

* Symbols:- Basic building blocks of any language which can be any character or token.

Example:- A, B, C, -- Z, a, b, c, -- z, (), \$, &, /, [], +, ;, -, *

* Alphabets:- An alphabet is a finite collection of symbols

$$\Sigma = \{ a, b, c, \dots, z, 0, 1, 2, 3, \dots, 9 \}$$

* Strings:- String is a finite collection of symbols from an alphabet set.

$$w = \{ \epsilon, a, b, ab, aa, ba, bb, aab, aab, aab, \dots \}$$

* Length of string ($|w|$) \Rightarrow Number of symbols involved in the string w is called length of string.

$$w = ab \quad |w| = 2 \quad w = aaaa \quad |w| = 8$$

$$w = \epsilon \quad |w| = 0 \text{ (null string)}$$

* Language:- Collection of strings is called Language.

$$\Sigma = \{ a, b \} \quad w_1 = \epsilon, w_2 = a, w_3 = b, w_4 = aa, \dots$$

$$L = \{ \epsilon, a, b, aa, ab, ba, bb, aaa, aab, aab, aab, baa, bbb, \dots \}$$

* formal language:- Collection of strings we put some conditions & restrictions in the formation of string is called formal language.

$$\Sigma = \{ a, b \}$$

length=1, $\Sigma^1 = L_1 = \{ a, b \}$ These all languages are formal language with length condition
 length=0 $\Sigma^0 = L_2 = \{ \epsilon \}$
 length=2 $\Sigma^2 = L_3 = \{ aa, ab, ba, bb \}$
 length=3 $\Sigma^3 = L_4 = \{ aaa, aab, aba, baa, abb, bab, bba, bbb \}$

* Grammar \Rightarrow Grammar is a set of rules (formal rules) which checks the correctness of the sentences (or) we can construct the correct sentence using these rules.
 OR

Language are associated with rules which helps to make a valid string (sentence) of input symbols.

Grammar is a collection of 4-tuples:-

$$G = (V_n, \Sigma, P, S)$$

where

$V_n \Rightarrow$ It is a finite non empty set whose elements are called variable or non variable.

$\Sigma \Rightarrow$ It is a finite non empty set whose elements are called terminals.

$S \Rightarrow$ It is a starting variable ($S \in V_n$) or special symbols.

$P \Rightarrow$ It is finite set of production whose elements are $\alpha \rightarrow \beta$ where $\alpha, \beta \in (V_n \cup \Sigma)$.

* Types of grammar [chomsky classification] There are mainly four types of grammar

- Type-0 grammar (REC) (unrestricted grammar / Regular & unrestricted lang.)
- Type-1 grammar (CSG) (context sensitive grammar / context sensitive lang.)
- Type-2 grammar (CFG) (Context free grammar / context free language)
- Type-3 grammar (RLA) (Regular grammar / Regular language)

* → Type-0 Grammar $\Rightarrow \emptyset A \psi \rightarrow \emptyset \alpha \psi$ (At least one variable on LHS of production)

(i) $\underline{ab} A \underline{bcd} \rightarrow \underline{ab} A \underline{bcd}$ compare with $\emptyset A \psi \rightarrow \emptyset \alpha \psi$
 $\emptyset = ab, \psi = bcd, A = A \& \alpha = AB$
left context right context

(ii) $A C \rightarrow A$
 $\emptyset = A, \psi = \lambda$ [left & right context are $\lambda/e/d$ (null)]
 $A = C, \alpha = \lambda$

(iii) $C \rightarrow i$
 $\emptyset = \psi = \lambda$ (left & right context are null λ)
 $A = C, \alpha = \lambda$

A production without any restriction is called type-0 grammar (or) unrestricted grammar (or) Recursive enumerable grammar.

→ Type-1 Grammar $\Rightarrow \phi A \psi \rightarrow \phi \alpha \psi$

where (i) $\alpha \neq \lambda$

(ii) $S \rightarrow \lambda$ is allowed in type-1 grammar but S is not appear in RHS of any production.

$S \rightarrow Aa$ But $A \rightarrow \lambda$ is not allowed because variable A is appear in RHS of S production

$S \rightarrow Aa \& A \rightarrow a$ is allowed.

→ Type-2 Grammar $\Rightarrow \phi A \psi \rightarrow \phi \alpha \psi$ [context free grammar]

where (i) $\phi \& \psi$ are null ($\lambda/e/d$) on left side of production means left side of is only single variable on left side of production

(i) $S \rightarrow Aa \& A \rightarrow a$ are allowed in type-2 grammar but not in type-1 grammar.

→ Type-3 Grammar (Regular grammar) \Rightarrow

where (i) start with only single variable (Production start)

(i) $A \rightarrow aB$ (Right side of any production start with terminal & after that multiple terminals are not variables allowed on RHS of the production)

Example $A \rightarrow abc$ X)

(ii) $S \rightarrow a$ is allowed but S is not appear in RHS of any production.

Q.1) check the production type?

(i)

$S \rightarrow Aa$ type-2

$A \rightarrow Ba$ type-2

$B \rightarrow abc$ type-2

$A \rightarrow c$ type-3

(iii)

$S \rightarrow aS$ type-3

$S \rightarrow AsR$ type-2

$S \rightarrow d$ type-3

$A \rightarrow aA$ type-3

$S \rightarrow ab$ type-2

(iv)

$A \rightarrow aba$ is type-2.

* Context free Grammar (CFG) \Rightarrow As the name indicates context free means the grammar have no context (right & left) context is called context free grammar. It is type-2 grammar. This grammar is accepted by PDA (Push-down Automata). Context free languages are applied in parse design. It is also useful for describing block structure in programming languages.

A CFG is defined by 4-tuples

$$G = (V_n, \Sigma, P, S)$$

V_n is the set of variables

Σ is the set of input symbols

S is a special variable called start symbol.

P is a production rules

$\therefore G$ is a CFL (context free language). If every production is in the form of $A \rightarrow \alpha$, where $A \in V_n$ & $\alpha \in (V_n \cup \Sigma)^*$

It means that this is not contain left & right context.

Q.1) Construct CFG that generates the language

$$L = \{ w c w^R / w \in (a, b)^* \}$$

$$S \rightarrow A c A / B c B \quad \& \quad A \rightarrow aB / B a / \lambda - \text{null symbol}$$

$$B \rightarrow bA / A b / \lambda \leftarrow \text{null symbol}$$

Given $G = \{ \dots \}$. $G = (S, \{a, b\}, P, S)$

where $P = \{ S \rightarrow aSa, S \rightarrow bSb \ \& \ S \rightarrow \lambda \}$

~~$L = \{ aabb, aaabbb, \dots \} \quad \& \quad L = \{ a^9, b^b, ab^5a, ba^9b, \dots \}$~~

$$S \rightarrow aSa$$

$$S \rightarrow \lambda \quad \therefore S \rightarrow a^9$$

$$S \rightarrow bSb$$

$$S \rightarrow \lambda \quad \therefore S \rightarrow b^b$$

$$S \rightarrow aSa$$

$$S \rightarrow bSb \quad \& \quad S \rightarrow \lambda \quad \therefore S \rightarrow a^9b^b a^9 = a^9b^b a^9$$

$$S \rightarrow bSb \quad S \rightarrow aSa \quad \& \quad S \rightarrow \lambda \quad \therefore S \rightarrow b^9a^9b^9 = b^9a^9b^9$$

