Unit-IV

* **Fundamental Principal of counting** ⇒ The fundamental counting principle is a rule used to count the total number of possible outcomes in a situation.
It states that if there are n ways of doing something & m ways of doing another thing, after that there are $n \times m$ ways to perform both of the actions.

Q.1) Suppose you have 3 shirts (call A, B, C) & 4 pairs of pants (call w, x, y, z) then we have
$3 \times 4 = 12$ possible ways (outcomes) ⇒ 12
Ex:- Aw, Ax, Ay, Az, Bw, Bx, By, Bz, Cw, Cx, Cy, Cz

Q.2) Suppose you roll a 6 sided die & draw a card from a deck of 52 cards. There are 6 possible outcomes on the die & 52 possible outcomes from the deck of card. so total possible outcomes are
$6 \times 52 = 312$

Ex:-

* **Permutation & combinations** ⇒
(Arrangement) (Selection)

⇒ Counting Principale ⇒ (Read fundamental principal of counting)
Ex:- {1, 2, 3, 4} Number से three digit number बनाएं without repeatation of any no.

4 choice no. | 3 choice no. | 2 choice no. (without repeatation) $= 4 \times 3 \times 2 = 24$ No.

4 choice no. | 4 choice no. | 4 choice no. (Repeatition possible) $= 4 \times 4 \times 4 = 64$ No.

{A, B, C, D, E}
= 5 alphabets

5 $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Combinatorics ⇒ Combinatorics is that branch of discrete mathematics which is concerned with the counting problems.

Counting rules ⇒

Sum rule :- $m|A+B| = |A| + |B|$

Product rules $|A*B| = |A| * |B|$

⇒ **Permutation (P)** ⇒ Selection + order [select + Arrangement]

An ordered argument (or ordered selection) of objects from a finite set of objects is called permutation

[इसमें सबसे पहले चुनते हैं और चुन के order में रखना होता है]

$${}^{n}P_{r} = \frac{\lfloor n}{\lfloor n-r} \quad \left\{ \begin{array}{l} \text{Here } n \text{ no. of items \&} \\ r \text{ no. of selection/available} \end{array} \right.$$

If repeatation {Permutation with repeation} $= \dfrac{\lfloor n}{\lfloor p \lfloor q \lfloor r}$

Circular permutation $= \lfloor n-1$ {1 fixed point/item}

Ex:- How many numbers can be formed taking only 3 out of 5 digits {3, 4, 5, 6, 7}?

Sol^n Required number $= {}^{5}P_{3} = \dfrac{\lfloor 5}{\lfloor 5-3} = \dfrac{\lfloor 5}{\lfloor 2} = \dfrac{5\times4\times3\,\cancel{\lfloor 2}}{\cancel{\lfloor 2}} = 60$

Ex:- Word "Medicaps" be arrange.

Sol^n Total no. of arrangements $= \lfloor 8$ or ${}^{8}P_{8} = 40320$

If vowels are never separated

$\boxed{eia}$ Mdcps total word length $= 6$

${}^{6}P_{6} = \lfloor 6 = 720$ arrangements

but vowels being three (a, e, i) characters & it may be arrangement together is $\lfloor 3 = 6$

∴ total no. of arrangement $= 720 \times 6 = 4320$ ways

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Example Word "Allahabad" or "ALLAHABAD"

Here No. of A's = 4 & No. of L's = 2

$$\therefore \frac{\lfloor 9}{\lfloor 4 \lfloor 2} = \frac{9 \times 8 \times 7 \times \cancel{6}^{3} \times 5 \cancel{\lfloor 4}}{\cancel{\lfloor 4} \times \cancel{2} \times 1} = 7560$$

Ex:- How many ways can be 9 persons be seated at round table.

Sol^n $\lfloor 9-1 = \lfloor 8 = 40320$ { Here circular permutation concept used.

⇒ Combination ⇒ { only selection not arrangement/order

An unordered arrangement or unordered selection of objects from a finite set of objects is called combination.

[only selection (चुनना) करना]

$$^{n}C_{r} = \frac{\lfloor n}{\lfloor r \lfloor n-r}$$

Q.1) Find the no. of diagonals which can be drawn by joining the angular points of a hexagon.

