HIGHER SECONDARY FIRST YEAR
MATHEMATICS
MODEL QUESTION PAPER

Time Allowed: 2.30 Hours] **[Maximum Marks:90**

Instructions: (a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

SECTION – I

Note: (i) **All** questions are **compulsory.** $20\times1=20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

1. If two sets A and B have 17 elements in common, then the number of elements common to the set $A\times B$ and $B\times A$ is
   (1) 2^{17} (2) 17^2 (3) 34 (4) insufficient data
2. If $\mathbb{R}$ is the set of all real numbers and if $f:\mathbb{R}-\{3\}\to\mathbb{R}$ is defined by $f(x)=\dfrac{3+x}{3-x}$ for $x\in\mathbb{R}-\{3\}$, then the range of f is
   (1) $\mathbb{R}$ (2) $\mathbb{R}-\{1\}$ (3) $\mathbb{R}-\{-1\}$ (4) $\mathbb{R}-\{-3\}$
3. If the sum and product of the roots of the equation $2x^2+(a-3)x+3a-5=0$ are equal, then the value of a is
   (1) 1 (2) 2 (3) 0 (4) 4
4. Which one of the following is not true?
   (1) $|\sin x|\le 1$ (2) $|\sec x|<1$
   (3) $|\cos x|\le 1$ (4) $\operatorname{cosec} x\ge 1$ or $\operatorname{cosec} x\le -1$
5. $\cos 1^\circ+\cos 2^\circ+\cos 3^\circ+\cdots+\cos 179^\circ$ is
   (1) 0 (2) 1 (3) -1 (4) 89
6. If 10 lines are drawn in a plane such that no two of them are parallel and no three are concurrent,then the total number of points of intersection are
   (1) 45 (2) 40 (3) 10! (4) 2^{10}
7. The remainder when 2^{2020} is divided by 15 is
   (1) 4 (2) 8 (3) 1 (4) 2
8. The harmonic mean of two positive numbers whose arithmetic mean and geometric mean are 16, 8 respectively is
   (1) 10 (2) 6 (3) 5 (4) 4
9. In the equation of a straight line $ax+by+c=0$, if a,b,c are in arithmetic progression then the point on the straight line is
   (1) $(1,2)$ (2) $(1,-2)$ (3) $(2,-1)$ (4) $(2,1)$
10. If two straight lines $x+(2k-7)y+3=0$ and $3kx+9y-5=0$ are perpendicular to each other then the value of k is
    (1) 3 (2) $\dfrac{1}{3}$ (3) $\dfrac{2}{3}$ (4) $\dfrac{3}{2}$

11. If $|\vec{a}|=13$, $|\vec{b}|=5$ and $\vec{a}\cdot\vec{b}=60^\circ$ then $|\vec{a}\times\vec{b}|$ is

(1) 15 (2)35 (3)45 (4) 25

12. A vector $\overrightarrow{OP}$ makes 60° and 45° with the positive direction of the x and y axes respectively. Then the angle between $\overrightarrow{OP}$ and the z-axis is

(1) 45° (2) 60° (3) 90° (4) 30°

13. A vector perpendicular to both $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$ is,

(1) $2\hat{i}+\hat{j}-\hat{k}$ (2) $2\hat{i}-\hat{j}-\hat{k}$ (3) $3\hat{i}+\hat{j}+2\hat{k}$ (4) $3\hat{i}+\hat{j}-2\hat{k}$

14. $\lim_{x\to 0}\frac{\sin|x|}{x}$ is

(1) 1 (2) -1 (3)0 (4) does not exist

15. If $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=\lfloor x-3\rfloor+|x-4|$, $x\in\mathbb{R}$, then $\lim_{x\to 3^-}f(x)$ is equal to

(1) -2 (2) -1 (3) 0 (4) 1

16. If $f(x)=\begin{cases} x^3, & x<0 \\ 3a+x^2, & x\geq 0\end{cases}$ is continuous at $x=0$, then a is

(1) -2 (2) -1 (3) 0 (4) 1

17. The derivative of $f(x)=x|x|$ at $x=-3$ is

(1) 6 (2) -6 (3) does not exist (4) 0

18. $\int\frac{dx}{x(x+1)}$ is

(1) $\log\left|\frac{x+1}{x}\right|+c$ (2) $\log\left|\frac{x}{x+1}\right|+c$ (3) $\log\left|\frac{x-1}{x}\right|+c$ (4) $\log\left|\frac{x}{x-1}\right|+c$

