# Distributed Computing and Introduction to High Performance Computing

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#### Introduction

- In communications, exchanged data have different datatypes: MPI\_INTEGER, MPI\_REAL, MPI\_COMPLEX, etc.
- We can create more complex data structures by using subroutines such as
- MPI\_TYPE\_CONTIGUOUS(), MPI\_TYPE\_VECTOR(), MPI\_TYPE\_INDEXED() or MPI\_TYPE\_CREATE\_STRUCT()
- Derived datatypes allow exchanging non-contiguous or non-homogenous data in the memory and limiting the number of calls to communications subroutines

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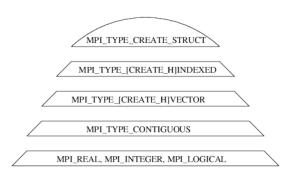


Figure: Hierarchy of the MPI constructors

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Contiguous datatypes : MPI\_TYPE\_CONTIGUOUS()

 MPI\_TYPE\_CONTIGUOUS() creates a data structure from a homogenous set of existing datatypes contiguous in memory.

```
1 MPI_TYPE_CONTIGUOUS(count,old_type,new_type,code)
2 
3 integer, intent(in) :: count, old_type
4 integer, intent(out) :: new_type,code
```

```
1 Datatype.Create_contiguous(self, int count)
```

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#### MPI\_TYPE\_COMMIT and MPI\_TYPE\_FREE()

 Before using a new derived datatype, it is necessary to validate it with the MPI\_TYPE\_COMMIT() subroutine.

```
1 MPI_TYPE_COMMIT(new_type,code)
2
3 integer, intent(inout) :: new_type
4 integer, intent(out) :: code
```

```
1 new_type.Commit()
```

The freeing of a derived datatype is made by using the MPI\_TYPE\_FREE() subroutine.

```
1 MPI_TYPE_FREE(new_type,code)
2 integer, intent(inout) :: new_type
3 integer, intent(out) :: code
```

```
1 new_type.Free()
```

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Contiguous datatypes : MPI\_TYPE\_CONTIGUOUS()

Example :

```
count = 5
    size = 10
 3
    type_ligne = MPI.DOUBLE.Create_contiguous(count)
    type ligne.Commit()
 6
7
    if rank=0:
8
        data = np.array([i for i in range(size)], dtype=np.float64)
g
         comm. Send([data.1.tvpe ligne].dest=1)
10
         print ("Original data", data, rank)
11
12
    elif rank == 1.
13
        data = -1*np.ones(size, dtype=np.float64)
14
         comm. Recv([data, 1, type_ligne], source=0)
15
         print ("Received data", data, rank)
16
17
   type_ligne.Free()
```

```
mpirun -n 2 python3 create_contiguous.py
Original data [0. 1. 2. 3. 4. 5. 6. 7. 8. 9.] 0
Received data [ 0. 1. 2. 3. 4. -1. -1. -1. -1. -1.] 1
```

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Constant stride: MPI\_TYPE\_VECTOR()

 MPI\_TYPE\_VECTOR() creates a data structure from a homogenous set of existing datatypes separated by a constant stride in memory. The stride is given in number of elements

```
1 MPI_TYPE_VECTOR(count,block_length,stride,old_type,new_type,code)
2 
3 integer, intent(in) :: count,block_length
4 integer, intent(in) :: stride
5 integer, intent(in) :: old_type
6 
7 integer, intent(out) :: new_type,code
```

```
Datatype.Create_vector(self, int count, int blocklength, int stride)
```

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#### Constant stride : MPI\_TYPE\_VECTOR()

Example :

```
stride = 2
    count = 5
    blocklen = 1
    size = 10
    type colum = MPI.DOUBLE.Create vector(count.blocklen.stride)
 7
    type colum.Commit()
 8
 9
    if rank=0:
10
        data = np.array([i for i in range(size)], dtype=np.float64)
11
        comm.Send([data,1,type_colum],dest=1)
12
         print ("Original data", data, rank)
13
    elif rank=1.
14
        data = -1*np.ones(size, dtype=np.float64)
15
        comm.Recv([data,1,type_colum],source=0)
16
         print ("Received data", data, rank)
17
18
    type_colum.Free()
```

```
mpirun -n 2 python3 create_vector.py
Original data [0. 1. 2. 3. 4. 5. 6. 7. 8. 9.] 0
Received data [0. -1. 2. -1. 4. -1. 6. -1. 8. -1.] 1
```