Sol^n A hexagon, six vertices & six sides

No. of vertex → $^{6}C_{2} = \frac{\lfloor 6}{\lfloor 2 \lfloor 6-2} = \frac{\lfloor 6}{\lfloor 2 \lfloor 4} = \frac{\cancel{6}^{3} \times 5 \times \cancel{\lfloor 4}}{\cancel{2} \times 1 \cancel{\lfloor 4}} = 3 \times 5 = 15$

No. of vertex required for a single edge

Q.2) A man has 7 friends, 3 of them are gentalman & 4 ladies his wife has 7 friends 4 of them are gentalman & 3 ladies.

In how many ways can they invite a dinner party of 3 gentalman & 3 ladies so that there are 3 of man's friends & 3 of wife's friends.

Soln

Q.1 Prove that $^{n}C_{r} = {}^{n}C_{n-r}$

Sol^n LHS⇒ $^{n}C_{r} = \frac{\lfloor n}{\lfloor r \lfloor n-r}$

RHS = $^{n}C_{n-r} = \frac{\lfloor n}{\lfloor n-r \lfloor n-(n-r)} = \frac{\lfloor n}{\lfloor n-r \lfloor \cancel{n}-\cancel{n}+r} = \frac{\lfloor n}{\lfloor n-r \lfloor r}$

$= \frac{\lfloor n}{\lfloor n} \quad \frac{\lfloor n}{\lfloor r \lfloor n-r} = {}^{n}C_{r}$ ∴ LHS = RHS

Q.2 Prove that $^{n-1}C_{r} + {}^{n-1}C_{r-1} = {}^{n}C_{r}$

RHS = $^{n}C_{r} = \frac{\lfloor n}{\lfloor r \lfloor n-r}$ or $\frac{\lfloor n}{\lfloor r \lfloor n-r} \times 1$

$= \frac{\lfloor n}{\lfloor r \lfloor n-r} \times \left[\frac{n-r}{n} + \frac{r}{n} \right]$

$= \frac{(n-r)\lfloor n}{n \lfloor r \lfloor n-r} + \frac{r \lfloor n}{n \lfloor r \lfloor n-r}$

$\Rightarrow \frac{(\cancel{n-r})\cancel{n}\lfloor n-1}{\cancel{n}\lfloor r\,(\cancel{n-r})\lfloor n-r-1} + \frac{\cancel{r}\cdot\cancel{n}\lfloor n-1}{\cancel{r}\cdot\cancel{n}\lfloor r-1 \lfloor n-r} \Rightarrow \frac{\lfloor n-1}{\lfloor r \lfloor n-r-1} + \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r}$

$^{n-1}C_{r} + {}^{n-1}C_{r-1} = LHS$

{ $n-1(r-1)$
$(n-1-r+1)$
↓

LHS = $^{n-1}C_{r} + {}^{n-1}C_{r-1} \Rightarrow \frac{\lfloor n-1}{\lfloor r \lfloor n-1-r} + \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-\cancel{1}-r+\cancel{1}}$

$\Rightarrow \frac{\lfloor n-1}{\lfloor r \lfloor n-1-r} + \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r}$

$= \frac{\lfloor n-1}{r \lfloor r-1 \lfloor n-1-r} + \frac{\lfloor n-1}{\lfloor r-1\,(n-r)\lfloor n-1-r} = \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r-1}\left[\frac{1}{r} + \frac{1}{n-r} \right]$

$= \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r-1}\left[\frac{n-\cancel{r}+\cancel{r}}{r(n-r)} \right] = \frac{n \lfloor n-1}{r \lfloor r-1\,(n-r)) \lfloor n-r-1}$

$= \frac{\lfloor n}{\lfloor r \lfloor n-r} = {}^{n}C_{r}$ = RHS ∴ LHS = RHS

Q.3) Prove that $^nP_r = \lfloor r \; {}^nC_r$

Sol^n RHS $= \lfloor r \; {}^nC_r = \cancel{\lfloor r}\,\frac{\lfloor n}{\cancel{\lfloor r}\lfloor n-r} = \frac{\lfloor n}{\lfloor n-r} = {}^nP_r =$ LHS

LHS $= {}^nP_r = \frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor r}{\lfloor r} \times \frac{\lfloor n}{\lfloor n-r} = \lfloor r \, \frac{\lfloor n}{\lfloor r \lfloor n-r} = \lfloor r \; {}^n$

