Contents

1 Introduction

The generation of a pair of entangled photons from the biexcition-exciton cascade is complicated by the presence of fine structure splitting (FSS)[1] that leads to the degradation of the degree of entanglement. Many methods were implemented to overcome the problem with varying degrees of success, but the problem continues to persist.

This research proposal aims to take a different approach to the problem by implementing a scheme for the restoration of the degree of entanglement of the photon pairs by fast polarization manipulation [2, 3].
This scheme can be potentially used as a method for fast photon rerouting in integrated photonics.

2 Theroritical Background

2.1 QD 'Artificial Atoms'

Self-assembled quantum dots (QDs) are localized nano-scale semiconductor structures that creates a three-dimensional(3D) potential well, both in the valence and conduction bands,which trap charge carriers. Due to their small confinement length relative to the particle's wavelength, the energy levels of the QD are quantized with properties similar to atoms which lead them to be described as "Artificial Atoms" [4].

In recent years, semiconductor QDs were thoroughly investigated as technology-compatible single photon sources, providing a quantum source of "flying qubits" on demand.[5, 6, 7, 8] Moreover, recently, it has been shown that QDs can emit pair of entangled photons [9, 10] and that an emitted photon can be entangled with the remaining spin in the QD. [11, 12, 13] These achievements form the required building blocks for quantum information processing. [14, 15]

2.2 Entangled photons

2.2.1 Definition of Entanglement

Quantum entanglement is a phenomenon in quantum mechanics where two or more particles can become correlated in such a way that the state of each particle cannot be described independently of the state of the others. In other words, The overall state cannot be expressed as a product of the states of their subsystems. In other words, an entangled system is not separable.

If, on the other hand, the system is prepared in a separable state, then the overall state can be expressed as a product of the states of the individual particles. In this case, the particles are not entangled, and their states can be described independently of each other.

2.2.2 The role of entangled photon s in quantum information

Entanglement is a crucial resource in quantum information processing, enabling key advantages over classical information processing and making possible many of the most exciting and promising applications of quantum technologies.

Quantum communication Quantum communication is all about transferring a quantum state from one place to the other. Entanglement has a big role in doing that. Entanglement between light and matter is used to perform quantum state transfer and entanglement distribution among distant nodes. Cirac [10] suggested a protocol in which atoms in a cavity are excited by a laser, mapping the atom state to a photon wave packet that enters a cavity at the receiving node. Quantum repeaters [11] correct errors in an entanglement that increase with distance and transmit entangled quantum states over large distances.

2.2.3 Fundamental excitations in QD

In semiconductor QDs, at low temperatures, the basic optical excitation, in which an electron is optically excited from the full valence band, thereby leaving there a hole, to the empty conduction band, is called a bright exciton(BE). In self-assembled QDs, the BE consists of an electron-heavy-hole pair. This optical excitation preserves the excited electron's spin orientation; therefore, the electron and hole have anti-parallel spins. The total angular momentum projection of the BE in the direction of the exciting light is ± 1 (like that of the absorbed photon), reflecting the orbital momentum difference between the electron in the valence band to that in the conduction band. If the excited conduction electron's spin projection is in addition opposite to the spin projection of the ground valence band electron, the electron, and the 2 holes have parallel spins, with a total angular momentum projection of ± 2. This type of excitation is optically forbidden since the light electric field cannot change the electronic spin; therefore, it is called dark exciton (DE). Excitons in the QD can also be generated while the QD contains additional charges, such as a single electron or a single hole. Similarly, many other types of multiple carrier configurations can populate the QD simultaneously. For example, a biexciton is two excitons (two electrons and two holes), and a positively (negatively) charged exciton (trion) is an exciton with the addition of one hole (electron)

2.2.4 The Λ and Π systems

Λ system is a three-level physical system in which the two eigenstates of a qubit are connected to a third auxiliary level via an optical transition. An example of a relevant to this proposal Λ system is presented in Figure 1(a), where the two (degenerate) BE eigenstates are coupled to the ground (nondegenerate) biexciton state. Similarly, one can also think about the BE and the (non-degenerate) vacuum level as an Inverted Λ system or a V system (not shown).

