* <u>Graph coloring</u> ⇒ It gives welch & Powell scientist. ~~Consider a graph G.~~

Graph coloring is the procedure of assignment of color of graph eighter vertex color or edge color

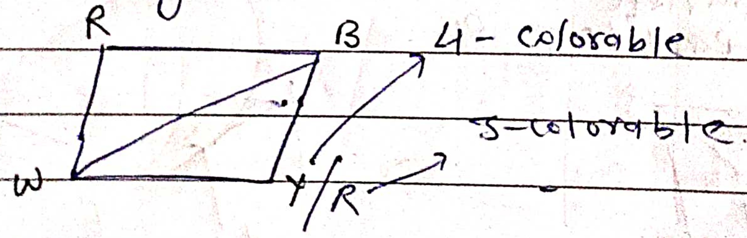
Graph coloring may be two types :-

- → Vertex coloring → Adjacent vertices
- → Edge coloring → Adjacent edges
- → Region coloring in Planar graph

Note:- जहाँ दो vertex के बीच का connection important होगा वहा edge coloring करते हैं & जहाँ दो vertex के बीच का connection का कोई importance नहीं हो वहां vertex coloring करते हैं। (only vertex identify करना)

⇒ Adjacent vertices (or) adjacent edges must have different color is called K-coloring / K-colorable / proper coloring.

Example

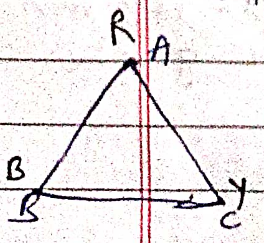


X
साई

<u>Chromatic number [X(G)]</u> ⇒ Minimum colors with which vertices of graph can be colored.

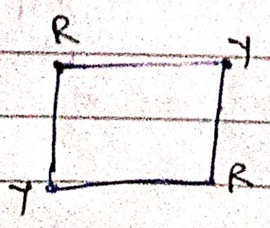
(OR)

The minimum number of colors needed to paint a graph G is called the chromatic number of graph G. It is denoted by sai (X(G) symbol.



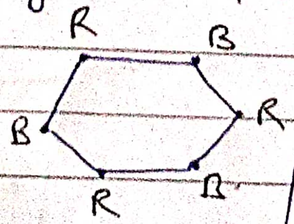
odd cycle

$X(G)=3$



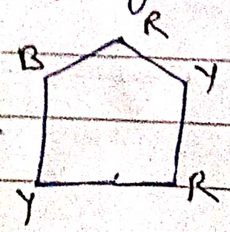
~~even cycle~~

$X(G) = 2$



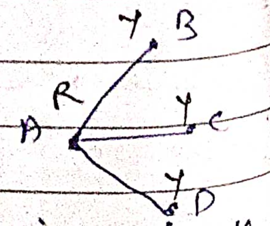
~~even graph~~

$X(G) = 2$



~~odd cycle~~

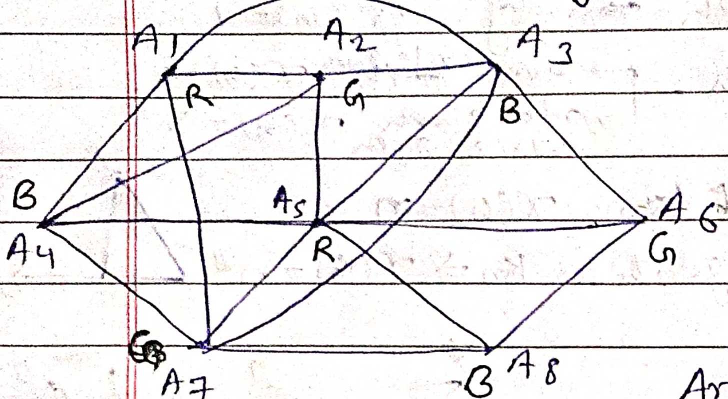
$X(G)=3$



~~Bi-partite~~ $K_{1,3}$

$X(G)=2$

Q. Consider a Graph G, we use welch-powel algo. to obtain a coloring of graph



$D(A_1) = 4$, $D(A_2) = 4$, $D(A_3) = 5$

$D(A_4) = 4$, $D(A_5) = 6$

$D(A_6) = 3$, $D(A_7) = 5$

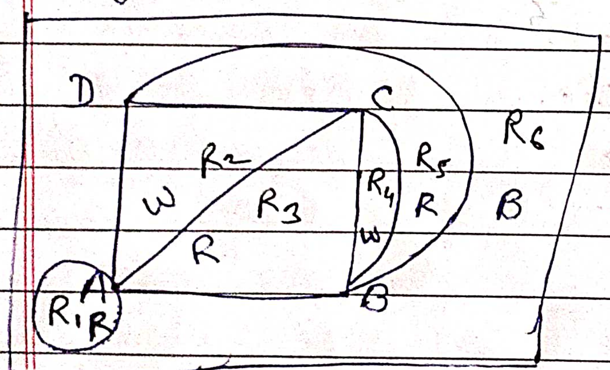
$D(A_8) = 3$

Arrange Degree in descending (decreasing) order

vertices	A_5	A_3	A_7	A_1	A_2	A_4	A_6	A_8
Degree	6	5	5	4	4	4	3	3
color	R	B	G	R	G	B	G	B

$\therefore \chi(G) = 3$

Q. Region color →



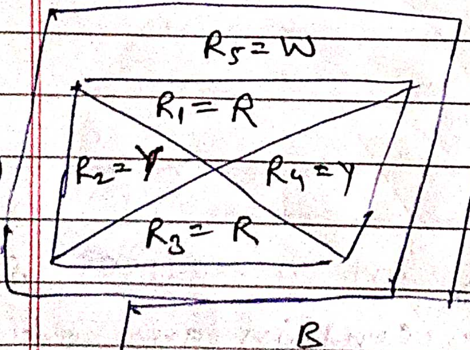
$D(R_1) =$ $D(R_2) = 3$, $D(R_3) = 3$

$D(R_4) = 2$, $D(R_5) = 3$ $D(R_6) = 4$

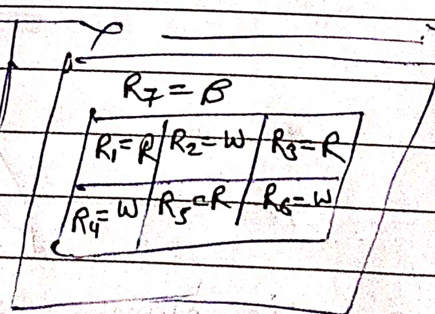
R_1	R_2	R_3	R_4	R_5	R_6
1	3	3	2	3	4
R	W	R	W	R	B

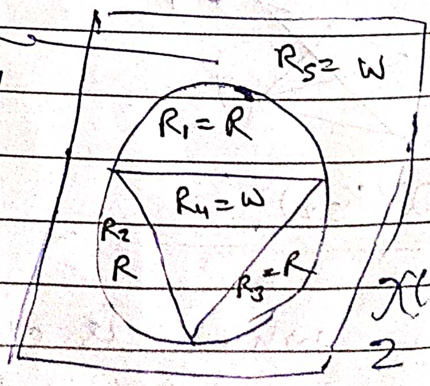
$\chi(G) = 3$

Q.