19. $\int 2^{3x+5}dx$ is

(1) $\frac{3(2^{3x+5})}{\log 2}+c$ (2) $\frac{2^{3x+5}}{2\log(3x+5)}+c$ (3) $\frac{2^{3x+5}}{2\log 3}+c$ (4) $\frac{2^{3x+5}}{3\log 2}+c$

20. If X and Y be two events such that $P(X/Y)=\frac{1}{2}$, $P(Y/X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$, then $P(X\cup Y)$ is

(1) $\frac{1}{3}$ (2) $\frac{2}{5}$ (3) $\frac{1}{6}$ (4) $\frac{2}{3}$

SECTION – II

Note: (i) Answer any **SEVEN** questions. $7\times 2=14$

(ii) Question number **30 is compulsory.**

21. From the graph $y=\cos x$, draw $|y|=\cos x$.

22. If $\frac{\log(x)}{y-z}=\frac{\log(y)}{z-x}=\frac{\log(z)}{x-y}$, then prove that $xyz=1$.

23. Show that $\tan(45^\circ-A)=\frac{1-\tan A}{1+\tan A}$

24. How many ways are there to arrange the letters of the word "GARDEN" with vowels in the alphabetical order.

25. Find the sum $1+\frac{4}{5}+\frac{7}{25}+\frac{10}{125}+\cdots$

26. Show that the points whose position vectors are $2\hat{i}+3\hat{j}-5\hat{k}$, $3\hat{i}+\hat{j}-2\hat{k}$ and $6\hat{i}-5\hat{j}+7\hat{k}$ are collinear.

27. Examine the continuity of the function $\dfrac{x^2-16}{x+4}$

28. Find the derivative of $y=\log_{10}x$ with respect to x.

29. Evaluate: $\displaystyle\int \frac{\sin x}{1+\cos x}dx$

30. If $A=\begin{bmatrix} 4 & 2 \\ -1 & x \end{bmatrix}$ and $(A-2I)(A-3I)=O$, find the value of x.

SECTION – III

Note: (i) Answer any **SEVEN** questions. $7\times3=21$

(ii) Question number **40 is compulsory.**

31. Check the relation $R=\{(1,1),(2,2),(3,3),\ldots,(n,n)\}$ defined on the set $S=\{1,2,3,\ldots,n\}$ for the three basic relations.

32. Prove that $\dfrac{\cot(180^\circ+\theta)\sin(90^\circ-\theta)\cos(-\theta)}{\sin(270^\circ+\theta)\tan(-\theta)\operatorname{cosec}(360^\circ+\theta)}=\cos^2\theta\cot\theta$.

33. In an examination a student has to answer 5 questions out of 9 questions, in which 2 are compulsory. In how many ways a student can answer the questions?

34. Find the coefficient of x^{15} in $\left(x^2+\dfrac{1}{x^3}\right)^{10}$.

35. Find the equations of the straight lines, making the y-intercept of 7 and angle between the line and the y-axis is 30°.

36. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}=(x-y)(y-z)(z-x)$.

37. If $\vec{a},\vec{b}$ and $\vec{c}$ are vectors with magnitudes 3,4 and 5 respectively and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

38. Evaluate : $\int x\log x\,dx$.

39. If A and B are mutually exclusive events $P(A)=\dfrac{3}{8}$ and $P(B)=\dfrac{1}{8}$, then find

(i) $P(\overline{A})$ (ii) $P(A\cup B)$ (iii) $P(\overline{A}\cap B)$

40. Evaluate : $\displaystyle\lim_{x\to0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$.

SECTION – IV

Note: Answer **all** the questions. $7\times5=35$

41. (a) If $f,g:\mathbb{R}\to\mathbb{R}$ are defined by $f(x)=|x|+x$ and $g(x)=|x|-x$, find $g\circ f$ and $f\circ g$.

(OR)

(b) Solve the linear inequalities and exhibit the solution set graphically:

$x+y\geq 3, 2x-y\leq 5, -x+2y\leq 3$.

42. (a) If $A+B+C=\pi$, prove that $\cos A+\cos B+\cos C=1+4\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$

(OR)

(b) In a ΔABC, prove that $a\cos A+b\cos B+c\cos C=2a\sin B\sin C$.

43. (a) Prove by the principle of mathematical induction, the sum of the first n non-zero even numbers is n^2+n.

(OR)

(b) The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2^{nd} hour, 4^{th} hour and the n^{th} hour?