$\therefore$ LHS=RHS

Q.4) Prove that $^nC_r = {}^nC_{n-r}$

Sol^n LHS $= {}^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$

RHS $= {}^nC_{n-r} = \frac{\lfloor n}{\lfloor (n-r) \lfloor n-(n-r)} = \frac{\lfloor n}{\lfloor (n-r) \lfloor r} = \frac{\lfloor n}{\lfloor r \lfloor n-r} = {}^nC_r$

$\therefore$ LHS=RHS

Q.5) Prove that $^nC_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$

$^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r} = \frac{\lfloor n}{\lfloor r \lfloor n-r}\left[\frac{n-r}{n} + \frac{r}{n}\right]$

$= \frac{(n-r)\lfloor n}{n\lfloor r \lfloor n-r} + \frac{r\lfloor n}{n \lfloor r \lfloor n-r} \Rightarrow \frac{\cancel{(n-r)}\,\cancel{n}\lfloor n-1}{\cancel{n}\lfloor r\,\cancel{(n-r)}\lfloor n-r-1} + \frac{\cancel{r}\,\cancel{n}\lfloor n-1}{\cancel{n}\,\cancel{r}\lfloor r-1 \lfloor n-r}$

$\Rightarrow \frac{\lfloor n-1}{\lfloor r \lfloor n-r-1} + \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r} \Rightarrow {}^{n-1}C_r + {}^{n-1}C_{r-1} =$ RHS

RHS $= {}^{n-1}C_r + {}^{n-1}C_{r-1}$

$\Rightarrow \frac{\lfloor n-1}{\lfloor r \lfloor n-1-r} + \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-\cancel{1}-r+\cancel{1}} \Rightarrow \frac{\lfloor n-1}{\lfloor r \lfloor n-r-1} + \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r}$

$\Rightarrow \frac{\lfloor n-1}{r\lfloor r-1 \lfloor n-r-1} + \frac{\lfloor n-1}{\lfloor r-1 \,(n-r) \lfloor n-r-1} \Rightarrow$

$\Rightarrow \frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r-1}\left[\frac{1}{r} + \frac{1}{n-r}\right] = \left[\frac{n-\cancel{r}+\cancel{r}}{r(n-r)}\right]\frac{\lfloor n-1}{\lfloor r-1 \lfloor n-r-1}$

$= \frac{n\lfloor n-1}{r\lfloor r-1 \,(n-r)\lfloor n-r-1} = \frac{\lfloor n}{\lfloor r \lfloor n-r} = {}^nC_r =$ LHS

$\therefore$ LHS=RHS.

* Binomial Theorem ⇒

$(x+a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + \cdots + {}^nC_n x^{n-n} a^n$

or

$(x+n)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + \cdots + {}^nC_n x^0 a^n$

$$\boxed{(x+a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + \cdots + {}^nC_n a^n = \sum_{r=1}^{n} {}^nC_r x^{n-r} a^r}$$

Q.1) $(1+z)^n = {}^nC_0 + {}^nC_1 z + {}^nC_2 z^2 + \cdots + {}^nC_r z^r + \cdots + {}^nC_n z^n$ —— (1)

(i) $C(n,0) + C(n,1) + C(n,2) + \cdots + C(n,r) + \cdots + C(n,n) = 2^n$ or $\sum_{k=0}^{n} C(n,k) = 2^n$

Puting $z=1$ in equation (1), we get

${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_r + \cdots + {}^nC_n$ —— (2)

$\cancel{1+n}\ (1+1)^n = 2^n$

(ii) $C(n,0) + C(n,2) + C(n,4) + \cdots = C(n,1) + C(n,3) + \cdots = 2^{n-1}$

Solⁿ: Put $z=-1$ in equation (1), we get

${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + {}^nC_4 - {}^nC_5 + \cdots + (-1)^r\, {}^nC_r + \cdots + (-1)^n\, {}^nC_n = 0$ —— (3)

${}^nC_0 + {}^nC_2 + {}^nC_4 + {}^nC_6 + \cdots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \cdots$ —— (4)

${}^nC_0 + {}^nC_2 + {}^nC_4 + \cdots = \frac{2^n}{2} = 2^{n-1}$ —— (5)

from equation (4) & (5)

