CENG 223

Discrete Computational Structures

Fall '2023-2024

Take Home Exam 5

Due date: January 10 2024, Wednesday , 23:55
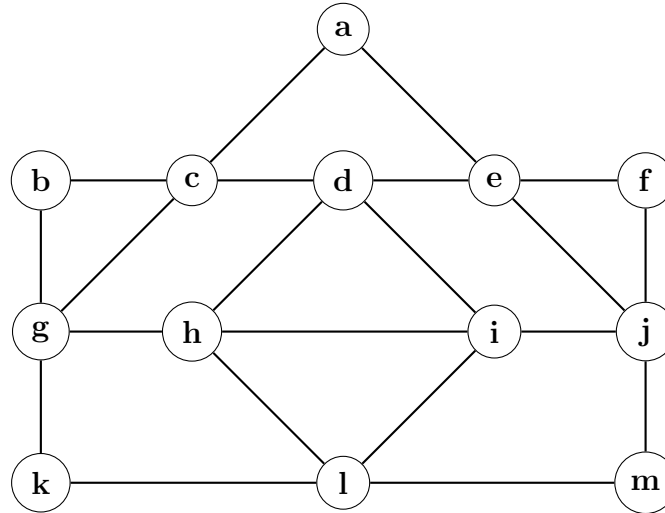
Question 1

Figure 1: Graph G in Q1.

Consider the graph G in Figure 1 to answer the following questions. Explain all the answers.

a) Is there an Eulerian circuit in G?

b) Is there an Eulerian path (that is not a circuit) in G?

c) Is there a Hamilton circuit in G?

d) Is there a Hamilton path (that is not a circuit) in G?

e) Determine the chromatic number of G, $\chi(G)$ and show your coloring on the graph given in Figure 1.

f) Is G bipartite? If your answer is yes, give the partioning of the set of vertices. If not, determine the min number of edges to be deleted from the set of edges in G to make it bipartite. List these edges.

g) Does G have a complete graph with at least four nodes as a subgraph? If yes, draw this subgraph. If no, state an edge which should be added to the graph in order for G to have a complete graph with at least four nodes.

Question 2

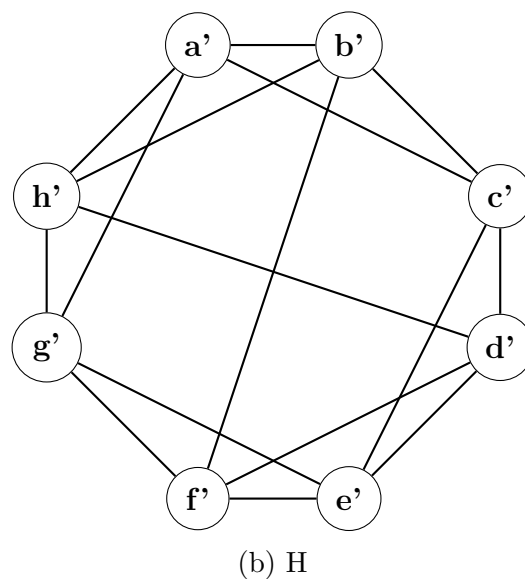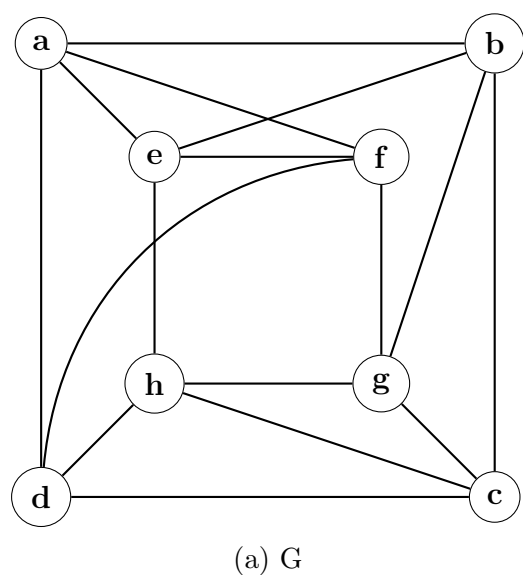Given the graphs G and H in Figure 2.

(a) G

(b) H

Figure 2: Graph G and H in Q2.

Determine whether G and H are isomorphic, or not. Explain your answer.

Question 3

Determine the graph chromatic number for each of the following graphs and also state whether it is bipartite, or not.

a) $n \geq 3$ C_n (cycle graph). Justify your answer.

b) $n \geq 1$ Q_n (cube graph). Justify your answer.

Question 4

Use either Kruskal's or Prim's algorithm to find a minimum spanning tree for the graph G given below (Figure 3). Please state the algorithm you choose at the beginning of your solution.
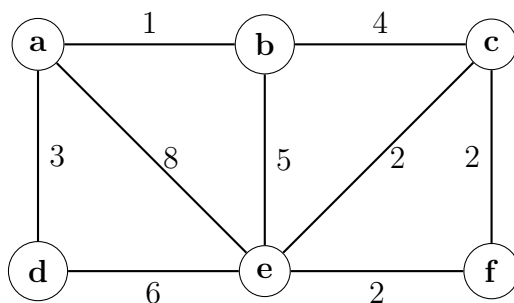

Figure 3: Graph G in Q4.

a) Write the order in which the edges are added to the tree.

b) Draw the minimum spanning tree.

c) Is the minimum spanning tree unique? Justify your answer.

Question 5

Explain all the answers.

a) Show that a full binary tree (a binary tree where each branch/internal vertex has exactly two children) with n vertices has $\frac{n+1}{2}$ leaf vertices.

b) Determine the chromatic number of a tree.

c) Give an upper bound on the height of a full m-ary tree with n vertices in terms of m and n.

1 Regulations

1. Your submission should be a single vector-based PDF document with the name "the5.pdf".

2. **Late Submission:** Not allowed!

3. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.

4. **Newsgroup:** You must follow the newsgroup (odtuclass.metu.edu.tr) for discussions and possible updates on a daily basis.

5. **Evaluation:** Your pdf file will be checked for plagiarism automatically using "black-box" technique and manually by assistants.

2 Submission

Submission will be done via odtuclass. You will submit a single PDF file, the5.pdf.