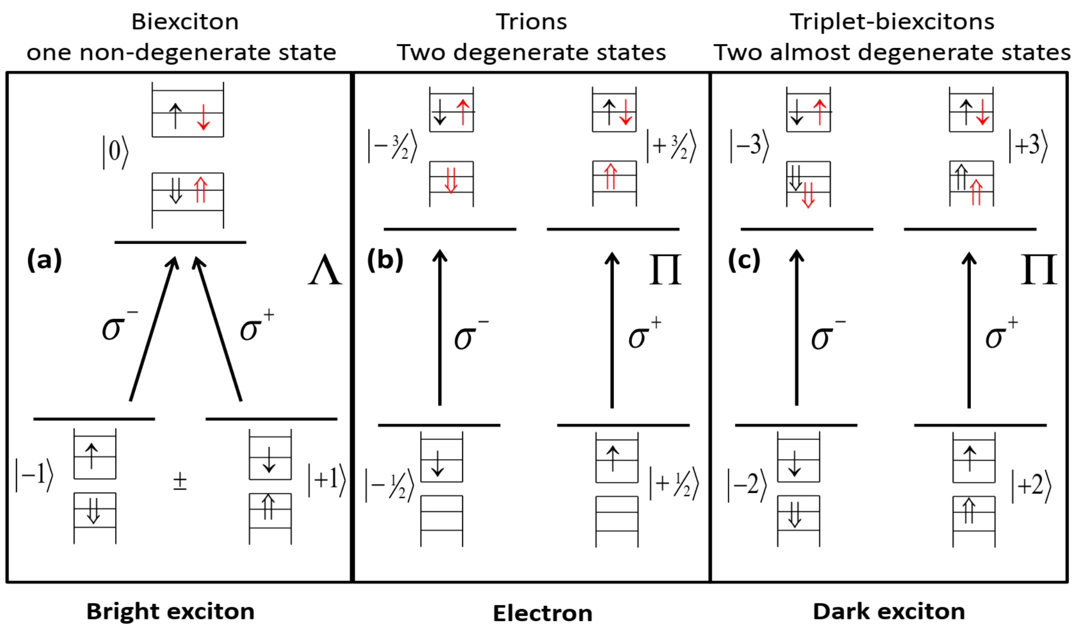


Figure 1: Schematic description of the energy level and selection rules for optical transition in Λ and Π systems. (a) The BE and the ground biexciton form a Λ system. (b) A (degenerate) electron spin forms a Π qubit system with the excited trion states. (c) The (degenerate) DE forms a Π qubit system with the excited biexciton state. The states are described by their total angular momentum projections. The arrows symbolize optical transitions, and σ+ (σ-) symbolizes right (left) hand circular polarization.

A Λ system has physical properties which are very useful for QIP. Notably, since the radiative decay of the non-degenerate auxiliary level to the qubit has two paths, the lack of ”which-path” information leads to entanglement between the emitted photon polarization and the remaining matter qubit spin. This kind of entanglement between a photon and matter spin qubit – the exciton was first demonstrated by Akopian et al

2.2.5 QD as a source of single photon

2.2.6 exciton-biexciton cascade (QD as a source of entangled photon pair)

The projection of the spin of the electron on the z-axis (growth axis) of the QD can be either 1/2 or -1/2, while for the heavy hole, the spin projection is either 3/2 or -3/2 such that the two spin states of the bright exciton are $\left|\frac{1}{2}, -\frac{2}{3}\right\rangle$ and $\left|-\frac{1}{2}, \frac{2}{3}\right\rangle$ with a total spin in the z direction of ± 1. While in the case of electron-hole pair with parallel spin, the spin states are $\left|\frac{1}{2}, \frac{2}{3}\right\rangle$ and $\left|-\frac{1}{2}, -\frac{2}{3}\right\rangle$ with a total spin of ± 2. In figure 2, we describe two simple configurations we can have in a QD. We start with an empty dot denoted by $|0\rangle$.

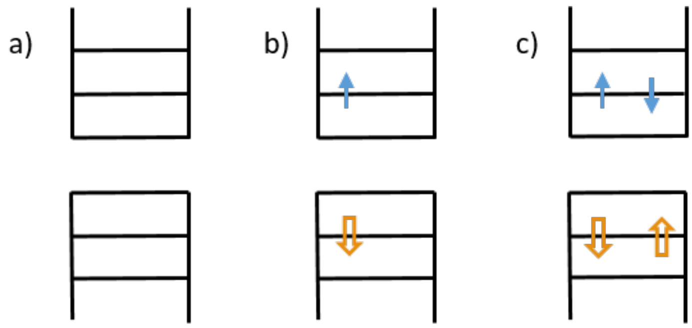


Figure 2: Schematic of the energy levels in the quantum dot for empty quantum dot (a), exciton (b), and a biexciton (c)

while having two electron-hole pairs falling in the dot, a biexciton is formed, and we denote it by $|XX_0\rangle$. The total energy of the biexciton differs from twice the energy of the exciton due to the interaction between all the particles.

In an ideal QD, the radiative decay path of the biexciton back to the ground state goes via one path. But due to the anisotropy of the QD, the energy of exciton is split in what is defined as the fine structure splitting (FSS) as seen in figure **??**b. Here we refer to the up (down) spin of the electron as $|\uparrow\rangle$ ($|\downarrow\rangle$ and the up (down) spin of the hole as $|\Uparrow\rangle$ ($|\Downarrow\rangle$ such as the two eigenstates of the exciton are $|\uparrow\Downarrow\rangle$ and $|\downarrow\Uparrow\rangle$, and the decay from these states result in the emission of photons with co-linear polarization. Here we refer to them as horizontal $|H\rangle$ and vertical $|V\rangle$. We can represent these two rectilinear bases using the Bloch sphere, where the poles in the sphere are $|H\rangle$ and $|V\rangle$ bases.