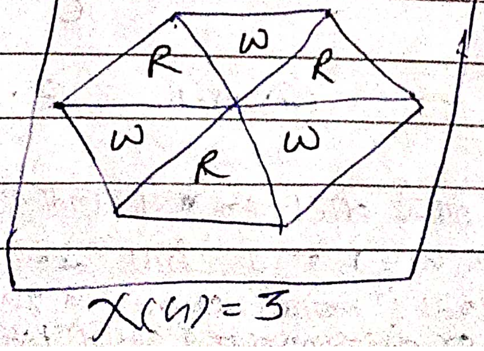


$\chi(G) = 3$





$\chi(G) = 2$

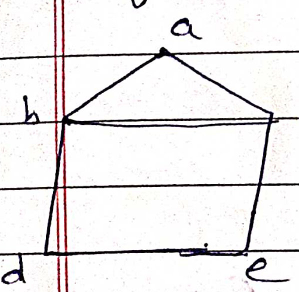


$\chi(G) = 3$

* Chromatic Partition ⇒

The Problem of Partitioning all the vertices of a connected graph G into the smallest possible number of disjoint, independent sets is called as chromatic partitioning.

Independent set of vertices ⇒ A set of vertices in a graph G is independent if not (no) two vertices in the set are adjacent.



Adjacent vertices pair ⇒ {a,d}, {b,e}, {a,e}, {c,d}

These are ~~Maxi~~ ~~minimal~~ maximal independent sets. of this

Solⁿ Solution by enumerating all maximal independent set & then selecting the smallest no. of sets that includes all vertices of the graph.

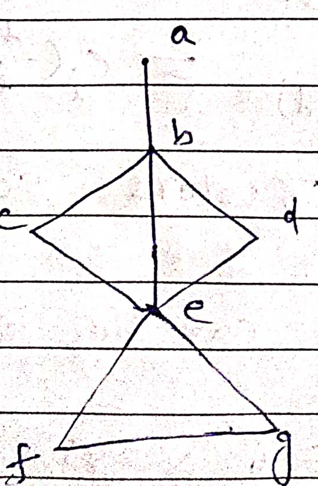
Different methods :- [Algorithm]

(i) Using boolean arithmatic on vertices ⇒ Logical sum $x+y$ denote the operation of including vertex x/y or both

(ii) Logical multiplication ⇒ $x \cdot y$ denote the operation of including vertex x & y.

(iii) Complement ⇒ $\bar{x}$ denote that vertex x is not included.

Example



An independent is called a maximum independent set if no other vertex can be added without destroying it's independent property. Here {a,d} is an independent set but is not a maximum independent set still we can add two more vertices c & d i.e. {a,g, c,d} is a maximal independent set other maximal independent sets are {b,f} or {b,g}

Independent / independendance number (β) ⇒ The number of vertices in the longest independent sets is called independence number.

In privious graph Independance number = $\{a, g, b, d\}$

$\therefore \beta = 4$

⇒ Definition [Chromatic Partitioning]

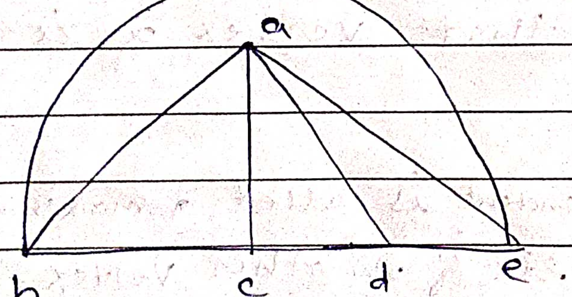
A partitioning all the vertices into smallest possible number of disjoint independent sets is called chromatic partitioning.

* Chromatic Polynomials ⇒ For a given G, The number of ways of coloring the vertices with x or fewer colors is denoted by $P(G, x)$ & is called the chromatic polynomial of Graph G (in terms of x)

$$\boxed{P_n(\lambda) = \sum_{i=1}^{n} C_i \binom{\lambda}{i}}$$

n is no. of vertices

Q.1) Find chromatic polynomial for the following graph



$$P_n(\lambda) = \sum_{i=1}^{n} C_i \binom{\lambda}{i} \quad \text{so}$$

$n = 5$ $$P_5(\lambda) = \sum_{i=1}^{5} C_i \binom{\lambda}{i}$$

$\triangle abc$ we required at least 3 colors. so $n = 3$ $\{C_1, C_2, C_3\}$

$\therefore C_1 = C_2 = 0$

Now the calculate 3 colors can be assigned to 3 vertices suppose a, b, c so different ways $\lfloor 3 = 3 \times 2 \times 1 = 6$ ways

$\therefore c_3 = 6$

for $c_4 \Rightarrow$ out of 4 colors 3 can be selected & assigned to vertex a, b, c $\therefore$ ${}^4P_3 = \frac{\lfloor 4}{\lfloor 4-3} = \frac{4\times3\times2\times1}{1} = 24$ ways

if consider 4th vertex D then two choices — fourth color / take the color as b

$c_4 = 24 \times 2 = 48$ ways

~~for c_5~~ $= \lfloor 5 = 120$

$P_5(d) = c_1 + c_2 + c_3 + c_4 + c_5$

$$= c_1 \frac{d}{\lfloor 1} + c_2 \frac{d(d-1)}{\lfloor 2} + c_3 \frac{d(d-1)(d-2)}{\lfloor 3} + c_4 \frac{d(d-1)(d-2)(d-3)}{\lfloor 4}$$

$$+ c_5 \frac{d(d-1)(d-2)(d-3)(d-4)}{\lfloor 5}$$

put the value of all c_1 to c_5

$$P_5(d) = 0 + 0 + \frac{\cancel{6}\, d(d-1)(d-2)}{\cancel{3\times2\times1}} + \frac{\cancel{48}\, d(d-1)(d-2)(d-3)}{\cancel{4\times3\times2\times1}} + \frac{\cancel{120}\, d(d-1)(d-2)(d-3)(d-4)}{\cancel{5\times4\times3\times2\times1}}$$

$\cancel{P_5(d) = d(d-1) + 2d(d-1)(d-2) + d(d-1)(d-2)(d-3)}$

$\cancel{= d(d-1)[1 + 2(d-2) + (d-2)(d-3)]}$

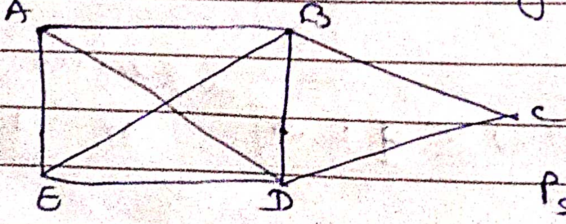
$\cancel{= d(d-1)[1 + 2d - 2 + d^2 - 5d + 6]} \Rightarrow \cancel{d(d-1)[d^2 - 3d + 5]}$

$P_5(d) = d(d-1)(d-2)[1 + 2(d-3) + (d-3)(d-4)]$

$P_5(d) = d(d-1)(d-2)[1 + 2d - 6 + d^2 - 7d + 12]$

$P_5(d) = d(d-1)(d-2)[d^2 - 5d + 7]$

Q.2) Find the chromatic polynomial for the following graph



$$P_n(d) = \sum_{i=1}^{n} c_i \binom{d}{i}$$

$$P_5(d) = \sum_{i=1}^{5} c_i \binom{d}{i}$$

find the value of c_1, c_2, c_3, c_4, c_5

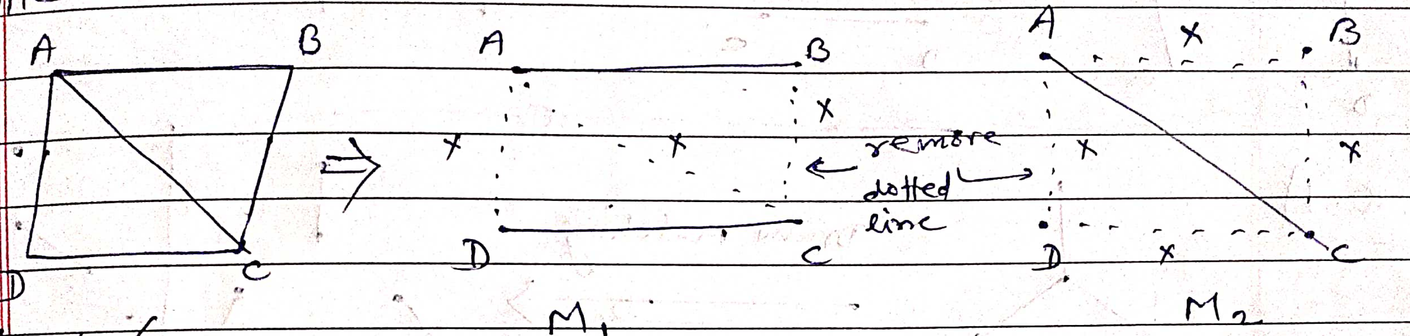
$$P_5(d) = c_1 \frac{d}{\lfloor 1} + c_2 \frac{d(d-1)}{\lfloor 2} + c_3 \frac{d(d-1)(d-2)}{\lfloor 3} + c_4 \frac{d(d-1)(d-2)(d-3)}{\lfloor 4} +$$