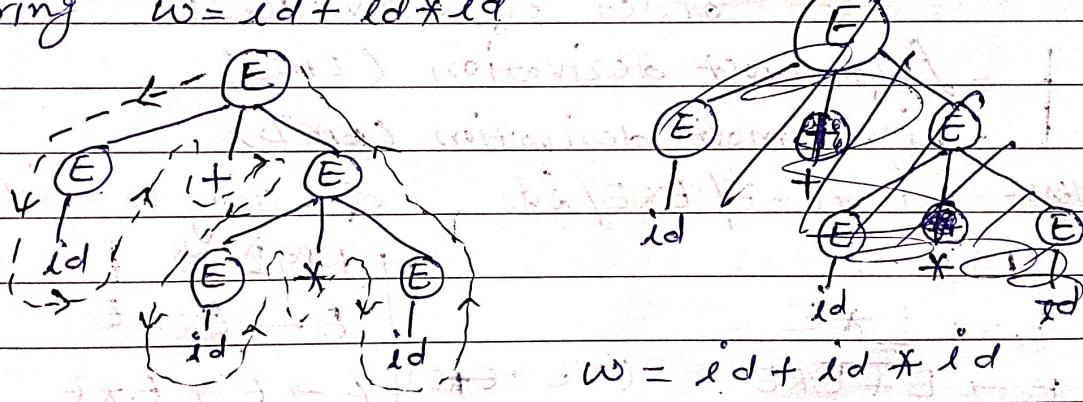
* Derivation tree / Parse tree \Rightarrow Parse tree is also called a derivation tree.
 When deriving a string w from s , if every derivation is considered to be a step in the tree construction then we get the graphical representation of derivation of string w as a tree. This is called derivation tree or parse tree of strings.

Rules:-

- All the leaf nodes of the tree are labeled by terminal of the grammar.
- The root node of the tree is labeled by the start symbol of the grammar.
- The interior nodes are labeled by the variables.

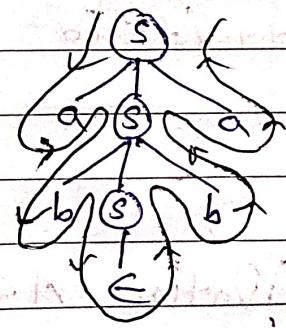
Example :- (i) $E \rightarrow E+E \mid E*E \mid id$

String $w = id + id * id$



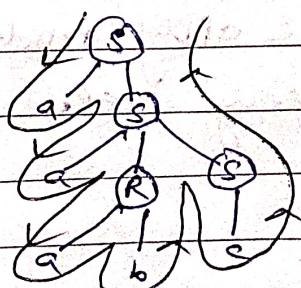
(ii) $S \rightarrow aSa \mid bSb \mid c$

$w = abba$

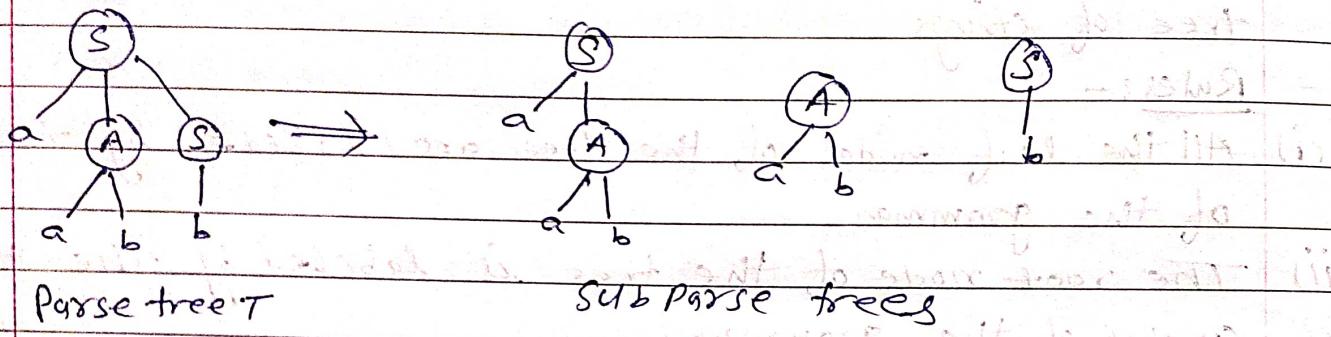


* Yield \Rightarrow The yield of a parse tree is the concatenation of the labels of the leaves without repetition in the left or right ordering.

* Yield will be = aabc



* Subtree of a parse tree \Rightarrow A subtree of a parse tree T is a tree whose root is same vertex V of T & whose vertices are the descendants of V together with their labels & whose edges are those connecting the descendants of V .



* Derivation \Rightarrow There are mainly two types of order of derivations:-

- Left most derivation (LMD)
- Right most derivation (RMD)

Examples:- $E \rightarrow E+E / E*E / id$ & string $w = id + id * id$.

LMD \Rightarrow

$$E \rightarrow E * E$$

$$E \rightarrow E + E * E \quad (\because E \rightarrow E * E)$$

$$E \rightarrow id + id * id \quad (\because E \rightarrow id)$$

RMD \Rightarrow

$$E \rightarrow E + E$$

$$E \rightarrow E + E * E \quad (\because E \rightarrow E * E)$$

$$E \rightarrow id + id * id \quad (\because E \rightarrow id)$$

or

$$E \rightarrow E + E * id \quad | \quad E \rightarrow E + id * id$$

$$\quad | \quad E \rightarrow id + id * id$$

LMD

A derivation $A \xrightarrow{*} w$ is called a LMD if we apply a production only to the left most variable at every step.

RMD

A derivation $A \xrightarrow{*} w$ is a right most derivation if we apply production to the right most variable at every step.

Q.1

Let G be the grammar $S \rightarrow aB/bA$, $A \rightarrow a/as/bAA$ & $B \rightarrow b/bS/aBB$ for string "aaabbabbba" then find LMD & also construct parse tree

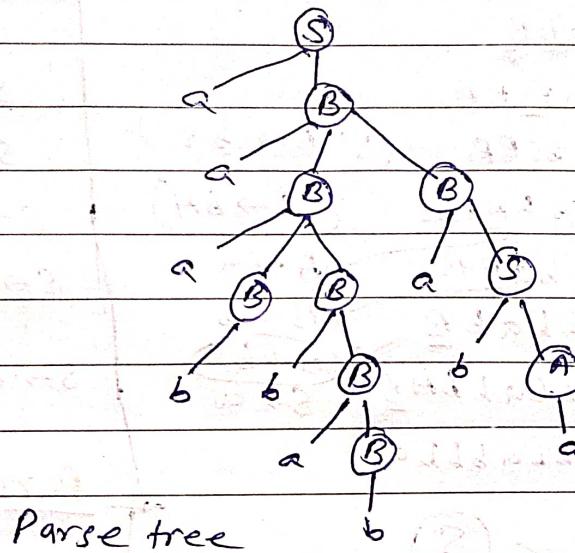
Solⁿ

$$S \rightarrow aB$$

$$S \rightarrow aaB$$

$$S \rightarrow aaaB$$

$$S \rightarrow aaabB$$



Parse tree

Q.2

Let G be the grammar $S \rightarrow oB/1A$, $A \rightarrow o/os/1AA$ & $B \rightarrow 1/1s/oBB$ for the string "00110101" then find LMD similarly Q.1