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$^nC_0 + {}^nC_2 + {}^nC_4 + \cdots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \cdots = 2^{n-1}$

* Properties of Binomial coefficients:-

(i) $C(n,r) = C(n, n-r)$

(ii) $C(n,0) + C(n,1) + \cdots + C(n,n) = 2^n$

(Hint use $(1+1)^n$

(iii) $C(n,0) - C(n,1) + C(n,2) - C(n,3) + \cdots \pm C(n,n) =$

{ Hint $(1-1)^n$ use

(iv) $C(n+1, r) = C(n,r) + C(n, r-1)$

* Multimonial coefficient ⇒

An expression of the form $x_1 + x_2 + x_3 + \cdots + x_n$ where $n \geq 3$ is called a multimonial

$(x_1 + x_2 + x_3 + \cdots + x_r)^n$ & $x_1^{n_1}, x_2^{n_2}, x_3^{n_3} \cdots x_n^{n_r}$ then

$$\binom{n}{n_1, n_2, n_3 \cdots n_r} = \frac{\lfloor n}{\lfloor n_1 \lfloor n_2 \lfloor n_3 \cdots \lfloor n_r}$$

The quantity $\binom{n}{n_1\, n_2\, n_3 \cdots n_r}$, ~~where~~.

where $n_1 + n_2 + n_3 + \cdots + n_r = n$ is called the multimonial coefficient.

Q.1) Find the coefficient of $x^5 y^3 z^2$ in $(x+y+z)^{10}$

Sol^n coefficient of $x^5 y^3 z^2$ in $(x+y+z)^{10}$ is

$$\binom{10}{5\ 3\ 2} = \frac{\lfloor 10}{\lfloor 5 \lfloor 3 \lfloor 2} = 2520$$

Q.2) Find the coefficient of $xy^2z^2u^2$ in $(x+y+z+u)^7$.

Sol^n required coefficient $= \binom{7}{1\ 2\ 2\ 2} = \frac{\lfloor 7}{\lfloor 1 \lfloor 2 \lfloor 2 \lfloor 2} = 630$

Q.3) Find the coefficient of $x^5 y^2 z^2$ in $(x+y+z)^9$

Sol^n Required coefficient $= \binom{9}{5\ 2\ 2} = \frac{\lfloor 9}{\lfloor 5 \lfloor 2 \lfloor 2} = 756$

* Mathematical induction method ⇒

Q.1) $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$ is divisible by 25 for $\forall n \in N$

Sol^n $P(n) = 7^{2n} + 2^{3n-3} \cdot 3^{n-1}$

for $n=1$ $\quad P(1) = 7^{2\cdot 1} + 2^{3\cdot 1-3} \cdot 3^{1-1} \Rightarrow 7^2 + 2^0 \cdot 3^0 = 7^2 + 1 = 49+1$

$= 50$

$= 50 = 25 \times 2$

m^{th} terms $\quad \therefore P(m) = 7^{2m} + 2^{3m-3} \cdot 3^{m-1} = 25 \cdot K$ —(1)

where K is some +ve integer no.

$\therefore (m+1)^{th}$ terms

$P(m+1) = 7^{2(m+1)} + 2^{3(m+1)-3} \cdot 3^{(m+1)-1}$

$= 7^{2m} \cdot 7^2 + 2^{(3m-3)} \cdot 2^3 \cdot 3^{(m-1)} \cdot 3^1 \Rightarrow 49 \cdot 7^{2m} + 24 \cdot 2^{3m-3} \cdot 3^{(m-1)}$

$= (50-1) \cdot 7^{2m} + (25-1) \cdot 2^{3m-3} \cdot 3^{(m-1)}$

$= 25\left[2 \cdot 7^{2m} + 2^{3m-3} \cdot 3^{(m-1)}\right] - \left(7^{2m} + 2^{3m-3} \cdot 3^{(m-1)}\right)$

$= 25\left(2 \cdot 7^{2m} + 2^{3m-3} \cdot 3^{(m-1)}\right) - 25 \cdot K$ ← put value from eq^n (1)