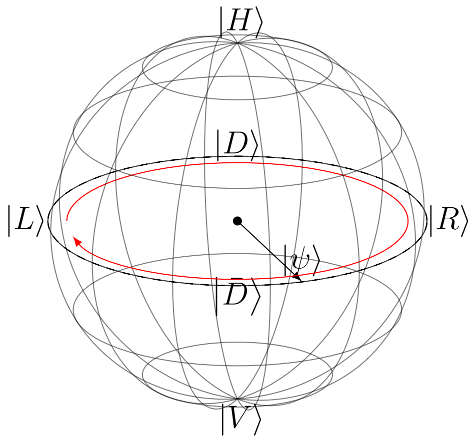
In addition to the rectilinear polarization states, We can define the diagonal linear and the circular polarization bases:

$$\begin{aligned} |L\rangle &= (|H\rangle + i\,|V\rangle)/\sqrt{2} \\ |R\rangle &= (|H\rangle - i\,|V\rangle)/\sqrt{2} \end{aligned} \tag{1}$$

Where $|R\rangle$ and $|L\rangle$ are the left and right circular bases, respectively, and:

$$\begin{aligned} |D\rangle &= (|H\rangle + |V\rangle)/\sqrt{2} \\ \left|\bar{D}\right\rangle &= (|H\rangle + |V\rangle)/\sqrt{2} \end{aligned} \tag{2}$$

$|D\rangle$ and $\left|\bar{D}\right\rangle$ are the diagonal and anti-diagonal bases respectively



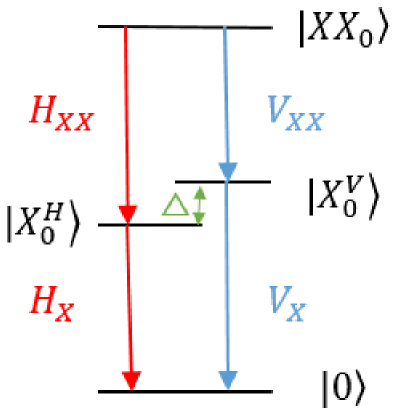


Figure 3: a) A Bloch sphere representation of the spin state. A point on the sphere represents an arbitrarily polarized spin state. b) Decay paths of the biexciton back to the ground state where the red arrows represent the emission of an H-polarized photon and the blue arrows represent the emission of a V-polarized photon.

When the biexciton spontaneously radiatively decays, it emits a photon leaving in the QD, an exciton in a coherent superposition of its two eigenstates. The optical selection rules for the biexciton radiative recombination and the lack of information by "which path" the recombination proceeds result in the entanglement between the exciton state and the polarization state of the emitted photon. Their mutual wave function is given by:

$$|\psi_{X^0}\rangle = \frac{1}{\sqrt{2}}(\left|H_1 H_H^1\right\rangle + \left|V_1 V_V^1\right\rangle) \tag{3}$$

here $|H_1\rangle$ and $|V1\rangle$ are the two rectilinear polarization states of the first (biexciton) photon. Since the two exciton eigenstates are not degenerate, the relative phase between these eigenstates precesses in time with a period of $T_P = h/\triangle$, where h is the Planck constant and $\triangle$ is the exciton FSS [1] This precession is schematically described on the exciton Bloch sphere in the figure. $3a$. The precession “stops” when the exciton recombines, and the radiative cascade is completed with the emission of a second photon. The two photons are thus entangled. Their mutual wave function depends on the recombination time and is given by:

$$|\psi_{X^0}\rangle = \frac{1}{\sqrt{2}}(|H_1H_2\rangle + e^{-i2\pi t/T_P}\,|V_1V_2\rangle) \tag{4}$$

where $|H_2\rangle$ and $|V_2\rangle$ are the second (exciton) photon polarization states and $t = t_{X_0} - t_{XX_0}$ is the time between the emission of the biexciton photon t_{XX_0} and that of the exciton t_{X_0}.

2.2.7 QD spin as a generator for a string of entangled photons

2.3 Optimized Platforms for the emission from a QD (Grating/Open Cavity)

3 Research Proposal

3.1 Deterministic generation of a time-independent polarization entangled photon pairs

The main goal of my research is to provide a solution to the problem described in section 2.2.6, where the existence of FSS in the two bright exciton eigenstates results in time-dependent phase oscillations and the reduction of entanglement in te biexciton-exciton radiative cascade. Here, I propose a method to perform manipulation over the emitted photons instead of using widespread ideas and approaches to eliminate the FSS [many refs], which, so far, are unable to do so. My method is unique in the sense that it does not try to fine-tune the emitter's physical properties in order to restore the entanglement.

In this work I will do the following steps in order to restore the entanglement:

1. I will improve and characterize the interferometer for this experiment and I will search for better methods to perform the required manipulations over the emitted photons.

2. Using the interferometer, I will perform unitary rotation of the exciton's emitted photon polarization, as theoretically described in Appendix A.

3. I will extend the idea of manipulating the exciton emitted photon to both photons of the biexction-exciton radiative cascade. Simultaneous manipulation of the to photons will results in restoring the polarization entanglement of the pair, as theoretically described in B.