Solⁿ

The given grammar G is $S \rightarrow aA/bB$, $A \rightarrow a/bA/aA$ & $B \rightarrow a/bS$ for string "aaabbba" then find LMD & parse tree

Solⁿ

$$S \rightarrow aA$$

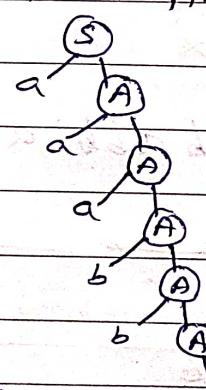
$$S \rightarrow aaA$$

$$S \rightarrow aaaA$$

$$S \rightarrow aaabA$$

$$S \rightarrow aaabbA$$

$$S \rightarrow aaabba$$



RMD

Q.4

Let G be the given grammar $S \rightarrow oB/1A$, $A \rightarrow o/os/1AA$ & $B \rightarrow 1/1s/oBB$ for string "00110101" Find RMD & parse tree

$$S \rightarrow oB$$

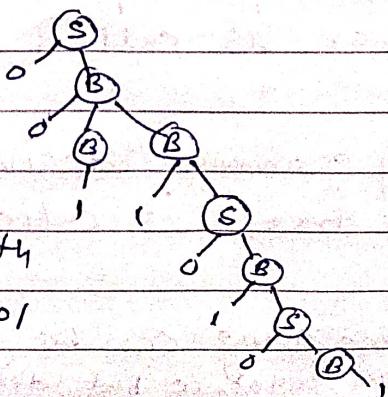
$$S \rightarrow ooB$$

$$S \rightarrow oob$$

$$S \rightarrow oob1oB$$

Parse tree with

yield = 00110101



Q.5) The given grammar is $S \rightarrow aB/bA$, $A \rightarrow a/ab/bAA$ & $B \rightarrow b/bS/abBB$. Find LMD & RMD for the string "aaabbabbba".

LMD

$$S \rightarrow aB$$

$$S \rightarrow aaB$$

$$S \rightarrow aaaB$$

$$S \rightarrow aaabs$$
 (since $B \rightarrow bS$)

$$S \rightarrow aaab$$

$$S \rightarrow aaabb$$

$$S \rightarrow aaabbab$$

$$S \rightarrow aaabbabbS$$

$$S \rightarrow aaabbabbba$$

RMD

$$S \rightarrow aB$$

$$S \rightarrow aBB$$

$$S \rightarrow aaB$$

$$S \rightarrow aaBbb$$

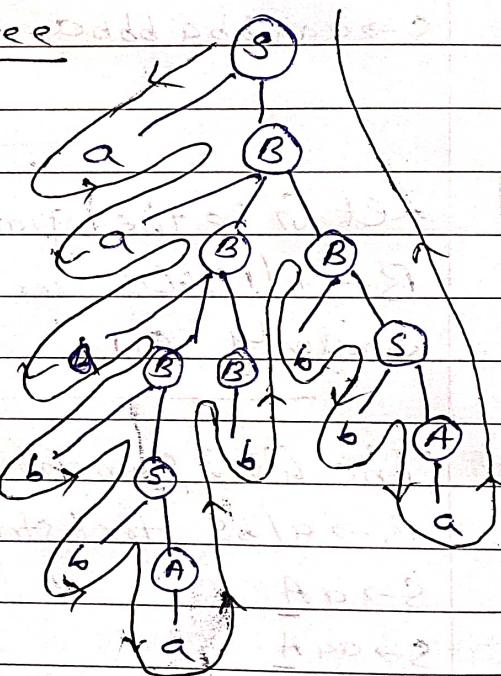
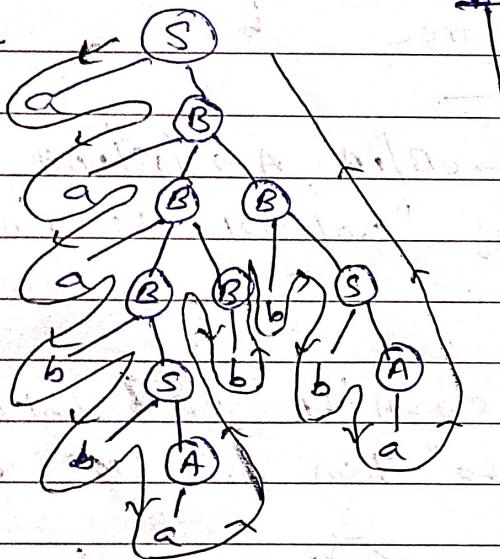
$$S \rightarrow aaBbbba$$

$$S \rightarrow aaBbbbaa$$

$$S \rightarrow aaBbbbaa$$

$$S \rightarrow aaabbabbba$$

Parse tree



string $w = aaabbabbba$

string $w = aaabbabbba$

* Ambiguous Grammar

Ambiguity in CFG \Rightarrow

Any grammar construct more than one parse tree for a single string (grammar) then these grammar is called ambiguous grammar.

(OR)

If any grammar derived more than one derivation tree (parse tree) is called ambiguous grammar.

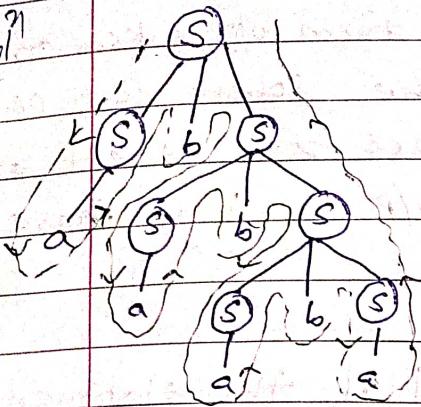
(OR)

If same string can be generate in two different ways (LMD/RMD) derivation sequence but derivation will be different (LMD/RMD).

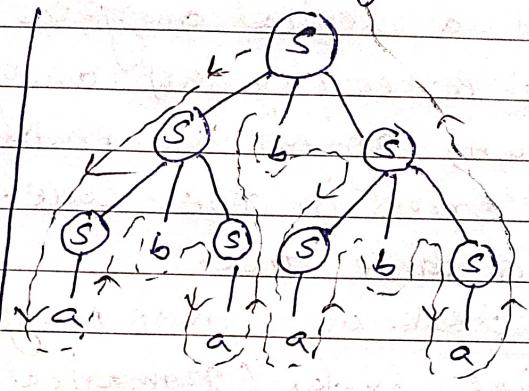
Q.1)

If the productions are $S \rightarrow SBS/a$ & string $w = abababa$
 then show that G (grammar) is ambiguous.

Sol:



$w = abababa$



$w = abababa$

So given grammar is ambiguous grammar.