Q.2) Use mathematical induction to prove that

$2 \cdot 7^n + 3 \cdot 5^n - 5$ is divisible by 24 for $\forall n > 0$

Sol^n Let $P(n) = 2 \cdot 7^n + 3 \cdot 5^n - 5$

Put $n=1$

$P(1) = 2 \cdot 7^1 + 3 \cdot 5^1 - 5 = 14 + 15 - 5 = 24$

Put $n=2$

$P(2) = 2 \cdot 7^2 + 3 \cdot 5^2 - 5 = 98 + 75 - 5 = 168 = 24 \times 7$

$\therefore$ m^{th} terms $P(m) = 2\cdot 7^m + 3\cdot 5^m - 5 = 24\cdot K$ ——— ① where $(K \in N)$

$(m+1)^{th}$ terms

$P(m+1) = 2\cdot 7^{m+1} + 3\cdot 5^{m+1} - 5$ ——— ②

$\because P(m+1) - P(m) = (2\cdot 7^{m+1} + 3\cdot 5^{m+1} - 5) - (2\cdot 7^m + 3\cdot 5^m - 5)$

$= 2(7^{m+1} - 7^m) + 3(5^{m+1} - 5^m) \cancel{(-5+5)} - 5 + 5$

$= 2\cdot 7^m(7-1) + 3\cdot 5^m(5-1) - 0 = 2\cdot 7^m(6) + 3\cdot 5^m(4)$

$= 12\cdot 7^m + 12\cdot 5^m = 12(7^m + 5^m)$

$\because 7^m + 5^m = 2P$ (let)

$\therefore P(m+1) - P(m) = 12\cdot 2P$

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* Combinatorial number ⇒ A combinatorial number is formed when we put two positive integers n & k one on top of the other within brackets.

i.e. a combinatorial number will looks like $\binom{n}{k}$ {Here $n \geq k$

Note:- The only one restriction that the one on top number must always be greater or equal than bottom one

The combinatorial number $\binom{n}{k}$ read as n choose k

Example:- $\binom{5}{3}$ is read as 5 choose 3.

Meaning of a combinatorial number $\binom{n}{k}$ gives the number of ways we can choose k items from a pool of n items disregarding their order (अव्यवस्थित क्रम)

^nC_k (or) $\binom{n}{k} = \dfrac{\lfloor n}{\lfloor k \, \lfloor n-k}$ $\lfloor n = n\cdot(n-1)\cdot(n-2)\cdot(n-3) \cdots 3\cdot 2\cdot 1$

Q.1 $\binom{10}{4} = \dfrac{\lfloor 10}{\lfloor 4 \, \lfloor 10-4} = \dfrac{\lfloor 10}{\lfloor 4 \, \lfloor 6} = \dfrac{10\times \overset{3}{\cancel{9}}\times \cancel{8}\times 7 \, \cancel{\lfloor 6}}{\cancel{4}\times \cancel{3}\times \cancel{2}\times 1 \, \cancel{\lfloor 6}} = 10\times 3\times 7$

$= 210$

⇒ Counting Principle (Principle of Ex-clusion) ⇒
Consider two finite sets A & B with $n(A)$ & $n(B)$ elements then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) \quad \text{or}$$
$$|A \cup B| = |A| + |B| - |A \cap B|$$

Theorem-I ⇒ If A & B are disjoint finite sets then
$$n(A \cap B) = \phi$$
$$\therefore n(A \cup B) = n(A) + n(B) \quad \text{or} \quad |A \cup B| = |A| + |B|$$

Theorem-II ⇒ If A & B are non-disjoint finite sets then
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

If three finite sets A, B & C with $n(A)$, $n(B)$ & $n(C)$ elements then
$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Q.1) A survey of 1000 customers reading news paper conducted & reported that 720 customers read times of India & 450 likes Hindustan times. what is the least number that reads both the news papers.

$n(x) \rightarrow$ times of India, $n(y)$ → Hindustan times

Solⁿ Let x is number of persons which read times of India & y is number of persons which reads Hindustan times