44. (a) Show that $\begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}=0$

(OR)

(b) Show that the vectors $\hat{i}-2\hat{j}+3\hat{k}, -2\hat{i}+3\hat{j}-4\hat{k}, -\hat{j}+2\hat{k}$ are coplanar.

45. (a) Describe the interval(s) on which the function $h(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$ is continuous.

(OR)

(b) If $\sin y=x\sin(a+y)$, then prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}, a\neq n\pi$.

46. (a) Using the substitution $2x+1=t^2$, show that $\int \frac{6x}{\sqrt{2x+1}}dx = 2(x-1)\sqrt{2x+1}+c$.

(OR)

(b) A construction company employs 2 executive engineers. Engineer-1 does the work for 60% of jobs of the company. Engineer-2 does the work for 40% of jobs of the company. It is known from the past experience that the probability of an error when engineer-1 does the work is 0.03, whereas the probability of an error in the work of engineer-2 is 0.04. Suppose a serious error occurs in the work, which engineer would you guess did the work?

47. (a) At a particular moment, a student needs to stop his speedybike to avoid a collision with the barrier ahead at a distance 40 metres away from him. Immediately he slows (retardation) the bike under braking at a rate of 8 metre/second^2. If the bike is moving at a speed of 24m/s,when the brakes are applied, would it stop before collision?

(OR)

(b) Find the separate equations of the pair of straight lines $2x^2-xy-3y^2-6x+19y-20=0$.

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HIGHER SECONDARY FIRST YEAR

MATHEMATICS

MODEL QUESTION PAPER - SOLUTIONS

PART - I

(1) (2) 17^2

(2) (3) $R-\{-1\}$

(3) (2) 2

(4) (2) $|\sec x|<1$

(5) (1) 0

(6) (1) 45

(7) (3) 1

(8) (4) 4

(9) (2) $(1,-2)$

(10) (1) 3

(11) (4) 25

(12) (2) 60°

(13) (2) $2\hat{i}-\hat{j}-\hat{k}$

(14) (4) Does not exist

(15) (3) 0

(16) (3) 0

(17) (1) 6

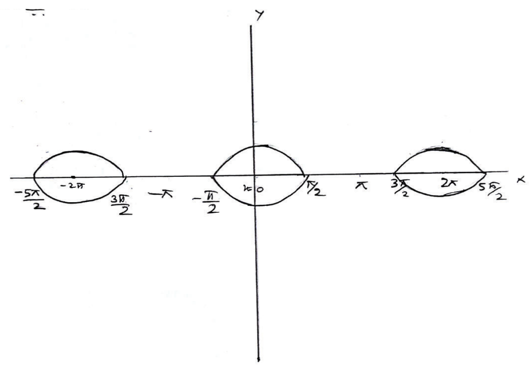
(18) (2) $\log\left|\frac{x}{x+1}\right|$

(19) (4) $\frac{2^{3x+5}}{3\log 2}+c$

(20) (4) $\frac{2}{3}$

PART - II

(21) $|y| = \cos x$



(22) Let $\dfrac{\log x}{y-z}=\dfrac{\log y}{z-x}=\dfrac{\log z}{x-y}=k$

$$\log x = k(y-z) \Rightarrow x = e^{k(y-z)} \quad \text{... (1)}$$

$$\log y = k(z-x) \Rightarrow y = e^{k(z-x)} \quad \text{... (2)}$$

$$\log z = k(x-y) \Rightarrow z = e^{k(x-y)} \quad \text{... (3)}$$

$(1)\times(2)\times(3)$

$$\begin{aligned} xyz &= e^{k(y-z)} \times e^{k(z-x)} \times e^{k(x-y)} \\ &= e^{k(y-z+z-x+x-y)} \\ &= e^{0} = 1 \\ xyz &= 1 \end{aligned}$$

(23)
$$\begin{aligned} \tan(45^\circ - A) &= \frac{\tan 45^\circ - \tan A}{1+\tan 45^\circ \tan A} \\ &= \frac{1-\tan A}{1+\tan A} \end{aligned}$$

(24)
$$\begin{aligned} \text{The required no. of ways} &= \frac{6!}{2!} \\ &= \frac{6\times5\times4\times3\times2!}{2!} \\ &= 360 \end{aligned}$$