Q.2)

Check whether the following grammar is ambiguous

$S \rightarrow iCtS / iCtSeS / b/a$ & string $w = ibtibtaea$

LMD \Rightarrow

RMD

$S \rightarrow iCtSeS$

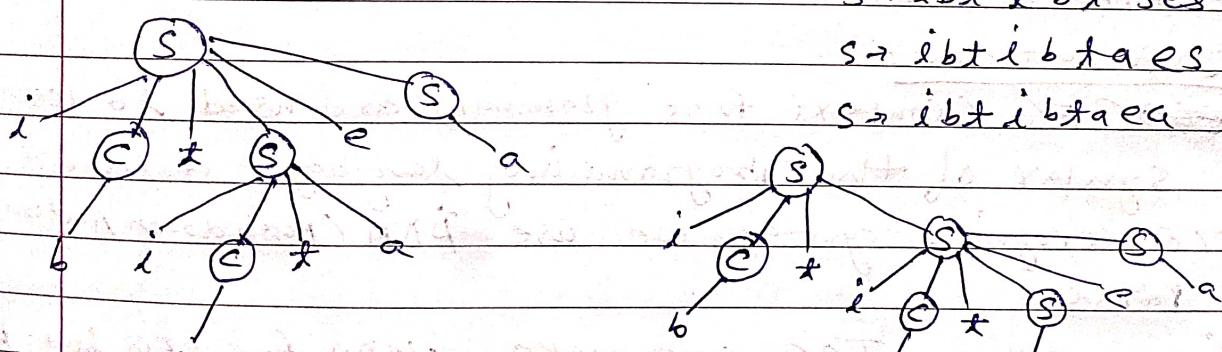
$S \rightarrow iCtS$

$S \rightarrow ibt iCtSeS$

$S \rightarrow ibtS$

$S \rightarrow ibtibtaea$

$S \rightarrow ibtiCtSeS$



$w = ibtibtaea$

$w = ibtibtaea$

So given grammar is ambiguous grammar.

Q.3)

$S \rightarrow SAS/b$, $A \rightarrow ba$ string $w = bba bbbbab$

Q.4)

$S \rightarrow aAs/a$, $A \rightarrow SbA/SS/ba$ string $w = aabaaabbbaaa$

Q.5)

* Syntax analyzer (Parser) \Rightarrow A syntax analyzer or parser takes the input from a lexical analyzer in the form of token stream. The parser (syntax) analyzes the source code (token stream) against the production rules to detect any errors in the code. The o/p of this phase is a parse tree.

Main task of Syntax Analyzer:-

- Verifies Syntax (grammatical checking) of the statement.
- It constructs the parse tree.
- It detects syntax errors.
- Error recovery.

Syntax Errors :-

- Less number of tokens required for the statement.
Example:- $a = b + c$
- More number of tokens
Example:- $a = (b + c) * (d * e))$
- Tokens are not arranged properly.
Example:- ; int a, b

Important point :-

- ~~CFG~~ (Context free grammar) are used to describe syntax of the programming language statement.
- To recognize syntax we use PDA (Pushdown Automata) Parser
- Generally in TOC, we make NPDA for CFG but here we make DPDA which are very difficult for this we restrict in CFG.

Example ①	Input: a, b;
	$S \rightarrow TL;$
	$T \rightarrow \text{int} / \text{float}$
	$L \rightarrow L, id / id$

Example ②
$a = b + c * d;$
$S \rightarrow id = E;$
$E \rightarrow E + E / E * E / id$

GIRIRAT

Example (3) I am talking to him

$S \rightarrow \text{subject} \rightarrow \text{verb} \rightarrow \text{object}$

$S \rightarrow I/\text{we}/\text{you}$

verb \rightarrow talking

object \rightarrow he/she/her/him

→

* Parser \Rightarrow A Syntax analyzer or parser is nothing but a program that takes string w & CFG grammar G and verifies whether the string derivable from the grammar or not.

If it is derivable we construct derivation tree for that string otherwise, it returns syntax error to the error handler.

OR

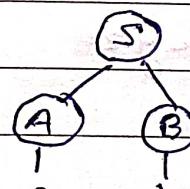
A Syntax analyzer or parser is a program that performs Syntax analysis.

A Parser obtains a string of tokens from the lexical analyzer & verifies whether or not the string is valid construct of the source language.

Example:- String $w = ab$

Grammar:- $S \rightarrow AB$

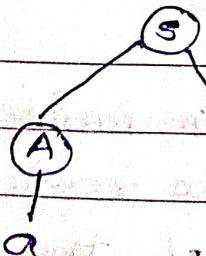
$A \rightarrow a$ & $B \rightarrow b$



↓ Top-down parsing

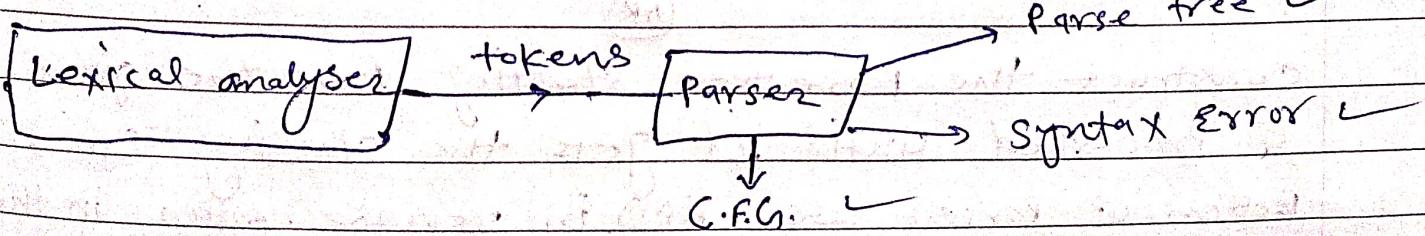
∴ string ab is a member of given grammar

parse tree



write string 1st if you see the terminal & it is in CFG, replace it by its LHS part [it in RMD]

↳ Bottom-up Parsing:



* Parsing ➔ Parsing is the technique or process used to determine whether string of tokens can be generated by grammar.

Classification of Parsing (Parser) :-

TOP-Down Parsing

Bottom-Up Parsing

→ TOP-Down Parsing ➔ TOP-down Parsing attempts to find the left most derivations for generating parse tree from root to leaf nodes.

(OR)

In top-down parsing, we will construct the parse tree starting from the starting symbol symbol the required string. At every steps will replace LHS by non-terminal by its corresponding RHS part.

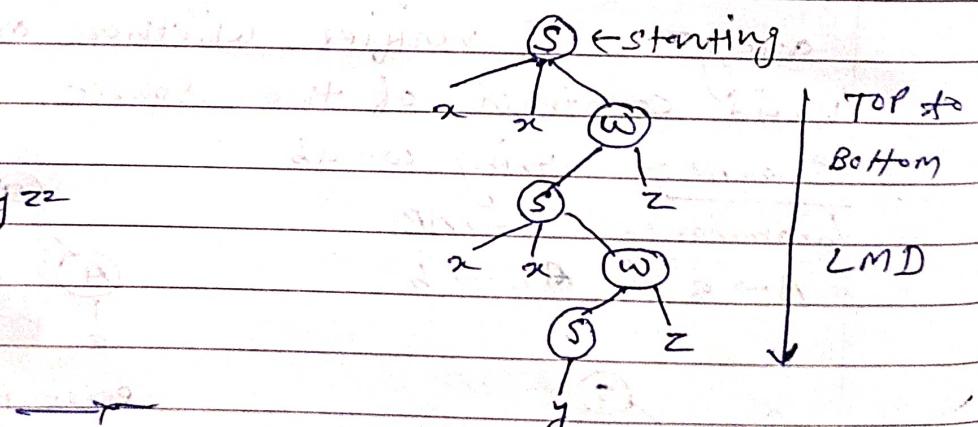
TOP-Down Parser uses LMD to construct the parse tree.