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#### Constant stride: MPI\_TYPE\_CREATE\_HVECTOR()

- MPI\_TYPE\_CREATE\_HVECTOR() creates a data structure from a homogenous set of existing datatype separated by a constant stride in memory. The stride is given in bytes.
- This call is useful when the old type is no longer a base datatype (MPI\_INTEGER, MPI\_REAL,...) but a more complex datatype constructed by using MPI subroutines, because in this case the stride can no longer be given in number of elements.

Datatype.Create\_hvector(self, int count, int blocklength, Aint stride)

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#### Homogenous datatypes of variable strides

- MPI\_TYPE\_INDEXED() allows creating a data structure composed of a sequence of blocks containing a variable number of elements separated by a variable stride in memory. The stride is given in number of elements.
- MPI\_TYPE\_CREATE\_HINDEXED() has the same functionality as MPI\_TYPE\_INDEXED() except that the strides separating two data blocks are given in bytes. This subroutine is useful when the old datatype is not an MPI base datatype(MPI\_INTEGER, MPI\_REAL, ...). We cannot therefore give the stride in number of elements of the old datatype.
- For MPI\_TYPE\_CREATE\_HINDEXED(), as for MPI\_TYPE\_CREATE\_HVECTOR(), use MPI\_TYPE\_SIZE() or MPI\_TYPE\_GET\_EXTENT() in order to obtain in a portable way the size of the stride in bytes.

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Homogenous datatypes of variable strides: MPI\_TYPE\_INDEXED()

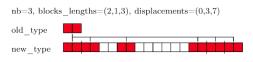


Figure: The MPI\_TYPE\_INDEXED constructor

Datatype.Create\_indexed(self, blocklengths, displacements)

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Homogenous datatypes of variable strides: MPI\_TYPE\_INDEXED()

Example :

```
size = 10
    count = 3
    counts = [2, 1, 3]
    displacements = [0, 3, 7]
    indexedtype = MPI.INT64_T.Create_indexed(counts, displacements)
    indexedtype.Commit()
 g
10
    if rank=0:
11
        data = np.array([i for i in range(size)], dtype=np.float64)
12
        comm. Send ([data.1.indexedtype].dest=1)
13
         print ("Original data", data, rank)
14
    elif rank=1:
15
         data = -1*np.ones(size. dtvpe=np.float64)
        comm. Recv([data.1.indexedtype].source=0)
16
17
         print ("Received data", data, rank)
18
19
    indexedtype.Free()
```

```
mpirun -n 2 python3 create_indexed.py
Original data [0. 1. 2. 3. 4. 5. 6. 7. 8. 9.] 0
Received data [0. 1. -1. 3. -1. -1. 7. 8. 9.] 1
```

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Homogenous datatypes of variable strides: MPI\_TYPE\_CREATE\_HINDEXED()



Figure: The MPI\_TYPE\_CREATE\_HINDEXED constructor

Datatype.Create\_hindexed(self, blocklengths, displacements)

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Homogenous datatypes of variable strides: MPI\_TYPE\_INDEXED()

Example : triangular matrix

In the following example, each of the two processes :

- 1. Initializes its matrix (positive growing numbers on process 0 and negative decreasing numbers on process 1).
- Constructs its datatype: triangular matrix (superior for the process 0 and inferior for the process 1).
- 3. Sends its triangular matrix to the other process and receives back a triangular matrix which it stores in the same place which was occupied by the sent matrix.
- 4. Frees its resources.