(25)
$$\begin{aligned} S &= 1+\frac{4}{5}+\frac{7}{25}+\frac{10}{125}+\cdots \\ a &= 1, d=3, r=\frac{1}{5} \\ s_\infty &= \frac{a}{1-r}+\frac{dr}{(1-r)^2} \\ &= \frac{1}{1-\frac{1}{5}}+\frac{3\times\frac{1}{5}}{\left(1-\frac{1}{5}\right)^2}=\frac{5}{4}+\frac{3}{5}\times\frac{25}{16}=\frac{35}{16} \end{aligned}$$

(26)
$$\begin{aligned} \overrightarrow{OA} &= 2\hat{i}+3\hat{j}-5\hat{k} \\ \overrightarrow{OB} &= 3\hat{i}+\hat{j}-2\hat{k} \\ \overrightarrow{OC} &= 6\hat{i}-5\hat{j}+7\hat{k} \end{aligned}$$

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{OB}-\overrightarrow{OA}=\hat{i}-2\hat{j}+3\hat{k}\\ \overrightarrow{BC} &= \overrightarrow{OC}-\overrightarrow{OB}=3\hat{i}-6\hat{j}+9\hat{k}\\ &= 3\left(\hat{i}-2\hat{j}+3\hat{k}\right)\\ \overrightarrow{BC} &= 3\overrightarrow{AB}\end{aligned}$$

Therefore, $\overrightarrow{BC}$ and $\overrightarrow{AB}$ are parallel vectors but B is the Common point
Hence, A,B,C are collinear

(27) $f(x)=\dfrac{x^2-16}{x+4}$

$f(x)$ is not defined at $x=-4$

Therefore $f(x)$ is continuous for all $x\in R-\{-4\}$

(28)
$$\begin{aligned} y &= \log_{10}^{x}=\log_{e}^{x}\log_{10}^{e}\\ \frac{dy}{dx} &= \log_{10}^{e}\frac{1}{x}\\ \frac{dy}{dx} &= \frac{\log_{10}^{e}}{x}\end{aligned}$$

(29) $\displaystyle\int\frac{\sin x}{1+\cos x}dx$

$\displaystyle=\int\frac{-du}{u}=-\log u+c$

$=-\log|1+\cos x|+c$

put
$1+\cos x=u$
$-\sin x\,dx=du$
$\sin x dx=-du$

(30) $A=\begin{bmatrix}4 & 2\\ -1 & x\end{bmatrix}$

$$\begin{aligned}(A-2I)(A-3I) &= \left[\begin{pmatrix}4 & 2\\ -1 & x\end{pmatrix}-\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}\right]\times\left[\begin{pmatrix}4 & 2\\ -1 & x\end{pmatrix}-\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}\right]\\ &= \begin{pmatrix}2 & 2\\ -1 & x-2\end{pmatrix}\begin{pmatrix}1 & 2\\ -1 & x-3\end{pmatrix}\end{aligned}$$

Given $(A-2I)(A-3I)=0$

$$\begin{pmatrix}2 & 2\\ -1 & x-2\end{pmatrix}\begin{pmatrix}1 & 2\\ -1 & x-3\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

$$\begin{pmatrix}2-2 & 4+2(x-3)\\ -1-(x-2) & -2+(x-2)(x-3)\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

$$\begin{aligned}2x-2 &= 0\\ x &= 1\end{aligned}$$

PART - III

(31) $R=\{(1,1)\ (2,2)\cdots\ (n,n)\}$

$S=\{1,2,3,\cdots n\}$

R is reflexive since $(a,a)\in R\ \forall\, a\in S$

R is symmetric since there is no pair $(a,b)\in R$ such that $(b,a)\in R$

R is transitive since (a,b) and $(b,c)\notin R \Rightarrow (a,c)\notin R$.

Since R is reflexive, symmetric and transitive

the relation R is an equivalence relation.

(32) L.H.S

$$=\frac{\cot(180+\theta).\sin(90-\theta)\cos(-\theta)}{\sin(270+\theta).\tan(-\theta).\text{cosec}(360+\theta)}$$

$$=\frac{\cot\theta\times\cos\theta\times\cos\theta}{(-\cos\theta)\times(-\tan\theta)\times\text{cosec}\,\theta}$$

$$=\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta\times\sin\theta}{\frac{\sin\theta}{\cos\theta}}=\cot\theta\cos^2\theta\ =\text{R.H.S}$$

Hence proved.

(33) 5 questions to be attempted out of 9 questions.

2 questions are compulsory.