Example:- $S \rightarrow xxw$

$S \rightarrow y$

$w \rightarrow sz$

String $w = x x x x y z z$



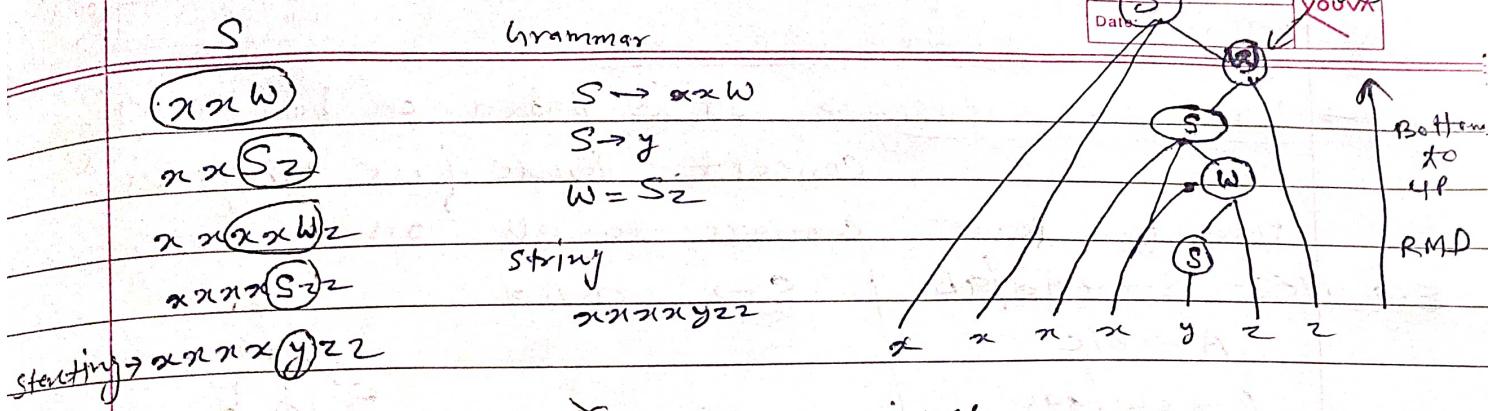
→ Bottom-up Parsing ➔ Bottom-up parser or parsing can be defined as attempt to reduce the input string (w) to the starting symbol of the grammar by tracking of the RHS (RMD) of string (w) in reverse!

(OR)

construct the parse tree starting from the given string & processed until it gets the starting symbol.

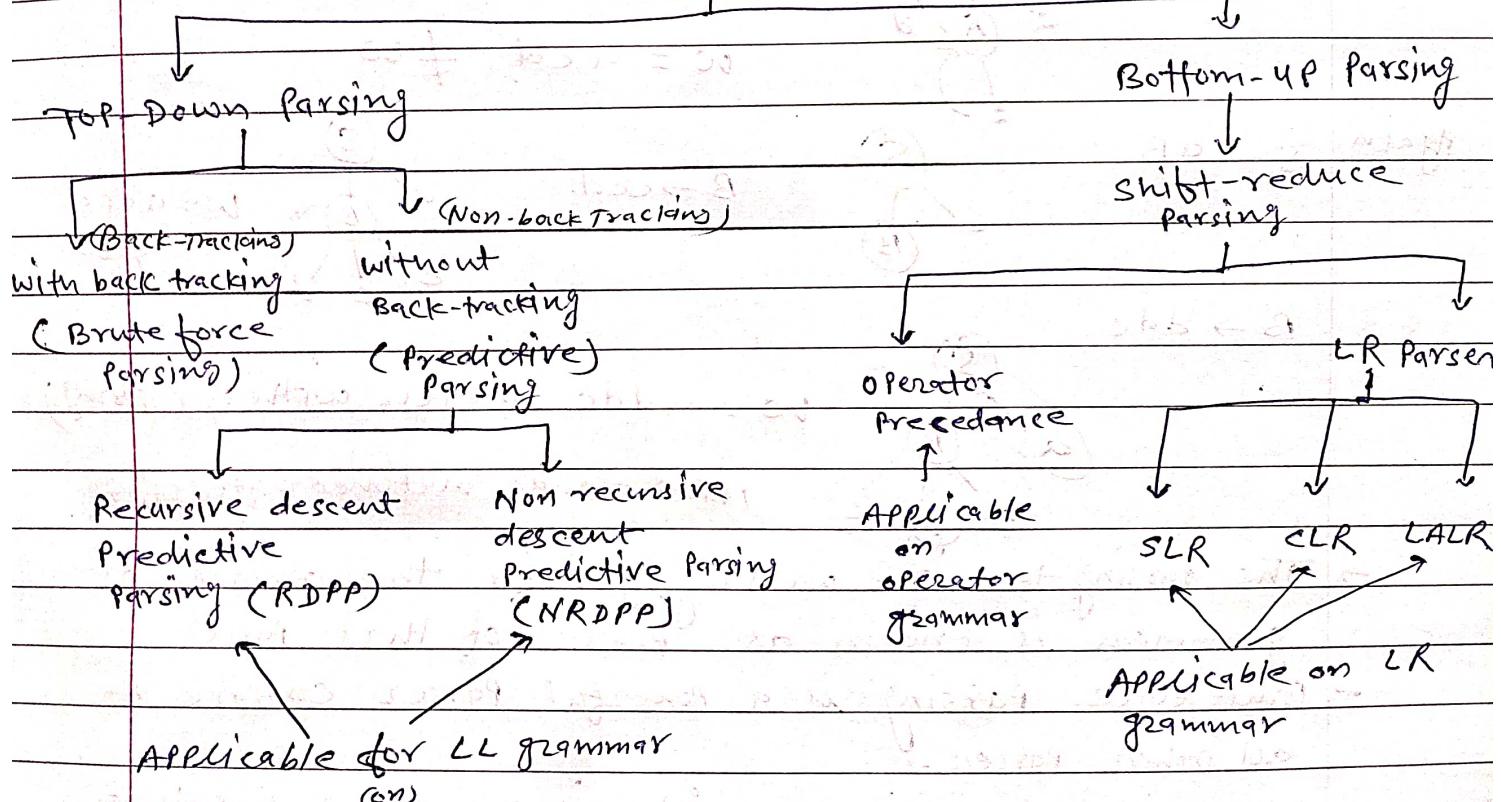
Bottom-up parser uses RMD in reverse order in the construction of the parse tree.

Bottom-up parser design from leaf node to root node.