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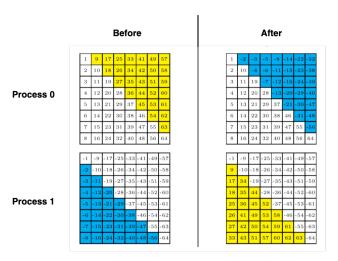


Figure: – Exchange between the two processes

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Homogenous datatypes of variable strides: MPI\_TYPE\_INDEXED()

Example :

```
from mpi4py import MPI
     import numpy as np
     comm = MPI.COMM_WORLD
     nb_procs = comm.Get_size()
    rank = comm.Get rank()
    n = 8; sign = -1
     if rank = 0: sign = 1
     a = [sign*i for i in range(1,n*n+1,1)]
10
     Matrix = np.array(a)
11
     Matrix = np.reshape(Matrix. (n.n)).transpose()
12
13
     if rank == 0:
14
         displacements = [n*i for i in range(n)]
15
         block lengths = [i \text{ for } i \text{ in } range(n)]
16
     else:
17
         displacements = [n*i+i+1 \text{ for } i \text{ in } range(n)]
         block_lengths = [n-i-1 \text{ for } i \text{ in } range(n)]
18
19
20
     type triangle = MPI.DOUBLE.Create indexed(block lengths, displacements)
21
     type triangle.Commit()
22
23
     num_proc = (rank+1)\%2
24
     comm. Send ([Matrix.1.tvpe triangle].dest=num proc)
25
     comm.Recv([Matrix,1,type_triangle],source=num_proc)
26
27
     type triangle. Free()
28
     print (Matrix . rank)
```

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Homogenous datatypes of variable strides: MPI\_TYPE\_INDEXED()

Example : Matrix after permutation

```
mpirun -n 2 python3 matrixExchange.py
      -2 -3 -5 -8 -14 -22 -32]
     10 -4 -6 -11 -15 -23 -38]
    11 19 -7 -12 -16 -24 -391
     12 20 28 -13 -20 -29 -40]
     13 21
           29
                37 -21 -30 -47]
    14 22
           30 38 46 -31 -481
  7 15 23 31
                39 47 55 -56]
     16 24
            32 40 48 56 64]] 0
  -1 -9 -17 -25 -33 -41 -49 -57]
  9 -10 -18 -26 -34 -42 -50 -58]
     34 -19 -27 -35 -43 -51 -591
Γ 18
     35 44 -28 -36 -44 -52 -60]
Γ 25
     36 45 52 -37 -45 -53 -61]
Γ 26
     41 49 53 58 -46 -54 -62]
[ 27 42 50 54
                59 61 -55 -63]
Γ 33
     43 51 57
                60
                    62 63 -64]] 1
```

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#### Introduction

The purpose of communicators is to create subgroups on which we can carry out operations such as collective or point-to-point communications. Each subgroup will have its own communication space.

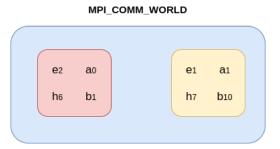


Figure: Communicator partitioning

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#### Example

For example, we want to broadcast a collective message to even-ranked processes and another message to odd-ranked processes.

- Looping on send/recv can be very detrimental especially if the number of processes is high. Also a test inside the loop would be compulsory in order to know if the sending process must send the message to an even or odd process rank.
- A solution is to create a communicator containing the even-ranked processes, another containing the odd-ranked processes, and initiate the collective communications inside these groups.

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#### Default communicator

- A communicator can only be created from another communicator. The first one will be created from the MPI\_COMM\_WORLD.
- After the MPI\_INIT() call, a communicator is created for the duration of the program execution.
- Its identifier MPI\_COMM\_WORLD is an integer value defined in the header files.
- This communicator can only be destroyed via a call to MPI\_FINALIZE().
- By default, therefore, it sets the scope of collective and point-to-point communications to include all the processes of the application.

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#### Groups and communicators

- A communicator consists of :
  - ☐ A group, which is an ordered group of processes.
  - A communication context put in place by calling one of the communicator construction subroutines, which allows determination of the communication space.
- The communication contexts are managed by MPI (the programmer has no action on them: It is a hidden attribute).
- In the MPI library, the following subroutines exist for the purpose of building communicators: MPI\_COMM\_CREATE(), MPI\_COMM\_DUP(), MPI\_COMM\_SPLIT()
- The communicator constructors are collective calls.
- Communicators created by the programmer can be destroyed by using the MPI\_COMM\_FREE() subroutine.