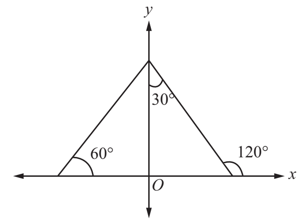
$$\begin{aligned}\therefore \text{No. of ways} &= 7C_3\\ &= \frac{7\times6\times5}{3\times2\times1}\\ &= \boxed{35}\ .\end{aligned}$$

(34) $a=x^2,\qquad b=\dfrac{1}{x^3},\qquad n=10$

$$\begin{aligned}T_{r+1} &= nC_r\ a^{n-r}\times b^r\\ &= 10C_r(x^2)^{10-r}\times\left(\frac{1}{x^3}\right)^r\\ &= 10C_r x^{20-2r}\times x^{-3r}\\ T_{r+1} &= 10C_r x^{20-5r}\\ 20-5r &= 15\\ -5r &= -5\\ &\boxed{r=1}\\ \therefore T_{1+1} &= 10C_1\ \ x^{20-5}\\ &= 10C_1\ \ x^{15}\end{aligned}$$

Here the coefficient of x^{15} is $\boxed{10}$.

(35) From the figure,



$$m_1 = \tan 60° = \sqrt{3}$$

$$m_2 = \tan 120° = \tan(180-60°)$$

$$= -\sqrt{3}$$

$\therefore$ Equation of lines are

$$y = m_1 x + c,$$

$$\boxed{y=\sqrt{3}x+7}\ ,\ \boxed{y=-\sqrt{3}x+7}$$

(36) L.H.S. $= \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$

$c_2 \to c_2 - c_1$

$c_3 \to c_3 - c_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \end{vmatrix}$$

$$= (y-x)(z-x)\begin{vmatrix} 1 & 0 & 0 \\ x & 1 & 1 \\ x^2 & y+x & z+x \end{vmatrix}$$ Taking $(y-x)$ and $(z-x)$ from c_2 and c_3

$$= (y-x)(z-x)[z+x-y-x]$$

$$= (x-y)(y-z)(z-x)$$

$$= \text{R.H.S.}$$

(37) Given : $|\vec{a}|=3\ \ |\vec{b}|=4\ \ |\vec{c}|=5$

$$|\vec{a}+\vec{b}+\vec{c}| = 0$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$$

$$0 = 9+16+25+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$$

$$-50 = 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$$

$$\boxed{-25 = \vec{a}\cdot\vec{b}+\vec{b}\cdot.\vec{c}+\vec{c}\cdot\vec{a}}$$

(38) $\int x \log x \, dx$

$u = \log x \qquad \int dv = \int x\,dx$

$du = \dfrac{1}{x}dx \qquad v = \dfrac{x^2}{2}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\log x\,dx = \frac{x^2}{2}\log|x| - \int \frac{1}{x}\times\frac{x^2}{2}\,dx$$

$$= \frac{x^2}{2}\log|x| - \frac{x^2}{4} + c$$

(39) $P(A) = \frac{3}{8}$ $\qquad P(B) = \frac{1}{8}$

$$P(\bar{A}) = 1 - P(A)$$

$$= 1 - \frac{3}{8} = \boxed{\frac{5}{8}}$$

$P(A\cup B) = P(A) + P(B)$ $\quad(\therefore A$ and B are mutually exclusive $P(A\cap B) = 0)$

$$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

$$P(\bar{A}\cap B) = P(B) - P(A\cap B)$$

$$= \frac{1}{8} - 0 = \boxed{\frac{1}{8}}.$$

(40) $\lim\limits_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}\times\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$

$$= \lim_{x\to 0}\frac{x+2-2}{x\left(\sqrt{x+2}+\sqrt{2}\right)} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}.$$

PART - IV

(41)(a) $|x| = \begin{cases} -x & \text{if } x \le 0 \\ x & \text{if } x > 0 \end{cases}$

$$f(x) = \begin{cases} -x+x & \text{if } x \le 0 \\ x+x & \text{if } x > 0 \end{cases}$$

$$f(x) = \left| \begin{matrix} 0 & \text{if } x \le 0 \\ 2x & \text{if } x > 0 \end{matrix} \right.$$

$$g(x) = \begin{cases} -x-x & \text{if } x \le 0 \\ x-x & \text{if } x > 0 \end{cases}$$

$$g(x) = \begin{cases} -2x & \text{if } x \le 0 \\ 0 & \text{if } x > 0 \end{cases}$$