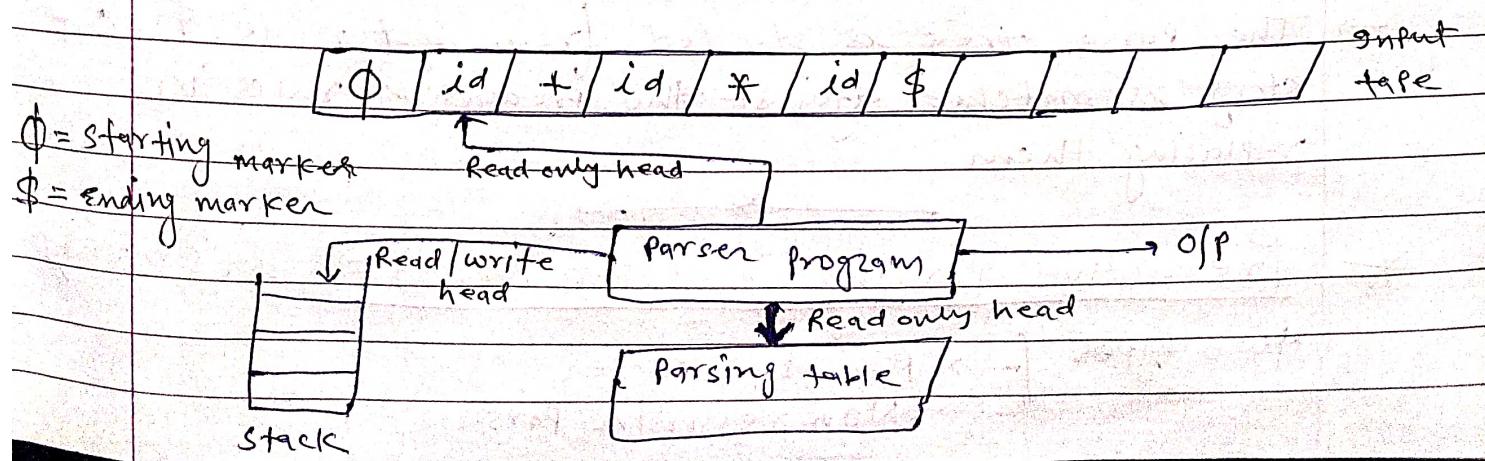


* Parsing techniques (Parsing classification)

Parsing techniques



* Basic Architecture of Parser \Rightarrow All the parser (Top-down & bottom-up) used same architecture.



~~TOP-Down Parsing techniques~~ \Rightarrow

\rightarrow Brute force Parsing \Rightarrow It is based on back-tracking concept. Brute force parsing is a powerful parser compare to all other parsers.

Example:- $S \rightarrow aAd/aB$ | $S \rightarrow aAd.$

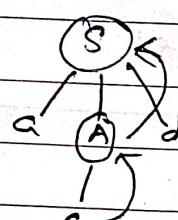
$$A \rightarrow b/c$$

$$B \rightarrow ccd/ddc$$

$$w = addc$$



Again



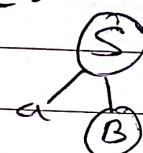
$$A \rightarrow b/c$$

$$w = abd \notin w$$

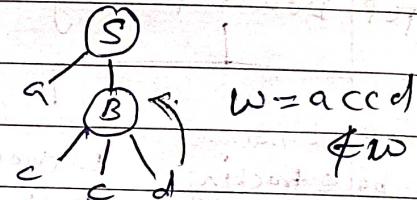
\rightarrow back track

Again

$$S \rightarrow aB$$

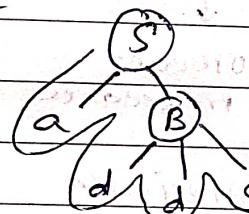


$$B \rightarrow ccd.$$



$$w = accd \notin w$$

$$B \rightarrow ddc$$



$$w = addc \text{ (match with } w \text{ (string))}$$

\rightarrow yield = addc matched with string.

\rightarrow The parsing technique working good for the number of grammars is known as power of that parser

\rightarrow Brute force Parsing is a powerful parser compare to all other parser.

time complexity $O(n^3)$ it takes exponential time

\rightarrow Back tracking require exponential time complexity, hence no practical compiler is design with back tracking.

\rightarrow The parse tree is started from root node &. If string is matched against the production rules in replacing them

* Predictive Parsing [predictive parser] \Rightarrow it's mainly

two types

\rightarrow Recursive parser

\rightarrow Non recursive parser

Predictive Parser \Rightarrow whenever a non-terminal having multiple production on the RHS part, we can predict using production to select by using look ahead symbol. To construct Predictive Parser grammar should not contain left recursion & it should be left factoring (factored).

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Non Recursive Descent Predictive Parser (NR DPR) \Rightarrow LL(1)

We discuss the Non-recursive Descent Predictive parser in two ways:-

- ① Check the validity of string assume predictive table is given
- ② Grammar given & we have to design predictive table

Q.1) Grammar $\Rightarrow E \rightarrow TE' \quad E' \rightarrow +TE'/\epsilon$
 $T \rightarrow FT' \quad T' \rightarrow *FT'/\epsilon \quad \& \quad F \rightarrow (E)/id$
 string $\Rightarrow id + id * id$

Using LMD (left most derivation) \Rightarrow

$E \rightarrow TE'$	put $T \rightarrow FT'$	$E \rightarrow id + FT'E' \quad F \rightarrow id$
$E \rightarrow FT'E'$	$F \rightarrow id$	$E \rightarrow id + id * FT' \quad T' \rightarrow *FT'$
$E \rightarrow id T'E'$	$T' \rightarrow \epsilon$	$E \rightarrow id + id * id T' \quad F \rightarrow id$
$E \rightarrow id E'$	$E' \rightarrow +TE'$	$E \rightarrow id + id * id \epsilon \quad T' \rightarrow \epsilon$
$E \rightarrow id + TE'$	$T' \rightarrow FT'$	$E \rightarrow id + id * id$

-- | | ϕ | id | + | id * | id | \$ | | -- input tape

Read/Write pointer

Parser program

Stack	V	T	id	+	*	()	\$
	E	E $\rightarrow TE'$				E $\rightarrow TE'$		
	T	T $\rightarrow FT'$				T $\rightarrow FT'$		
	E'		E' $\rightarrow +TE'$			E' $\rightarrow \epsilon$	E' $\rightarrow \epsilon$	
	T'		T $\rightarrow \epsilon$	T' $\rightarrow *FT'$		T' $\rightarrow \epsilon$	T' $\rightarrow \epsilon$	
	F	F $\rightarrow id$				F $\rightarrow (E)$		

\rightarrow Two markers put in input tape :- starting marker (ϕ) & end marker ($\$$)

\rightarrow Stack contains initial two elements $\$. (dollar)$ &
 Starting variable E

Shift operation means:- Push into stack corresponding entry from parsing table.

Example:- $E \text{ vs } id \Rightarrow$ different value & corresponding entry will be $E \rightarrow TE'$ then push E' onto stack first push E' , second T . similarly all entry.

Reduce operation means:- Pop from stack & head move one position right hand side.

$(\phi | id | + | id | * | id | \$)$ initial

Step-I :- $E \text{ vs } id \Rightarrow E \rightarrow id \quad E \rightarrow TE' \quad | E$
Push $E \rightarrow TE'$ into stack $| \$$

T
E'
\$

E
\$

Step-II :- $T \text{ vs } id \Rightarrow FT' \quad$ push into stack $| F$

T
T'
E'
\$

Step-III :- $F \text{ vs } id \Rightarrow F \rightarrow id \quad | id$
Push into stack $| T'$

id
T'
E'
\$

Step-IV :- $id \text{ vs } id \Rightarrow$ pop id from the stack $| T'$

T'
E'
\$

& head move one position Right hand side

Step-V :- $\phi | id | + | id | * | id | \$ \quad | E'$
 $T' \text{ vs } + \Rightarrow T' \rightarrow E'$ push $+ | \$$

E'
\$

+
T
E'
\$

Step-VI :- $E' \text{ vs } + \Rightarrow E' \rightarrow +TE' \quad$ push into stack $| F$

T
E'
\$
F

$+ \text{ vs } + \Rightarrow$ pop from stack top $| T$
head more Right side $| E'$

