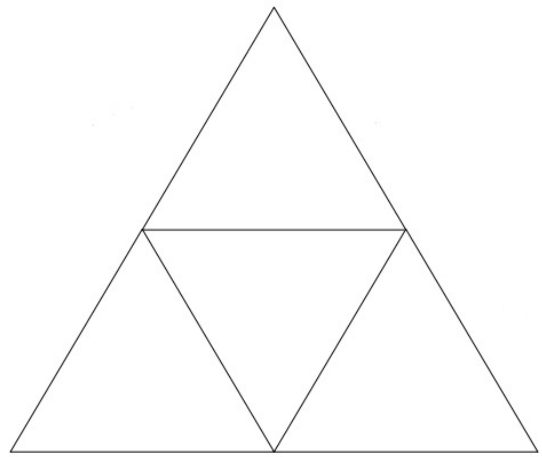
b-)

- The requirement is at least one Statistical Methods textbook and at least one Discrete Mathematics. So, take one from each of them.
- After, there remains 4 Statistical Methods book and 3 Discrete Mathematics book.
- Then, in order to complete 4 books. We need to choose 2 books more.
- Books are identical, so the remaining choices are SS DS DD.
- Finally, the choices are SDSS SDDS SDDD.
- The answer is 3.

c-)

- We can find this by substracting the number of no-onto functions from the number of all functions.
- Each five item in domain, there are three options in co-domain. So, the number of all functions is $3^5 = 243$.
- We must look the number of cases which some items is unused in co-domain. (not onto functions)
- The number of cases which some items is unused is $\binom{3}{1}.2^5$ - $\binom{3}{2}.1^5$. Because, while counting, actually we count the cases which two items are unused twice. So, we need to substract them. (Inclusion/Exclusion Principle)
- Finally, the number of onto functions is 3^5- ($\binom{3}{1}.2^5$ - $\binom{3}{2}.1^5$) = 243-(96-3)= 150.

Answer 3



- We can divide the main equilateral triangle into 4 equilateral triangle with side 250 meters. Then, we can use Pigeonhole principle. The pigeons are 5 kids and the pigeonholes are 4 equilateral triangle. By Pigeonhole Principle, there are two kids in same triangle. In the same triangles, the maximum distance is 250meters. It means that there are two kids within 250 meters of each other.

Answer 4

a-) $a_n - 3a_{n-1} = 0$ (for particular solution)

- Characteristic equation is $\alpha - 3 = 0$. α=3 (multiplicity 1)
- The solution is $a_n^{(h)}$=A.3^n

b-) $a_n - 3a_{n-1} = 5^{n-1} = \frac{1}{5}.5^n$ (for homogeneous solution)

- The solution is $a_n^{(p)}$=B.5^n
- Substitute this solution into original equation so as to find B.
- $B.5^n - 3.B.5^{n-1} = 5^{n-1}$
- $B.5^n = \frac{(3B+1)}{5}.5^n$
- 5B=3B+1 and B=1/2, $a_n^{(p)}$=$\frac{5^n}{2}$
- Total solution is a_n=A.3^n+$\frac{5^n}{2}$. In order to find A, substitute initial condition $a_1 = 4$.
- 3A+5/2=4, A=1/2.

- The total solution is $a_n = \frac{3^n + 5^n}{2}$

c-)

- Let's say P(n) is our statement. ($a_n = \frac{3^n + 5^n}{2}$ is a solution to the recurrence relation)
- Base step P(1) is true. We've already checked for the initial condition in previous part. It holds for a_1.
- Inductive Hypothesis: Assume that P(k) is true for k≥1.
- It means that $a_k = \frac{3^k + 5^k}{2}$ is a solution. (eqn. 1)
- Now, we must check whether P(k+1) is true for k≥1.
- Substitute $n = k + 1$ into original equation. It gives $a_{k+1} - 3a_k = 5^k$. Then, substitute a_k from (eqn. 1) into this equation.
- $a_{k+1} - 3(\frac{3^k + 5^k}{2}) = 5^k$
- $a_{k+1} = \frac{3^{(k+1)} + 5^{(k+1)}}{2}$
- Finally, it means that the statement holds for a_{k+1} and P(k+1) is true.
- By induction, our statement is true for all integers n≥1.