Assignment - 1

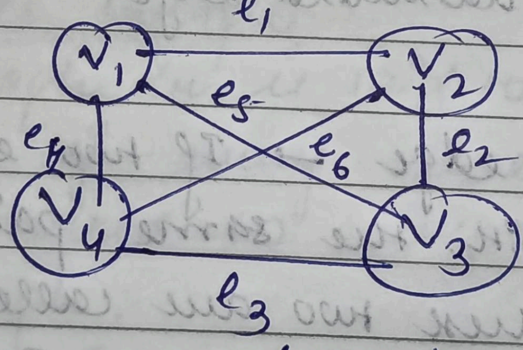
1. What is graph? Define self loop and edges.

→ A graph $G = (V, E)$ is the collection of set of vertices $V = \{v_1, v_2, v_3 ---\}$ and set of edges $E = \{e_1, e_2 ---\}$

There are two types of graphs →

1) Undirected graph → Undirected graph G consists of set of vertices and set of edges $G = (V, E)$. Undirected graph contains set of edges by unordered pair of vertices

E.g

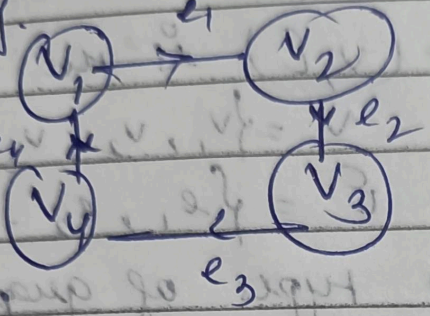


$G = (V, E)$
$V = \{v_1, v_2, v_3, v_4\}$
$E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$

2) Directed graph → Directed Graph consists of vertices & set of edges $G = (V, E)$
In directed graph set of edges contains the

ordered pair of vertices and hence order is important.

E.g.

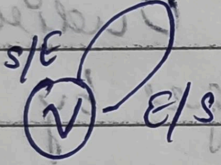


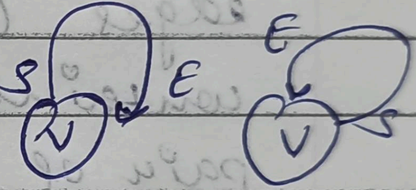
$G = (V, E)$

$V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4\}$

* Self loop → If an edge start and end on the same vertex, it is called self loop.

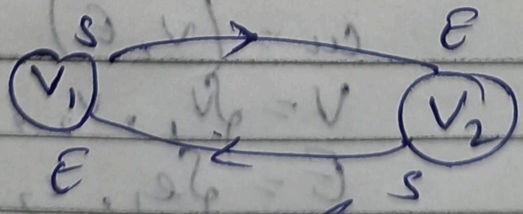




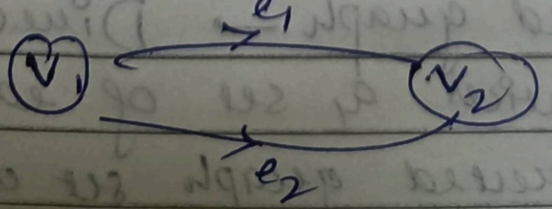
Undirected edge

Directed edge.

* Parallel edge → If two edges start & end with the same pair of vertices then these two are called parallel edges



$e_1 \parallel e_2$



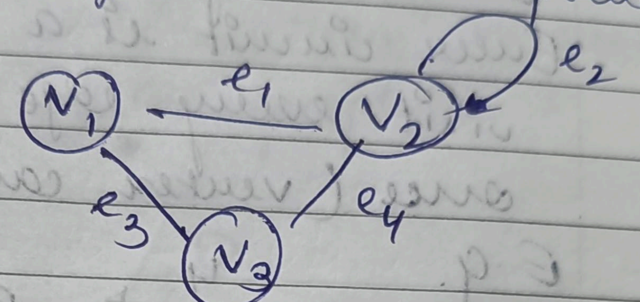
$e_1 \parallel e_2$

2. Define Pseudo graph & complete graph.

→ Pseudo Graph

A graph that have only self loop but not parallel edges is called pseudo graph.

E.g.



$G = (V, E)$

$V = \{v_1, v_2, v_3\}$

$E = \{e_1, e_2, e_3, e_4\}$

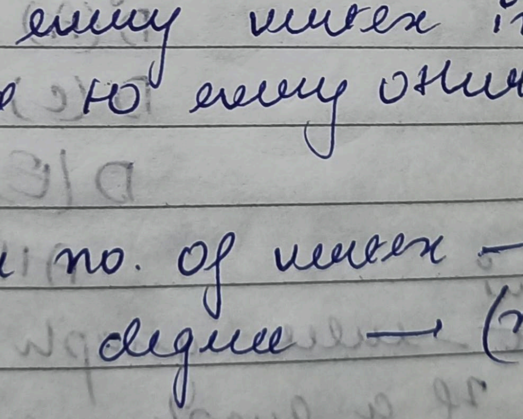
→ Complete Graph

A graph G is said to be complete graph if every vertex in graph G is connected to every other vertex in graph.