$$c_5 \frac{d(d-1)(d-2)(d-3)(d-4)}{\lfloor 5} \quad \text{--- (1)}$$

* **Matching** ⇒ A subgraph M of a Graph G is called matching if every vertex of graph G ~~is~~ is incident with at most one vertex in M and no two edges are adjacent ⇒

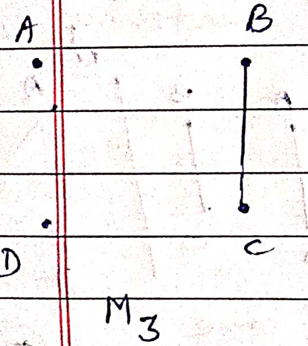
$D(V) \le 1$, or degree of $(V) \le 1$.

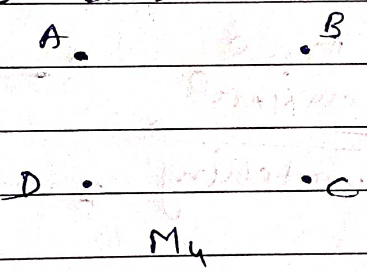
Example



Graph G

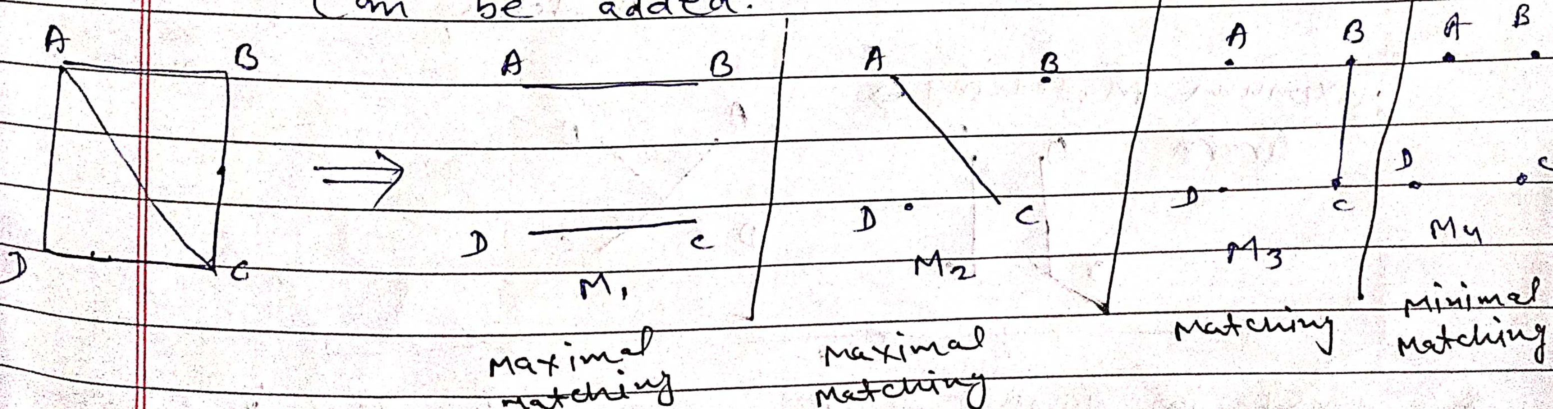
No adjacent ~~ver~~ edge & No degree of vertex is $d(V) \le 1$ ⟶



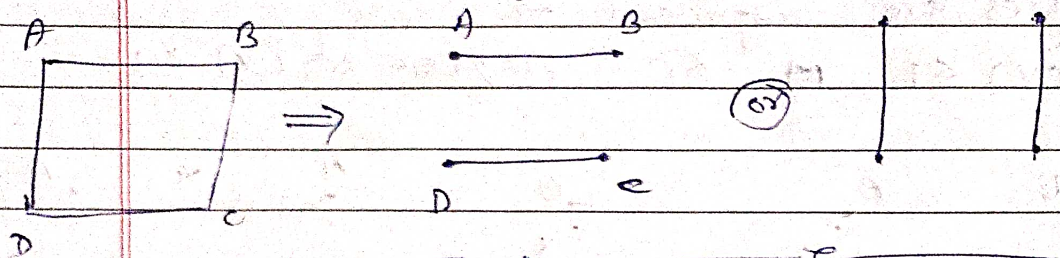


Matching graphs are M_1, M_2, M_3 & M_4

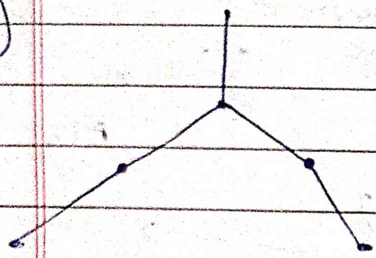
** **Maximal Matching** ⇒ Matching M is called maximal if no other edges of graph G can be added.



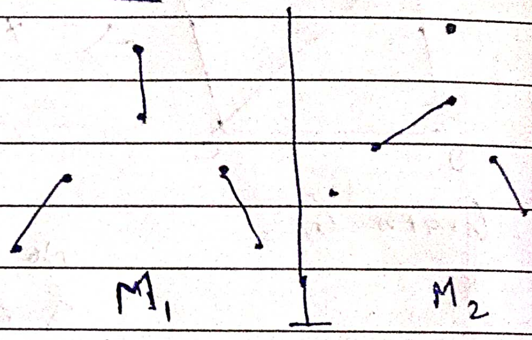
* **Maximum Matching** ⇒ Matching with maximum no. of edges i.e. (that is) most edge-matching from maximal matching.