3.2 Additional proposed advances

3.2.1 Combining the radiative cascade into the knitting machine.

I'm not sure about the subject; requires more discussion.

3.2.2 Using the experimental system for feed-forward operations in 1D cluster states.

I propose using our constructed device to perform feed-forward operations by rerouting each photon based on information gained about the polarization of the previous emitted photon.

4 Preliminary Results

4.1 Experimental setup

Figure 4 presents the schematic of the experimental setup. The sample is held in a cryostat at a temperature of 5K. The PL is collected using an objective lens with a numerical aperture of 0.85. The same objective also focuses the exciting lasers into the sample. We used a tunable, 76 MHz pulsed laser for the lifetime measurement and a CW, 632.8 nm HeNe laser, to perform the correlation measurements.

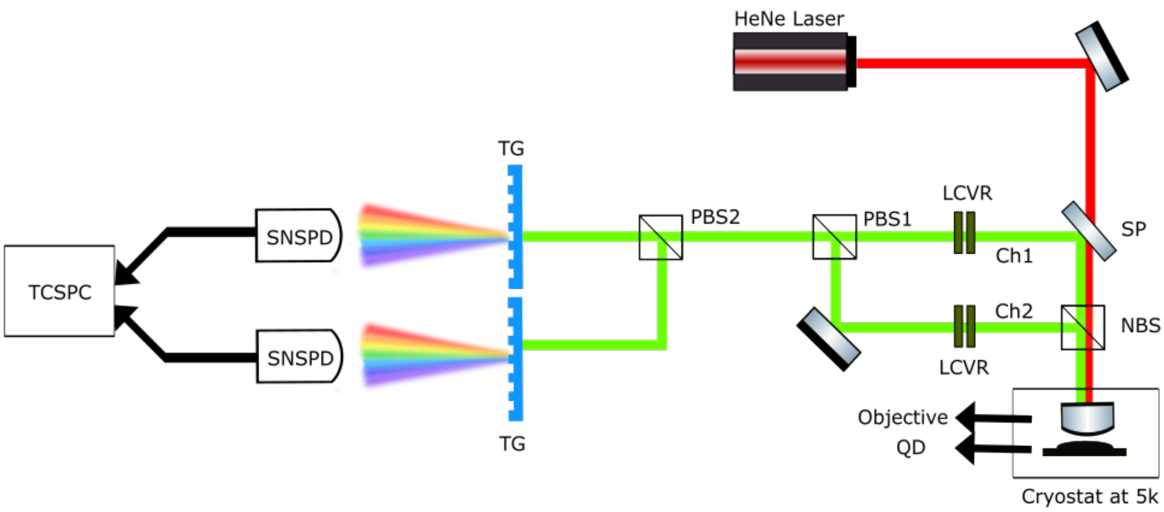


Figure 4: Schematics of the measurement setup.

The PL collected from the dot is split into two channels using a non-polarizing beam splitter (BS), wherein one of the channels, a short pass filter (LP), is used for transmitting the laser while reflecting the PL.

In both channels, the PL passes through a liquid crystal variable retarder (LCVR), which projects its polarization onto the axis of the first polarized beam splitter (PBS), where both channels recombine such as the H(V) polarization of the first(second) channel combine with the V(H) polarization from the second(first) channel and vice versa. Finally, the PL reaches the transmission grating (TG), where the PL is dispersed, and only photons with specific energy reach the superconducting nanowire single photon detector (SNSPD). The signal emitted from the detector is sent to a time-correlated single photon counter (TCSPC) that registers the time information about the events and saves it for analysis.

4.2 Photoluminescence Spectroscopy

In photoluminescence (PL) spectroscopy, emitted light from the QD sample is spectrally dispersed in a transmission grating. Several optically-active transitions of the QD system can be studied using a CCD camera as seen in figure 5. The excitation laser power can be varied, changing the number of carrier pairs that occupy the dot. The QD is most likely occupied by single neutral or charged excitons at low excitation power. At higher excitation power, biexciton and states with similar configurations become visible. Measuring the spectrum as a function of power can yield some of the information necessary to identify the observed transitions. Changing the illumination source, or varying the sample temperature, results in the appearance of emission lines corresponding to charged states and enables examination of QD charging processes.