For no. of vertex → n

degree → $(n-1)$

E.g.



$G = (V, E)$

$V = \{v_1, v_2, v_3\}$

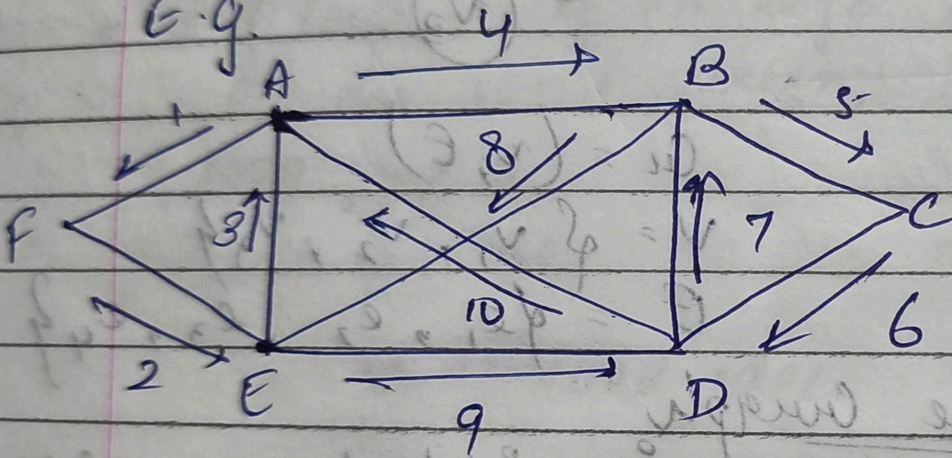
$E = \{e_1, e_2, e_3\}$

3. Write short note on Euler graph.

→ Euler graph

A graph G, which contains euler circuit or cycle is called euler graph. Euler circuit is a closed walk which visit every edge of the graph exactly once (vertex can be repeated).

E.g.



Degree of

D(A) = 4 D(B) = 4

D(C) = 2 D(D) = 4

D(E) = 4 D(F) = 2

All vertices are even

→ ~~open~~ semi - euler graph

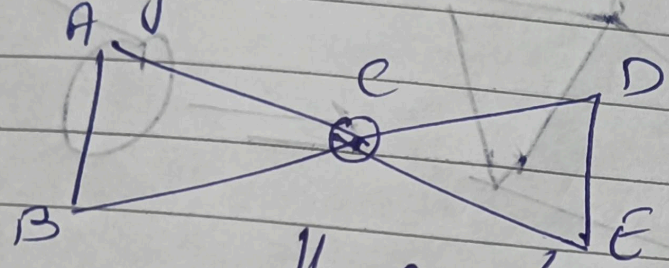
If a graph contains an open euler graph i.e. an open walk which to visit every edge of the graph only once is called open euler graph

4. Define Isomorphism 1 & 2.

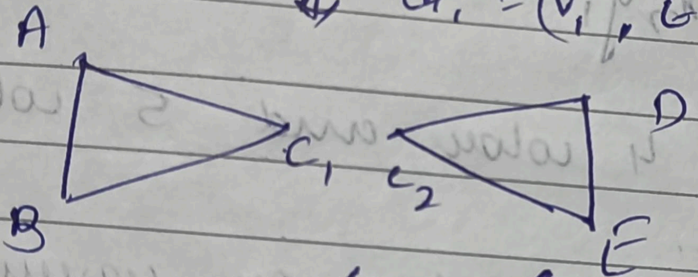
→ **Isomorphism 1**

Split a cut (articulation point) vertex into two vertices to produce two disjoint sub graph. Also, the vertex connectivity for the graph is 1.

E.g.