$n(x) = 720 \qquad n(y) = 450, \quad n(x \cup y) = 1000, \quad n(x \cap y) = ?$

$n(x \cup y) = n(x) + n(y) - n(x \cap y)$

$1000 = 720 + 450 - n(x \cap y) \qquad \cancel{-}n(x \cap y) = \cancel{-}170$

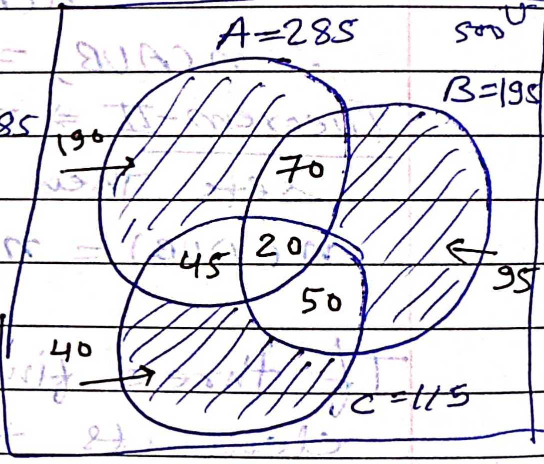
$n(x \cap y) = 170$ Ans

Q.2) A survey of 500 television views produced the following information. 285 watch news, 195 watch songs, 115 watch sports, 45 watch news & sports, 70 watch news & songs, 50 watch songs & sports & 50 donot watch any of the three kinds of channels.

(i) How many people in the survey watch all three kinds of channel.

(ii) How many people watch exactly one channel out of three

Sol^n A = news, B = songs, C = sports



$|U| = \text{all} = 500$ $\quad |A| = |\text{news}| = 285$

$|B| = |\text{songs}| = 195$

$|C| = |\text{sports}| = 115$

$|A \cap C| = 45, \quad |A \cap B| = 70$

$|B \cap C| = 50$

$|A \cap B \cap C| = ?$

$|\overline{A \cup B \cup C}| = 50$ $\quad \therefore |A \cup B \cup C| = 500 - 50 = 450$

(i) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$450 = 285 + 195 + 115 - 45 - 70 - 50 + |A \cap B \cap C|$

$|A \cap B \cap C| = 20$

(ii) $n(A) + n(B) + n(C) = \text{only news} + \text{only songs} + \text{only sports}$

only news $= 285 - [(70 - 20) + (45 - 20) + (20)]$

$= 285 - (50 + 25 + 20) = 190$

only songs $= 195 - [(70 - 20) + (50 - 20) + 20] = 195 - (50 + 30 + 20)$

$= 195 - 100 = 95$

only sports $= 115 - [(45 - 20) + (50 - 20) + 20] = 115 - (25 + 30 + 20)$

$= 115 - 75 = 40$

$n(A) + n(B) + n(C) = 190 + 95 + 40 = 325$ Ans

* Principle of Inclusion & Exclusion ⇒

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

~~डि- अरेंजमेंट~~

* Derangement ⇒ An arrangement of object in which no object appear at it's correct place.

ex:- [4 Friend & Not use it's self bike, use only other friend bike]

No. of derangement of n distinct objects.

$$\boxed{D_n = \lfloor n \left[\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} + \frac{1}{\lfloor 6} - \cdots \frac{(-1)^n}{\lfloor n} \right]}$$

De-rangement

if $n = 2$ { Two person }

$n=2$ $D_2 = 1$ $\{ \lfloor 2 \left(\frac{1}{\lfloor 2} \right) = 1 \}$

$n=3$ $D_3 = 2$ $\{ \lfloor 3 \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} \right) = \cancel{\lfloor 3} \left(\frac{\lfloor 3 - \lfloor 2}{\lfloor 2 \cancel{\lfloor 3}} \right) = \frac{3 \times 2 \times 1 - 2 \times 1}{2 \times 1} \} = \frac{6-2}{2} = \frac{4}{2} = 2$

$n=4$ $D_4 = 9$ $\{ \lfloor 4 \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} \right) = \cancel{\lfloor 4 \left(\frac{\lfloor 6 - \lfloor 4 + \lfloor 3}{\lfloor 12} \right)}$ $\lfloor 4 \left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} \right))$

$n=5$ $D_5 = 44$ $\{ \lfloor 5 \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} \right) = \cancel{\lfloor 5} \left(\frac{60 - 20 + 5 - 1}{\cancel{\lfloor 5}} \right) = 44$

$n=6$ $D_6 = 265$ $\{ \lfloor 6 \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} + \frac{1}{\lfloor 6} \right) = \cancel{\lfloor 6} \left(\frac{360 - 120 + 30 - 6 + 1}{\cancel{\lfloor 6}} \right) = 265$

$n=1$ $\boxed{D_1 = 0}$

Q.1 Suppose 6 students write a Quiz, to purpose of evolution students are told to exchange their paper such no students should get their own paper

Sol^n $n = 6$ $D_6 = \lfloor 6 \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} + \frac{1}{\lfloor 6} \right) = 265$ Ans

Q.2 Suppose we have 5 letters (L_1, L_2, L_3, L_4, L_5) & 5 Envelopes (E_1, E_2, E_3, E_4, E_5), one letter per envelope. How many ways we can place such that.