Step-VII :- $T \text{ vs } id \quad | id$
 $T \rightarrow FT' \quad | E'$ push into stack $| T'$

id
T'
E'
\$

\rightarrow Step-VIII :- $F \text{ vs } id \Rightarrow F \rightarrow id \quad$ push into stack $| T'$

Step-IX :- $id \text{ vs } id \Rightarrow$ pop from stack top & head more right side $| id$ $| T'$

id
T'
E'
\$

Step-X :- $T' \text{ vs } * \Rightarrow T' \rightarrow *FT'$ $| F$

F
\$

Step-XI :- $* \text{ vs } * \Rightarrow$ pop stack $| T'$

Top & head move Right side $| E'$

E'
\$

: | id | + | id | * | id | \$ |

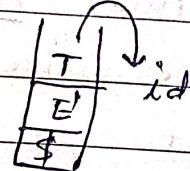
Step-XII :- $F \text{ Vs } id \Rightarrow F \rightarrow id$ push into stack

Step-XIII :- $id \text{ Vs } id \Rightarrow \text{Pop from stack TOP}$

head move one position right side

Step-XIV :- $\phi | id | + | id | * | id | $ |$

$T' \text{ Vs } \$ \Rightarrow T' \rightarrow \epsilon$ push ϵ



Step-XV :- $E' \text{ Vs. } \$ \Rightarrow E' \rightarrow \epsilon$ push ϵ

Step-XVI :- $\$ \text{ Vs } \$ \Rightarrow \text{String is valid}$

Note :- Invalid string \Rightarrow (i) If no corresponding entry found in table for stack top Vs I/p tape

(ii) if it contains different terminals like

$+ \text{ Vs } id$ or $* \text{ Vs } id$ or $+ \text{ Vs } *$ or $* \text{ Vs } +$

(iii) Both above case comes (Existence) then string is invalid.

Consider a string $(id) + id$ find the given string is valid or invalid. Grammars are:

$E \rightarrow TE'$ $E' \rightarrow +TE'/\epsilon$ $T \rightarrow FT'$

$T' \rightarrow *FT'/\epsilon$ $F \rightarrow (E)/id$

Self \Rightarrow Parsing table first question of use \Rightarrow 1

Recursive Descent predictive Parser (RDPP)

Working steps :-

- ① Remove left recursion from grammar if it is present in the grammar
- ② Do left factoring if needed
- ③ calculate the first & follow of given grammar
- ④ Construct predictive parsing table.

\Rightarrow Eliminating Left Recursion \Rightarrow If a grammar contains a pair of productions of the form $A \rightarrow A\alpha / \beta$ then the grammar is called left recursive grammar.

If we don't remove the left recursion then top-down parser enters into an infinite loop during the parsing process.

There are following types of recursions

- ① Recursive ② left recursion ③ right recursion
- ④ Direct recursion ⑤ Indirect recursion

Rule :- $E \rightarrow E\alpha / \beta$ or $E \rightarrow E\alpha \& E \rightarrow \beta$

then $E \rightarrow \beta E'$ & $E' \rightarrow \alpha E'/\epsilon$

In general form:-

$A \rightarrow A\alpha_1 / A\alpha_2 / A\alpha_3 / \dots / A\alpha_n / \beta_1 / \beta_2 / \beta_3 / \dots / \beta_m$
then

$A \rightarrow \beta_1 A' / \beta_2 A' / \beta_3 A' / \dots / \beta_m A'$

$A' \rightarrow \alpha_1 A' / \alpha_2 A' / \alpha_3 A' / \dots / \alpha_n A' / \epsilon$

Q.1)

Consider the following grammar

$A \rightarrow ABB / Aa / a / b$ $B \rightarrow Bb / b$ remove left recursion

$A \rightarrow aA' \& A' \rightarrow BbA'/\epsilon$ $A \rightarrow aA' / ba'$
 $A \rightarrow bA' \& A' \rightarrow aa'/\epsilon$ $A' \rightarrow Bba' / aA'/\epsilon$

Similarly $B \rightarrow Bb / b$

$B \rightarrow bB' \& B' \rightarrow bB'/\epsilon$

Q.2)

$A \rightarrow ABd / Aa / a$ Soln $A \rightarrow aA' \& A' \rightarrow bdA' / aA'/\epsilon$

Q.3)

$S \rightarrow Aa / b \& A \rightarrow Ac / Sd / \epsilon$

$A \rightarrow Ac / Sd / \epsilon$ put $S \rightarrow Aa / b$ $A \rightarrow Ac / bd / \epsilon$

$A \rightarrow Ac / Aad / \epsilon$ then

$A \rightarrow EA' \text{ or } A \rightarrow A'$

$A' \rightarrow AadA'/\epsilon$

$A \rightarrow bdA'$

$A' \rightarrow CA'/\epsilon$

then $A \rightarrow bdA'/\epsilon \& A' \rightarrow CA' / AadA'/\epsilon$

Remove left recursion

Date:

(Q.4) $E \rightarrow E + T / T \quad T \rightarrow T * F / F \quad \& \quad F \rightarrow (E) / \text{id}$

sol: $E \rightarrow E + T \quad \{ \Rightarrow E \rightarrow TE' \quad \& \quad E' \rightarrow +TE'/\epsilon$

$E \rightarrow T$

$T \rightarrow T * F \quad \{ \Rightarrow T \rightarrow FT' \quad \& \quad T' \rightarrow *FT'/\epsilon$

$T \rightarrow F \quad \& \quad F \rightarrow (E) / \text{id}$

(Q.5) $S \rightarrow (L) / a \quad \& \quad L = L_1 S / S$

$L \rightarrow L_1 S \quad \{ \Rightarrow L \rightarrow S L'$

$L \rightarrow S \quad \{ \Rightarrow L' \rightarrow S L'/\epsilon$

Left factoring \Rightarrow If $A \rightarrow \alpha / \alpha A$ generate "aaaa" we design the grammar that has no production starting symbol same called left factoring grammar Result of given grammar $A \rightarrow \alpha A' \quad \& \quad A' \rightarrow A/\epsilon$

(OR)

If you have more than one string on the grammar are started with the same symbol then left factoring.

In parser we can not see forward otherwise we have to use look ahead

Q.1 Calculate the left factoring [Do the left factoring]

(i) $A \rightarrow aA / aB / a \quad \& \quad B \rightarrow b / bB$

sol: $A \rightarrow aA' \quad \& \quad B \rightarrow bB'$

$A' \rightarrow A / B / \epsilon \quad \& \quad B' \rightarrow B / \epsilon$

(ii) $A \rightarrow aB / aG \quad \& \quad B \rightarrow bB / b$

$A \rightarrow aA' \quad \& \quad B \rightarrow bB'$

$A' \rightarrow B / G \quad \& \quad B' \rightarrow B / \epsilon$

$G \rightarrow cG / \epsilon \quad \& \quad G' \rightarrow C / \epsilon$

(iii) $A \rightarrow aB / aA / aBA / \alpha \quad \& \quad B \rightarrow bB / b$

$A \rightarrow aA' \quad \& \quad B \rightarrow bB'$

$A' \rightarrow B / A / BA / \epsilon \quad \& \quad B' \rightarrow B / \epsilon$

(or)

