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Answer 1

- In order to construct Q_n, we can use two cube graphs of Q_{n-1}. Add 1 to binary string of one of them, 0 to binary string of other one. With this two cube graph, we have $2a_{n-1}$ edges. In addition there is new edges between the two cube graphs. The number of the new constructed edges is same with the number of nodes in the Q_{n-1} which is 2^{n-1}. Finally, we have $a_n = 2a_{n-1} + 2^{n-1}$, $n \geq 2$ and $a_1 = 1$.

Answer 2

- $< 1, 1, 1, 1, , ..., 1^n, ... > \longleftrightarrow \dfrac{1}{1-x}$ (Table 1 Section 8.4)
- $< 1, 2, 3, 4, ..., (n+1), ... > \longleftrightarrow \dfrac{1}{(1-x)^2}$ (Take derivative)
- $< 0, 1, 2, 3, , ..., n, ... > \longleftrightarrow \dfrac{x}{(1-x)^2}$ (Shift one right = multiply by x^1)
- $< 0, 3, 6, 9, ..., 3n, ... > \longleftrightarrow \dfrac{3x}{(1-x)^2}$ (Multiply by 3)
- $< 1, 4, 7, 10, ..., 3n+1, ... > \longleftrightarrow \dfrac{3x}{(1-x)^2} + \dfrac{1}{1-x} = \dfrac{1+2x}{(1-x)^2}$ (Summation with $< 1, 1, 1, 1, .. >$)
- The closed form of the desired sequence is $\dfrac{1+2x}{(1-x)^2}$

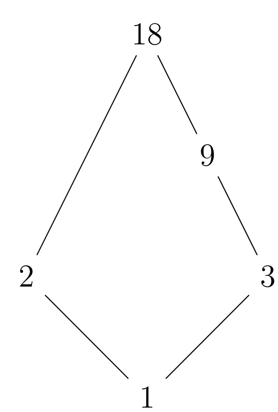
Answer 3

- $a_n = a_{n-1} + 2^n \ n \geq 1 \ a_0 = 1$
- $F(x) = \sum\limits_{n=0}^{\infty} a_n x^n$
- Let's write the equation:
- $\sum\limits_{n=1}^{\infty} a_n x^n = \sum\limits_{n=1}^{\infty} (a_{n-1} + 2^n) x^n$

- $\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} a_{n-1} x^n + \sum_{n=1}^{\infty} 2^n x^n$
- Substitute $\sum_{n=1}^{\infty} a_n x^n = F(x) - a_0$, $\sum_{n=1}^{\infty} 2^n x^n = \sum_{n=0}^{\infty} 2^n x^n$ - 2^0, $\sum_{n=0}^{\infty} 2^n x^n \longleftrightarrow \frac{1}{1-2x}$(Table 1 Section 8.4)
- $F(x) - a_0 = x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + \frac{1}{1-2x} - 1$
- Substitute $x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} = xF(x)$
- $F(x) - a_0 =$ xF(x) $+ \frac{1}{1-2x} - 1$
- $F(x) = \frac{1}{(1-2x)(1-x)}$
- By partial fractions, we can write F(x) in this form:
- $F(x) = \frac{2}{1-2x} + \frac{-1}{(1-x)}$
- $\frac{2}{1-2x} \longleftrightarrow < 2^1, 2^2, 2^3, 2^4, ..., 2^{n+1}, ... >$ (Table 1 Section 8.4 and multiple by 2)
- $\frac{-1}{1-x} \longleftrightarrow < -1, -1, -1, -1, ..., -(1)^n, ... >$ (Table 1 Section 8.4)
- $F(x) \longleftrightarrow < 2^1 - 1, 2^2 - 1, 2^3 - 1, 2^4 - 1, ..., 2^{n+1} - 1, ... >$
- Finally, the answer is $a_n = 2^{n+1} - 1$

Answer 4

a-) The hasse diagram of R:



b-) M_R :

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c-) In order to be lattice, it should be partial order relation and for every pair of elements in A there must be unique greatest lower bound and least upper bound. First check whether it is partial ordering or not.