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#### Partitioning of a communicator

In order to solve the problem example :

- Partition the communicator into odd-ranked and even-ranked processes.
- Broadcast a message inside the odd-ranked processes and another message inside the even-ranked processes.

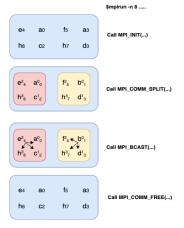


Figure: Communicator creation/destruction

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#### Partitioning of a communicator with MPI\_COMM\_SPLIT()

The MPI\_COMM\_SPLIT() subroutine allows :

- Partitioning a given communicator into as many communicators as we want.
- Giving the same name to all these communicators: The process value will be the value of its communicator.
- Method :
  - 1. Define a colour value for each process, associated with its communicator number.
  - 2. Define a key value for ordering the processes in each communicator
  - 3. Create the partition where each communicator is called new\_comm

```
1     MPI_COMM_SPLIT(comm,color,key,new_comm,code)
2     integer, intent(in) :: comm, color, key
4     integer, intent(out) :: new_comm, code
```

```
Comm.Split(self, int color=0, int key=0)
```

A process which assigns a color value equal to MPI\_UNDEFINED will have the invalid communicator MPI\_COMM\_NULL for new\_com.

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Example

Let's look at how to proceed in order to build the communicator which will subdivide the communication space into odd-ranked and even-ranked processes via the MPI\_COMM\_SPLIT() constructor.

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Partitioning of a communicator with MPI\_COMM\_SPLIT() Example :

```
1
    world rank = comm.Get rank()
    world_size = comm.Get_size()
    m = 5
    a = np.zeros(m)
 8
    if world rank=2: a[:] = 2.
    if (world rank=5): a[:] = 5.
10
11
    key = world_rank
12
    if world rank=2 or world rank=5 :
13
        key=-1
14
    color = world rank%2
15
16
    newcomm = comm.Split(color. kev)
17
18
    row rank = newcomm.Get rank()
19
    row size = newcomm. Get size()
20
21
    print("WORLD RANK/SIZE: {RANK}/{SIZE} \t ROW RANK/SIZE: {ROW_RANK}/{ROW_SIZE}". ←
          format(RANK=world rank, SIZE=world size, ROW RANK=row rank, ROW SIZE=↔
          row_size))
22
23
    newcomm.Bcast(a. root=0)
24
25
    newcomm.Free()
```

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Partitioning of a communicator with MPI\_COMM\_SPLIT()

#### Results:

```
mpirun -n 6 python3 split_communicator.py
```

```
WORLD RANK/SIZE: 0/6 ROW RANK/SIZE: 1/3
WORLD RANK/SIZE: 1/6 ROW RANK/SIZE: 1/3
WORLD RANK/SIZE: 2/6 ROW RANK/SIZE: 0/3
WORLD RANK/SIZE: 3/6 ROW RANK/SIZE: 2/3
WORLD RANK/SIZE: 4/6 ROW RANK/SIZE: 2/3
WORLD RANK/SIZE: 5/6 ROW RANK/SIZE: 0/3
```

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#### Communicator built from a group

- We can also build a communicator by defining a group of processes: Call to MPI\_COMM\_GROUP(), MPI\_GROUP\_INCL(), MPI\_COMM\_CREATE(), MPI\_GROUP\_FREE()
- This process is however far more cumbersome than using MPI\_COMM\_SPLIT() whenever possible.

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#### **Topologies**

- In most applications, especially in domain decomposition methods where we match the calculation domain to the process grid, it is helpful to be able to arrange the processes according to a regular topology.
- MPI allows defining virtual cartesian or graph topologies.
  - Cartesian topologies :
    - I Each process is defined in a grid.
    - I Each process has a neighbour in the grid.
    - I The grid can be periodic or not.
    - I The processes are identified by their coordinates in the grid.
  - ☐ Graph topologies :
    - I Can be used in more complex topologies.