The PL spectrum can also be measured with variable polarization, and these measurements also provide information about the type of optical transitions observed. For example, the neutral excitonic and biexcitonic optical transitions display a fine structure HV-polarized doublet. Charged excitonic transitions are, on the other hand, unpolarized but have strong polarization memory [60]. Studies of the polarization behavior of spectral lines can aid in the identification of the transitions [3, 12, 19, 48, 20]

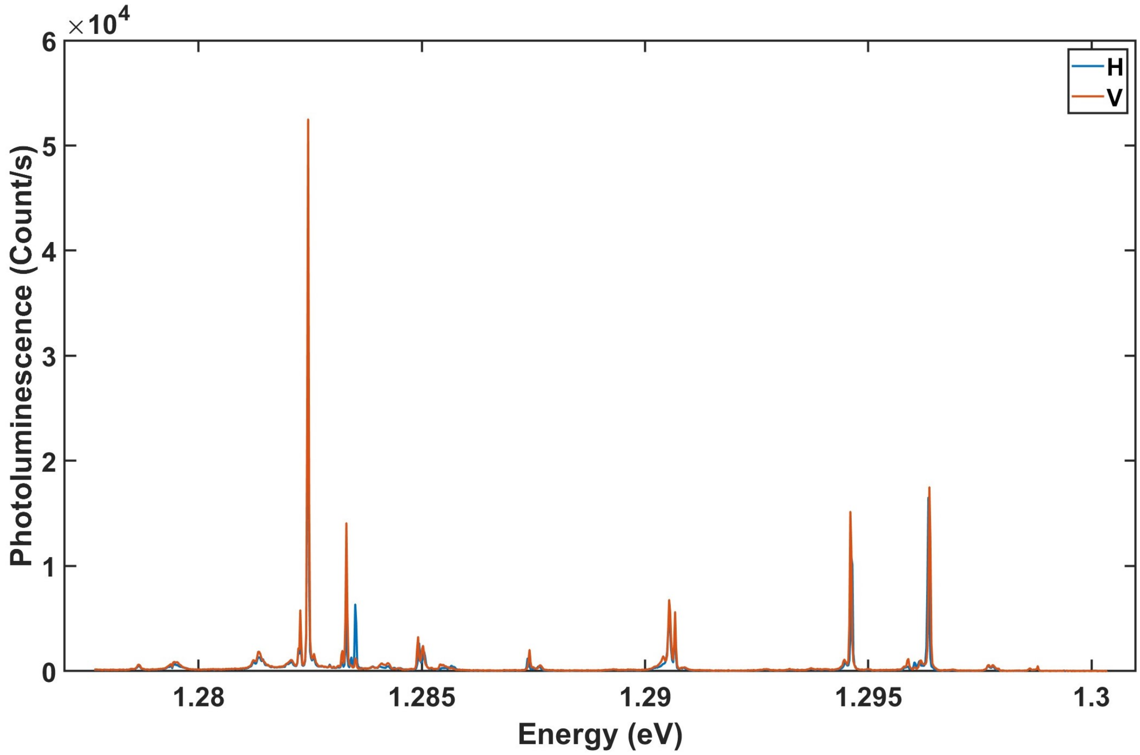


Figure 5: Polarization-dependent photoluminescence spectra from a single quantum dot in a microcavity under non-resonant excitation using HeNe laser for horizontal (H) and vertical (V) polarization.

4.3 Exciton-biexciton cascade under CW excitation

The second-order intensity correlation function between the emission lines of the biexciton-exciton can be described as:

$$g_2(\tau) = \frac{\langle I_{XX}(t) \cdot < I_X(t+\tau) \rangle}{\langle I_{XX}(t) \rangle \cdot \langle I_X(t) \rangle} \tag{5}$$

where $I_{XX}(t)$ ($I_X(t)$) is the emission intensity of the biexciton (exciton), averaged over the time t.

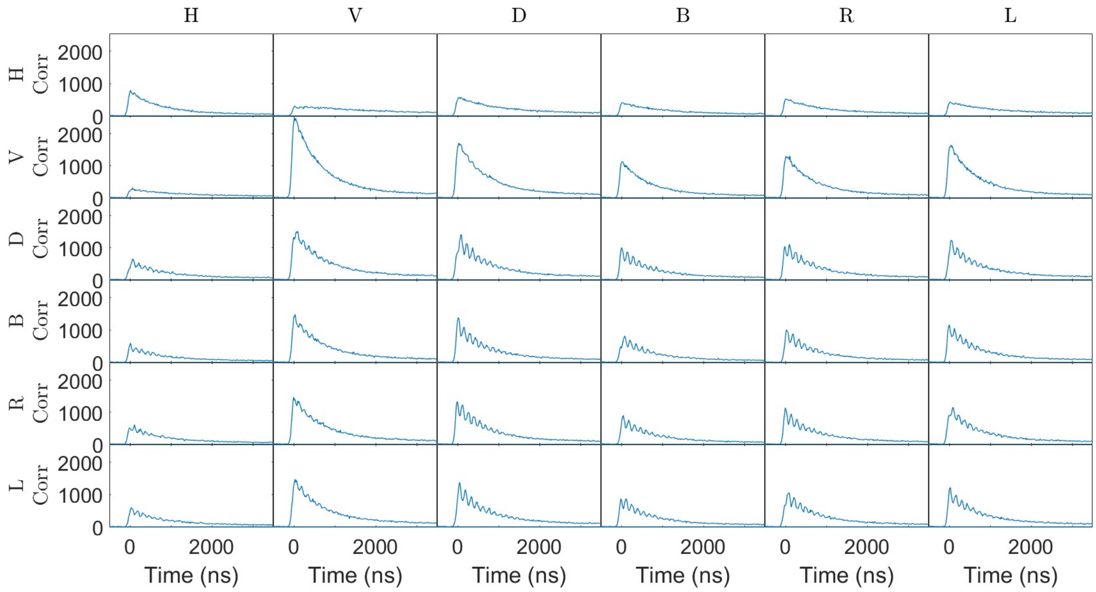


Figure 6: The second-order correlations of the biexciton-exciton radiative cascade., where each row (column) represents a biexciton (exciton) photon polarization (H,V,D,B,R,L).