Q.1)

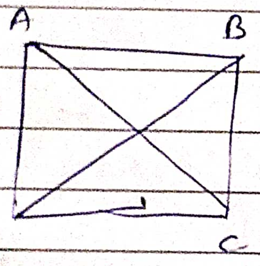


Draw matching graph ⇒



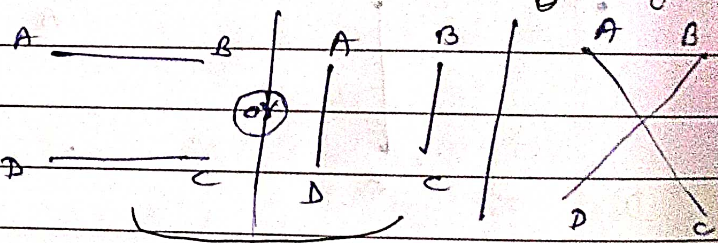
Maximal matching diagram

Q.2)



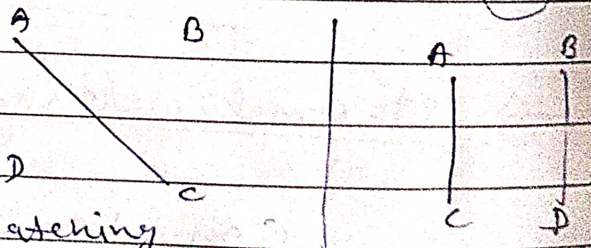
K_4 - complete graph

Draw maximal matching ⇒



both are same

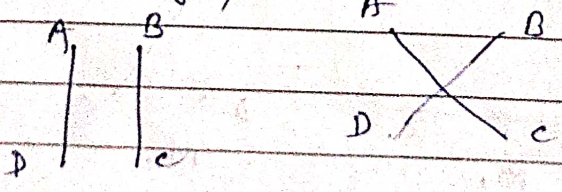
or



Matching

Maximal Matching

Maximum Matching ⇒
Graph



* **Matching Number** ⇒ The number of edges in a maximum matching of graph G.

$$\alpha_1(G) = |\text{Max}\{M \text{ is a matching of graph } G\}|$$

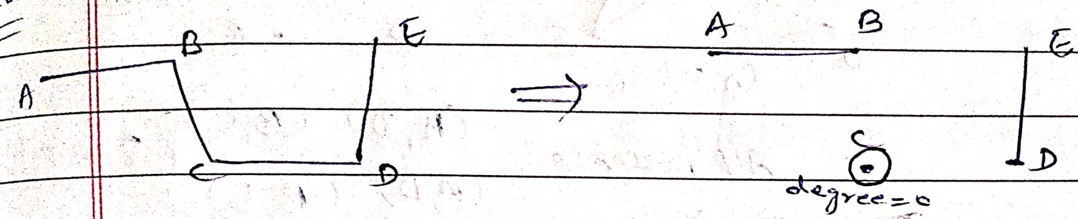
* **Perfect Matching ⇒** A matching in which every vertex is saturated (सैचुरेटेड)

i.e. (that is) has an edge

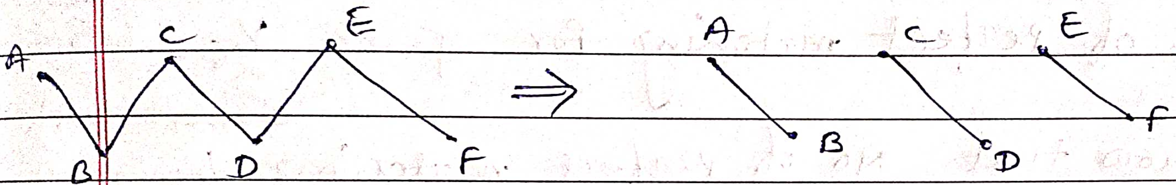
$\forall v \in V(G)$ for all vertex $\in V(G)$

degree $(v) = 1$.

Example



not perfect matching.

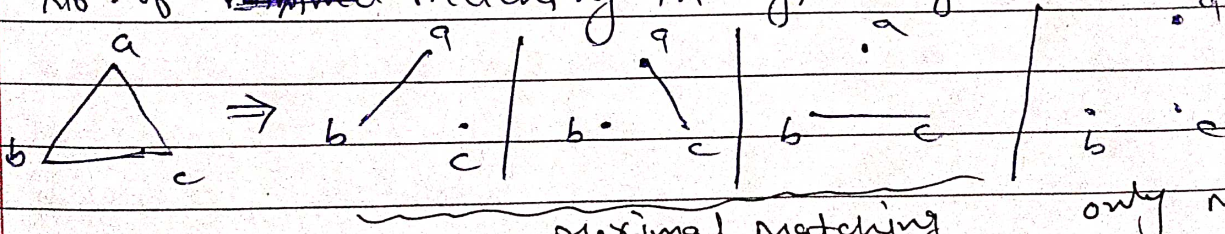


Perfect matching
all vertex degree is one

Properties of perfect matching ⇒

1. All perfect matchings are maximum matching & maximal matching
2. perfect matching has always even no. of vertices.
3. All maximum or maximum matching is not always perfect matching.
4. No. of perfect matching in $K_{n,n} = \lfloor n$ [in Bi-Partite graph]
5. No. of perfect matching in $K_{even} = \dfrac{\lfloor 2n}{\lfloor n \, 2^n}$

Q.1 No. of ~~minimal~~ maximal matching in given graph



Maximal Matching

only matching

* **Covering** ⇒ A covering graph is a sub graph which contains either all the vertices or all the edges corresponding to some other graph. covering - [→ edge, → vertex]

edge covering ⇒ A sub ~~graph~~ graph which contains all the vertices is called a line/edge covering.

vertex covering ⇒ A sub graph which contains all the edges is called a vertex covering.

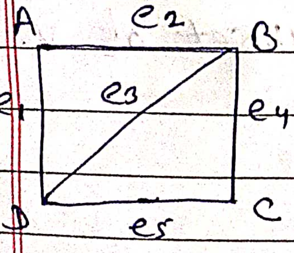
vertex cover is at least Equal to matching number.

|vertex cover| or $|Q| \geq \alpha_1(G)$ ← matching

⇒ **Edge covering** ⇒ A sub graph which contains all the vertices is called line/edge covering.

or

A subset is called a line/edge covering of graph G if every vertex of graph G is incident with at least one edge.



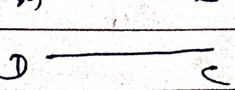
$G = (V, E)$

$V = \{A, B, C, D\}$

$E = \{e_1, e_2, e_3, e_4, e_5\}$ or

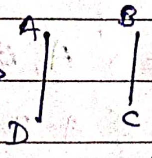
$E = \{(A,D), (A,B), (B,D), (B,C), (C,D)\}$

Subset (covering) S_1 or $Q_1(G) = \{(A,B), (C,D)\}$ →

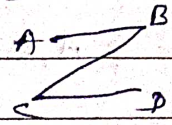


cover all vertex

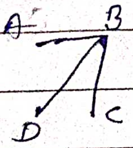
similarly $Q_2(G) = \{(A,D), (B,C)\}$ →



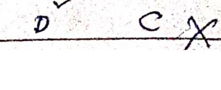
$Q_3(G) = \{(A,B), (B,D), (C,D)\}$ →

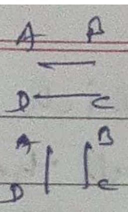


$Q_4(G) = \{(A,B), (B,C), (B,D)\}$ →



$Q_5(G) = \{(A,B), (B,D)\}$ it's not covering

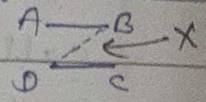


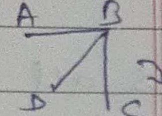


$Q_1(G) = \{(A,B),(C,D)\}$ it's minimal line covering

$Q_2(G) = \{(A,D),(B,C)\}$ it's minimal line covering

$Q_3(G) = \{(A,B),(B,D),(C,D)\}$ Delete edge (B,D) so it's full fill the edge covering condition so given ~~set~~ edge set is not minimal edge covering



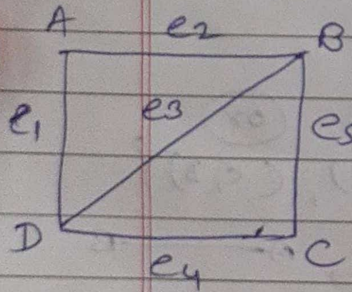


$Q_4(G) = \{(A,B),(B,C),(B,D)\}$ it's minimal edge covering

⇒ Minimum line covering ⇒ Minimal line/edge covering set मे से minimum edge set cover को minimum edge/line covering है।

from ~~edg~~ minimal edge covering subsets are Q_1, Q_2, Q_4 so minimum edge covering sets are Q_1 & Q_2

⇒ Vertex covering ⇒ A subset of V is called a vertex covering of graph G if every edge of graph G is incident with or ~~coverted~~ covered by a vertex in subset of V.