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#### Cartesian topologies

- A Cartesian topology is defined from a given communicator named comm\_old, calling the MPI\_CART\_CREATE() subroutine.
- We define :
  - ☐ An integer ndims representing the number of grid dimensions.
  - An integer array dims of dimension ndims showing the number of processes in each dimension.
  - ☐ An array of ndims logicals which shows the periodicity of each dimension.
  - $\ \square$  A logical reorder which shows if the process numbering can be changed by MPI.

```
1 MPI_CART_CREATE(comm_old, ndims,dims,periods,reorder,comm_new,code
    )
2
3 integer, intent(in) :: comm_old, ndims
4 integer, dimension(ndims),intent(in) :: dims
5 logical, dimension(ndims),intent(in) :: periods
6 logical, intent(in) :: reorganization
7
8 integer, intent(out) :: comm_new, code
```

```
Intracomm.Create_cart(self, dims, periods=None, bool reorder=False)
```

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#### 2D Example

Example on a grid having 4 domains along x and 2 along y, periodic in y.

```
from mpi4py import MPI
 2
    comm = MPI.COMM WORLD
    nb procs = comm.Get size()
    rank = comm.Get_rank()
    periods = tuple([False.False])
    reorder = False
    dims = [2,2]
10
11
12
    cart2d = comm.Create_cart(
13
    dims
           = dims.
14
    periods = periods.
15
    reorder = reorder
16
```

- If reorder = .false. then the rank of the processes in the new communicator (comm\_2D) is the same as in the old communicator (MPI\_COMM\_WORLD).
- If reorder = .true., the MPI implementation chooses the order of the processes.

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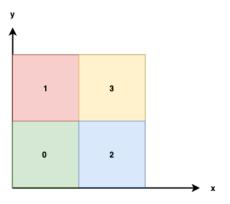


Figure: A 2D non-periodic Cartesian topology

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#### 3D Example

Example on a 3D grid having 2 domains along x, 2 along y and 2 along z, non periodic.

```
from mpi4py import MPI
    comm = MPI.COMM_WORLD
    nb_procs = comm.Get_size()
    rank = comm.Get_rank()
    periods = tuple([False, False, False])
    reorder = False
    dims = [2, 2, 2]
10
11
12
    cart3 = comm.Create_cart(
13
    dims = dims.
14
    periods = periods,
15
    reorder = reorder
16
```

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#### 3D Example



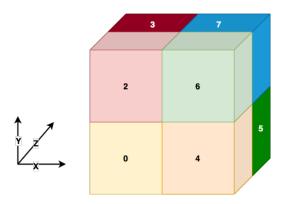


Figure: A 3D non-periodic Cartesian topology

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#### Process distribution

The MPI\_DIMS\_CREATE() subroutine returns the number of processes in each dimension of the grid according to the total number of processes.

Remark: If the values of dims in entry are all 0, then we leave to MPI the choice of the number of processes in each direction according to the total number of processes.

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#### Rank od a process

In a Cartesian topology, the MPI\_CART\_RANK() subroutine returns the rank of the associated process to the coordinates in the grid.

```
1 MPI_CART_RANK(comm,coords,rank,code)
2
3 integer, intent(in) :: comm
4 integer, dimension(ndims),intent(in) :: coords
5 integer, intent(out) :: rank, code
```

```
Cartcomm.Get_cart_rank(self, coords)
```

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#### Coordinates of a process

In a cartesian topology, the MPI\_CART\_COORDS() subroutine returns the coordinates of a process of a given rank in the grid.

```
Cartcomm.Get_coords(self, int rank)
```

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Example: MPI\_CART\_COORDS()

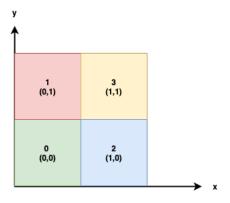


Figure: A 2D non-periodic Cartesian topology

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#### Example : MPI\_CART\_COORDS()

```
from mpi4py import MPI
    comm = MPI.COMM_WORLD
    rank = comm. Get rank()
    periods = tuple([False, False])
    reorder = False
    dims = [2, 2]
10
    cart2d = comm.Create cart(
11
    dims
           = dims.
12
    periods = periods,
13
    reorder = reorder
14
15
16
    coord2d = cart2d.Get_coords(rank)
17
18
    print("l'm rank", rank, "my 2d coords are", coord2d)
```