4.4 Exciton-biexciton cascade under pulsed excitation

4.5 Mach-Zehnder Interferometer

The Mach-Zehnder interferometer consists of a light source that produces a coherent beam of light, which is then split into two separate paths by a beam splitter. Each path contains a series of mirrors reflecting the light back toward the beam splitter. The two reflected beams are recombined at the beam splitter and directed toward a detector.

When the two beams are in phase, they will constructively interfere and produce a bright spot on the detector. When they are out of phase, they will destructively interfere and produce a dark spot. By varying the path length of one of the arms of the interferometer, the relative phase between the two beams can be changed, causing the interference pattern to shift from bright to dark or vice versa.

4.5.1 Schematic

The basic description of the interferometer is presented in figure 7. The photons emitted from the quantum dot (blue) are split using a polarized beam splitter (PBS), where the vertical component is reflected. Then couples to polarization maintaining (PM) fiber attached to the phase modulator controlled using an arbitrary wavefunction generator. In contrast, the horizontal component is transmitted and then couples to another PM fiber. Afterward, The vertical and horizontal components recombine at the interferometer's output using a second PBS. The two paths must be the same length for recombining the two components.

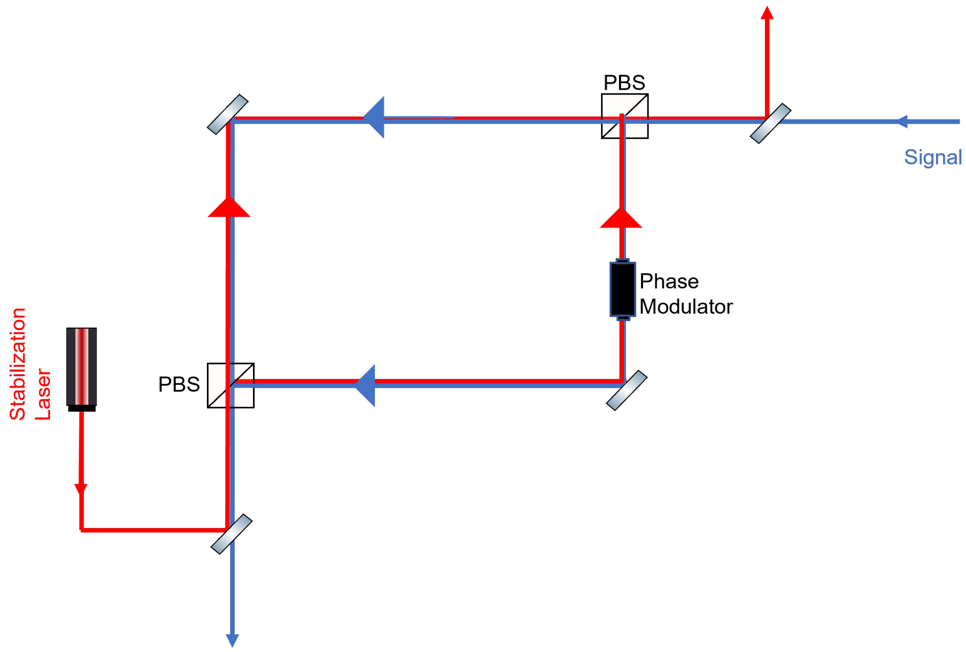


Figure 7: Schematics of the Mach-Zehnder Interferometer.The path of the signal from the quantum dot in blue and the laser for the active stabilization in red

The introduction of fibers into the Mach-Zehnder introduces phase instability to it. This can be due to temperature variations, vibrations, or airflow around the fibers. While these factors can be minimized or eliminated, A completely stable interferometer remains difficult; Therefore, we must construct an active stabilization method using a feedback loop for a more stable interferometer.

The feedback loop is constructed using a $4°$ wedge mounted on a slider that can be displaced using a piezo motor. This wedge is added to one of the arms of the interferometer, which allows us to adjust the path length. Next, we introduce a laser signal to the interferometer as seen in figure 7 in red. This laser is combined and separated from the quantum dot signal using Neutral Density (ND) filters or dichroic mirrors at the in and out ports, propagating in the opposite direction from the signal emitted from the quantum dot.

After the signal from the laser passes through the interferometer, we collect it using a detector which allows us to observe the stability of the interferometer. The signal will drift with time, representing the phase difference between the two arms, where the minimum and maximum counts observed represent destructive and constructive interference. To stabilize the interferometer, We move the piezo motor in 0.3 intervals based on the count measured at the detector until we achieve destructive interference.