$\Downarrow$ $G_1 = (V_1, E_1)$



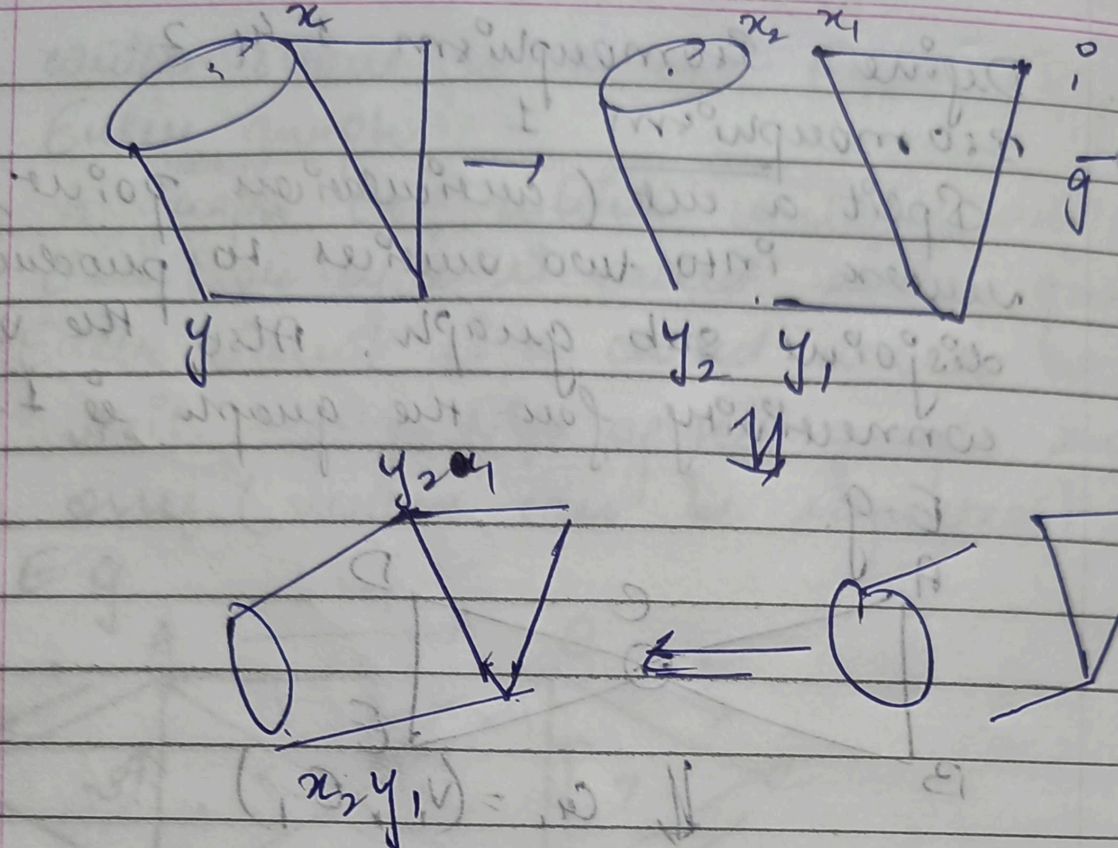
$G_2 = (V_2, E_2)$

→ **Isomorphism 2**

It's the extended form of 1- Isomorphism

Split the vertex x into x_1 and x_2 and vertex y into y_1 and y_2 such that graph G is split into g & $\bar{g}$ ~~or~~

Let vertices x_1 and y_1 go with g & x_2 & y_2 with $\bar{g}$, now rejoin the graph ~~to~~ g and $\bar{g}$ by merging x_1 with y_2 and y_1 with x_2



5. Explain 4 colour and 5 colour problem.

→ In greedy colouring or sequential colouring is a colouring of the vertices of a graph formed by a greedy algorithm that considers the vertices of the graph in sequence & assigns each vertex its first available colour.

Steps:

1) Choose an order for the vertices.
2) Choose a list of colors also in same order
3) In order color each vertex, making sure

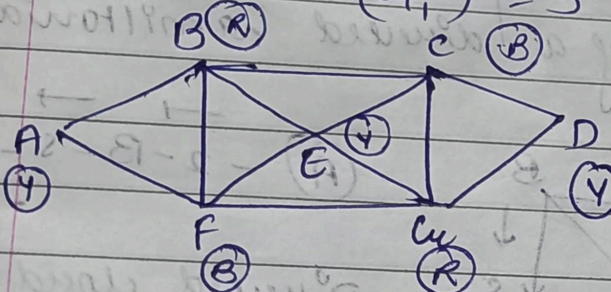
that no two adjacent vertices are in same colour.

4) Continue in this way until each vertex is coloured.

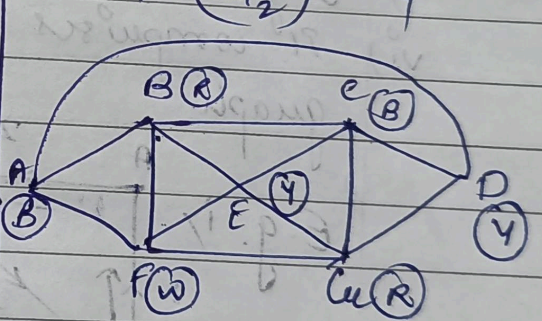
Four colour problems

The four colour problem theorem is the chromatic number $\chi(G)$ of a ~~g~~ planar graph G is not greater than 4.