- Reflexivity : $\forall x \in A$, $x|x$ because all numbers can divide themselves. It is reflexive.
- Antisymmetry : $(xRy) \wedge (yRx) \rightarrow x = y$. It is also satisfied because if $x|y$, it means that $y \geq x$, so $y \nmid$ x except $y = x$. It is antysymmetric also.
- Transitivity : The rule is $(aRb) \wedge (bRc) \rightarrow (aRc)$. If $a|b$ then $b = ka$ for some p $\in$ Z, $b|c$ then $c = tb$. We can infer that $c = tka$ and $a|c$. The requirement is satisfied. It is transitive.
- The relation is reflexive, antisymmetric and transitive. So, it is partial ordering.
- Let's look at the LUB's and GLB's of all distinct pair of elements in A.
  - 18 and 9 : LUB = 18 , GLB = 9
  - 18 and 3 : LUB = 18 , GLB = 3
  - 18 and 2 : LUB = 18 , GLB = 2
  - 18 and 1 : LUB = 18 , GLB = 1
  - 9 and 3 : LUB = 9 , GLB = 3
  - 9 and 2 : LUB = 18 , GLB = 1
  - 9 and 1 : LUB = 9 , GLB = 1
  - 3 and 2 : LUB = 18 , GLB = 1
  - 3 and 1 : LUB = 3 , GLB = 1
  - 2 and 1 : LUB = 2 , GLB = 1
- It can be seen that all pairs in A have unique greatest lower bound and unique least upper bound. It means that (A,R) is lattice.

d-) The symmetric closure of R = R $\cup$ R^{-1}. In the symmetric closure of R, the all pairs of entries which is diagonally symmetric to each other should be (1,1) or (0,0). For example, if the entry a_{ij} is 1, then a_{ji} must be 1 also. Thus, R_S is :

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

e-) 2 and 9 are not comparable because there are no elevation between them. We cannot reach 2 from 9 or 9 from 2. However, 3 and 18 are comparable because there is a path from 3 to 18. There are elevation and so they are comparable.

Answer 5

a-) For reflexivity: Diagonal entries must be 1. For symmetricity: There are $n^2 - n$ entries i.e., there are $\dfrac{n^2-n}{2}$ entry pairs which is diagonally symmetric to each other. These pairs must be (0,0) or (1,1) for symmetricity. So, there are $2^{\dfrac{n^2-n}{2}}$ different binary relations.

$$\begin{bmatrix} 1 & . & 0 & . & . \\ . & 1 & . & 1 & . \\ 0 & . & 1 & . & . \\ . & 1 & . & 1 & . \\ . & . & . & . & 1 \end{bmatrix}$$

b-) For reflexivity: Diagonal entries must be 1. For antisymmetricity: There are $n^2 - n$ entries i.e., there are $\frac{n^2-n}{2}$ entry pairs which is diagonally symmetric to each other. These pairs must be (0,1), (1,0) or (0,0) for antisymmetricity. So, there are $3^{\frac{n^2-n}{2}}$ different binary relations.

$$\begin{bmatrix} 1 & . & 0 & . & . \\ . & 1 & . & 1 & . \\ 0 & . & 1 & . & . \\ . & 0 & . & 1 & . \\ . & . & . & . & 1 \end{bmatrix}$$

Answer 6

- Transitive closure of R = $R^* = R \cup R^2 \cup ... \cup R^{n-1}$
- Let's prove this by contradiction.
- Let R = {(a,b),(b,c),(c,a)} which is antisymmetric.
- $R^* = R \cup R^2 =$ {(a,b),(b,c),(c,a),(a,c),(b,a),(c,b)}
- It can be seen that in the transitive closure of R, there are (a,b) and (b,a), (c,b) and (b,c). So, it is not antisymmetric.
- This is the counterexample for the statement.