4.5.2 Measurements of stability

To assess the stabilization quality, We measure the laser signal for a period of time for the case with active stabilization and one without. The data is plotted in a histogram as shown in figure 8.

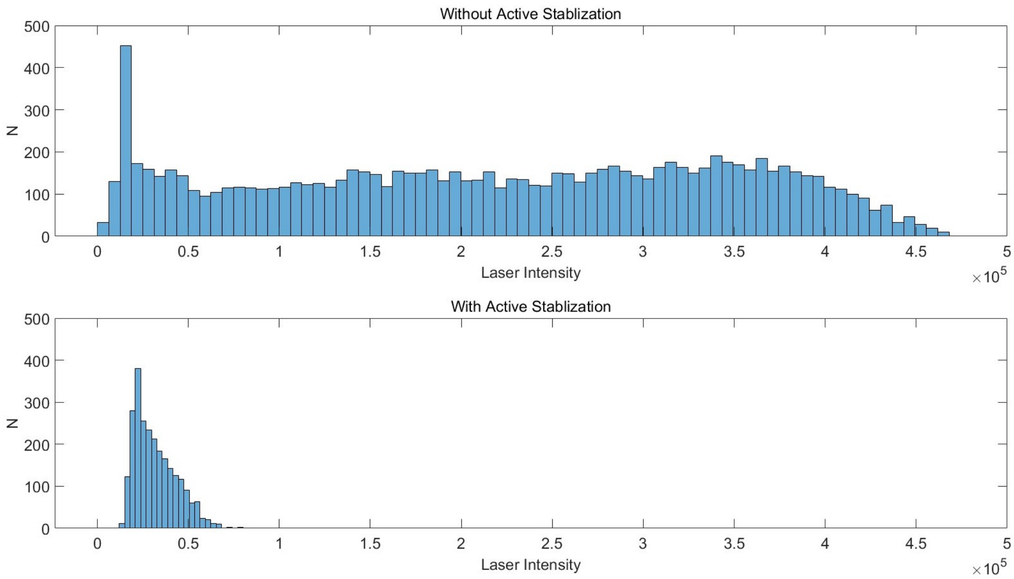


Figure 8

First, the stability of the interferometer is tested without the correction feedback. The data collected is presented in figure 8a. as seen, the count from the detector is evenly distributed at all intervals. In active stabilization, the count is distributed around the minimum.

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A Appendix I

Theoretical formulation (exciton).

We start by writing the exciton's wavefunction after a short laser π-pulse excitation with polarization $|P\rangle = \alpha |H_L\rangle + \beta |V_L\rangle$, where $\alpha^2 + \beta^2 = 1$:

$$|\Psi_X(t)\rangle = \alpha |X_H\rangle \cdot e^{\frac{-iE_H t}{\hbar}} + \beta |X_V\rangle \cdot e^{\frac{-iE_V t}{\hbar}} \tag{6}$$

Here $|X_H\rangle$ and $|X_V\rangle$ are the bright exciton eigenstates, and $E_{H,V}$ are the related eigenenergies. By defining $\overline{h}\omega = \Delta E = E_V - E_H$, dropping global phase, and rearranging we get:

$$|\Psi_X(t)\rangle = \alpha |X_H\rangle + \beta |X_V\rangle \cdot e^{-i\omega t} \tag{7}$$

The instantaneous state of the emitted photon's polarization is directly related to the excitonic wavefunction [ref]:

$$|\Psi_P(t)\rangle = \alpha |H_X\rangle + \beta |V_X\rangle \cdot e^{-i\omega t} \tag{8}$$

Here, $|H_X\rangle$ and $|V_X\rangle$ are the photon's polarization state. We note that in eqs. 7 and 8 we split the exciton and photon wavefunction and did not took into account the radiative decay. We further note that the instantaneous polarization state of the emitted photon change in time with the same angular frequency as the exciton.

Rotation of the exciton's polarization.

If we construct a device that allows us to induce a time-dependent phase shift to the exciton polarization, then we can write phase relations as:

$$\begin{aligned}\Phi_{H_X}(t, t_{prop}) &= K_{H_X} \cdot (t - t_{prop}) + \Phi^0_{H_X} \\ \Phi_{V_X}(t, t_{prop}) &= K_{V_X} \cdot (t - t_{prop}) + \Phi^0_{V_X}\end{aligned} \tag{9}$$

Here the K's are the different slopes that introduce a shift to the photons' polarizations, and Φ^0's are the initial phase of the photons. t_{prop} is the propagation times of the photons from the quantum dot to the device. Since it's constant time, we can simplify the function by including it in the constant phase Φ^0.

$$|\Psi(t)\rangle = \alpha(|H_X\rangle \cdot e^{i\Phi_{H_X}(t_X - t^x_{start})}) + \beta(|V_X\rangle \cdot e^{i\Phi_{V_X}(t_X - t^x_{start})} \cdot e^{-i\omega t}) \tag{10}$$