P.T.O.

(i) All are wrongly (गलत) placed.

Ans: $D_5 = \lfloor 5 \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} \right) = \cancel{\lfloor 5} \left(\frac{60-20+5-1}{\cancel{\lfloor 5}} \right) = 44$

(ii) At least one letter is wrongly placed.

Ans: Total ~~correctly~~ way – one letter wrongly placed

$\lfloor 5 - 1 = 120 - 1 = 119$

(iii) At least one letter is correctly placed.

Ans: Total way – all are wrongly placed.

$\lfloor 5 - D_5 = \lfloor 5 - 44 = \cancel{76}\ 120 - 44 = 76$

(iv) Exactly two letters are correctly placed $\boxed{D_3 = 2}$

$^5C_2 \cdot 1 \cdot D_3$ – remaining ~~three~~ three letters wrongly

5 में से 2 सही ~~लेटर~~ selecte करने correctly का way

correct ways

$\Rightarrow \frac{\lfloor 5}{\lfloor 2 \lfloor 5-3} \times 1 \times 2$

$= \frac{5 \times 4 \times 3 \times \cancel{\lfloor 2}}{2 \times 1 \times \cancel{\lfloor 3}} = \frac{60}{2} = 30$

$\Rightarrow 30 \times 1 \times 2 = 60$

(v) Only one letter is wrongly placed

Ans:- 4 letter ~~wrongly~~ correctly but 1 is wrongly it's not possible so total way is zero

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Q.3)

1	2	3	4	5	
↓	↓	↓	↓	↓	
1	2	3	4	5	~~correct~~

But not bellow the same no. ~~and~~ on it's self

↓	↓	↓	↓	↓
2	1	4	5	3

$n=5$ there fore $D_5 = \lfloor 5 \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} \right)$

$= \cancel{\lfloor 5} \left(\frac{60-20+5-1}{\cancel{\lfloor 5}} \right) = 44$

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or $\lfloor 5 - \binom{5}{1} \lfloor 4 + \binom{5}{2} \lfloor 3 - \binom{5}{3} \lfloor 2 + \binom{5}{4} \lfloor 1 - \binom{5}{5} \lfloor 0$

$= \lfloor 5 - \frac{\lfloor 5}{\lfloor 1 \lfloor 5-1} \lfloor 4 + \frac{\lfloor 5\ \lfloor 3}{\lfloor 2 \lfloor 5-2} + \frac{\lfloor 5\ \lfloor 2}{\lfloor 3 \lfloor 5-3} + \frac{\lfloor 5}{\lfloor 4 \lfloor 5-4} \lfloor 1 - \frac{\lfloor 5}{\lfloor 5 \lfloor 5-5} \cdot \lfloor 0$

$= \lfloor 5 - \frac{\lfloor 5}{\lfloor 1 \cancel{\lfloor 4}} \cdot \cancel{\lfloor 4} + \frac{\lfloor 5 \cdot \cancel{\lfloor 3}}{\lfloor 2 \cancel{\lfloor 3}} \mp \frac{\lfloor 5}{\lfloor 3 \cancel{\lfloor 2}} \cdot \cancel{\lfloor 2} + \frac{\lfloor 5}{\lfloor 4 \cancel{\lfloor 1}} \cdot \cancel{\lfloor 1} - \frac{\lfloor 5}{\lfloor 5 \cancel{\lfloor 0}} \cdot \cancel{\lfloor 0}$

$\Rightarrow \lfloor 5 - \frac{\lfloor 5}{1} + \frac{\lfloor 5}{\lfloor 2} \mp \frac{\lfloor 5}{\lfloor 3} + \frac{\lfloor 5}{\lfloor 4} - \frac{\cancel{\lfloor 5}}{\cancel{\lfloor 5}} 1$ बाकी