When $x \le 0$

$$(f\circ g)(x) = f[g(x)] = f(-2x)$$

when $x > 0$ $f\circ g(x) = f[g(x) = f(0) = 0$

When $x > 0$

$$(g \circ f)(x) = g[f(x)] = g[2x] = 0$$

when $x \le 0$

$$(g \circ f)(x) = g[f(x)] = g(0) = 0$$

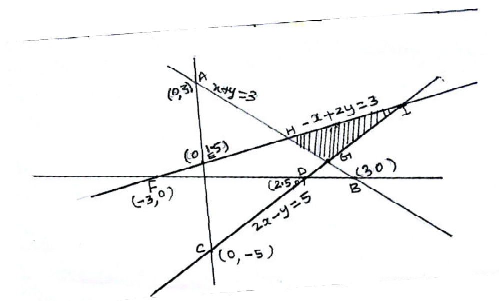
(b) $x + y \ge 3$ $\quad$ $2x - y \le 5$ $\quad$ $-x + 2y \le 3$

$x + y = 3$ $\quad$ $2x - y = 5$ $\quad$ $-x + 2y = 3$

x	0	3
y	3	0

x	0	2.5
y	-5	0

x	0	-3
y	1.5	0



$A(0,3), B(3,0)$ are the 2 points on the line $x + y = 3$ and $(0,0)$ is not in the solution region of $x + y \ge 3$.

$C(0,-5), D(2.5,0)$ are the 2 points on the line $2x - y = 5$ and $(0,0)$ is in the solution region of $2x - y \le 5$.

$E(0,1.5), F(-3,0)$ are the 2 points on the line $-x + 2y = 3$ and $(0,0)$ is in the solution region of $-x + 2y \le 3$.

$\therefore \Delta$ GHI is the required solution set of the given linear inequalities.

(42) Given $A + B + C = \pi$

(a) L.H.S. $= \cos A + \cos B + \cos C$

$$= 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) + \cos C \qquad \left[\because \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}\right]$$

$$= 2\cos\left(\frac{\pi}{2} - \frac{C}{2}\right)\cdot\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \cos C$$

$$= 2\sin\left(\frac{C}{2}\right)\cdot\cos\left(\frac{A}{2} - \frac{B}{2}\right) + 1 - 2\sin^2\left(\frac{C}{2}\right)$$

$$= 1 + 2\sin\left(\frac{C}{2}\right)\left[\cos\left(\frac{A}{2} - \frac{B}{2}\right) - \sin\left(\frac{C}{2}\right)\right]$$

$$= 1 + 2\sin\left(\frac{C}{2}\right)\left[\cos\left(\frac{A}{2} - \frac{B}{2}\right) - \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)\right]$$

$$= 1 + 2\sin\left(\frac{C}{2}\right)\left[\cos\left(\frac{A}{2} - \frac{B}{2}\right) - \cos\left(\frac{A}{2} + \frac{B}{2}\right)\right]$$

$$= 1 + 2\sin\left(\frac{C}{2}\right) \cdot 2\sin\left(\frac{B}{2}\right) \cdot \sin\left(\frac{A}{2}\right)$$

$$= 1 + 4\sin\left(\frac{A}{2}\right) \cdot \sin\left(\frac{B}{2}\right) \cdot \sin\left(\frac{C}{2}\right)$$

$$= \text{R.H.S}$$

(b) L.H.S. $= a\cos A + b\cos B + c\cos C$

$$= 2R\sin A\cos A + 2R\sin B\cos B + 2R\sin C\cos C$$

$$\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \ .$$

$$= R[\sin 2A + \sin 2B + \sin 2C]$$

$$= R\left[2\sin\left(\frac{2A+2B}{2}\right) \cdot \cos\left(\frac{2A-2B}{2}\right) + \sin 2C\right]$$

$$= R\left[2\sin(A+B) \cdot \cos(A-B) + \sin 2C\right]$$

$$= R\left[2\sin(\pi - c)\cos(A-B) + \sin 2C\right]$$

$$= R[2\sin C.\cos(A-B) + 2\sin C\cos C]$$

$$= 2R\sin C[\cos(A-B) + \cos C]$$

$$= 2R\sin C[\cos(A-B) + \cos[\pi - (A+B)]$$

$$= 2R\sin C \times [\cos(A-B) - \cos(A+B)]$$

$$= 2R\sin C \times 2\sin A\sin B$$

$$= 2R\sin A 2\sin B\sin C$$

$$= 2a\sin B\sin C$$

L.H.S $= R.H.S$

Hence proved.

(43) (a) Let $P(n) = 2 + 4 + 6 + \cdots + 2n = n^2 + n \quad \forall n \in N$

Step (1)

Put $n = 1$

$$P(1) = 2 \times 1 = 1^2 + 1$$

$$2 = 1$$

$\therefore P(1)$ is true.