Where t_x is the random emission time of the exciton photon, and t_{start} is the time when ramping of the differential phase begins. Substituting relations (9) and reorganizing the terms, we get:

$$|\Psi(t)\rangle = \alpha\,|H_X\rangle + \beta\,|V_X\rangle \cdot e^{i\Phi} \tag{11}$$

where $\Phi(t)$:

$$\begin{aligned}\Phi(t) =& (K_{V_X} - K_{H_X} + \omega) \cdot t_x - \\ & t^X_{Start} \cdot (K_{V_X} - K_{H_X}) + \\ & (\Phi^0_{V_X} - \Phi^0_{H_X})\end{aligned} \tag{12}$$

For the function to be independent of the time t_x, the following condition must be met:

$$K_{V_X} - K_{H_X} = -\omega \tag{13}$$

Since we are interested only in the relative phase, we can further reduce the condition by introducing phase shift to only one direction:

$$K_{V_X} = -\omega \tag{14}$$

B Appendix II

Theoretical formulation (biexciton-exciton)

The state of the two photons emitted from the radiative cascade is described as follows:

$$|\Psi(t)\rangle = \alpha(|H_{XX} \otimes H_X\rangle \cdot e^{\frac{-iE_H t}{\hbar}} + |V_{XX} \otimes V_X\rangle \cdot e^{\frac{-iE_V t}{\hbar}}) \tag{15}$$

Where HH_X and HH_V are the horizontal and vertical polarization of the biexciton, α is a global phase. Same with the exciton, we can simplify the function to:

$$|\Psi(t)\rangle = \alpha(|H_{XX} \otimes H_X\rangle + |V_{XX} \otimes V_X\rangle \cdot e^{-i\omega t}) \tag{16}$$

Restoring the entanglement of the photons in the biexciton-exciton radiative cascade.

Similar to the treatment of the exciton treatment, We can construct two devices that allow us to induce phase shift to both the biexciton and exciton polarizations, The time-dependent phase relations to both photons are:

$$\begin{aligned}
\Phi_{H_{XX}}(t, t_{prop}) &= K_{H_{XX}} \cdot (t - t_{prop}) + \Phi^0_{H_{XX}} \\
\Phi_{V_{XX}}(t, t_{prop}) &= K_{V_{XX}} \cdot (t - t_{prop}) + \Phi^0_{V_{XX}} \\
\Phi_{H_X}(t, t_{prop}) &= K_{H_X} \cdot (t - t_{prop}) + \Phi^0_{H_X} \\
\Phi_{V_X}(t, t_{prop}) &= K_{V_X} \cdot (t - t_{prop}) + \Phi^0_{V_X}
\end{aligned} \tag{17}$$

Here the K's are the different slopes that introduce the shift to the photons' polarizations, and Φ^0's are the initial phase of the photons. t_{prop} is the propagation times of the photons from the quantum dot to the device. Since it's constant time, we can simplify the function by including it in the constant phase Φ^0. By taking the starting time of our system as the biexciton excitation time, we can write the state using the t_x and t_{xx} (where t_{xx} and t_x are the random emission times of the biexciton and exciton respectively), as follows:

$$\begin{aligned}
|\Psi(t)\rangle = \alpha(|H_{XX}\rangle \cdot e^{i\Phi_{H_{XX}}(t_{XX} - t^{xx}_{start})} \otimes |H_X\rangle \cdot e^{i\Phi_{H_X}(t_{XX} + t_X - t^{x}_{start})} + \\
|V_{XX}\rangle \cdot e^{i\Phi_{V_{XX}}(t_{XX} - t^{xx}_{start})} \otimes |V_X\rangle \cdot e^{i\Phi_{V_X}(t_{XX} + t_X - t^{x}_{start})} \cdot e^{-i\omega t})
\end{aligned} \tag{18}$$

Substituting relations (17) and reorganizing the terms, we get:

$$\Psi(t) = |H_{xx} H_x\rangle + e^{i\Phi} |V_{xx} V_x\rangle \tag{19}$$

where $\Phi(t)$:

$$\begin{aligned}\Phi(t) =&(K_{V_{XX}} - K_{H_{XX}} + K_{V_X} - K_{H_X}) \cdot t_{xx} + (K_{V_X} - K_{H_X} + \omega) \cdot t_x - \\ &t^{XX}_{Start} \cdot (K_{V_{XX}} - K_{H_{XX}}) - t^{X}_{Start} \cdot (K_{V_X} - K_{H_X}) + \\ &(\Phi^0_{V_{XX}} - \Phi^0_{H_{XX}} + \Phi^0_{V_X} - \Phi^0_{H_X})\end{aligned} \tag{20}$$

For the function to be independent of the times t_{xx} and t_x, the following condition must be met:

$$K_{V_X} - K_{H_X} = -\omega \tag{21}$$

$$(K_{V_{XX}} - K_{H_{XX}}) = -(K_{V_X} - K_{H_X}) \tag{22}$$

The same with the exciton rotation treatment, We can further simplify the conditions by inducing a shift in one direction:

$$K_{V_X} = -\omega \tag{23}$$

$$K_{V_{XX}} = -K_{V_X} \tag{24}$$