# mpirun -n 4 python3 coordinate\_2d\_cart.py I'm rank 0 my 2d coords are [0, 0] I'm rank 1 my 2d coords are [0, 1] I'm rank 2 my 2d coords are [1, 0] I'm rank 3 my 2d coords are [1, 1]

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#### Rank of neighbours

In a Cartesian topology, a process that calls the MPI\_CART\_SHIFT() subroutine can obtain the rank of a neighboring process in a given direction.

```
Cartcomm.Shift(self, int direction, int disp)
```

- The direction parameter corresponds to the displacement axis (xyz).
- The step parameter corresponds to the displacement step.
- If a rank does not have a neighbor before (or after) in the requested direction, then the value of the previous (or following) rank will be MPI\_PROC\_NULL.

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Example : MPI\_CART\_SHIFT()

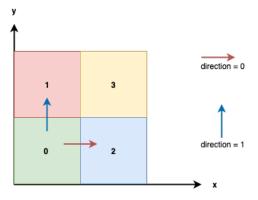


Figure: Call of the MPI\_CART\_SHIFT() subroutine

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#### Example : MPI\_CART\_SHIFT()

```
from mpi4pv import MPI
    comm = MPI.COMM_WORLD
    rank = comm.Get_rank()
    periods = tuple([False,False])
    reorder = False
    dims = [2,2]
10
    cart2d = comm.Create_cart(
11
    dims = dims.
12
    periods = periods.
13
    reorder = reorder
14
15
16
    left,right = cart2d.Shift(direction = 0, disp=1)
17
    low, high = cart2d. Shift (direction = 1, disp=1)
18
19
    print("I'm rank", rank, "my (left, right) neighbours are",( left, right),
20
    "my (low, high) neighbours are", (low, high))
```

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Example : MPI\_CART\_SHIFT()

```
mpirun -n 4 python3 neighbours_2d_cart.py

I'm rank 0
my (left,right) neighbours are (-2, 2) my (low,high) neighbours are (-2, 1)

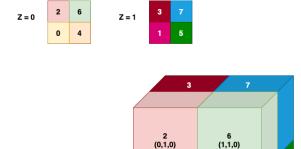
I'm rank 1
my (left,right) neighbours are (-2, 3) my (low,high) neighbours are (0, -2)

I'm rank 2
my (left,right) neighbours are (0, -2) my (low,high) neighbours are (-2, 3)

I'm rank 3
my (left,right) neighbours are (1, -2) my (low,high) neighbours are (2, -2)
```

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3D Example: coordinates and neighbours



(0,0,0) (1,0,0)

Figure: 3D Coordinates and Neighbours

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#### 3D Example: coordinates and neighbours

```
from mpi4py import MPI
    comm = MPI.COMM WORLD
    rank = comm.Get_rank()
    periods = tuple([False,False,False])
    reorder = False
    dims = [2, 2, 2]
10
    cart3d = comm.Create_cart(
11
    dims
           = dims,
12
    periods = periods.
13
    reorder = reorder
14
15
    coord3d = cart3d.Get_coords(rank)
left,right = cart3d.Shift(direction = 0, disp=1);
16
17
18
    low, high
                    = cart3d.Shift(direction = 1, disp=1)
19
                    = cart3d.Shift(direction = 2. disp=1)
    ahead.before
20
21
    print("I m rank", rank, "my 3d coords are", coord3d, "my (left, right) neighbours ↔
          are",(left, right), "my (low, high) neighbours are", (low, high), "my (ahead ←
          , before ) neighbours are", (ahead, before))
```

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3D Example: coordinates and neighbours

```
mpirun -n 8 --oversubscribe python3 create_3d_cart.py
I'm rank 0 my 3d coords are [0, 0, 0]
my (left, right) neighbours are (-2, 4)
my (low, high) neighbours are (-2, 2)
my (ahead, before) neighbours are (-2, 1)
I'm rank 1 my 3d coords are [0, 0, 1]
my (left, right) neighbours are (-2, 5)
my (low, high) neighbours are (-2, 3)
my (ahead, before) neighbours are (0, -2)
I'm rank 2 my 3d coords are [0, 1, 0]
my (left, right) neighbours are (-2, 6)
my (low, high) neighbours are (0, -2)
my (ahead, before) neighbours are (-2, 3)
. . .
```

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