Step (2)

Put $n = k$

Let $P(k) = 2 + 4 + \cdots + 2k = k^2 + k$ be true.

Step (3)

Put $n = k + 1$

$$P(k+1) = 2+4+\cdots+2(k+1) = (k+1)^2+(k+1) = (k+1)(k+2)$$
$$\text{L.H.S.} = 2+4+\cdots+2k+2(k+1)$$
$$= k(k+1)+2(k+1)$$
$$= (k+1)(k+2)$$

$\therefore P(k+1)$ is true.

Hence by principle of mathematical induction, $P(n)$ is true for all $n \in N$.

(b) Let the initial amount of bacteria, $a = 30$.

Since the amount of bacteria is doubled in every one hour, the sequence is

$30, 30\times 2^1, 30\times 2^2 \cdots$

It is in G.P.

$$\text{End of } 2^{\text{nd}} \text{ hour} = 30\times 2^2 = 120$$
$$\text{End of } 4^{\text{th}} \text{ hour} = 30\times 2^4 = 480$$
$$\therefore \text{End of } n^{\text{th}} \text{ hour} = 30\times 2^n .$$

(44) (a) L.H.S.

$$\begin{vmatrix} \log x & \log y & \log z \\ \log 2x & \log 2y & \log 2z \\ \log 3x & \log 3y & \log 3z \end{vmatrix}$$

$R_2 \to R_2 \to R_1, \quad R_3 \to R_3 - R_1$

$$\begin{vmatrix} \log x & \log y & \log z \\ \log 2 & \log 2 & \log 2 \\ \log 3 & \log 3 & \log 3 \end{vmatrix} = \log 2 \times \log 3 \begin{vmatrix} \log x & \log y & \log z \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \quad \therefore R_2 \cong R_3$$
$$= 0$$
$$\text{L.H.S} = \text{R.H.S}$$

Hence proved.

(b)
$$\vec{a} = \hat{i}-2\hat{j}+3\hat{k}$$
$$\vec{b} = -2\hat{i}+3\hat{j}-4\hat{k}$$
$$\vec{c} = -\hat{j}+2\hat{k}$$
$$\vec{a} = l\vec{b}+m\vec{c}$$
$$\hat{i}-2\hat{j}+3\hat{k} = l(-2\hat{i}+3\hat{j}-4\hat{k})+m(-\hat{j}+2\hat{k})$$
$$1 = -2l \qquad \Rightarrow l = -\frac{1}{2}$$
$$-2 = 3l-m \qquad \text{... (2)}$$
$$3 = -4l+2m \qquad \text{... (3)}$$

Solving (1) and (2)

$$3\left(-\frac{1}{2}\right)-m = =-2$$

$$-\frac{3}{2}+2 = \text{m}$$

$$\boxed{m=\frac{1}{2}}$$

Put the values of l and m in equation (3)

$$-4\times-\frac{1}{2}+2\times\frac{1}{2} = 3$$

$$2+1 = 3$$

Thus one vector is a linear combination of other 2 vectors.

$\therefore$ The given vectors are coplanar.

(45) (a) The function $h(x)$ is defined at all points of the real line $R=(-\infty,\infty)$, for any $x_0\neq 0$.

$$\lim_{x\to x_0} h(x)=\lim_{x\to x_0} \; x\sin\frac{1}{x}$$

$$= x_0\frac{1}{\sin x_0}=h(x_0)$$

$$\text{For } x_0 = 0,\ h(x)=x\sin\frac{1}{x}$$

$$-x \leq x\sin\frac{1}{x}\leq x$$

$$g(x) = -x,\ \ f(x)=\frac{1}{x}\sin\frac{1}{x},\ \ h(x)=x$$

$$\lim_{x\to 0} g(x) = 0,\ \ \lim_{x\to 0} h(x)=0 \text{ and } \lim_{x\to 0} x\sin\frac{1}{x}=0$$

$\therefore$ By Sandwich theorem

$$\lim_{x\to 0}\left(x\sin\frac{1}{x}\right) = 0=h(0)$$

$\therefore h(x)$ is continuous in the entire interval.