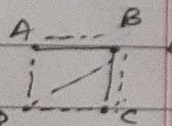


$G \Rightarrow (V, E)$

$V = \{A, B, C, D\}$

$E = \{e_1, e_2, e_3, e_4, e_5\}$

$Q_1(G) = \{B, D\}$ All edges coverted by these two vertex



$Q_2(G) = \{A, B, C\}$ All edges covers by these vertex

$Q_3(G) = \{A, D, C\}$ All edges cover by these vertex

$Q_4(G) = \{A, C\}$ Not all edges covered $\{e_3$ or BD edge$\}$

Example $Q_1(G) = \{B, D\}$ - Minimal vertex covering

$Q_2(G) = \{A, B, C\}$ - minimal vertex covering

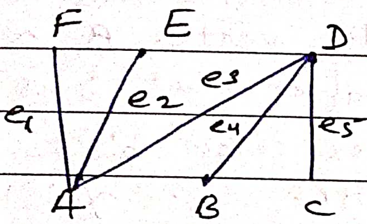
$Q_3 = \{A, D, C\}$ - Minimal vertex covering.

$Q_5 = \{A, B, C, D\}$ → it's not minimal vertex covering.

Minimum vertex covering ⇒ Minimal vertex covering set में से minimum vertex set को minimum vertex covering subset कहते हैं

∴ Minimum vertex covering subset is $Q_1(G) = \{B, D\} = 2$

Q.1)



Minimal edge covering :-

$Q_1(E) = \{e_1, e_2, e_4, e_5\}$ → MEC

$Q_2(E) = \{e_1, e_2, e_3, e_4, e_5\}$ → it's not MEC

~~$Q_3(E) = \{$~~

it's minimum edge covering is $Q_1(E) = \{e_1, e_2, e_4, e_5\} = 4$

Minimal vertex covering ⇒

$Q_1(V) = \{A, D\}$ it's minimal vertex covering (MVC)

$Q_2(V) = \{A, B, C\}$ it's MVC

$Q_3(V) = \{E, F, D\}$ it's MVC

$Q_4(V) = \{E, F, B, C\}$ it's MVC

Minimum vertex covering is $Q_1(V) = \{A, D\} = 2$

* Greedy Coloring Algorithm ⇒ In greedy coloring or sequential coloring is a coloring of the vertices of a graph formed by a Greedy algorithm that considers the vertices of the graph in sequence & assigns each vertex it's first available color.

Algorithm steps ⇒

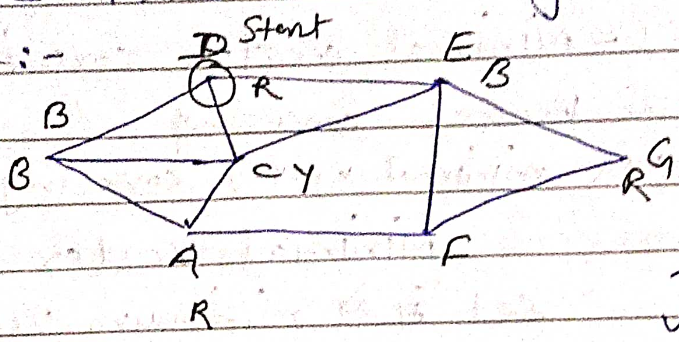
steps ① choose an order for the vertices

② choose a list of colors, also in some order

③ In order, color each vertex using the first available color on the list, making sure that no two adjacent verices are the same color

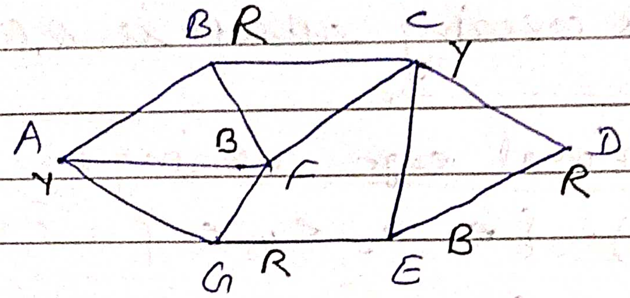
(4) continue in this way until each vertex is colored.

Example :-



total $\chi(G) = 3$

vertex D select & color it after that all non adjacent vertex with this color this process continue

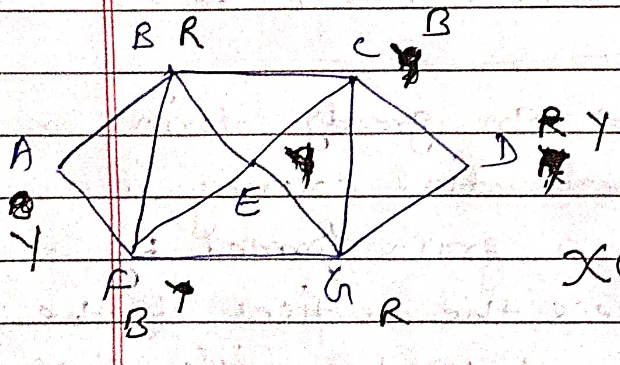


select B vertex with red color After this vertex next order vertex B is yellow color after that D is Red, E vertex color black continue this steps $\therefore \chi(G) = 3$

* four color problem ⇒ The four color problem theorem, The chromatic number ($\chi(G)$) of a planar graph G is not greater than 4

Example :- G_1 graph

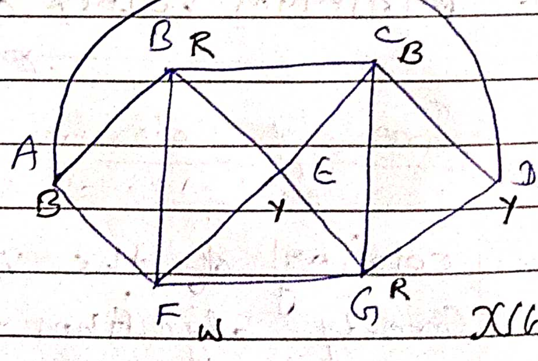
$\chi(G_1) = 3$



$\chi(G_1) = 3$

graph G_2

$\chi(G_2) = 4$



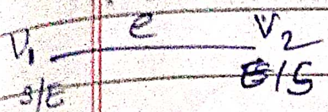
$\chi(G_2) = 4$

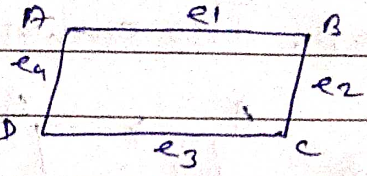
If the region of a planar graph are colored so that adjacent regions have different colors then no more than 4 colors are required

$\therefore$ (that is [i.e.]) $\chi(G) \leq 4$

* Directed Graph ⇒

A Graph consists of an object (V, E) & edges (E) does not have any direction is called undirected graph

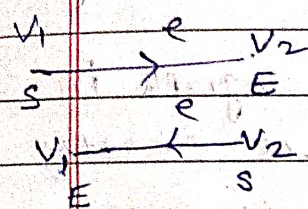


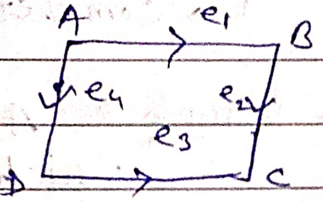


$G = (V, E)$
$V = \{A, B, C, D\}$
$E = \{e_1, e_2, e_3, e_4\}$

$e_1 = \{A, B\} / \{B, A\}$
$e_2 = \{B, C\} / \{C, B\}$
$\{A, B\} = \{B, A\}$

A Graph consists of an object (V, E) & an edge (E) have a direction that from which vertex it is started & which vertex is end.