E.g. G_1 graph $\chi(G_1) = 3$



G_2 graph $\chi(G_2) = 4$



If the region of a planar graph are coloured so that adjacent regions have different colours then ~~th~~ no more than 4 colours are required.

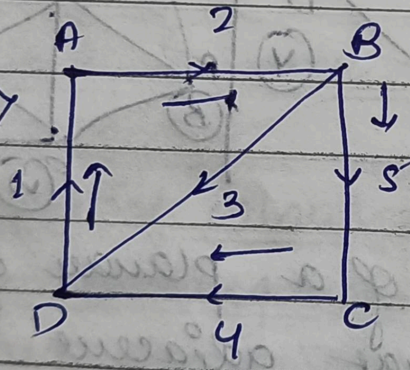
Assignment - 2

1. What is a directed hamiltonian graph?

→ The directed graph can be defined using the following conditions →

i) Digraph should be connected.
ii) It comprises of a closed directed graph
iii) The initial and final vertex are same
iv) In this graph, every vertex is traversed exactly once.
v) Edges cannot be repeated.
vi) It comprises of a directed hamiltonian graph.

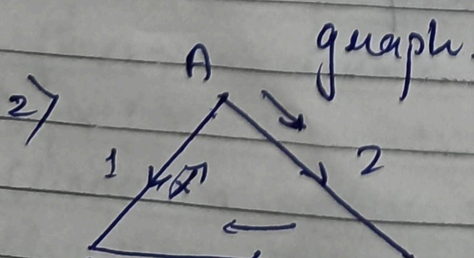
E.g. 1)



A - 2 - B - 5 - C - 4 - D - 1 - A

Directed closed walk present. All vertex traverse & no edge repeated. Starting and ending vertex are same so given graph is directed hamiltonian graph.

2)



A - 2 - B - 3 - C - 1 - A

Here only closed walk is not directed closed walk so given graph is not hamiltonian directed graph

2. Define fundamental counting principles.

→ The fundamental counting principle is a rule used to count the total number of possible outcomes in a situation.

It states that if there are n ways of doing some thing and m ways of doing another thing, after that there are n×m ways to perform both of the actions.

E.g. {1, 2, 3, 4} make a three digit number using the above digits without any repeatation.

$\boxed{}$ $\boxed{}$ $\boxed{}$

4 choices 3 choices 2 choices

So → 4 × 3 × 2 → $\underline{\underline{24}}$

3. Explain Binomial theorem with example.

→ The binomial expansion formula involves ~~bion~~ binomial coefficients which are of the form ^nC_k and it is calculated using the formula $\frac{n!}{(n-k)!\,k!}$. The binomial expansion is also known as binomial theorem.

Binomial theorem is given by the formula

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} + {}^nC_2 x^{n-2} y^2 \dots {}^nC_n x^0 y^n$$

E.g. $(a+b)^3$

By, $(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 \dots {}^nC_n x^0 y^n$.

$$(a+b)^3 = {}^3C_0 a^3 + {}^3C_1 a^{3-1} b + {}^3C_2 a^1 b^2 + {}^3C_3 a^{(3-3)} b^3$$

$$= a^3 (1) + (3) a^2 b + (3) a^1 b^2 + (1) a^0 b^3$$

$$= a^3 + 3a^2 b + 3ab^2 + b^3.$$

4. What is Summation operator?

→ The summation operator (Σ) is an instruction to sum over a series of values.

– For instance, if we have the set of values for the variable, $X = \{X_1, X_2, X_3, X_4, X_5\}$

then,

$$\sum_{i=1}^{5} X_i = X_1 + X_2 + X_3 + X_4 + X_5$$

Literally the expression $\sum_{i=1}^{5} X_i$, says: beginning with $i=1$ and ending with $i=5$, sum over the variables X_i.

E.g. $x_1 = 8$, $x_2 = 10$, $x_3 = 11$, $x_4 = 15$, $x_5 = 16$
then $n = 5$ (no. of cases) and

$$\sum_{i=1}^{n=5} x_i = 8 + 10 + 11 + 15 + 16$$
$$= \underline{\underline{60}}$$

5. What is generating function with example.
→ Generating functions provide us an alternative way to represent discrete numeric functions and solving recurrence relations. Generating functions also provide us an alternative way to solve combinatorial problems.

Let $(a_0, a_1, a_2 \dots a_r \dots) = a$ be a discrete numeric function then an infinite series in terms of Z parameter

$$A(z) = a_0 + a_1 z + a_2 z^2 \dots + a_r z^r \dots$$

E.g.

$$a_r = 1$$

$$A(z) = 1 + z + z^2 + z^3 \dots z^r + \dots$$
$$= \underline{\underline{\frac{1}{1-z}}}$$