(b)

$$\sin y = x\sin(a+y)$$

$$x = \frac{\sin y}{\sin(a+y)}$$

Differentiating

$$1 = \frac{\sin(a+y)\cdot\cos y\cdot\frac{dy}{dx}-\sin y\times\cos(a+y)\cdot\frac{dy}{dx}}{\sin^2(a+y)}$$

$$\sin^2(a+y) = \frac{dy}{dx}[\sin(a+y)\cdot\cos y - \cos(a+y)\cdot\sin y]$$

$$= \frac{dy}{dx}[\sin(a+y-y)]$$

$$\boxed{\frac{\sin^2(a+y)}{\sin a} = \frac{dy}{dx}}$$

(46) (a)

$$I = \int \frac{6x}{\sqrt{2x+1}}\cdot dx.$$

$$\boxed{\begin{aligned} &\text{Put } \sqrt{2x+1} = t \\ &2x+1 = t^2 \\ &2dx = 2tdt \\ &\text{Now } x = \frac{t^2-1}{2} \end{aligned}}$$

$$= 6\int \frac{t^2-1}{2t}\times t\,dt$$

$$= 3\int (t^2-1)dt$$

$$= 3\left[\frac{t^3}{3}-t\right]$$

$$= 3\left[\frac{t^3-3t}{3}\right]+c$$

$$= t(t^2-3)+c$$

$$= \sqrt{2x+1}(2x+1-3)+c$$

$$= \sqrt{2x+1}(2x-2)+c$$

$$= 2(x-1)\sqrt{2x+1}+c$$

(b) Let A_1 and A_2 be the events of job done by engineer I an II.

Let B be the event that the error occurs in the work.

$$P(A_1) = \frac{60}{100} \qquad P(B/A_1) = 0.03 = \frac{3}{100}$$

$$P(A_2) = \frac{40}{100} \qquad P(B/A_2) = 0.04 = \frac{4}{100}$$

Applying Baye's theorem,

$$P(A_1/B) = \frac{P(A_1)\cdot P(B/A_1)}{P(A_1)\cdot P(B/A_1)+P(A_2)\cdot P(B/A_2)}$$

$$= \frac{\frac{60}{100}\times\frac{3}{100}}{\frac{60}{100}\times\frac{3}{100}+\frac{40}{100}\times\frac{4}{100}} = \frac{\frac{18}{1000}}{\frac{18}{1000}+\frac{16}{1000}}$$

$$= \frac{18}{1000}\times\frac{1000}{34} = \frac{9}{17}$$

$$P(A_2/B) = \frac{\frac{40}{100}\times\frac{4}{100}}{\frac{34}{1000}}$$

$$= \frac{16}{1000}\times\frac{1000}{34} = \frac{8}{17}$$

$$P(A_1/B) > P(A_2/B)$$

$\therefore$ The serious error would have been done by Engineer I.

(47) (a) Let a, v and s be the acceleration, velocity and distance respectively.

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = -8m/s^2$$

$$\int dv = \int -8\ dt$$

$$\boxed{v = -8t + c_1}$$

When the brakes are applied, $t = 0$ and $v = 24\,m/s$

$$24 = -8\times 0 + c_1$$

$$c_1 = 24$$

$$\therefore v = -8t + 24$$

$$\frac{ds}{dt} = -8t + 24$$

$$\therefore ds = \int(-8t + 24)dt$$

$$s = -\frac{8t^2}{2} + 24t + c_2$$

$$s = -4t^2 + 24t + c_2$$

$$\text{when } t = 0\ ,\quad s = 0$$

$$0 = c_2$$

$$\therefore s = -4t^2 + 24t$$

when the bike stops, $v = 0$

$$-8t + 24 = 0$$

$$\boxed{t = 3}$$

when $t = 3,\ s = -4\times 3^2 + 24\times 3$

$$= -36 + 72$$

$$s = 36 \text{ metres}$$

The bike stops at a distance 4 metres to the barriers.

(b) The equation of pair of straight lines is $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$.

Consider

$$2x^2 - xy - 3y^2 = (2x - 3y)(x + y)$$

$$2x^2 - xy - 3y^2 - 6x + 19y - 20 = (2x - 3y + l)(x + y + m)$$

Equating the coefficient of $x; y$, constant term

$$-6 = l + 2m \quad \text{... (1)}$$

$$19 = l - 3m \quad \text{... (2)}$$

$$-20 = lm \quad \text{... (3)}$$

Solving (1) and (2),

$$l + 2m = -6$$

$$l - 3m = 19$$

$$5m = -25$$

$$\boxed{m = -5}$$

$$\therefore l + (-10) = -6$$

$$l = 4$$

$\therefore$ Separate equations are

$$2x - 3y + 4 = 0$$

$$x + y - 5 = 0$$