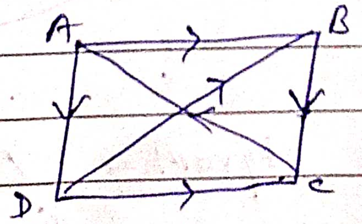


$G = (V, E)$
$V = \{A, B, C, D\}$
$E = \{e_1, e_2, e_3, e_4\}$

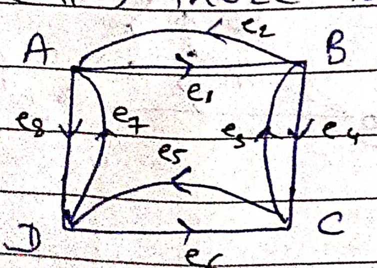
$e_1 = (A, B)$
$e_2 = (B, C)$
$(A, B) \neq (B, A)$

* Types of directed graph ⇒ (Types of Digraph) ⇒

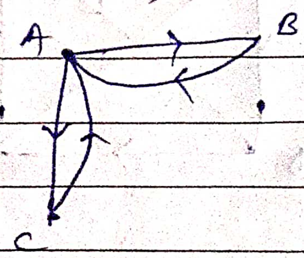
⇒ Simple Digraph ⇒ A digraph that have no self loop & no parallel edges is called simple digraph.



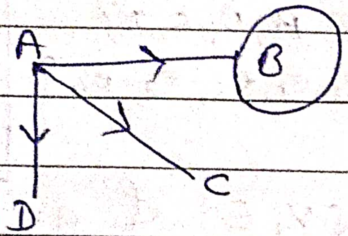
⇒ Symmetric Directed graph (Digraph) ⇒ Digraph in which for every edge (a, b) there is an edge (b, a)



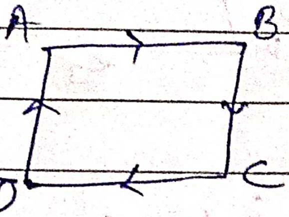
(or)

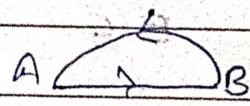


⇒ **Asymmetric Digraph** ⇒ Digraph that have at most one directed edge b/w a pair of vertices but are allowed to have self loop. Also known as Antisymmetric



or

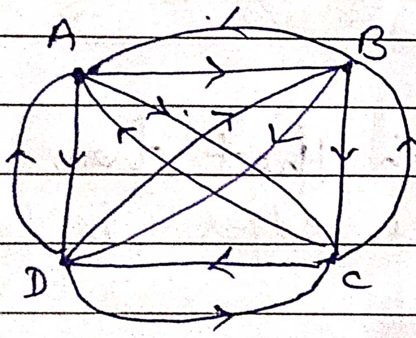


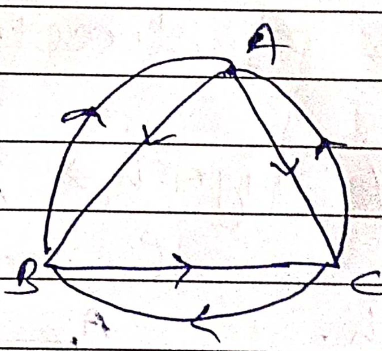


X not allow.

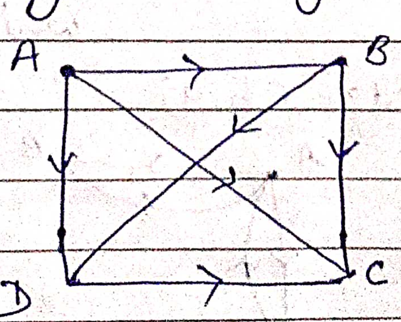
⇒ **Complete Digraph** ⇒
- → Complete symmetric digraph
- → complete asymmetric digraph

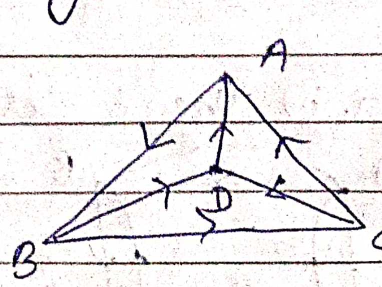
(i) **Complete symmetric digraph** ⇒ A simple digraph in which there is exactly one edge directed from every vertex to every other vertex.

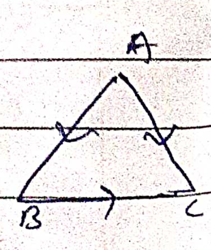




(ii) **Complete Asymmetric Digraph** ⇒ It is an asymmetric digraph in which there is exactly one edge b/w every pair of vertices.







* **Isomorphic Digraph** ⇒ Two graphs to be isomorphic not only must their corresponding undirected graphs be isomorphic but the directions of the corresponding edges must also agree.

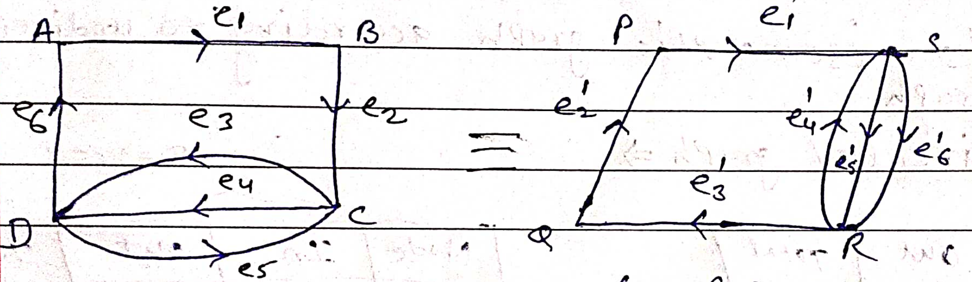
Condition ⇒ (i) proof Isomorphic graph (ii) Directed sequence

undirected A —— B   C —— D

Directed A → B   C ← D

G_1 G_2

Q.



$G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$

conditions: Isomorphic graph → |V|, → |E|, → Degree, → Degree sequence

$V_1 = \{A, B, C, D\} = 4$

$E_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\} = 6$

$D(A) = 2$, $D(B) = 2$

$D(C) = 4$ $D(D) = 4$

$D(A) = D(B) = 2$ {Two vertices}

$D(C) = D(D) = 4$ {Two vertices}

$V_2 = \{P, Q, R, S\} = 4$

$E_2 = \{e_1', e_2', e_3', e_4', e_5', e_6'\} = 6$

$D(P) = 2$, $D(Q) = 2$

$D(R) = 4$ $D(S) = 4$

$D(P) = D(Q) = 2$ {Two vertices}

$D(R) = D(S) = 4$ {Two vertices}

~~D(A) = D(P) = 2   D(B) = D(Q) = 2~~

Degree sequence

A → B → C → D

Q → P → S → R

it's isomorphic graph according to undirected graph condition

According to digraph ⇒

G_1 ⇒

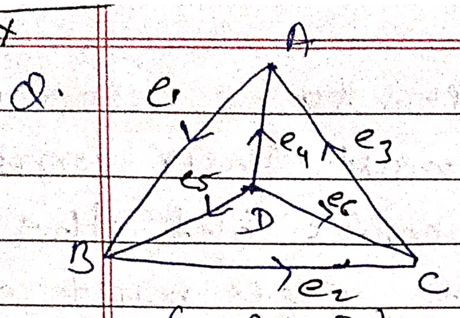
node	In	out	total degree
A	1	1	2
B	1	1	2
C	2	2	4
D	2	2	4

G_2 ⇒

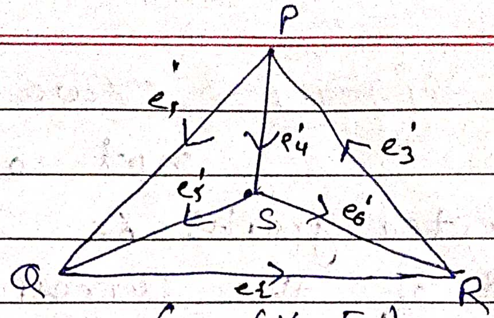
node	In	out	total
P	1	1	2
Q	1	1	2
R	2	2	4
S	2	2	4

So given graph is isomorphic digraph

Q.