$A' \rightarrow B / BA \quad \& \quad A'' \rightarrow BA''$

$A' \rightarrow BA'' \quad \& \quad A'' \rightarrow A / \epsilon \quad \text{or} \quad \epsilon / A$

(iv) $A \rightarrow aA' \quad \& \quad A' \rightarrow A / AB / ABC$

$A'' \rightarrow \epsilon / B / BG \quad \& \quad A''' \rightarrow B / BG \quad \& \quad A''' \rightarrow BA''' / \epsilon$

$A'' \rightarrow \epsilon / C \quad \& \quad A''' \rightarrow C / \epsilon$

Q1 Calculate the first & follow \Rightarrow

First :- Rules for first :- If $\alpha = XYZ$

Rule-1 :- if $\text{first}(\alpha) = \text{first}(XYZ) = \{\epsilon\}$ if X is terminal otherwise

$\text{first}(\alpha) = \text{first}(X) \cup \text{first}(XY) \cup \dots \cup \text{first}(XYZ)$ if X does not derive to an empty string (ϵ / id)

If X derives to an empty string (ϵ) ($\because X \rightarrow \epsilon$) then

$$\text{first}(\alpha) = \text{first}(XYZ) = \text{first}(X) - \{\epsilon\} \cup \text{first}(YZ)$$

$P \rightarrow$ Production Variable, $T \rightarrow$ Terminal, $V \rightarrow$ Variable

(i) $P \rightarrow TV$ then $\text{first}(P) = \{T\}$

(ii) $P \rightarrow PVT$ & $V \rightarrow T'$ then $\text{first}(P) = \text{first}(V) = \{T'\}$

(iii) $P \rightarrow VT$ & $V \rightarrow T'(\epsilon)$ then $\text{first}(P) = \text{first}(V) - \{\epsilon\} \cup \text{first}(T)$
 $= \{T'\} - \{\epsilon\} \cup \{T\} \Rightarrow \{T, T'\}$

Q.1 Consider the following grammar & calculate the first

$$S \rightarrow aABh$$

$$A \rightarrow c/\epsilon \quad B \rightarrow d/\epsilon$$

$$\text{first}(S) = \{a\} \quad \text{or} \quad \text{first}(S) = \text{first}(aABh) = \{a\}$$

$$\text{first}(A) = \text{first}(c) \cup \text{first}(\epsilon) = \{c, \epsilon\}$$

$$\text{first}(B) = \text{first}(d) \cup \text{first}(\epsilon) = \{d, \epsilon\}$$

Q.2 Calculate the first of the following grammar

$$E \rightarrow TE' \quad E' \rightarrow +TE'/\epsilon \quad T \rightarrow FT' \quad T' \rightarrow *FT'/\epsilon$$

$$F \rightarrow (E)/\text{id}$$

$$\text{first}(E) = \text{first}(T) = \text{first}(F) = \{\epsilon, \text{id}\}$$

$$\text{first}(E') = \{+, \epsilon\}$$

$$\text{first}(T') = \{* \}, \epsilon\}$$

W	first
E	$\{\epsilon, \text{id}\}$
E'	$\{+, \epsilon\}$
T	$\{\epsilon, \text{id}\}$
T'	$\{* \}, \epsilon\}$
F	$\{\epsilon, \text{id}\}$

\Rightarrow Follow \Rightarrow

- (i) follow of starting variable is $\$$
- (ii) $P \rightarrow VT$ or $P \rightarrow T'VT$
 $\text{follow}(V) = \{T\}$
- (iii) $P \rightarrow VV'$ or $P \rightarrow T'VV'$ & $V' \rightarrow T'$
 $\text{follow}(V) = \text{first}(V') = \{T'\}$
- (iv) $P \rightarrow VV'$ & $V' \rightarrow T'/\epsilon$
 $\text{follow}(V) = \text{first}(V') - \{\epsilon\} \cup \text{follow}(P)$
- (v) $P \rightarrow VV'$
 $\text{follow}(V') = \text{follow}(P)$

Note:- We can not use ϵ in follow of any variable.

Q.1) calculate the follow

$$S \rightarrow aABb \quad A \rightarrow c/\epsilon \quad B \rightarrow d/\epsilon$$

$$\text{follow}(S) = \{\$\}$$

$$\begin{aligned}\text{follow}(A) &= \text{first}(B) - \{\epsilon\} \cup \text{first}(b) \\ &= \{d, c\} - \{\epsilon\} \cup \{b\} = \{d, b\} \text{ or } = \{b, d\}\end{aligned}$$

$$\text{follow}(B) = \text{first}(b) = \{b\}$$

Q.2) calculate the follow for the following grammar

$$E \rightarrow TE' \quad E' \rightarrow +TE'/\epsilon \quad T \rightarrow FT'$$

$$T' \rightarrow *FT'/\epsilon \quad F \rightarrow (E) / \text{id}$$

$$\text{follow}(E) = \{\$\} \cup \text{first}(+) = \{+, \$\}$$

$$\text{follow}(E') = \text{follow}(E) = \{+, \$\}$$

$$\begin{aligned}\text{follow}(T) &= \text{first}(E') - \{\epsilon\} \cup \text{follow}(E) \\ &= \{+, \epsilon\} - \{\epsilon\} \cup \{+, \$\} \Rightarrow \{+, \), \$\}\end{aligned}$$

$$\text{follow}(T') = \text{follow}(T) = \{+, \), \$\}$$

$$\text{follow}(F) = \text{first}(T') - \{\epsilon\} \cup \text{follow}(T')$$

$$= \{* , \epsilon\} - \{\epsilon\} \cup \{+, \), \$\} = \{+, *, \), \$\}$$

Q.3) $S \rightarrow AGB / GB / BA$

$$A \rightarrow da / BG$$

$$B \rightarrow g/\epsilon$$

$$f \quad G \rightarrow h/\epsilon$$

calculate the first & follow

Predictive

\Rightarrow Construct Parsing Table \Rightarrow

$$\begin{array}{l} E \rightarrow TE' \quad E' \rightarrow +TE' / \epsilon, \quad T \rightarrow FT' \\ T \rightarrow *FT' / \epsilon \quad F \rightarrow (E) / id \quad F \rightarrow (E) / id \end{array}$$

Variable	First	Follow
E	$\{\epsilon, c, id\}$	$\{, \), \$\}$
E'	$\{+, \epsilon\}$	$\{\, , \$\}$
T	$\{c, id\}$	$\{+, \), \$\}$
T'	$\{* , \epsilon\}$	$\{+, \), \$\}$
F	$\{c, id\}$	$\{+, *, \), \$\}$

$\checkmark T$	id	*	*	*	*	*	\$
E	$E \rightarrow TE'$				$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$				$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$				$T \rightarrow FT'$		
T'		$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$			$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$				$F \rightarrow (E)$		

(Q.2)

calculate first & follow & construct predictive table
 $S \rightarrow iETSS, / a$ $S_1 \rightarrow eS / \epsilon$ $E \rightarrow b$

Variable	First	Follow
S	$\{a, i\}$	$\{\epsilon, \$\}$
S_1	$\{\epsilon, e\}$	$\{\epsilon, \$\}$
E	$\{b\}$	$\{+\}$

$\checkmark T$	a	b	e	i	*	t	\$
S	$S \rightarrow a$						
S_1				$S_1 \rightarrow eS$			$S_1 \rightarrow \epsilon$
E		$E \rightarrow b$					

time complexity $O(n)$