$G_1 = (V_1, E_1)$



$G_2 = (V_2, E_2)$

$V_1 = \{A, B, C, D\} = 4$ $\qquad$ $V_2 = \{P, Q, R, S\} = 4$

$E_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\} = 6$ $\qquad$ $E_2 = \{e_1', e_2', e_3', e_4', e_5', e_6'\} = 6$

$D(A) = 3 \quad D(B) = 3$ $\qquad$ $D(P) = 3 \quad D(Q) = 3$

$D(C) = 3 \quad D(D) = 3$ $\qquad$ $D(R) = 3 \quad D(S) = 3$

$D(A) = D(B) = D(C) = D(D) = 3$ $\qquad$ $D(P) = D(Q) = D(R) = D(S) = 3$

it's isomorphic graph according to undirected graph

According to directed graph ⇒

G_1 ⇒

Node	In	out	Total
A	2	1	3
B	2	1	3
C	2	1	3
D	0	3	3

G_2 ⇒

Node	In (Degree)	out (Degree)	Total (Degree)
P	1	2	3
Q	2	1	3
R	2	1	3
S	1	2	3

Not No. of in degree / out degree are same so given graph is not isomorphic digraph.

* Walk / Path / circuit in Digraph ⇒

⇒ walk ⇒
- → Open walk
- → close walk

} Undirected graph

walk is alternativ sequence of vertex & edges

Open walk ⇒ start with vertex x & End with y vertex

$x e_1 x_2 e_2, x_3 e_3 \ldots y$

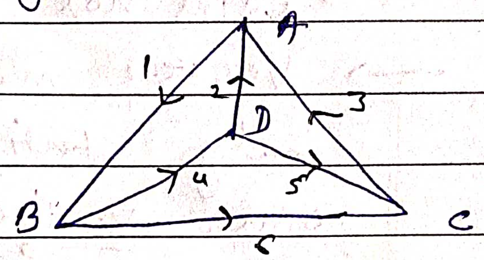
close walk ⇒ start & end with x vertex (same vertex)

$x e_1 x_2 e_2 x_3 e_3 \ldots x$

According to digraph ⇒
- → Directed open walk
- → Directed close walk

Directed ~~Open~~ close/ open walk ⇒ Alternative sequence of vertices & (or) Directed walk edges.

⇒ Vertex may be repeat but ~~edges can not be repeat~~



(1) $\overrightarrow{A \to 1 \to B \to 6 \to C}$ (or) $\overrightarrow{A\,1\,B\,6\,C}$
Start End

It's directed open walk

Same direction होना चाहिये सभी की

Open directed walk

(2) $\overrightarrow{B\,4}\,\overrightarrow{D\,5}\,\overleftarrow{C\,6}\,B$ Closed walk
Start End

semi-close directed walks.

(3) D 2 A 1 B 6 C 5 D
close walk

$\overrightarrow{D-2}-\overrightarrow{A-1}-\overleftrightarrow{B-6}-\overleftarrow{C-5}-D$

it's semi-close directed walk
(or) close-semi walk

(4) $\overrightarrow{A-1-B}-\overrightarrow{4-D}$ open walk
open directed walk (or) Directed walk.

(5) $\overrightarrow{A \to 1-B}-\overrightarrow{4-D}-\overrightarrow{2-A}$ Close directed walk

⇒ Path ⇒ Path is a open walk.
It alternative sequence of vertices & edges

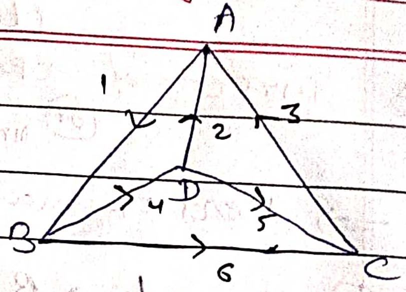
⇒ Not vertex & Not edges repeat.

(1) $\overrightarrow{A-1-B}-\overrightarrow{6-C}-\overleftarrow{5-D}$ it's path or open walk.
Start End

it's semi-open (or) semi-path.

(2) $\overrightarrow{A \to 1-B}-\overrightarrow{6-C}$ It's Directed path

⇒ Circuit ⇒ – Directed circuit / semi-circuit



Conditions :-

→ Edges not repeat
→ Vertex may be repeat but only starting & ending vertex may be repeat not other vertices.

Example ① A – 1 – B – 6 – C – 3 – A

start — End — same start/end vertex

It's circuit & it's directed circuit

② A – 1 – B – 4 – D – 5 – C – 3 – A

start — end.

It's semi-~~directed~~ circuit

③ D – 2 – A – 3 – C – 5 – D

same

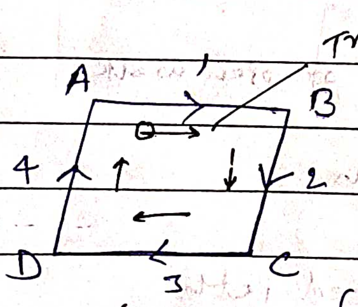
~~It's semi circuit~~

✗ Euler Digraph ⇒ It's follow some conditions :-

→ Closed directed walk
→ Traverse Every edge
→ No edge repeat & Vertex can be repeat
→ Directed Euler line.
→ vertex can be repeat

or

Example ①

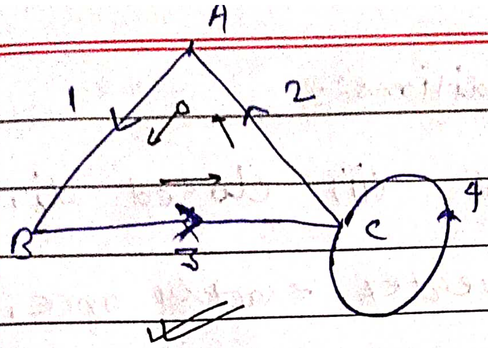


A – 1 – B – 2 – C – 3 – D – 4 – A

check :-

(i) closed Directed walk :- Yes
(ii) Traverse every edge :- Yes
(iii) No edge repeation :- Yes
(iv) vertex can be repeat :- Yes A
(v) Directed euler line :- Directed closed walk present means euler line / directed euler lines ~~are~~ is present so given Digraph is euler digraph.

(2)



~~A—1—B~~

B - 3 - C - 2 - A - 1 - B

edge 4, is missing

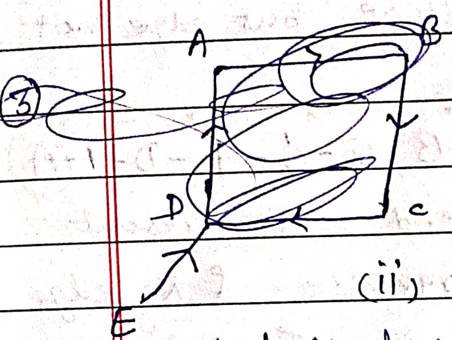
~~if we use 4 means vertex c is repeat~~ then

B — 3 — C → 2 — A — 1 — B - 3 - C

repeat edge

so given diagram is not ~~di~~ euler digraph.

(3)



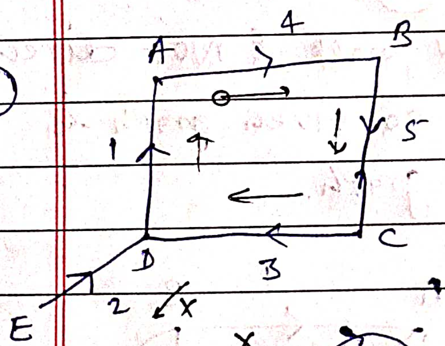
Again use another path :-

(B) → 3 — C — 4 — C - 2 — A - 1 — (B)

(i) closed directed walk is present (ii) Traverse every edge. (iii) No edge repeat but vertex may be repeat. (iv) if directed closed walk is present means Euler line (directed euler line is also present)

so given graph is Euler digraph

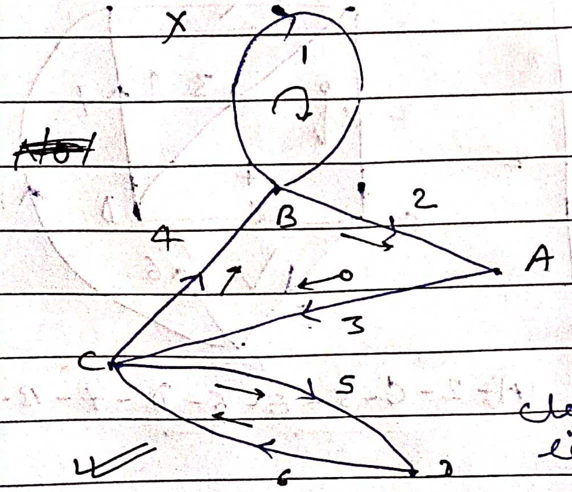
(3)



(A) → 4 — B — 5 — C - 3 — D - 1 - (A)

edge 2 is not traverse so given graph is not euler digraph

(4)



(A) - 3 - C - 5 - D - 6 - C - 4 - B - 1 - B - 2 - (A)

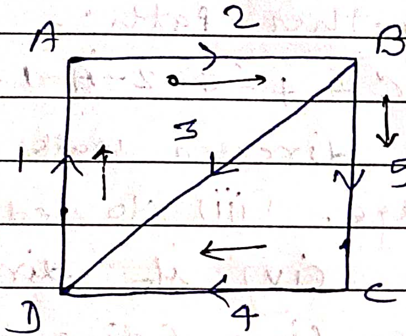
(i) closed directed walk is present (ii) Traverse every edge (iii) No edge repeat ~~but vertex may be repeat~~ (iv) if directed closed walk is present then directed euler line is also present, so given graph is euler directed graph

* Hamiltonian Digraph ⇒ Conditions:-

⇒ (i) Digraph should be connected. (ii) closed directed walk (iii) Traverse every vertex exactly once.
(iv) Initial & final vertex are same.
(v) Edges can not be repeated.
(vi) Directed Hamiltonian Circuit.

Example ⇒ Note:- Not needed all edges traverse but edge not repeat.

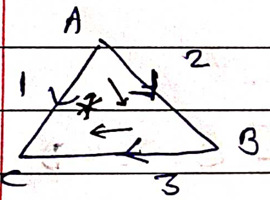
(1)



A - 2 - B - 5 - C - 4 - D - 1 - A

Directed closed walk present. All vertex traverse & No edge repeat. Starting & ending vertex are same so given graph is directed Hamiltonian graph.

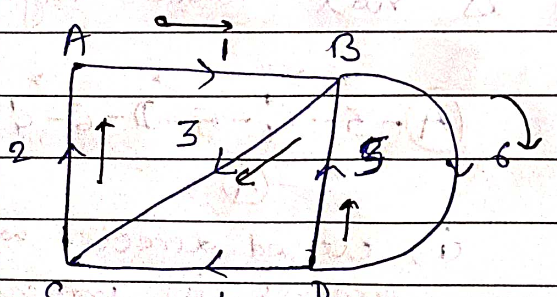
(2)



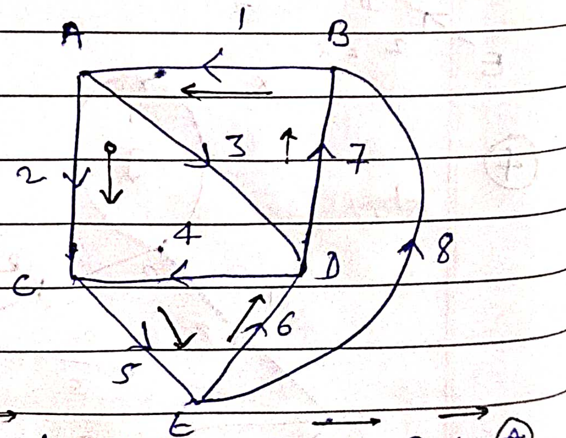
A - 2 - B - 3 - C - 1 - A

Here only closed walk & Not directed closed walk so given graph is not hamiltonian directed graph.

(3)



A - 1 - B - 6 - D - 5 - B - 3 - C - 2 - A



A - 2 - C - 5 - E - 6 - D - 7 - B - 1 - A

* **Digraph & Binary relation** ⇒ In a set of objects X where $X = \{x_1, x_2, x_3, \cdots\}$ a binary relation b/w pairs (x_i, x_j) may exist.

$$x_i R x_j \quad \text{or} \quad (x_i, x_j) \in R$$

Type of relation:-

- → parallel
- → orthogonal
- → congruent
- → greather than
- → ~~equal to~~
- → less than

⇒ A digraph is used ~~to~~ represent a binary relation on a set ~~X~~ X. Each $x_i \in X$ is represented by a vertex x_i & ~~edge~~ directed edges drawn

Example⇒ ~~X~~

$X = \{3, 4, 5, 7, 8\}$ & R is a relation "greaterthan"

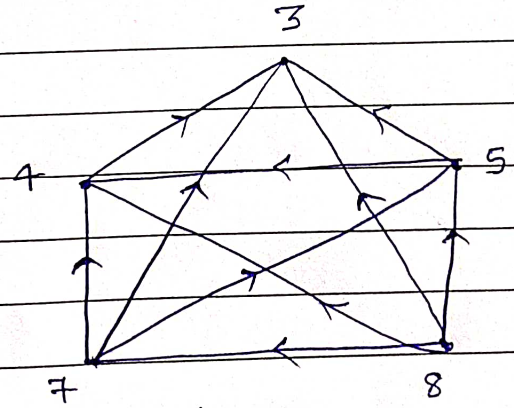
$(x_i > x_j) \in R$

$R = \{(4,3), (5,3), (7,3), (8,3), ~~(\ )~~ (5,4), (7,4), (7,5), (8,4), (8,5), (8,7)\}$

$$\begin{array}{c|ccccc} & 3 & 4 & 5 & 7 & 8 \\ \hline 3 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 1 & 0 & 0 & 0 \\ 7 & 1 & 1 & 1 & 0 & 0 \\ 8 & 1 & 1 & 1 & 1 & 0 \end{array}$$

outgoing edge → 1
incomming 0

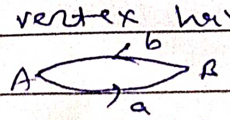
Relation matrix



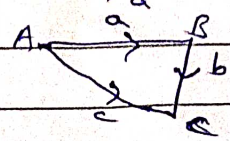
Graph

Reflexive Digraph → self loop aRa

Irreflexive digraph ⇒ No vertex have a self loop $a \not R a$

Symmetric digraph → 

aRb & bRa

Transitive digraph ⇒ 

aRb & $bRc \Rightarrow aRc$

~~for more details~~ use discrete mathematic subject